

**Code No:9HC12**

**Date: 31-August-2024 (FN)**

**B.Tech I-Year II- Semester External Examination, August-2024 (Regular & Supplementary)**  
**ADVANCED CALCULUS (COMMON to ALL)**

**Time: 3 Hours**

**Max.Marks:60**

**Note:** a) No additional answer sheets will be provided.  
b) All sub-parts of a question must be answered at one place only, otherwise it will not be valued.  
c) Missing data can be assumed suitably.

**Bloom's Cognitive Levels of Learning (BCLL)**

Remember	L1	Apply	L3	Evaluate	L5
Understand	L2	Analyze	L4	Create	L6

**Part - A**

**Max.Marks: 6x2=12**

**ANSWER ALL QUESTIONS, EACH QUESTION CARRIES 2 MARKS.**

- |  | BCLL | CO(s) | Marks |
|--|------|-------|-------|
| 1 Calculate the first order partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ where $u(x, y) = ye^{-2x} + x \log(xy) - x^2$ | L2   | CO1   | [2M]  |
| 2 Evaluate the integral $\int_1^2 \int_{-3}^4 \int_1^3 (x + xy + z^2) dz dy dx$ .  | L3   | CO2   | [2M]  |
| 3 Form the differential equation for $z = f(x - y) + g(x + y)$ where $f$ and $g$ are arbitrary functions.  | L3   | CO3   | [2M]  |
| 4 Define Fourier Series.   | L1   | CO4   | [2M]  |
| 5 Calculate $\text{curl}(\text{grad } u)$ where $u(x, y, z) = xy + yz + zx$  | L2   | CO5   | [2M]  |
| 6 State Gauss divergence theorem.  | L1   | CO6   | [2M]  |

**Part - B**

**Max.Marks: 6x8=48**

**ANSWER ALL QUESTIONS. EACH QUESTION CARRIES 8 MARKS.**

- |   | BCLL | CO(s) | Marks |
|---|------|-------|-------|
| 7. If $U = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ , $x^2 + y^2 + z^2 \neq 0$ , then prove that $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$ | L3   | CO1   | [8M]  |
| OR  |      |       |       |
| 8 Examine for extreme values, if any, of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$   | L4   | CO1   | [8M]  |
| 9. i) Evaluate the integral $\iint_R x^2 dx dy$ over the region 'R' bounded by $x = 0, y = 0, x + y = 2$  | L3   | CO2   | [8M]  |
| ii) Evaluate the integral $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dz dx dy$   | L3   |       |       |
| OR  |      |       |       |
| 10 Evaluate the triple integral $\int_0^1 \int_1^2 \int_2^3 x^2 y^2 z^3 dx dy dz$   | L3   | CO2   | [8M]  |
| 11 i). Form the partial differential equation by eliminating the arbitrary constants from $z = a \log \left\{ \frac{b(y-1)}{1-x} \right\}$  | L3   | CO3   | [8M]  |
| ii). Solve Lagrange's linear equation $x(y - z)p + y(z - x)q = z(x - y)$  | L3   |       |       |
| OR  |      |       |       |
| 12 Solve the differential equation $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ .   | L3   | CO3   | [8M]  |

- 13 Find Fourier series expansion of the function  $f(x) = x - x^2$  in  $[-\pi, \pi]$  Hence show that  $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$ . L3 CO4 [8M]
- OR
- 14 Obtain Half range Fourier cosine series expansion of  $f(x) = x \sin x, 0 < x < \pi$  and hence show that  $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi-2}{4}$  L3 CO4 [8M]
- 15 i) Find the directional derivative of  $f(x,y,z) = xy^2 + xyz + zx$  along the direction of the vector  $i + 2j + 2k$  at the point  $(1, 2, 0)$ . L3 CO5 [8M]  
 ii) Show that the vector function  $\vec{F} = (2x + yz)i + (2y + xz)j + (2z + xy)k$  is irrotational. L3
- OR
- 16 Show that the vector field defined by  $F = yzi + xzj + xyk$  is irrotational and find its scalar potential. L3 CO5 [8M]
- 17 Make use of Green's theorem in a plane to evaluate the line integral  $\oint_C (xy + y^2)dx + x^2dy$ , where  $C$  encloses the region bounded by  $y = x$  and  $y = x^2$  L4 CO6 [8M]
- OR
- 18 Verify Stoke's theorem for  $\vec{f} = (x^2 + y^2)i - 2xyj$  taken round the rectangle bounded by the lines  $x=\pm a, y=0$  and  $y=b$ . L4 CO6 [8M]

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