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Signal & System

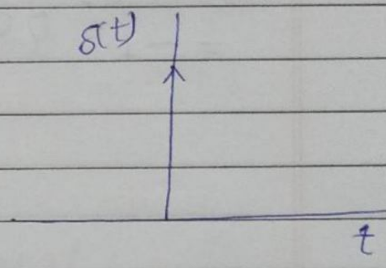
Signals
- ↳ If contains Informati
- ↳ Energy.

Elementry signal

(A) Unit Impulse function/ signal

$\delta(t) = \infty, \ t = 0$

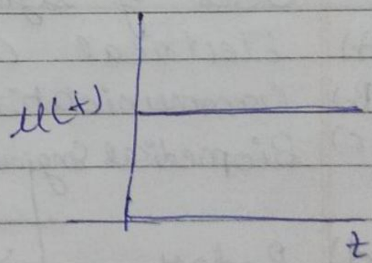
$\int_{-\infty}^{\infty} \delta(t) = 1$



(B) Unit step function

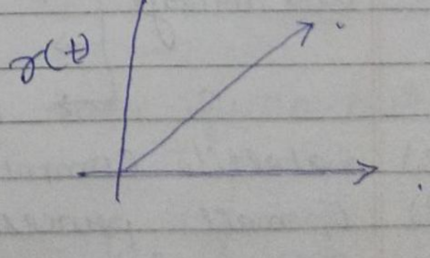
$u(t) = 1$

$t > 0$



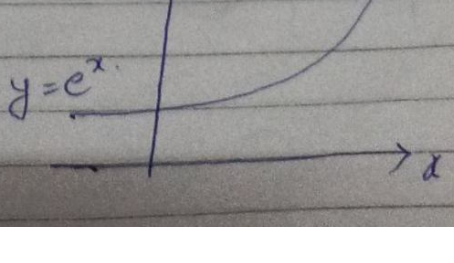
(C) Ramp signal

$r(t) = t \ , \ 0 < t < \infty$



(d) Exponential signal

$y = e^{x}$



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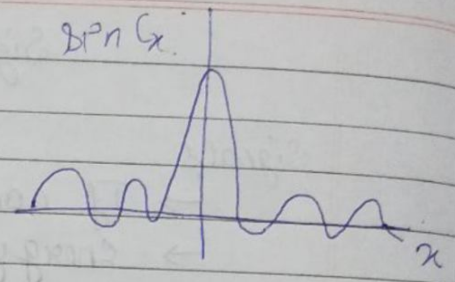
(e) Sampling Signal

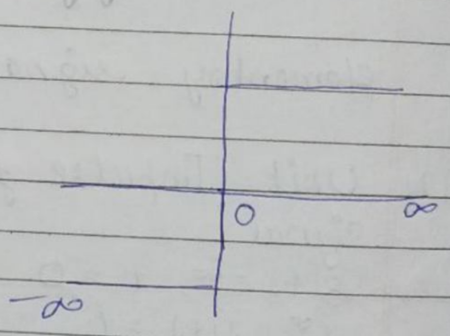
$\sin c_x = \frac{\sin x}{x}$



(f) Signum Signal

$u(t) = 1 \quad \{0, \infty$

$= -1$



Uses of signal system.

(A.) Electrical circuit design
(B.) Communication.
(C) Biomedical Engineering → transfer of message
→ modulation
(d) Radar and aircraft
→ face
→ MRI - CT Scan
→ EEG
→ ECG
(e.) Satellite communication
(f) E-mail processing
(g) video signal
(h) Space Signal

(a)

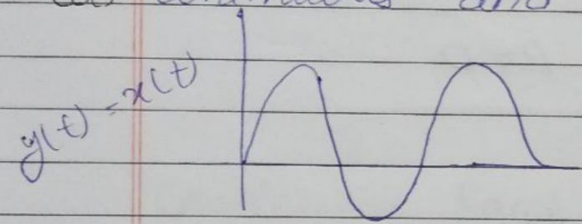
$y(t) = x$

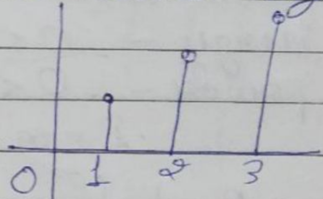
(b)

(c)

Classification of signal

(a) Continuous and discrete time signal



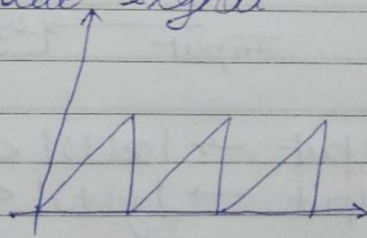


$x[n] = 1, 2, 3$

$n = 1, 2, 3$

(b) periodic and aperiodic signal

$x(t) = x(T+t)$

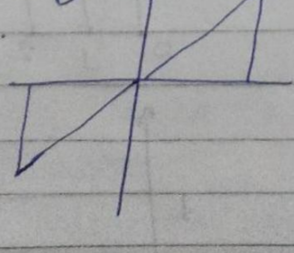


(c) Even and odd signal

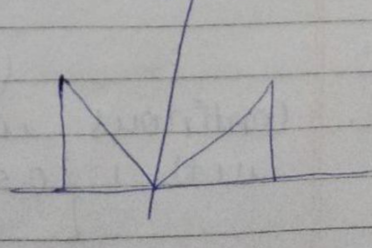
$x(t) = x(t)$ Even

$= -x(-t)$ odd.

odd symmetric



even symmetric



(d) Energy and power

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

Energy – $0 < E < \infty,\ P = 0$

power – $0 < P < \infty$

$E = \infty$

$$P = \frac{1}{T}\int_{\infty}^{\infty} x^2(t)\,dt$$

System

Entity manipulated

Input → system → output

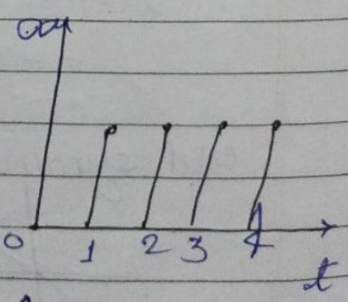
Input → $|x(t)| < \infty$

output → $|y(t)| < \infty$

Discrete signal

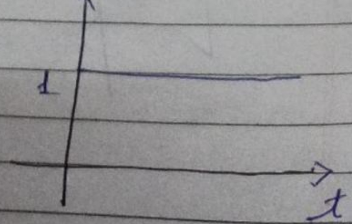
1. Discrete unit step signal

$u[n] = 1 \quad n = 0$



2. Continous unit signal

$u(t) = 1 \{0 < t < \infty$



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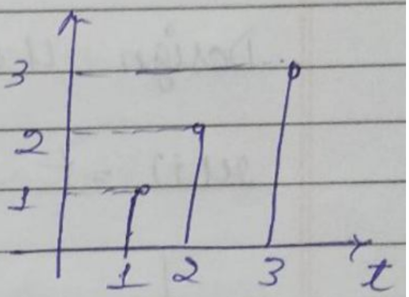
3. Discrete Ramp step signal

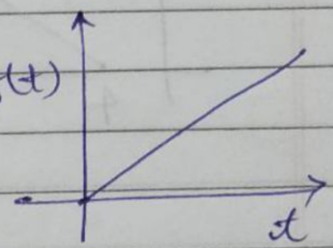
$x[n] = nu[n]$



Continuous Ramp signal

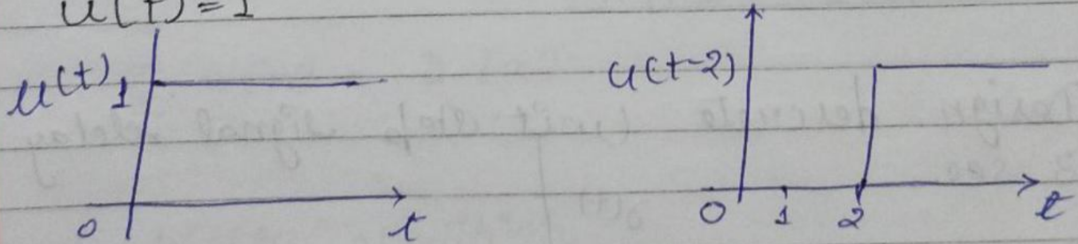
$r(t) = t$

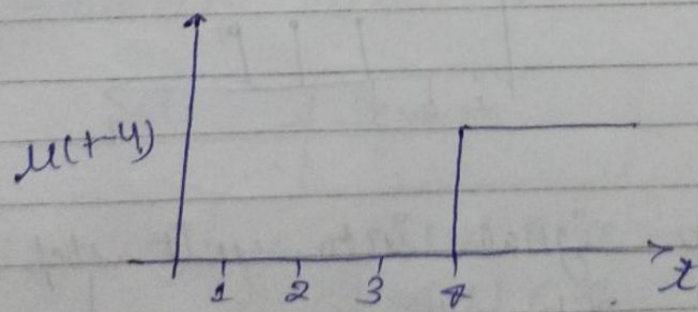
$y = x$



Delay Signal

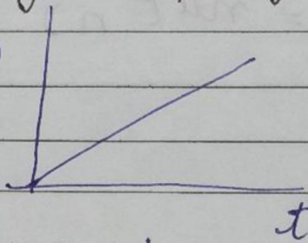
(1.) $u(t) = 1$

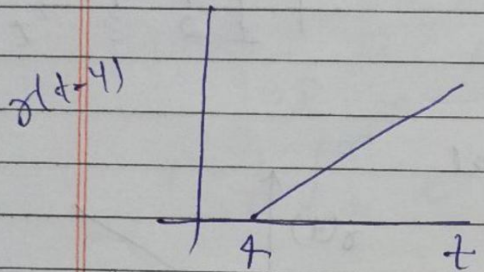


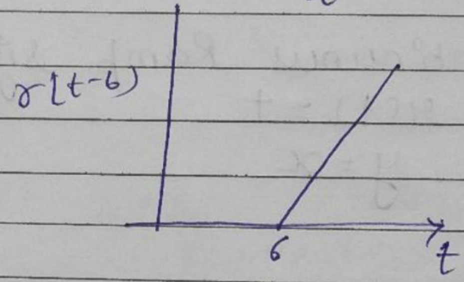


Design 4 sec delay ramp signal

$r(t) = t$







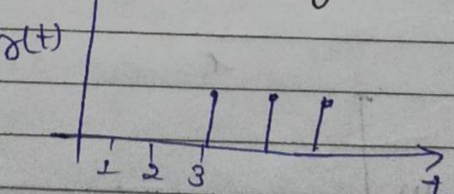
Time Period.

$\cos\theta, \sin\theta = 2\pi$

$\cos 7\theta = 2\pi/7$

$\sin n\theta = 2\pi/n$

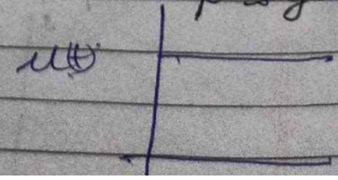
Q.1 Design descrete unit step signal delay with 3 sec



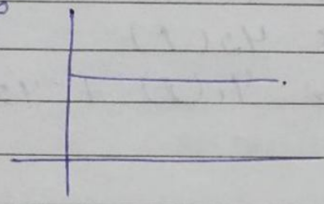
Q.2 Convert Ramp signal into unit step signal

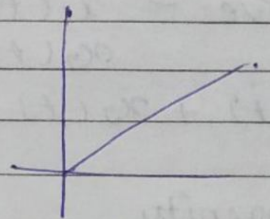
$r(t) = t$

$\frac{\partial(t)}{\partial t} = 1$



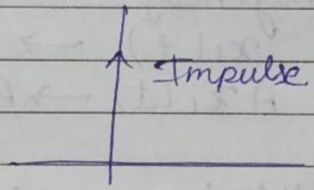
$$\int_{-\infty}^{t} u(t) = r(t)$$





Impulse

$$\frac{d^2 r(t)}{dt^2} = \delta(t)$$



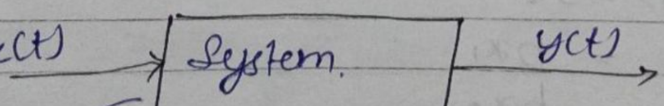
Impulse $$\int_{-\infty}^{\infty} \int_{-\infty}^{t} \delta(t)\,dt = r(t)$$

(3) Convert discrete ramp signal information

$$\sum_{-\infty}^{n} \delta[n] = u[n]$$

$$\sum_{-\infty}^{\infty} u[n] = r[n]$$

Stability condition



Stability → $|x(t)| < \infty$

$|y(t)| < \infty$

linearity → Additive

Homogenity

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Additive - $x_1(t) \longrightarrow y_1(t)$
$x_2(t) \longrightarrow y_2(t)$
$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

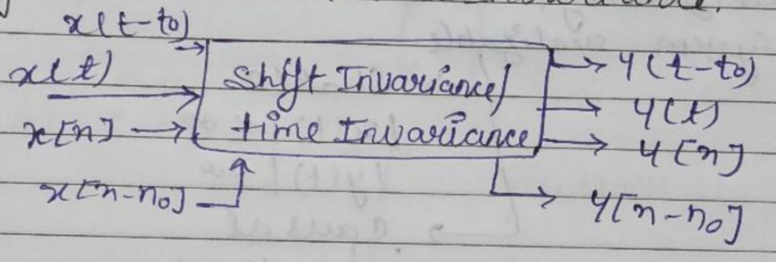
Homogenity
$x_1(t) \rightarrow y_1(t)$
$Ax_1(t) \rightarrow Ay_1(t)$

Ex (1) $y(x) = ax + b$
linear or not
$(t) \rightarrow$ continuous
$[n] \rightarrow$ Discrete
$x_1[n] = y_1[n]$
$x_2[n] = y_2[n]$
$x_1[n] + x_2[n] = y_1[n] + y_2[n]$
$x_1(t) = y_1(t) = ax_1 + b$ —— (I)
$x_2(t) = y_2(t) = ax_2 + b$ —— (II)
$x_1(t) + x_2(t) = y_1(t) + y_2(t) = a(x_1 + x_2) + b$
$(x_1 + x_2)t = a(x_1 + x_2) + 2b$
system nolineal

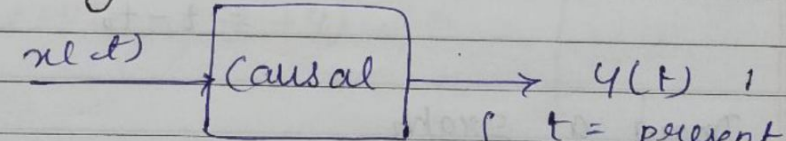
(2) $y(x) = bx$
$x_1 = bx_1$
$x_2 = bx_2$
$x_1 + x_2 = b(x_1 + x_2)$
$y(x_1 + x_2) = b(x_1 + x_2)$
System is lineal.

1.
2.
3.
4.
5.
~~5~~
6.)
7.)

Shift Invariance / time Invariance:



Causality



dependency: t = present; t - to = past; future ≠ t + to

1. $u(t-3)$ = causal
2. $u(t+4)$ = non causal
3. $r(t+6)$ = non causal
4. $\delta(t-3)$ = causal.
5. $y_1(t) = \int_{-\infty}^{t} x(t)\,dt$ = causal
6. $y_2(t) = \int_{-\infty}^{t+2} x(t)\,dt$ = non causal
7. $y_3(t) = \int_{-\infty}^{t-3} x(t)\,dt$ = causal.

causality of linear shift Invariance / LTI

$y(t) = x(t) * h(t)$

$= \int_{-\infty}^{t} x$

Realizability.

1 System realizable

$\rightarrow$ Stable

$|x(t)| < \infty$

$|y(t)| < \infty$

$\rightarrow$ Causal

$\rightarrow \cancel{t + t_0}$

$= t$

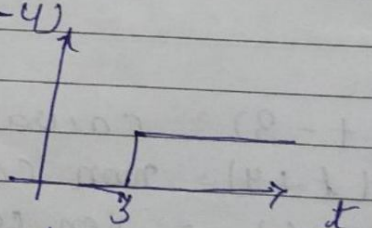
$= t - t_0$

Ques.1 Draw a graph.

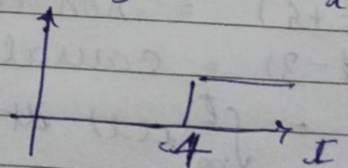
(i) $u(t + 4) \cdot u(-t + 4)$

(ii) $u(t - 3) \cdot u(t - 4)$

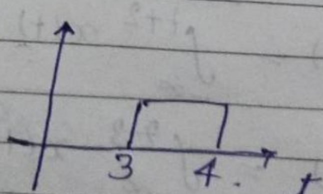
(ii) $u(t - 3)$



$u(t-4)$



$u(t-3) \cdot u(t-4)$



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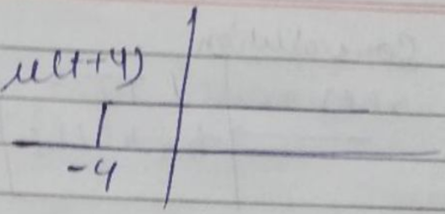
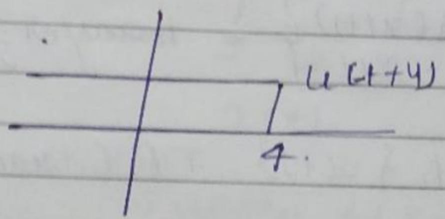
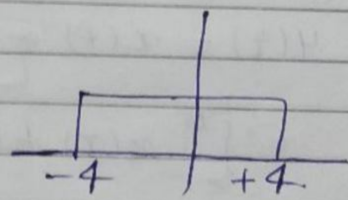
(i) $u(t+4)$



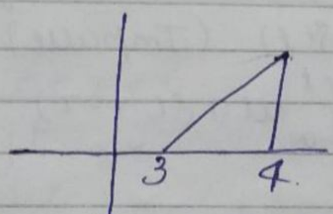
$u(-t+4)$



$u(t+4) \cdot u(-t+4)$



Q. 2 Find equation



~~x(t-3)-u(t~~) $(2t-6)[u(t-3)-u(t-4)]$.

Linear time Invariant / Linear shift Invariante

→ Linearity → Additive
→ Homogeneitye

⇒ Time variant

Ques Design Linear time Invariant function.

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Convolution

$$x(t) \longrightarrow \boxed{\begin{array}{c} LTI \\ h\{x(t)\} \end{array}} \longrightarrow y(t)$$

$$\left.\begin{array}{l} h\{x(t)\} \\ x(t) = \delta(t) \end{array}\right\} = \text{transfer function}$$

(2.5) $= h\{\delta(t)\}$ T.F. (transfer function)

$$y(t) = \underbrace{x(t) * h(t)}_{\text{convolution}}$$

$$= \int_{\infty}^{t} x(\tau)\, \underbrace{h(t-\tau)}_{\text{Torque}}\, d\tau$$

Shifting in time.

$h(t) = \delta(t)$ (Impulse)

$$y(t) = \int_{-\infty}^{t} x(\tau)\, \delta(t-\tau)$$

Q.1 $S(t) = \frac{1}{2} - e^{-2t} u(t)$

find Impulse Response.

$$\delta(t) = \frac{\partial u(t)}{\partial t}$$

$$= \frac{d}{dt}\,(\text{Step Response})$$

$$= \frac{d}{dt}\left(\frac{1}{2} - e^{-2t} u(t)\right)$$

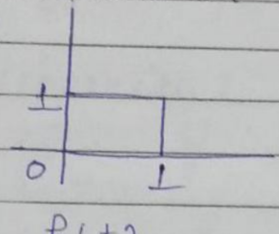
$$= \frac{1}{2} + 2e^{-2t} u(t)$$

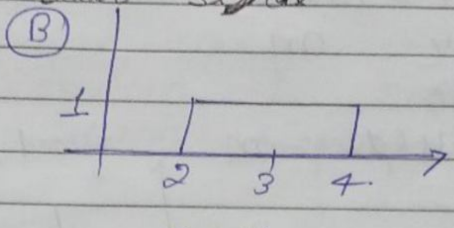
Q.2

(A)

Q. 3.

Q.2 Convolution of two signal

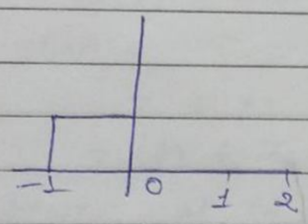


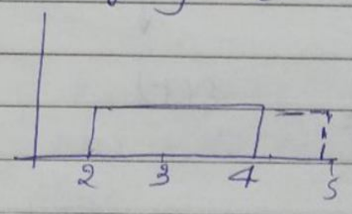


$h(t)$ $x(t)$

$y(t) = x(t) * h(t)$

↳ shifting $h(t-\tau)$



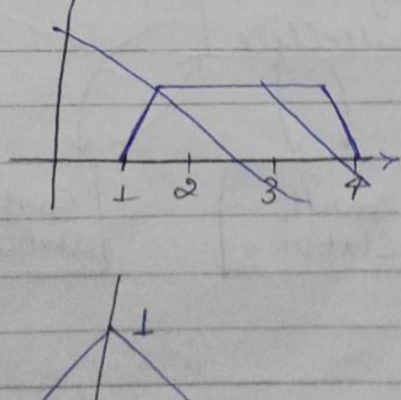


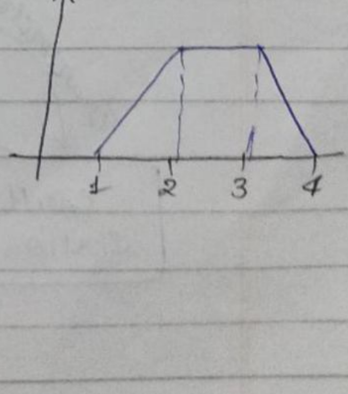
$h(-t)$

$1 < t < 2$ $2 < t < 3$ $t > 3$

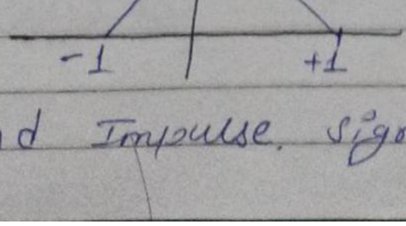
$t < 4.$

$1 < t < 2$	$2 < t < 3$	$3 < t < 4$
$= \int_1^t \partial\tau$	$= \int_{1-t}^{t} \partial\tau$	$= \int_{t-1}^{3} \partial\tau$
$= t - 1$	$= 1$	$= t - 4$





Q.3.



Find Impulse signal.

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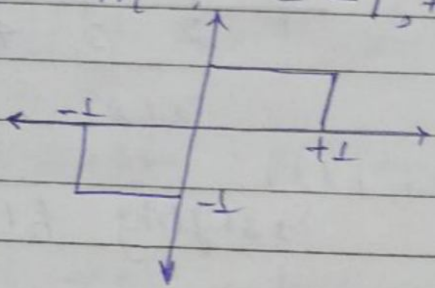
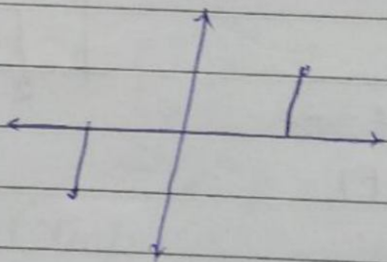
$$y = mx + c$$

$$\frac{\partial y}{dx} = m$$

$$u(t) = m \quad = -1, +1$$

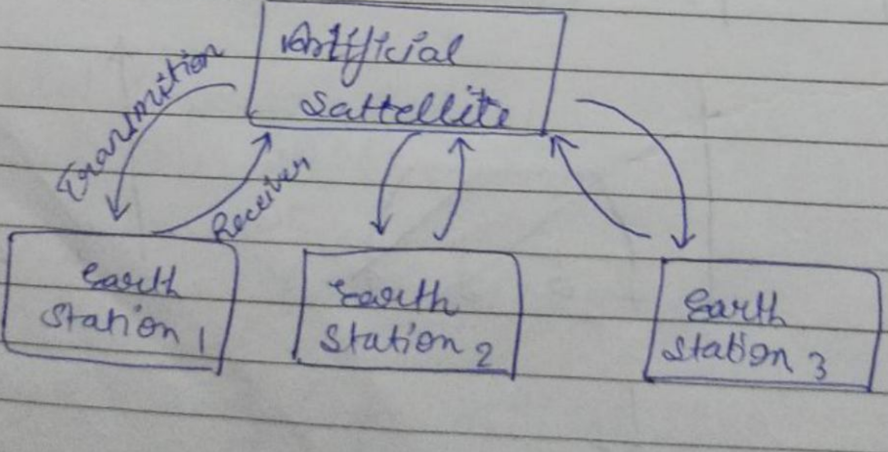


$$\frac{\partial^2 y}{\partial t^2} = \delta(t)$$



$$\delta(t+1) + \delta(t-1)$$

Signal System is sattellite



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System properties

(a) linearity: A system is said to be line if it satisfies to properties.

(i) additive.

$x_1(t) \xrightarrow{S} y_1(t)$

and

$x_2(t) \xrightarrow{S} y_2(t)$

Then,

$$x_1 + x_2 \xrightarrow{S} y_1(t) + y_2(t)$$

$$x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$$

terms of Discrete time.

$$x_1[n] + x_2[n] \longrightarrow y_1[n] + y_2[n]$$

(b.) Realized System
$\hookrightarrow$ Both Stable and causal

(A) $|x(t)| < \infty$

$|y(t)| < \infty$

(B) $y(t) = f(x(t), x(t-t_0))$

$y(t) \neq x(t + t_0)$

System property

1. Linearity $\left\langle \begin{array}{l} \text{Additive} \\ \text{Homogenity} \end{array} \right.$
2. Stability
3. Causal
4. Time Invariance
5. Realizability

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Q.1 Step Response $= e^{-at}u(t) + e^{at}u(-t)$
find Impulse response?

$$\frac{du(t)}{\partial(t)} = \delta(t)$$

$$I.R = \frac{\partial}{\partial t}(\cancel{e^{-at}} \text{ step Response})$$

$$= \frac{\partial}{\partial t}\left(e^{-at}u(t) + e^{at}u(-t)\right)$$

$$= -a e^{-at}u(t) + a e^{at}u(-t)(-1)$$

$$= -\left(a e^{-at}u(t) + a e^{at}u(-t)\right)$$

Note

☆ $\int_{-\infty}^{\infty}\int_{-\infty}^{t} \delta(t) = \gamma(t)$

☆ $\frac{\partial^2 \gamma(t)}{\partial(t)^2} = \delta(t)$

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Laplace Transform.

↳ convert time domain to s domain

⇒ Important role in study of stability of system.

Bilateral Laplace Transform

↳ $-\infty \leq t \leq \infty$

→ causal & non causal signal, stable

unilateral Laplace Transform.

↳ Causal signal only $0 < t < \infty$.

→ one sided Laplace Transform, stable

$$LT\{x(t)\} = x(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$s = \sigma + j\omega$$

Real

Imaginary

oscillation frequency in rad/s

Img {s}

$$x(t) \longrightarrow x(s)$$

Inverse laplace Transform of x(s) is given as

$$ILT\{x(s)\} = LT^{-1}[x(s)] = x(t)$$

$$x(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} x(s)\, e^{st}\, ds$$

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Imp

1. To find Laplace of $u(t)$

$x(t) = u(t)$

$$x(S) = \int_0^{\infty} u(t) e^{-St} dt \qquad \{u(t) = 1$$

$$= \int_0^{\infty} e^{-St} dt$$

$$= \left[\left\{\frac{-1}{S}\right\} e^{-St}\right]_0^{\infty}$$

$$= 0 - \left(\frac{-1}{S}\right)$$

$$= +\frac{1}{S}$$

Hence

$$L[u(t)] = 1/S \quad Re(S) > 0$$

$$L[\cos\omega t \cdot u(t)] = \frac{S}{S^2+\omega^2} \quad Re\{S\} > 0$$

$$L[\sin\omega t \; u(t)] = \frac{\omega}{S^2+\omega^2} \quad Re\{S\} > 0$$

2) $x(t) = e^{at} u(t)$

To find Laplace Transform of

$x(t) = e^{at} u(t)$

$$x(S) = \int_{-\infty}^{\infty} x(t) e^{-St} dt$$

$$= \int_0^{\infty} e^{-St} e^{at} dt \quad t > 0$$

$$= \int_0^{\infty} e^{-St} e^{at} dt = \frac{1}{S-a} \quad Re(S) > a.$$

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Unilateral-

$L[x(t)] \rightarrow X(s)$

$\Rightarrow \int_0^{\infty} x(t)\, e^{-st} dt$: unilateral

Bilateral.

$\Rightarrow \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$ Bilateral.

Properties.

$\Rightarrow$ 1. Additive

$L[x_1(t) + x_2(t)] \stackrel{L}{=} X_1(s) + X_2(s)$

2. Time shifting

$x(-t) \xrightarrow{L} X(-s)$

$x(t-t_1) \longrightarrow e^{-st_0} X(s)$

3. $\underset{\text{convolusion}}{x(t) * h(t)} \xrightarrow{L} X(s)\, H(s)$

4. $A(x(t)) \longrightarrow AX(s)$.

Q-1 Find $L[u(-t)]$?

$L[u(t)] = 1/s$ (shifting problem)

$L[u(-t)] = -1/s$

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2 Find Laplace $L[e^{-at}u(t)]$

$$x(s) \xrightarrow{L} \int_0^\infty x(t)\, e^{-st}\, dt$$

$$= \int_0^\infty e^{-at}\, e^{-st}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= \frac{-1}{s+a}\left[e^{-\infty} - e^{0}\right]$$

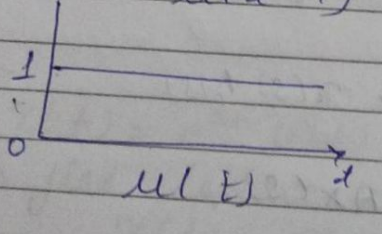
$$= \frac{-1}{s+a}\left[0 - 1\right] = \frac{1}{s+a}$$

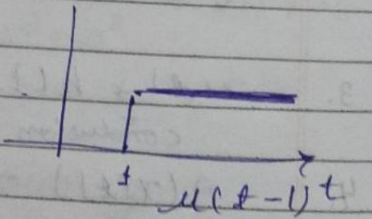
1.) Plot

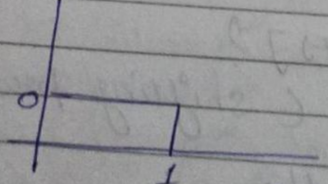
a) $u(t) - u(t-1)$

b) $u(t+1) - u(t-1)$

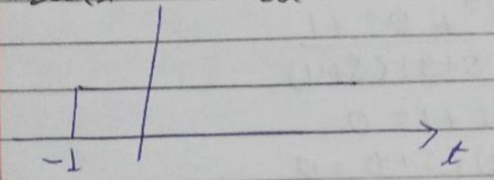
a) $u(t) - u(t-1)$



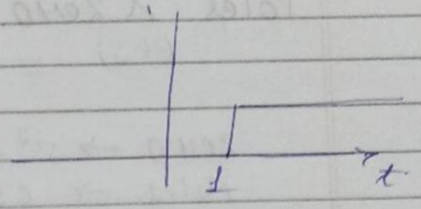




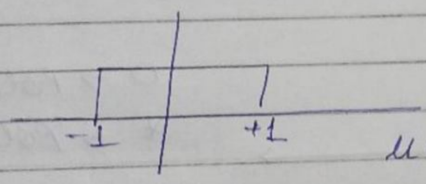
(b) $u(t+1) - u(t-1)$



$u(t+1)$



$u(t-1)$



$u(t+1) - u(t-1)$

Q.2. find Impulse.?

$x(t) = e^{at} u(t) + \sin t$.

$\text{Impulse} = \frac{d x(t)}{\partial(t)}$

$= a e^{at} (u(t)) + \cos(t)$

Q.3. a) $L[\sin \omega t] \longrightarrow \frac{\omega}{S^2 + \omega^2}$

b.) $L[\cos \omega t] \longrightarrow \frac{s\omega}{S^2 + \omega^2}$

c) $L[u(t)] \longrightarrow \frac{S}{S^2 + \omega^2}$

d.) $L[e^{-at} u(t)] \rightarrow \frac{1}{s+a}$.

e) $L[e^{at} u(t)] = 1/s - a$

Poles & Zero.

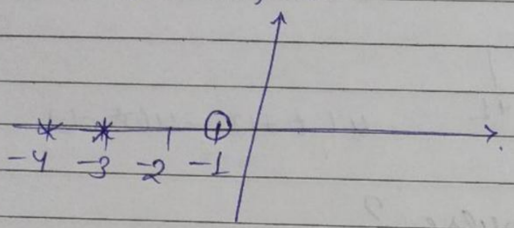
$$P(S) = \frac{S^2 + 2S + 1}{(S+3)(S+4)}$$

Zero $\rightarrow S^2 + 2S + 1 = 0$

Poles $\rightarrow (S+3)(S+4) = 0$

$P_1 P_2 = -3, -4$

$Z_1 Z_2 = -1, -1$



O → Poles zero.

* → Poles

~~1. System Stab.~~

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Transfer function

Different equation in LTI System.

Find $h(t) = ?$

$$x(t) \xrightarrow[\text{Excitation}]{} \boxed{h(t) = h(s) = \frac{Y(s)}{X(s)}} \xrightarrow[\text{Response}]{} \text{output } y(t)$$

Input = Excitation

$x(t)$ = Input Response in t

$y(t)$ = Output Response in t

given

$$\frac{dy(t)}{\partial t} + y(t) = x(t)$$

Taking Laplace on both side

$$f(Y(s)) + Y(s) = X(s)$$

$$Y(s)[s+1] = X(s)$$

$$e^{-at} \xrightarrow{L} \frac{1}{s+a}$$

$$e^{-t} \underset{\longleftarrow}{\xrightarrow{L}} \frac{1}{s+1}$$

Inverse Laplace

$$L[u(t)] = 1/s$$

$$L[\delta(t)] = 1$$

$$L[e^{-at}] = 1/s+a$$

$$x[z] = \sum_{-\infty}^{\infty} x[n] z^{-n}$$

$$x[n] = a^n u[n]$$

$$x[z] = \sum_{0}^{\infty} a^n z^{-n} u[n]$$

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$= 1 + az^{-1} + a^2 z^{-2} + a^3 z^{-3}$

$= \frac{1}{1 - az^{-1}}$

Differential Equation in LTI system.

Excitation $\rightarrow$ $\boxed{\text{System}}$ $\xrightarrow{\text{output}}$ Response

Input = Excitation.

$x(t)$ = Input Response in t

$y(t)$ = Output Response in t

given,

$$\frac{dy(t)}{d(t)} + 3y(t) = x(t)$$

here,

$$L\left[\frac{dy(t)}{d(t)}\right] = sY(s)$$

$$L\int_{-\infty}^{t} y(t)\,dt = \frac{Y(s)}{s}$$ Q.1

$$sY[s] + 3Y[s] = X[s]$$

$$\frac{Y[s]}{X[s]} = \frac{1}{s+3}$$

$H(s) \longrightarrow$ Transfer function

In terms of t,

$$h(t) = e^{-3t}\,u(t)$$

If Input Excitation

$x(t) = u(t)$ is given

find $y(t)$

here.

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$$\frac{dy(t)}{dt} + 3y(t) = x(t)$$

$$sY[s] + 3Y[s] = X(s)$$

$$Y(s) = \frac{X(s)}{s+3}$$

$$X(s) = 1/s$$

$$Y(s) = 1/s(s+3)$$

$$= \frac{1}{3}\left[\frac{1}{s} - \frac{1}{s+3}\right]$$

$$Y(s) = \frac{1}{3}\left[\frac{1}{s}\right] - \frac{1}{3}\left[\frac{1}{s+3}\right]$$

Taking Inverse Laplace Transform

$$y(t) = \frac{1}{3}[1 - e^{-3t}]u(t).$$

Q.1 find output $y(t)$ of a LTI system if Input $x(t)$ is $\delta(t)$

here $\frac{dy(t)}{dt} + 2y(t) = x(t)$ —— ①

$$L\left[\frac{\partial y(t)}{\partial t}\right] = sY(s)$$

$$y(t) \longrightarrow Y(s)$$

Then, Equation will be, taking Laplace on both side,

$$sY(s) + 2Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

$L[\delta(t)] = 1$ Note

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$$Y(s) = \frac{1}{s+2}$$

Taking Inverse Laplace transform

$$y(t) = e^{-2t} u(t)$$

$$\boxed{IL\left[\frac{1}{s+2}\right] \rightarrow e^{-2t} u(t)}$$

Z-transform properties.

(1.) Linearity

$$a_1 x_1[n] + a_2 x_2[n] \xrightarrow{z} a_1 x_1[z] + a_2 x_2[z]$$

(2) Time shifting

$$x(n-2) \xrightarrow{z} z^{-k} x[z]$$

(3.) Scaling

$$a^n x[n] \xrightarrow{z} x[z^{-1}]$$

(4) Convolution.

$$x_1[n] * x_2[n] \xrightarrow{z} x_1[z] x_2[z]$$

Notation :- Time domain $x(n)$

$x_1(n)$

$x_2(n)$

Z-domain $X[z]$

$X_1[z]$

$X_2[z]$

ROC : $r_1 < |z| < r_2$

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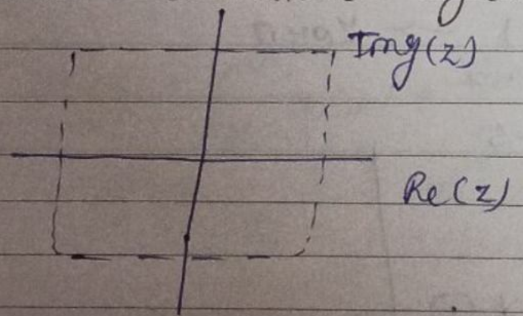
Z- transform
$\hookrightarrow$ Tools for Analyzing Discrete signal

$$x(z) = Z[x(n)] = \sum_{-\infty}^{\infty} x[n]\, z^{-n}$$

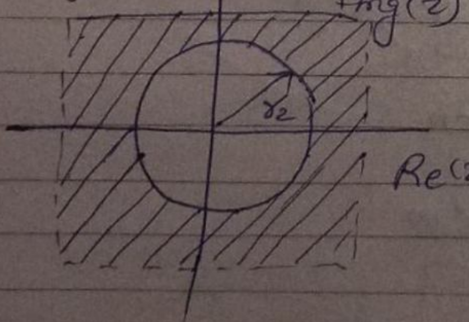
Region of convergence
$\hookrightarrow$ $x[z]$ is the set of all values of z for which attain a finite value.

ⓐ finite Duration signals

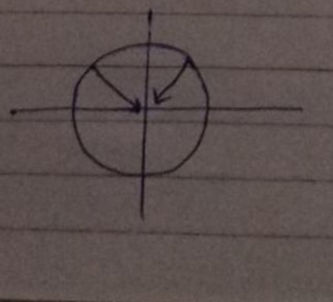


entire z plane
Except
$z = 0, \infty$

ⓑ Infinite Duration signal



causal $x(n) = 0$
$n < 0$.



Anti causal
$x(n) = 0 \quad n > 0$

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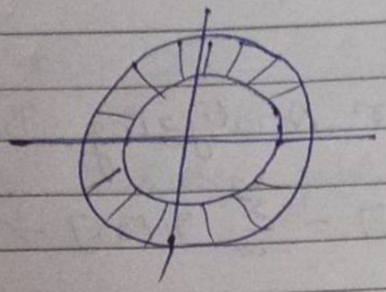


ROC $= r_1 < |z| < r_2$
causal & anticausal both.

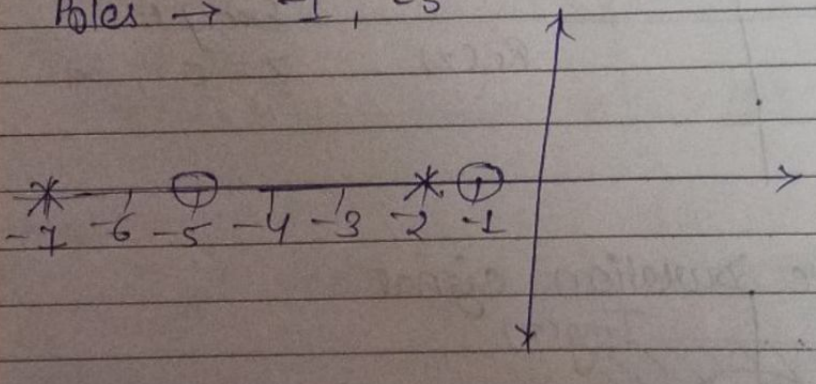
Poles & zero

$$F(s) = \frac{(s^2+4s+4)(s+7)}{(s+1)(s+5)}$$

$(s^2+4s+4)(s+7) \rightarrow$ zero

$s = -2, -2, -7$

Poles $\rightarrow -1, -5$



Fourier Condition

1.) $| \int_0^\infty |x(t)e^{-j\omega t}| < \infty$
2) maxima, minima
3.) discountin~~uous~~y

$|<r_2$

anticausal

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Application.

1. Distortion less
2. filter Design technique
3. Energy density.

find fourier expla transformation
convert $t \rightarrow$ frequency domain

~~1. $x(t) = e^{-a|t|}$~~

~~$= e^{-at}u(t) + e^{at}u(-t)$~~

$$X(\omega) = \int_0^\infty x(t)\, e^{-j\omega t}\, dt$$

Laplace : $S = \sigma + i\omega$ ($\sigma \to 0$)

time $\xrightarrow{FT}$ frequency

① $x(t) = e^{-a(t)}$

$$x(\omega) = \int_0^\infty x(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^\infty e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_0^\infty e^{-t(a+j\omega)} \cdot dt$$

$$= \frac{-1}{(a+j\omega)}\left[e^{-(a+j\omega)}\right]_0^\infty$$

$$= \frac{-1}{(a+j\omega)}[0-1]$$

$$= \frac{1}{(a+j\omega)}$$

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2) $e^{-at}u(t) + e^{at}u(-t)$

$e^{-at}u(t) \xrightarrow{FT} 1/j\omega + a$

$e^{at}u(-t) \xrightarrow{FT} 1/-j\omega + a$ (by shifting property)

$e^{-at}u(t) + e^{at}u(-t)$

$= \frac{1}{j\omega + a} + \frac{1}{-j\omega + a}$

$= \frac{2a}{a^2 + \omega^2}.$

Ques1 To find Inverse Laplace.

$f(s) = \frac{1}{(s+4)(s+2)(s+9)}$

$= L^{-1}\left[\frac{1}{(s+4)(s+2)(s+9)}\right]$

$\Rightarrow \frac{A}{(s+4)} + \frac{B}{(s+2)} + \frac{C}{(s+9)}$

Taking inverse laplace.

$\Rightarrow (Ae^{-4t} + Be^{-2t} + ce^{-9t})u(t)$

$= \frac{-1}{10}e^{-4t} + \frac{1}{10}e^{-2t} + \frac{1}{35}e^{-9t}u(t)$

Ques2 To find Inverse Laplace

(a) $f(s) = \frac{1}{(s+3)(s+5)}$

(b) $f(s) = \frac{1}{(s+1)(s+2)}$

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(a) $L^{-1}\left[\frac{1}{(S+3)(S+5)}\right]$

$= \frac{A}{(S+3)} + \frac{B}{(S+5)}$

taking inverse Laplace.

$\Rightarrow (Ae^{-3t} + Be^{-5t})u(t)$

$= \frac{1}{2(S+3)} - \frac{1}{2(S+5)}$

$A = \frac{(S+3)}{(S+3)(S+5)}$ $\boxed{S=-3}$

$B = \frac{(S+5)}{(S+3)(S+5)} \Rightarrow \boxed{S=-5}$

$L^{-1} = \left[\frac{1}{2}e^{-3t} - \frac{1}{2}e^{-5t}\right]u(t)$

(b) $f(S) = \frac{1}{(S+1)(S+2)}$

$= \frac{\cancel{A}}{\cancel{(S+}} \quad L^{-1}\left[\frac{1}{(S+1)(S+2)}\right]$

$= \frac{A}{(S+1)} + \frac{B}{(S+2)}$

taking inverse Laplace

$= Ae^{-t} + Be^{-2t}$

$S+1=0$ $\quad$ $S+2=0$

$S=-1$ $\quad$ $S=-2$

$\frac{1}{(-1+1)} = \cancel{A}\ 1$ $\quad$ $\frac{1}{-2+2} = 1$

$Ae^{-t} + Be^{-2t}$

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$$= e^{-t} + e^{-2t}.$$

Initial value Theorem.

$$\underset{t \to 0}{x(t)} = \underset{s \to \infty}{s\,f(s)}$$

final value theorem

$$\underset{t \to \infty}{x(t)} = \underset{s \to 0}{s\,f(s)}$$

Q1 find initial value.

$$f(s) = \frac{1}{(s+3)}$$

(1) $\underset{t \to 0}{x(t)} = ?$

(2) $\underset{s \to \infty}{f(s)} = \frac{s}{(s+3)}$ [L - Hospital. rule]

$$= \frac{1}{1} = 1$$

Q.2 $f(s) = \frac{1}{(s+1)}$

find $x(t)$

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Q.3. $f(s) = \frac{1}{(s+2)(s+3)}$

find Initial final value of $f(t)$

$$f(s) = \frac{1}{(s+2)(s+3)}$$

$$\underset{t \to 0}{x(t)} = \underset{s \to \infty}{s} \left(\frac{1}{(s+2)(s+3)} \right)$$

$$= \frac{s}{(s+2)(s+3)}$$

$$= \frac{1}{(s+2)+(s+3)} = \frac{1}{2s+5}$$

$$= \frac{1}{\infty}$$

$$= 0 \quad \text{(initial value)}$$

$$\underset{t \to 0}{x(t)} \quad s \to 0 \; \frac{1}{2s+5}$$

$$= \frac{1}{5} \quad \text{(final value)}$$