Indira Gandhi Delhi Technical University For Women

(Formerly Indira Gandhi Institute of Technology)

Kashmere Gate, Delhi-110006

PROBABILITY AND STATISTICS (BAS-103)

TUTORIAL SHEET -3

(Random Variable and Probability Distribution)

Q1. Consider a random variable X that is equal to 1, 2 and 3. If $p(1) = \frac{1}{2}, p(2) = \frac{1}{3}$ and $p(3) = \frac{1}{6}$.

(a) Check whether the given function is pmf or not?

(b) If yes, then determine its cdf and draw its graph also.

[Ans. (a) Yes (b) $F(x) = \begin{cases} 0\,, x < 1 \\ \frac{1}{2},\ 1 \le x < 2 \\ \frac{5}{6}, 2 \le x < 3 \\ 1,\ 3 \le x \end{cases}$]

Q2. A random variable X has the density function

$$f(x) = k.\frac{1}{1+x^2},\ -\infty < x < \infty$$

Determine k and the distribution function.

[Ans:- The value of k = $1/\pi$, $F(x) = \frac{1}{\pi} tan^{-1} x + \frac{1}{2},\ -\infty < x < \infty$]

Q3. Suppose that a random variable X has the density function

$$\text{f(x)} = \frac{1}{2} e^{-|x|},\quad -\infty < x < \infty$$

Find the value x_0 such that $F(x_0) = 0.5$. [Ans:- 0.5]

Q4. In a statistical survey, it was found that if X denote the daily numher of hours a child watches television and Y denotes the numher of hours he or she spends on the studies, then (X, Y) has the joint density function

$$f(x,y) = \begin{cases} xy\, e^{-(x+y)} & if \quad x > 0, y > 0 \\ 0 & otherwise \end{cases}$$

What is the probability that a child chosen at random spends at least twice as much time watching television as he or she does on studies? [Ans:- 7/27]

Q5. Find the joint density function of (X, Y) when its joint distribution function is given by

$$F_{x,y}(X,Y) = \begin{cases} (1 - e^{-\lambda y})(1 - e^{-\lambda x}) & if \quad x > 0, y > 0 \\ 0 & otherwise \end{cases}$$

[Ans:- $f_{X,Y}(x,y) = \begin{cases} \lambda^2 e^{-\lambda(x+y)} & if \quad x > 0, y > 0 \\ 0 & otherwise \end{cases}$]

Q6. Suppose (X, Y) has the joint density function

$$f(x,y) = \begin{cases} y^2\, e^{-y(x+1)}, & if \quad x \ge 0, y \ge 0 \\ 0 & otherwise \end{cases}$$

Determine the marginal densities $f_X(x)$ and $f_Y(y)$ of X and Y, respectively.

[Ans:- $f_X(x) = 0$, $f_Y(y) = 0$]

Q7. Suppose that the joint density function of (X, Y) is

$$f(x,y) = \begin{cases} C(x+y^2), & for \quad x > 0, y < 1 \\ 0 & otherwise \end{cases}$$

Find C. Also, find the conditional probability density function of X given Y = y for 0 < y < 1. Then compute P (X < $\frac{1}{2}$ | Y = $\frac{1}{2}$). [Ans:- 1/3]

Q8. Suppose (X, Y) is a random vector with the joint density function

$$f(x,y) = \begin{cases} 12xy(1-y), & for \quad 0 < x, y < 1 \\ 0 & otherwise \end{cases}$$

Show that X and Y are independent random variables.

Q9. Let X_1 be the score obtained on the first throw and X_2 be the score obtained on the second throw of an unbiased die. Define W = $X_1 - X_2$. Obtain the p.m.f. of W.

[Ans:- f(-5) = f(5) = 1/36, f(-4) = f(4) = 2/36, f(-3) = f(3) = 3/36, f(-2) = f(2) = 4/36, f(-1) = f(1) = 5/36, f(0) = 6/36]

Q10. The joint p.m.f. f(x, y) of two random variables X and Y is given in the following table.

x \ y	0	1	2	3
0	1/27	3/27	3/27	1/27
1	3/27	6/27	3/27	0
2	3/27	3/27	0	0
3	1/27.	0	0	0

Obtain

i. P[X = 2],
ii. P[Y = 0]
iii. P[X = 1, Y ≤ 2]
iv. P[X ≤ 2, Y = 0]
v. P[X = 2 | Y = 0].

[Ans:- (i) 2/9, (ii) 8/7, (iii) 4/9, (iv) 7/27, (v) 3/8]

Q11. Consider the function - f(x,y) = c(x + y + 1), x = 0,1,2, 3, y = 0,1,2.

a) Determine the value of c so that the function f(x,y) represent the joint p.m.f. of the r.vs. X and Y.
b) Obtain the marginal p.m.fs. of X and Y for the function f(x,y).
c) Examine if X and Y are independent for the function f(x,y).

[Ans:- (a) 1/42, (b) f(x) = (x+2)/14 for x = 0,1,2,3 ; f(y) = (2y+5)/21 for y = 0,1,2 (c) Dependent]