



Indira Gandhi Delhi Technical University For Women

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PROBABILITY AND STATISTICS (BAS-103)

TUTORIAL SHEET -4

(Expectation, Variance, Covariance & Moment Generating Function)

Q1. Show that $Cov(X, Y) = E(XY) - E(X)E(Y)$.

Q2. Let the p.m.f. of a random variable X be $f(x) = \frac{(3-x)}{10}$, $x = -1, 0, 1, 2$.

- Calculate $E(X)$.
- Calculate $E(X^2)$.
- Use the results of (a) and (b) to calculate $E[(4X + 5)^2]$.
- Obtain the m.g.f. and the moments of the random variable.

[Ans. (a) 0 (b) 1 (c) 41 (d) $\mu'_0 = 1, \mu'_1 = \frac{7}{2}, \mu'_2 = \frac{91}{12}, \dots$]

Q3. An unbiased die is rolled. We say that a success occurs if the score obtained is 1 or 2. Any other score (i.e. a score of 3, 4, 5 or 6) is called a failure. Let $X_k = 0$ or 1 according as the k^{th} trial results in a failure or success. Notice that $X_1 + \dots + X_n$ is the number of successes obtained in n rolls of the die.

- Obtain $E(X_k)$ and hence the expected number of successes in n rolls of the die.
- Obtain the variance of the total number of the successes under the assumption that X_1, X_2, \dots, X_n are independent random variables.

[Ans. (a) $1/3$ & $n/3$ (b) $2n/9$]

Q4. Suppose the joint p.m.f. of X and Y is given by the following table:

$y \backslash x$	0	1	2	$g(x)$
0	$3/28$	$9/28$	$3/28$	$15/28$
1	$3/14$	$3/14$	0	$3/7$
2	$1/28$	0	0	$1/28$
$h(y)$	$5/14$	$15/28$	$3/28$	1

Obtain $Var(X + Y)$.

[Ans. $45/112$]

Q5. Suppose the random variables X and Y have the joint p.m.f. $f(x, y)$ specified by the following table

$x \backslash y$	0	1
1	0.20	0.15
2	0.20	0.30
3	0.05	0.10

Compute the means, the variances and the covariances for the joint distribution.

[Ans. $E(X) = 2.1$, $E(Y) = 0.55$, $\text{Var}(X) = 4.19$, $\text{Var}(Y) = 0.2475$, $\text{Cov}(X, Y) = 0.395$]

Q6. Obtain the distribution of $X + Y$ when the joint p.m.f. of X and Y is defined as follows

a) $f(x, y) = \frac{x+y}{30}$ for $x = 0, 1, 2, 3$ and $y = 0, 1, 2$.

b)

$x \backslash y$	0	1	2	3	$g(x)$
0	0	1/6	1/12	1/12	1/3
1	1/24	1/24	1/8	0	5/24
2	5/24	4/24	1/24	1/24	11/24
$h(y)$	6/24	9/24	6/24	3/24	1

[Ans. (a) possible values of $X + Y$ are 0, 1, 2, 3, 4, 5 and its p.m.f is 0, 5/24, 1/3, 3/8, 1/24, 1/24

(b) possible values of $X + Y$ are 0, 1, 2, 3, 4, 5 and its p.m.f is 0, 1/15, 1/5, 3/10, 4/15, 5/30.]

Q7. Suppose the random vector (X, Y) has the joint density function given by

$$f(x, y) = \begin{cases} \frac{1}{\pi}, & x^2 + y^2 \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Then find

- The marginal density functions of X and Y .
- The covariance of X and Y .
- $E(X^2)$.

[Ans. (a) $\frac{2}{\pi}\sqrt{1-x^2}$ and $\frac{2}{\pi}\sqrt{1-y^2}$ (b) 0]