

### Assignment 3

$$\textcircled{1} \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix}$$

$$(-2-\lambda) [\lambda^2 - \lambda - 12] - 2[-2\lambda - 6] - 3[-4 + 1 - \lambda]$$

$$= -2\lambda^2 + 2\lambda + 24 - \lambda^3 + \lambda^2 + 12\lambda + 4\lambda + 12 + 3 + 3\lambda = 0$$

$$= -\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$$

$$= \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$= (\lambda + 3)^2 (\lambda - 5) = 0$$

$$\lambda = -3/5$$

$$\text{for } \lambda = -3$$

$$\begin{bmatrix} 2 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\text{Eigen values} = -3 \text{ and } -3/5$$

$$x + 2y - 3z = 0$$

$$2x + 4y - 6z = 0$$

$$-x - 2y + 3z = 0$$

$$x = k$$

$$\text{Eigen vector} = \begin{bmatrix} k \\ k \\ k \end{bmatrix}$$



for  $\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 6 \end{bmatrix}$$

$$-7x + 2y - 3z = 0$$

$$2x - 4y - 6z = 0$$

$$-x - 2y - 5z = 0$$

$$2x - 4y - 6z = 0$$

$$-2x - 4y - 10z = 0$$

$$-8y - 16z = 0$$

$$y = -2z$$

$$z = k$$

$$y = -2k$$

$$-x - 2y + 5z = 0$$

$$-x = -2(-2k) + 5(k)$$

$$\Rightarrow -x = -4k + 5k$$

$$x = -k$$

Eigen vector =  $\begin{bmatrix} -k \\ -2k \\ k \end{bmatrix}$

$$\textcircled{2} \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$$(4-\lambda) [(1-\lambda)^3 + (2(1-\lambda))] = 0$$

$$(1-\lambda) [(4-\lambda)(1-\lambda) + 2] = 0$$

$$\lambda = 1$$

$$4 - 4\lambda - \lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$$

$$\lambda(\lambda-3) - 2(\lambda-3)$$

$$\lambda = 3, \lambda = 2$$

Eigen values = 1, 2, 3



for  $\lambda = 1$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$3x + z = 0$$

$$-2x = 0$$

$$-2y = 0$$

$$\text{vector} = \begin{bmatrix} k \\ 0 \\ -3k \end{bmatrix}$$

for  $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$2x + z = 0$$

$$x = k \quad z = -2k$$

$$-2x - y = 0$$

$$y = -2k$$

$$-2x - z = 0$$

$$\text{vector} = \begin{bmatrix} k \\ -2k \\ -2k \end{bmatrix}$$

for  $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x + z = 0$$

$$x = k \quad z = -k$$

$$-2x - 2y = 0$$

$$y = -k$$

$$-2x - 2z = 0$$

$$\text{vector} = \begin{bmatrix} k \\ -k \\ -k \end{bmatrix}$$

$$\textcircled{3} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix} = 0$$

$$(5-\lambda)[(-\lambda)(3-\lambda)] = 0$$

$$\lambda = 0, 3, 5 \quad (\text{values})$$

for  $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$5x = 0$$

$$-x + 3z = 0$$

$$x = 3k \quad z = k$$

$$\text{vector} = \begin{bmatrix} 3k \\ 0 \\ k \end{bmatrix}$$

for  $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-2x = 0$$

$$3y = 0$$

$$-z = 0$$

$$\begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} = \text{vector}$$



for  $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{aligned} -5y &= 0 \\ -x - 2z &= 0 \\ y &= 0 \\ z &= k \\ x &= -2k \end{aligned}$$

$$\begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix} \text{ vector.}$$

$$\textcircled{4} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & 2-\lambda \end{bmatrix}$$

$$\begin{aligned} -\lambda((3-\lambda)(-2-\lambda)) &= 0 \\ \lambda = 0 \quad \lambda = -2 \quad \lambda = 3 \end{aligned}$$

for  $(\lambda = 0)$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{aligned} 3y + 4z &= 0 \\ -2z &= 0 \\ y = z &= 0 \end{aligned}$$

$x$  must be non zero  $x = k$

Eigen vector  $\begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}$

for  $(\lambda = 3)$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{aligned} -3x &= 0 \\ 4z &= 0 \\ -5z &= 0 \end{aligned}$$

$$x = z = 0$$

$y$  must be non zero

$$y = k$$

Eigen vector =  $\begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix}$

for  $(\lambda = -2)$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{aligned} 2x &= 0 \\ x &= 0 \end{aligned}$$

$$5y + 4z = 0$$

$$z = k$$

$$y = -\frac{4k}{5}$$

Eigen vector

$$\begin{bmatrix} 0 \\ -\frac{4k}{5} \\ k \end{bmatrix}$$