Assignment 2

(1) $[1,0,0]$ $[1,1,0]$ $[1,1,1]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$|A| = 1 \rightarrow$ Independent

(2) $\begin{bmatrix} 7 & -3 & 11 & -6 \\ -56 & 24 & -88 & 48 \end{bmatrix}$ $R_2 \rightarrow R_2 + 8R_1$ $\begin{bmatrix} 7 & -3 & 11 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) \neq \rho(AB) \Rightarrow$ no soln $\rightarrow$ independent

(3) $\begin{bmatrix} -1 & 5 & 0 \\ 16 & 8 & -3 \\ 64 & 56 & 9 \end{bmatrix}$ $R_2 \rightarrow R_2 + 16R_1$ $\begin{bmatrix} -1 & 5 & 0 \\ 0 & 88 & -3 \\ 0 & 24 & 21 \end{bmatrix}$ $R_3 \rightarrow R_3 - \dfrac{24}{88} R_2$

$R_3 \rightarrow R_3 - 4R_2$

$\begin{bmatrix} -1 & 5 & 0 \\ 0 & 88 & -3 \\ 0 & 0 & 21.82 \end{bmatrix}$ $\rho(A) = \rho(B) = n$ $\rightarrow$ dependent;

(4) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{array}{l} R_3 \rightarrow R_3 + R_2 \\ R_2 \rightarrow R_2 - R_1 \end{array}$ $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{array}{l} R_3 \leftrightarrow R_4 \\ R_3 \rightarrow R_3 - 2R_4 \end{array}$

$$\begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\$(A) \neq \$(AB)$

Linear independent.

(5) $\begin{bmatrix} 2 & -4 \\ 1 & 9 \\ 3 & 5 \end{bmatrix}$ $\begin{array}{l} R_2 \to R_2 - \frac{1}{2}R_1 \\[4pt] R_3 \to R_3 - 3R_2 \end{array}$ $\begin{bmatrix} 2 & -4 \\ 0 & 11 \\ 0 & -22 \end{bmatrix}$ $R_3 \to R_3 + 2R_2$ $\begin{bmatrix} 2 & -4 \\ 0 & 11 \\ 0 & 0 \end{bmatrix}$

$\$(A) \neq \$(AB) \to$ Linear Independent

(6) $\begin{bmatrix} 3 & -2 & 0 & 4 \\ 5 & 0 & 0 & 1 \\ -6 & 1 & 0 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix}$ $R_2 - R_3\left(\frac{5}{3}\right)$ $\begin{bmatrix} 3 & -2 & 0 & 4 \\ 0 & 10/3 & 0 & -17/3 \\ -6 & 1 & 0 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix}$

$\begin{array}{l} R_3 \to R_3 - (-2)R_1 \\[4pt] R_4 \to R_4 - \left(\frac{2}{3}\right)R_1 \end{array}$ $\begin{bmatrix} 3 & -2 & 0 & 4 \\ 0 & 10/3 & 0 & -17/3 \\ 0 & -3 & 0 & 9 \\ 2 & 0 & 0 & 3 \end{bmatrix}$ $R_4 \to R_4 - \left(\frac{2}{3}\right)R_3$ $\begin{bmatrix} 3 & -2 & 0 & 4 \\ 0 & 10/3 & 0 & -17/3 \\ 0 & 0 & 0 & 39/60 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\$(A) \neq \$(AB)$ Linear independent

(7) $\begin{bmatrix} 3 & 4 & -7 \\ 2 & 0 & 3 \\ 8 & 2 & 3 \\ 5 & 5 & 6 \end{bmatrix}$ $\begin{array}{l} R_3 \to R_3 - 4R_2 \\[4pt] R_2 \to R_2 - \frac{2}{3}R_1 \\[4pt] R_4 \to R_4 - \left(\frac{5}{3}\right)R_1 \end{array}$ $\begin{bmatrix} 3 & 4 & 7 \\ 0 & -8/3 & -5/3 \\ 0 & 2 & -9 \\ 0 & \frac{-5}{3} & \frac{-17}{3} \end{bmatrix}$ $R_3 \to R_3 - \left(\frac{13}{4}\right)R_2$

$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -8/3 & -5/3 \\ 0 & 0 & -41/4 \\ 0 & \frac{-5}{3} & \frac{17}{3} \end{bmatrix}$ $R_4 \to R_4 - \left(\frac{37}{82}\right)R_3$ $\begin{bmatrix} 3 & 4 & 7 \\ 0 & -8/3 & -5/3 \\ 0 & 0 & -41/4 \\ 0 & 0 & 0 \end{bmatrix}$

$\$(A) \neq \$(AB)$ Linear independent.

(8)

$$\begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \\ 0 & -1 & 2 & 7 & 0 & 5 \\ 12 & 3 & 0 & -19 & 8 & -11 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 2R_1$

$$\begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \\ 0 & -1 & 2 & 7 & 0 & 5 \\ 0 & 3 & -6 & -21 & 0 & -15 \end{bmatrix}$$

$R_3 \longrightarrow R_3 + 3R_2$

$$\begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \\ 0 & -1 & 2 & 7 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\mathcal{S}(A) \neq \mathcal{S}(AB)$

Linear independent.