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Applied Mathematics (BAS-101)

Tutorial Sheet -8

(Vector Calculus)

Q1. Find grad f, where f is given by $f = x^3 - y^3 + xz^2$, at the point (1, -1, 2).

Q2. Find the gradient and the unit normal to the level surface $x^2 + y - z = 4$ at the point (2,0,0).

Q3. Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

Q4. Find the equations of tangent plane and normal to the surface $z = x^2 + y^2$ at the point (2, -1, 5).

Q5 Find the divergence and curl of the vector $\vec{f} = (x^2 - y^2)\hat{\imath} + 2xy\,\hat{\jmath} + (y^2 - xy)\hat{k}$.

Q6. Find the constants a, b, c so that the vector $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\,\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.

Q7. Determine the constant $'a'$ so that the vector $\vec{V} = (x + 3y)\hat{\imath} + (y - 2z)\,\hat{\jmath} + (x + az)\hat{k}$ is solenoidal.

Q8. Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = (x^2 - y^2)\hat{\imath} + xy\,\hat{\jmath}$ and curve C is the arc of the curve $y = x^3$ from (0, 0) to (2, 8).

Q9. Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\,\hat{\jmath}$ moves a particle in xy-plane from (0, 0) to (1, 1) along the parabola $y^2 = x$.

Q10. If $\vec{F} = y\hat{\imath} - (x - 2xz)\,\hat{\jmath} - xy\,\hat{k}$, evaluate $\iint_S (\nabla \times \vec{F}).\hat{n}\,ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane.

Q11. If $\vec{F} = (2x^2 - 3z)\hat{\imath} - 2xy\,\hat{\jmath} - 4x\,\hat{k}$, then evaluate $\iiint_V \nabla.\vec{F}\,dV$ where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. Also evaluate $\iiint_V \nabla \times \vec{F}\,dV$.

Q12. Verify Green's theorem in the plane for $\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region defined by $x = 0, y = 0, x + y = 1$.

Q13. Use Gauss Divergence theorem to show that $\iint_S \left[(x^3 - yz)\hat{\imath} - 2x^2y\,\hat{\jmath} + 2\,\hat{k}\right].\hat{n}\,ds = a^5/3$

where S denotes the surface of the cube bounded by the planes $x = 0,\ x = a,\ y = 0,\ y = a,$

$z = 0,\ z = a.$

Q14. Verify Stoke's theorem for $\vec{F} = (x^2 + y - 4)\hat{\imath} + 3xy\,\hat{\jmath} + (2xz + z^2)\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$ and C is its boundary.

Q15. A fluid motion is given by $\vec{V} = (y\sin z - \sin x)\hat{\imath} + (x\sin z + 2yz)\hat{\jmath} + (xy\cos z + y^2)\hat{k}$. Is the motion irrotational ? If so, find the velocity potential.

Answer Key

Ans 1. $7\hat{\imath}$-$3\hat{\jmath}$+$4\hat{k}$

Ans 2. $4\hat{\imath} + \hat{\jmath} - \hat{k}$, $\frac{4\hat{\imath}+\hat{\jmath}-\hat{k}}{3\sqrt{2}}$

Ans 3. $\frac{37}{3}$

Ans 4. Equation of tangent plane is $4x - 2y - z = 5$

Equation of normal to surface is $\frac{x-2}{4} = \frac{y+1}{-2} = \frac{z-5}{-1}$

Ans 5. $div.\vec{f} = 4x$

$Curl\ \vec{f} = (2y - x)\hat{\imath} + y\,\hat{\jmath} + 4y\,\hat{k}$

Ans 6. a =4, b=2, c=-1

Ans 7. $a = -2$

Ans 8. $\frac{824}{21}$

Ans 9. $\frac{-2}{3}$

Ans 10. 0

Ans 11. $\frac{8}{3}\left(\hat{\jmath} - \hat{k}\right)$

Ans 15. Velocity Potential $u = xy\sin z + \cos x + y^2z + c$