Indira Gandhi Delhi Technical University For Women

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APPLIED MATHEMATICS-1(BAS-101)
(Differential Calculus)
TUTORIAL SHEET -7

1. Find the nth derivatives of the following :

i) $\sin^2 x \cos^3 x$

$(\frac{1}{16}(2\cos\left(x + \frac{n\pi}{2}\right) - 3^n \cos\left(3x + \frac{n\pi}{2}\right) - 5^n \cos\left(5x + \frac{n\pi}{2}\right)))$

ii) $e^x \cos x \cos 2x$

$(\frac{e^{ax}}{2}[10^{n/2}\cos(3x + n\tan^{-1}3) + 2^{n/2}\cos\left(3x + \frac{n\pi}{4}\right)])$

iii) $\frac{x}{(x+1)(2x+1)}$

$((-1)^n gamma\ (n)[\frac{1}{(x+1)^{n+1}} - \frac{2n}{(2x+1)^{n+1}}])$

iv) $\frac{x^2 + 4x + 1}{x^3 + 2x^2 - x - 2}$

$(y_n = (-1)^n n! \left[\frac{1}{(x-1)^{n+1}} + \frac{1}{(x+1)^{n+1}} - \frac{1}{(x+2)^{n+1}}\right])$

2. If y= $x^2 e^x$, prove that $y_n = \frac{n(n-1)}{2} y_2 - n(n-2)y_1 + \frac{(n-1)(n-2)}{2} y$

3. If y=$[x + \sqrt{(1 + x^2)}]^m$, show that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$

Also find $y_n(0)$.

4. If y = $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$

5. If x = tan(log y) , prove that $(1 + x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$

6. If y = $(1 - x)^{-\alpha}e^{-\alpha x}$, then prove that(1-x) $y_{n+1} - (n+\alpha x)y_n - n\alpha y_{n-1} = 0$

7. Find the limit and test for continuity of the function

$$f(x,y)=\begin{cases} \frac{x^3-y^3}{x+y} & \text{if } x+y \neq 0 \\ 0 & if\ x+y=0 \end{cases}$$

at the point (0,0)

(Ans. limit does not exist and function is not continuous)

8. Test continuity of the following

$$F(x,y)=\begin{cases} x^2+4y & \text{if } (x,y) \neq (1,2) \\ 0 & if\ (x,y)=(1,2) \end{cases}$$

(Ans. Not continuous)

9. If u= x^y,then show that (i) $u_{xy}=u_{yx}$ (ii) $u_{xxy}=u_{xyx}$