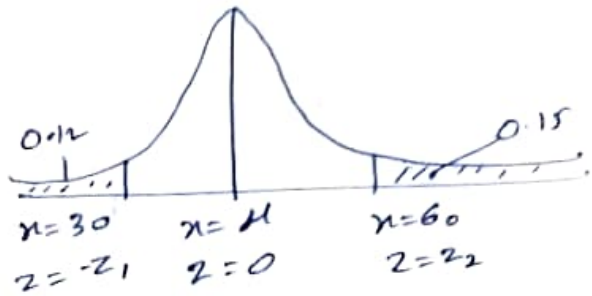
In a distribution which is exactly normal, 12% of the items are under 30 and 85% are under 60. find the mean & S.d of the distribution.

let $X \sim N(\mu, \sigma^2)$



Then
$P(X < 60) = 0.85$
or $P(X > 60) = 0.15$
and $P(X < 30) = 0.12$

Obviously $X = 30$ is to the left of $X = \mu$ &
$X = 60$ " right of $X = \mu$

when $X = 30$, $z = \frac{30-\mu}{\sigma} = -z_1$

" $X = 60$, $z = \frac{60-\mu}{\sigma} = z_2$

$P(z < z_1) = 0.12$
$z_1 = -1.17$
$P(z < z_2) = .85$
$z_2 = 1.04$

$\therefore P(0 < z < z_1) = 0.38$ & $P(0 < z < z_2) = 0.35$

From table, $z_1 \simeq 1.175$ and $z_2 = 1.0365$

$\therefore 30 - \mu = -\sigma z_1 = -1.175\sigma$

$60 - \mu = \sigma z_2 = 1.0365\sigma$

on solving them, $\sigma = 13.5655$ & $\mu = 45.9395$

J I P. 1972 Ex 19 c

Ex In an examination, the candidates are awarded the following grades depending on the marks scored by them:

distinction $\geq 80\%$
first class: $60\% \leq$ marks $< 80\%$
second class $45\% \leq$ " $< 60\%$

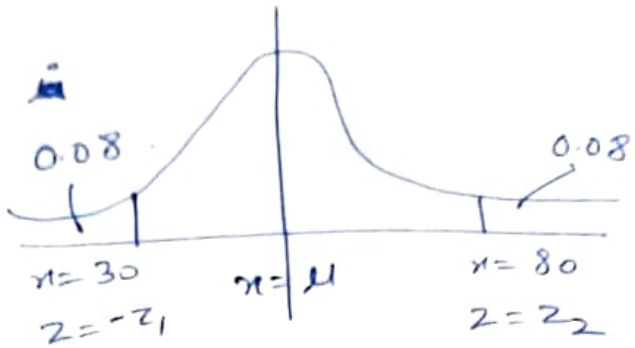
It was found that 8% of the students failed and 8% have scored distinction. Find the average marks obtained by the student/candidates.
Deduce the % of students placed in the second class.
Assume normal distribution of marks.

Let X denote the marks obtained by the candidates.

Let $X \sim N(\mu, \sigma^2)$

$P(X < 30) = 0.08$ & $P(X \geq 80) = 0.08$

$Z = \frac{X - \mu}{\sigma}$, the standard normal variate is



$X = 30$ is to the left of $X = \mu$ and

$X = 80$ is to the right of $X = \mu$

For $x = 30$, $Z = \frac{30 - \mu}{\sigma} = -z_1$

" $x = 80$, $Z = \frac{80 - \mu}{\sigma} = z_2$

Since $P(X < 30) = 0.08$ we have $P(Z < -z_1) = 0.08$

or $P(0 < Z < z_1) = 0.5 - 0.08 = 0.42$

Since $P(X \geq 80) = 0.08$ we have $P(0 < Z < z_2) = 0.42$

from table, we get $z_1 = 1.405 = z_2$

Hence $\frac{30 - \mu}{\sigma} = -1.405$, $\frac{80 - \mu}{\sigma} = -1.405$

Adding $110 - 2\mu = 0$ or $\mu = 55$

$\sigma = \frac{80 - \mu}{1.405} = \frac{25}{1.405} = 17.79$

Let p be the prob. that a candidate is placed in second class

Then $p = P(45 \leq X < 60) = P(-0.56 \leq Z < 0.28)$

$= P(-0.56 \leq Z < 0) + P(0 < Z < 0.28)$

$= P(0 < Z \leq 0.56) + P(0 < Z < 0.28)$

$= 0.2123 + 0.1103 = 0.3226$

$\therefore$ 32% candidates (approx) obtained second class.