

UNIT – IV

Dielectric materials and their properties

Electric Dipole, Dipole Moment, Dielectric Constant, Electric Susceptibility, Electronic and Ionic polarizability (Quantitative) Orientation Polarization (Qualitative), Internal fields in Solids, Clausius - Mossotti equation, Frequency and temperature effect on Dielectrics (Qualitative), Applications - Piezo-electricity, Pyro-electricity and Ferro-electricity.

Introduction:

Dielectrics are insulating materials which do not contain free electrons or the number of such electrons is too low to constitute the electric current. The electrons are tightly bound to the nucleus of the atom in dielectrics.

Dielectrics are the materials having electric dipole moment permanently.

All dielectrics are electrical insulators and they are mainly used to store electrical energy.

Ex: Mica, glass, plastic, water & polar molecules...

Electric Dipole:

A system of two equal and opposite charges separated by a distance is called electric dipole.



Dipole Moment(μ): The product of charge and distance between two charges is called dipole moment.

$\mu = qd$ Units: Coulomb-meter or Debye

Permittivity(ϵ): Permittivity is a quantity which represents the dielectric property of a medium. Permittivity of a material indicates the easily polarizable nature of the material.

For Vacuum or free space, $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$

Dielectric Constant(ϵ_r):

The dielectric constant or relative permittivity of a medium is defined as the ratio between the permittivity of that medium and the permittivity of free space.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{c}{c_0}$$

The characteristics of a dielectric material are determined by the dielectric constant and it has no units.

where ' ϵ ' is permittivity of the medium, ' ϵ_0 ' is permittivity of the free space, ' c ' is capacitance of a capacitor with dielectric and ' c_0 ' is the capacitance of a capacitor without dielectric.

Capacitance(c): The property of a conductor that describes its ability to store electric charge is called capacitance and is given by

$$c = \frac{q}{V} = \frac{A\epsilon}{d}$$

Where ' c ' is capacitance of the capacitor, ' q ' is charge on the capacitor plates, ' V ' is potential difference between the plates, ' A ' is area of a capacitor plate, ' ϵ ' is permittivity of the medium and ' d ' is distance between capacitor plates.

Units: Farad

Electric Polarization:

The process of producing electric dipoles by influence of an electric field is called electric polarization.

Polarizability(α):

The induced dipole moment per unit electric field is called Polarizability.

The induced dipole moment of a system is proportional to the applied electric field (E).

Therefore, $\mu \propto E$

$$\mu = \alpha E,$$

Where ' α ' is Polarizability

$$\alpha = \mu/E$$

Polarization Vector(P):

The average dipole moment per unit volume of the dielectric material is called polarization vector.

$$P = N\mu = N\alpha E$$

Where ' N ' is the number of atoms present per unit volume and ' μ ' is induced dipole moment.

Electric Flux Density (D):

Electric flux density is defined as charge per unit area and it has same units of dielectric polarization.

Electric flux density ' D ' at a point in a free space or air in terms of Electric field strength is

$$D_0 = \epsilon_0 E \quad \rightarrow (1)$$

At the same point in a medium is given by

$$D = \epsilon E \quad \rightarrow (2)$$

As the polarization measures the additional flux density arising from the presence of material as compared to free space

$$\text{i.e., } D = \epsilon_0 E + P \quad \rightarrow (3)$$

Using equations 2 & 3 we get

$$\epsilon E = \epsilon_0 E + P$$

$$(\epsilon - \epsilon_0) E = P$$

$$(\epsilon_r \cdot \epsilon_0 - \epsilon_0) E = P \quad \left(\because \epsilon_r = \frac{\epsilon}{\epsilon_0} \right)$$

$$\therefore (\epsilon_r - 1)\epsilon_0 E = P$$

Electric Susceptibility (χ_e):

The polarization vector P is proportional to the total electric flux density and is in the same direction of electric field.

Therefore, the polarization vector can be written

$$P = \epsilon_0 \chi_e E$$

$$\chi_e = \frac{P}{\epsilon_0 E}$$

$$= \frac{(\epsilon_r - 1)\epsilon_0 E}{\epsilon_0 E}$$

$$\chi_e = \epsilon_r - 1$$

Non-Polar molecule:

These molecules possess centre of symmetry. Here the centers of positive & negative charges coincide. Therefore, the net charge & net dipole moment of the molecules will be zero and non-polar molecules will not possess any dipole moment in it.

Ex: N_2 , H_2 , O_2 , CH_3 , CO_2 etc

Polar molecule:

These molecules will not have centre of symmetry. Here the centers of positive & negative charges will not coincide and hence it possesses a net dipole moment in it.

Ex: H_2O , N_2O , HCl , NH_3 , CO , CH_3OH etc

Various polarization processes:

When the specimen is placed inside an electric field, polarization is due to four types of processes....

- 1). Electronic polarization
- 2). Ionic polarization
- 3). Orientation polarization
- 4). Space charge polarization.

1). Electronic Polarization

When an Electric field is applied to an atom, +vely charged nucleus displaces in the direction of field and electron cloud in opposite direction. This kind of displacement will produce an electric dipole within the atom.

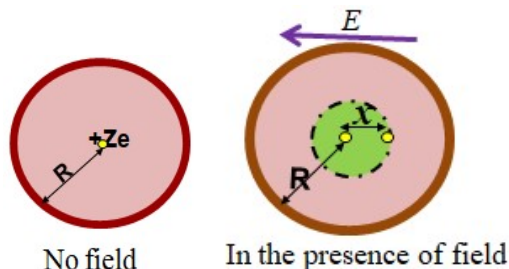
i.e., dipole moment is proportional to the magnitude of field strength and is given by

$$\begin{array}{c} \mu_e \propto E \\ \text{or} \\ \mu_e = \alpha_e E \end{array}$$

where ' α_e ' is called electronic Polarizability.

Calculation of Electronic Polarizability:

In a simplified classical model of an atom, nucleus of charge ' Ze ' is surrounded by an electron cloud of charge ' $-Ze$ ' distributed in a sphere of radius ' R '.



The charge density ' ρ ' is given by

$$\rho = \frac{-Ze}{\frac{4}{3}\pi R^3} = -\frac{3}{4} \left(\frac{Ze}{\pi R^3} \right)$$

When an electric field ' E ' is applied, nucleus and electron cloud are pulled apart due to Lorentz force of magnitude ' ZeE ' acting on them in opposite directions.

When they are separated, a coulomb force develops between them which tends to oppose the displacement. When equilibrium is reached, Lorentz force and coulomb forces are equal and opposite. Let ' x ' be the distance between nucleus and electron cloud under that condition.

Since nucleus is much heavier than the electron cloud, it is assumed that only the electron cloud is displaced when external field is applied.

$$\text{Lorentz force} = -ZeE$$

To determine the coulomb attraction on the nucleus, we divide the electron cloud into two regions. One region is inside the sphere of radius 'x' and the other is the annular region lying between the two spherical surfaces of radii 'x' & 'R'.

$$\text{Coulomb force} = \frac{1}{4\pi\epsilon_0} \frac{Ze \times (\text{Charge enclosed in the sphere of radius 'x'})}{x^2}$$

$$\begin{aligned} \text{Charge enclosed} &= \frac{4}{3}\pi x^3 \rho \\ &= \frac{4}{3}\pi x^3 \left[-\frac{3}{4} \left(\frac{Ze}{\pi R^3} \right) \right] \\ &= -\frac{Zex^3}{R^3} \end{aligned}$$

$$\begin{aligned} \text{Therefore, Coulomb force} &= \frac{Ze}{4\pi\epsilon_0 x^2} \left[-\frac{Zex^3}{R^3} \right] \\ &= -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3} \end{aligned}$$

$$\text{But, in the equilibrium position} \quad -ZeE = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$E = \frac{Zex}{4\pi\epsilon_0 R^3}$$

$$x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

Thus the displacement of the electron cloud is proportional to the applied field. Thus two electric charges +Ze and -Ze are separated by a distance 'x' under the action of the applied field thus constituting induced electric dipoles.

Induced electric dipole moment, $\mu_e = \text{Charge} \times \text{Distance}$
 $= Zex$

$$\mu_e = Ze \times \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

$$\begin{aligned} \mu_e &= 4\pi\epsilon_0 R^3 E \\ \mu_e &= \alpha_e E \end{aligned}$$

Where $\alpha_e = 4\pi\epsilon_0 R^3$ is called electronic Polarizability. Thus ' α_e ' is depending on the volume of the atom and is independent of temperature.

This kind of polarization is mostly exhibited in monatomic gases.

$$\alpha_e = \text{_____} \times 10^{-40} \text{F-m}^2$$

He	Ne	Ar	Kr	Xe
0.18	0.35	1.46	2.18	3.54

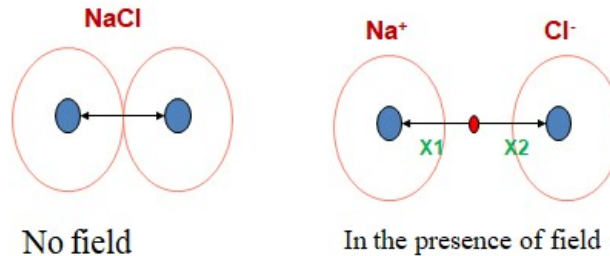
- It occurs only at optical frequencies (10^{15} Hz).
- It is independent of temperature.

2). Ionic Polarization

The ionic polarization occurs, when atoms form molecules and it is mainly due to a relative displacement of the atomic components of the molecule in the presence of an electric field.

When a Electric Field is applied to the molecule, the positive ions displaced by x_1 to the negative side electric field and negative ions displaced by x_2 to the positive side of field.

The resultant dipole moment $\mu = e(x_1 + x_2)$



Restoring force constant depends upon the mass of the ion and natural frequency and is given by

$$F = eE = m.\omega_0^2 x$$

or

$$x = \frac{eE}{m.\omega_0^2}$$

$$\text{and } x_1 = \frac{eE}{m.\omega_0^2}$$

$$x_2 = \frac{eE}{M.\omega_0^2}$$

$$\therefore x_1 + x_2 = \frac{eE}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

Where 'M' mass of anion and 'm' is mass of cat ion

$$\therefore \mu_{ionic} = e(x_1 + x_2) = \frac{e^2 E}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

$$\text{or } \alpha_{ionic} = \frac{\mu_{ionic}}{E} = \frac{e^2}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

- This polarization occurs at frequency **10^{13} Hz (IR)**.
- It is a slower process compared to electronic polarization.
- It is independent of temperature.

3). Orientation Polarization

It is also called dipolar or molecular polarization. The molecules such as H_2 , N_2 , O_2 , Cl_2 , CH_4 , CCl_4 etc., does not carry any dipole because centre of positive charge and centre of negative charge coincides. On the other hand molecules like CH_3Cl , H_2O , HCl , ethyl acetate (polar molecules) carries dipoles even in the absence of electric field.



However the net dipole moment is negligibly small since all the molecular dipoles are oriented randomly when there is no Electric Field. In the presence of the electric field these all dipoles orient themselves in the direction of field as a result the net dipole moment becomes enormous.

Expression for orientation polarization:

$$P_o = N \cdot \vec{\mu}_{ori} \Rightarrow \frac{N \cdot \mu_{ori}^2 \cdot E}{3kT} = N \cdot \alpha_{ori} \cdot E$$

$$\alpha_{ori} = \frac{\mu_{ori}^2}{3kT}$$

Thus the orientation polarization is inversely proportional to absolute temperature.

- It occurs at a frequency **10⁶ Hz to 10¹⁰ Hz**
- It is slow process compare to ionic polarization.
- It greatly depends on temperature.

Total Polarization:

$$P_{total} = P_e + P_i + P_o$$

$$\therefore \alpha = \alpha_{elec} + \alpha_{ionic} + \alpha_{ori} = 4\pi\epsilon_o R^3 + \frac{e^2}{w_0^2} \left[\frac{1}{M} + \frac{1}{m} \right] + \frac{\mu_{ori}^2}{3kT}$$

This is called **Langevin – Debye** equation for total Polarizability in dielectrics.

Frequency Dependence and Temperature effect on Dielectric:

The total Polarizability $\alpha = \alpha_e + \alpha_i + \alpha_o$ decreases by substantial amount as frequency increases. The decrease in ' α ' is not uniform, remarkable decrease occurs only in the microwave, infrared and ultraviolet regions.

This behavior of Polarizability can be understood from various polarization processes and from the concept of the relaxation time for each process. On application of **a.c field**, the polarization P(t) as a function of time 't' is given by

$$P(t) = P[-\exp(-t/t_r)]$$

Where **P** is maximum polarization attained and **t_r** is the relaxation time for the particular polarization process. It is the time taken for a polarization process to reach 0.63 of the maximum value. This varies widely for different polarization process.

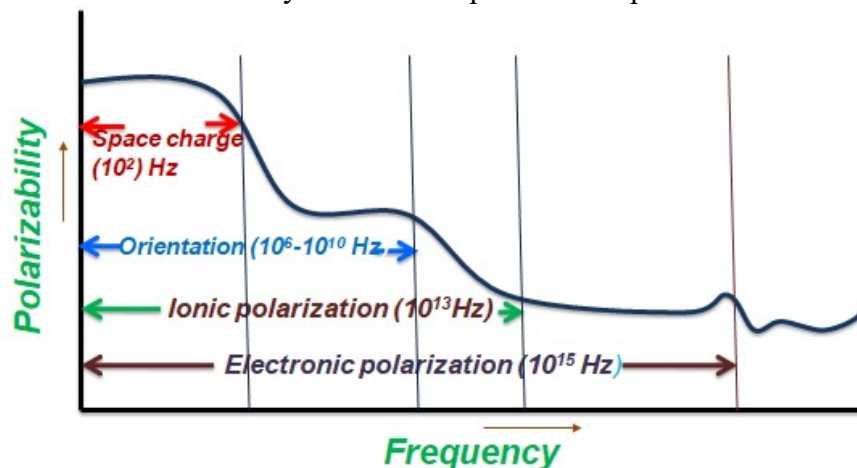


Fig. frequency dependence of various polarization processes

Electronic Polarization is extremely rapid and it's relaxation time is small.

It occurs only at optical frequencies range (~10¹⁵ Hz).

It is independent of temperature.

Ionic Polarization is a slower process compared to electronic polarization.

This polarization occurs at frequency ~10¹³ Hz (IR).

It is independent of temperature.

Orientation Polarization is a slower process compare to electronic and ionic polarization.

It occurs at a frequency 10⁶ Hz to 10¹⁰ Hz .

It greatly depends on temperature.

Effect of temperature on dielectric constant:

The electronic and the ionic polarizabilities are practically independent of temperature for normal temperatures where as orientation and space charge polarization is effected by temperature.

In orientation polarization, the randomizing action of thermal energy decreases the tendency of the permanent dipoles to align themselves in the applied field. This results in a decrease in the dielectric constant with increasing temperature.

Dielectric Loss:

When a dielectric is subjected to the a.c. voltage, the electrical energy is absorbed by the material and is dissipated in the form of heat. This dissipation of energy is called dielectric loss.

Dielectric Breakdown:

When a dielectric material loses its resistivity and permits very large current to flow through it, then the phenomenon is called dielectric breakdown.

Dielectric Strength:

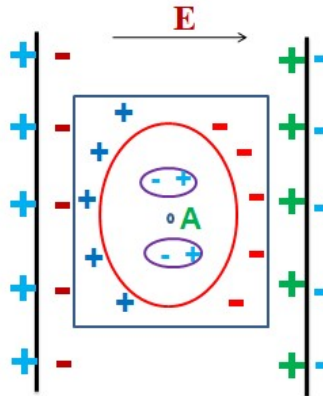
The minimum voltage required per unit thickness of the material to produce dielectric breakdown is called dielectric strength .

Unit is volt/meter

Ex: Breakdown voltage for air is 3 mV/meter.

Internal fields in Solids(Lorentz method):

When a dielectric is exposed to electric field then polarization takes place. Due to this polarization, different sets of dipoles are formed in the dielectric material. *'The total electric field present at a point or an atom in the dielectric is the sum of the electric fields produced by different dipoles. This total electric field at a point is known as **internal** or **local field**.'*

**Calculation of this field was done by Lorentz as follows:**

The dipole at the point 'A' is imagined to be surrounded by a spherical cavity of radius 'r' such that the radius 'r' is large compared with the intermolecular distance, so that the sphere contains many molecules but small compared to the dimensions of the whole dielectric. If this dielectric is placed between two capacitor plates, the electric field experienced by an atom of the dielectric, if assumed to be placed at the center of the cavity is given by

$$E_{\text{local}} = E_0 + E_P + E_S + E_m \text{ ----(1)}$$

Where,

E_0 is the field intensity at the point 'A' due to charge density on the plates of the capacitor (without dielectric).

E_P is the field at the point 'A' due to the polarized charges on the plane surfaces of the dielectric.

E_S is the field at the point 'A' due to polarized charges on the surface of the imagined spherical cavity.

E_m is the field at the point 'A' due to all the dipoles inside the spherical cavity.

Now,

$$D = \epsilon_0 E_0 + P$$

$$\frac{D}{\epsilon_0} = E_0 + \frac{P}{\epsilon_0}$$

$$E = E_0 + E_p \text{ -----(2) } (\because D/\epsilon_0 = E \text{ \& } P/\epsilon_0 = E_p)$$

At the point 'A', E_m is the field due to all dipoles present in the cavity. If the imagined sphere is highly symmetric then E_m becomes zero as the dipoles are distributed uniformly in all directions i.e., $E_m = 0$

$$\text{Therefore, } E_{\text{local}} = E + E_s \text{ -----(3)}$$

Calculation of E_s by Lorentz:

The magnified view of the imagined sphere is shown in the figure. If 'a' is the area of a area element on the surface of radius 'r' making angles between θ and $\theta + d\theta$ with field direction.

From the figure, area $da = 2\pi(PQ)(QR)$

$$\text{From } \Delta APQ, \sin\theta = PQ/r$$

$$PQ = r \sin\theta$$

$$\text{From Sector AQR } d\theta = QR/r$$

$$QR = r d\theta$$

$$\text{Therefore, } da = 2\pi r^2 \sin\theta d\theta$$

The charge dq on the surface da is equal to the normal component of the polarization multiplied by the surface area. Therefore,

$$dq = P \cos\theta da$$

$$dq = P \cos\theta 2\pi r^2 \sin\theta d\theta$$

$$= P 2\pi r^2 \sin\theta \cos\theta d\theta$$

The field due to charge dq is denoted by dE_s at 'A',

$$dE_s = \frac{dq \cos\theta}{4\pi\epsilon_0 r^2}$$

$$= \frac{P(2\pi r^2 \sin\theta \cos\theta d\theta) \cos\theta}{4\pi\epsilon_0 r^2}$$

$$dE_s = \frac{P}{2\epsilon_0} \cos^2\theta \sin\theta d\theta$$

Integration above equation

$$E_s = \int_0^\pi dE_s$$

$$= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2\theta \sin\theta d\theta$$

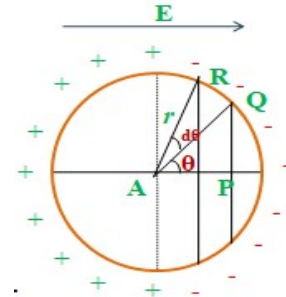
$$\text{Put } \cos\theta = x \Rightarrow -\sin\theta d\theta = dx$$

$$\text{when } \theta = 0 \text{ then } x = +1; \theta = \pi \text{ then } x = -1$$

$$E_s = -\frac{P}{2\epsilon_0} \int_{+1}^{-1} x^2 dx = -\frac{P}{2\epsilon_0} \left[\frac{x^3}{3} \right]_{+1}^{-1} = -\frac{P}{2\epsilon_0} \left[-\frac{1}{3} - \frac{1}{3} \right]$$

$$\therefore E_s = \frac{P}{3\epsilon_0}$$

Therefore, the local field at a point or on an atom in a dielectric (from eqn (3))



$$E_{\text{local}} = E + \frac{P}{3\epsilon_0}$$

This equation is known as Lorentz relation or local field or internal field. Here, the local field is to be greater than the electric field applied.

Clausius – Mosotti Equation:

The relation between the dielectric constant of dielectric and the Polarizability of atoms is known as Clausius – Mosotti equation.

The induced dipole moment of a single atom is proportional to the local field, i.e.,

dipole moment $\mu \propto E_{\text{local}}$

$$\mu = \alpha E_{\text{local}}$$

Where α is Polarizability

If there are ‘N’ number of atoms per unit volume, the electric dipole moment per unit volume which is called polarization vector, can be written as

$$P = N\mu = N\alpha E_{\text{local}} \text{ (Since } \mu = \alpha E_{\text{local}} \text{)}$$

But local field $E_{\text{local}} = E + P/3\epsilon_0$

$$\therefore P = N\alpha \left[E + \frac{P}{3\epsilon_0} \right]$$

$$P \left\{ 1 - \frac{N\alpha}{3\epsilon_0} \right\} = N\alpha E$$

$$P = \frac{N\alpha E}{\left\{ 1 - \frac{N\alpha}{3\epsilon_0} \right\}} \text{ ---- (1)}$$

But, electric flux density or Displacement vector ‘D’ is given as

$$D = P + \epsilon_0 E$$

$$P = D - \epsilon_0 E$$

$$\frac{P}{E} = \frac{D}{E} - \epsilon_0 = \epsilon - \epsilon_0 \text{ (}\because D = \epsilon E\text{)}$$

$$\frac{P}{E} = \epsilon_0 \epsilon_r - \epsilon_0 = \epsilon_0 (\epsilon_r - 1) \text{ ---- (2)}$$

From equations (1) & (2), we have

$$\begin{aligned} \epsilon_0 (\epsilon_r - 1) &= \frac{N\alpha}{\left\{ 1 - \frac{N\alpha}{3\epsilon_0} \right\}} \\ 1 - \frac{N\alpha}{3\epsilon_0} &= \frac{N\alpha}{\epsilon_0 (\epsilon_r - 1)} \\ 1 &= \frac{N\alpha}{3\epsilon_0} + \frac{N\alpha}{\epsilon_0 (\epsilon_r - 1)} = \frac{N\alpha}{3\epsilon_0} \left[1 + \frac{3}{(\epsilon_r - 1)} \right] \\ \frac{N\alpha}{3\epsilon_0} &= \frac{1}{\left[1 + \frac{3}{(\epsilon_r - 1)} \right]} = \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} \end{aligned}$$

This equation is known as Clausius - Mosotti equation.

$$\therefore \frac{N\alpha}{3\epsilon_0} = \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)}$$

Using this relation, one can determine the value of α knowing value of ϵ_r .

Note:

$$\epsilon_r - 1 = \frac{N\alpha_e}{\epsilon_0} \Rightarrow \epsilon_r = 1 + \frac{N\alpha_e}{\epsilon_0}$$

Problems:

1). A solid elemental dielectric, with density 3×10^{28} atoms/m³ shows an electronic Polarizability of 10^{-40} farad · m². assuming the internal electric field to be a Lorentz field, calculate the dielectric constant of the material.

(Hint: $\epsilon_0 = 8.85 \times 10^{-12}$ Fm⁻¹)

$$\epsilon_r - 1 = \frac{N\alpha_e}{\epsilon_0} \Rightarrow \epsilon_r = 1 + \frac{N\alpha_e}{\epsilon_0} \quad \text{Ans: } \epsilon_r = 1.39$$

2) The dielectric constant of He gas at NTD is 1.0000684. Calculate the electronic Polarizability of He atoms if the gas contains 2.7×10^{25} atoms per m³.

(Hint:

$$\epsilon_r - 1 = \frac{N\alpha_e}{\epsilon_0} \Rightarrow \alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N} \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1} \quad \text{Ans: } \alpha_e = 2.243 \times 10^{-41} \text{ Fm}^2$$

3) A parallel plate capacitor having a plate separation of 2×10^{-3} m across which a potential of 10V is applied. Calculate the dielectric displacement, when a material of dielectric constant 6.0 is introduced between the plates.

(Hint: $D = \epsilon E = \epsilon_0 \epsilon_r E$, Since $\epsilon_r = \epsilon / \epsilon_0$)

$$E = V/d, \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1} \quad \text{Ans: } D = 265.5 \times 10^{-9} \text{ Cm}^{-1}$$

Ferro electric materials or Ferro electricity:

Ferro electric crystals exhibit spontaneous polarization i.e., electric polarization without electric field.

- Ferro electric crystals possess high dielectric constant.
- **Ferro-electric materials exhibit the dielectric hysteresis**
- Each unit cell of a Ferro electric crystal carries a reversible electric dipole moment.

Examples: Barium Titanate (BaTiO₃), Sodium nitrate (NaNO₃), Rochelle salt etc..

Applications:

- Ferroelectric materials can be used to make capacitors to store electric charge.
- The spontaneous polarization of ferroelectric materials implies a hysteresis effect which can be used as a memory function, and ferroelectric capacitors are indeed used to make ferroelectric RAM for computers.
- Ferro electric semiconductors are used to make Posistors. Which are used to measure and control temperature.

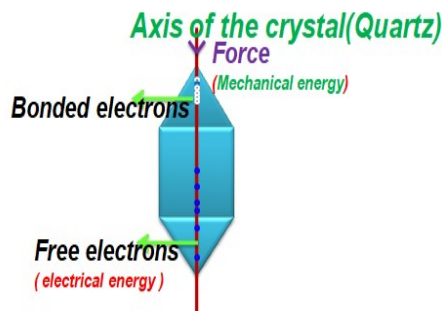
Piezo- electricity:

The process of creating electric polarization by mechanical stress is called as piezo electric effect.

This process is used in conversion of mechanical energy into electrical energy and also electrical energy into mechanical energy.

According to inverse piezo electric effect, when an electric stress is applied, the material becomes strained. This strain is directly proportional to the applied field.

Examples: **Quartz crystal, Rochelle salt etc.,**



From figure, it is clear that mechanical energy is converting into electrical energy. Hence, piezo-electrical crystals are used to construct the TRANSDUCERS

Applications:

- These crystals are used to build transducers.
- By using these materials we can construct crystal microphones, sound pick-ups etc.
- These are used to fabricate ultrasonic generators, detectors, pressure transducers & in stabilizing radio frequency oscillations.

Pyro-electricity:

Pyro-electric effect is the change in spontaneous polarization when the temperature of the specimen is changed.

Examples: BaTiO_3 , LiNbO_3 , TGS, NaNbO_3 and PZT ceramics etc.

Applications:

- The Pyro-electric materials such as BaTiO_3 , LiNbO_3 are used to make very good infra-red detectors which can operate at room temperature.
- Materials such as TGS, NaNbO_3 and PZT ceramics etc are used in the construction of Pyro-electric image tubes.

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