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APPLIED MATHEMATICS-1(BAS-101)

TUTORIAL SHEET -3

(MATRICES)

Q.1 Find the eigen values and the corresponding eigenvectors of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Q.2 Find the eigen values and the corresponding eigenvectors of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q.3Verify Cayley-Hamilton theorem for the given matrix A and hence, compute inverse.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Q.4 Show that $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ satisfies the matrix equation $A^3 - 4A^2 - 3A + 11I = 0$

Q.5 Find
$$A^8$$
, if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Q.6 Diagonalize A and find Modal Matrix

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

Q.7 Show that the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is diagonalizable.

Q.8 Find the eigenvalues, eigenvectors of

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Is A diagonalizable?

Answer key:

- 1. corresponding to eigenvalue $\lambda_1 = -3$ and $\lambda_2 = -3$, eigenvectors are (3, 0, 1) and (2, -1, 0) and corresponding to $\lambda_3 = 5$, eigenvector is (1, 2, -1)
- 2. corresponding to $\lambda_1 = 0$, eigenvector is (1, 2, 2), corresponding to $\lambda_2 = 3$, eigenvector is (2, 1, -2) and corresponding to $\lambda_3 = 15$, eigenvector is (2, -2, 1)
- 3. $\frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$
- 5. 625I, where I is and identity matrix
- 6. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{5} & 0 \\ 0 & 0 & \sqrt{-5} \end{bmatrix} and modal matrix is \begin{bmatrix} 1 & \sqrt{5} 1 & \sqrt{5} + 1 \\ 0 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$
- 8. Corresponding to $\lambda_1 = 1$, eigenvector is (1, 0, -1) corresponding to $\lambda_2 = 2$, eigenvector is (0, 1, 0) and corresponding to $\lambda_3 = 3$, eigenvector is (1, 0, 1). Since distinct eigenvalues, hence diagonalizable)