(5) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} = A$

when 2 rows are same in a matix, it value is zero. $|A| = 0$

$\lambda^3 + \lambda^2 + \lambda + |A| = 0$

$[\lambda^2 + \lambda + \quad] = 0$

and hence over eigen value is zero.

Semester 1 Assignment

① $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$

$R_3 \to R_3 - R_4$ $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$ $R_2 \leftrightarrow R_4$ $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$R_4 \to R_4 - R_3$ $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Rank = 3 (no. of non zeros in a row)

③ $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ Eigen values and eigen vectors of A^{-1} and $A+4I$.

$\left|\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}\right|$

$(2-\lambda)^2 - 1 = 0$

$\lambda^2 + 4 - 4\lambda - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

for

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda = 1, 3}$

(4) for $\lambda = 1$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$x - y = 0$

$-x + y = 0$

$$\begin{bmatrix} k \\ k \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$-x - y = 0$

$-x - y = 0$

$$\begin{bmatrix} k \\ -k \end{bmatrix}$$

Eigen values of $A = 1, 3$

Eigen values of $A^{-1} = 1, \frac{1}{3}$

Eigen values of $A + 4I = 5, 7$

If A is invertible:

Eigen vector A^{-1} = Eigen vector

Eigen vector of $(A + 4I)$ = "

f

(4) $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = 19.3$

$0.3x - 0.2y + 10z = 71.4$

$$x = \left(\frac{7.85 + 0.1y + 0.2z}{3}\right)$$

$$y = \left(\frac{-19.3 - 0.1x + 0.3z}{7}\right)$$

$$z = \left(\frac{71.4 - 0.3x - 0.2y}{10}\right)$$

$$x = \frac{7.85}{3} = 2.6167$$

$$y = \frac{-19.3 - 0.1(2.6167)}{7} = 19.5617$$

$$z = \frac{71.4 - 0.3(2.6167) ... (19.5 ...}{10}$$

$z = 6.67$

for second iteration

base case $(x, y, z) = (2.6167, 19.5617, 6.6703)$

$$x = \frac{7.85 + 0.1(19.5617) + 0.2(6.6703)}{3} = 7.716$$

$$y = \frac{-19.3 + 0.1(7.716) + 0.3(6.6703)}{7} = -2.36$$

$$z = \frac{71.4 - 0.3(7.716) - 0.2(-2.36)}{10} = 6.956$$

for third iteration,

base case $(x, y, z) = \{7.716, -2.36, 6.956\}$

$$x = \frac{7.85 + 0.1(-2.36) + 0.2(6.956)}{3} = x = 3.002$$

$$y = \frac{-19.3 + 0.1(3.002) + 0.3(6.956)}{7} = y = -2.456.$$

$$z = \frac{71.4 - 0.3(3.002) - 0.2(-2.455)}{10} = z = 7.099$$

(2) $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^3$. Rank and nullity.

$T: W \rightarrow P_2$

dimensions of w :-

symmetric 2×2 matrix has 2 independent value :-

The diagonal element and off-diagonal element. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $[b=c]$ $[a]$ $[d]$

$\dim(w) = 3$

Rank of T :-

T has all polynomials in P that can come by applying T to 2×2 mat (symmetric).

T mapping to all polynomial of degree at 2.

$\therefore$ Rank of T = 3

Using Rank-nullity Theorem, we have :-

(T) nullity $= \dim(W) - \text{rank}(T)$

$= 3 - 3 = 0$

(5) $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$ $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$

$R_4 \to R_4 + R_3$
& $R_3 \to R_3 - 2R_2$ $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\rho(A) = \rho(AB) \neq n$

inconsistent solution

(8) Jacob's method (perform 3 iterations).

(9)
$3x - 6y + 2z = 23$
$-4x + y - z = -15$
$x - 3y + 7z = 16$

initial value $x - \{0\} = 1$
$y - \{0\} = 1$
$z - \{0\} = 1$

$x = \dfrac{23 + 6y - 2z}{3}$

$y = -15 + 4x + z$

$z = \dfrac{16 - x + 3y}{7}$

$x_1 = \dfrac{23 + 6(1) - 2(1)}{3} = 9$

$y_1 = \dfrac{-15 + 4(1) + 1}{1} = -10$

$z_1 = \dfrac{16 - 1 + 3(1)}{7} = 2.$

iteration 2 :-

$x_2 = \dfrac{23 + 6(-10) - 2(2)}{3} = -25$

$y_2 = \dfrac{-15 + 4(-25) + 2}{1} = -102$

$z_2 = \dfrac{16 - (-25) + 3(-10)}{7} = -5$

Iteration 3:

$$x_3 = \frac{23 + 6(-102) - 2(-5)}{3} = 187$$

$$y_3 = \frac{-15 + 4(187) - (-5)}{1} = 753$$

$$z = \frac{16 - 187 + 3(-102)}{7} = -138$$

$x = 187$, $y = 753$, $z = -138$

(6) $T: P_2 \longrightarrow P_1$ is linear transformation or not.

$T(a+bx+cx^2) = (a+1) + (b+1)x + (c+1)x^2$

Additivity: $T(u+v) = T(u) + T(v)$ for all u, v in P_2

Homogeneity: $T(cu) = cT(u)$ for all u in P_2 and all scalars in c.

(1) $u = a + bx + cx^2$ and $v = a' + b'x + c'x^2$ in P_2

Then $u+v = (a+a') + (b+b')x + (c+c')x^2$

Now, $T(u) = (a+1) + (b+1)x + (c+1)x^2$ and

$T(v) = (a'+1) + (b'+1) + (c'+1)x^2$

$T(u+v) = (a+a'+1) + (b+b'+1)x + (c+c'+1)x^2$

~~$T(u) + T(v) = (a'+1) + (b+b'+1) + (c+c'+1)x^2$~~

$T(u) + T(v) = (a+1) + (b+1) + (c+1)x^2 + (a'+1) + (b'+1) + (c'+1)$

$= (a+a^2+1) + (b+b'+1) + (c+c'+1)x^2$

$T(u+v) = T(u) + T(v)$ satisfy additivity.

(2) $u = a+bx+cx^2 - P_2$, let c be scalar

$cu = ca + cbx + ccx^2$

$T(cu) = (ca+1) + (cb+1)x + (cc+1)x^2$

$cT(u) = c(a+1) + c(b+1)x + c(c+1)x^2$

~~coff~~ $c(a+1) + c(b+1)x + c(c+1)x^2$ satisfy homogeneity

Function T satisfies both additivity and homogeneity so it is a linear transformation.

(7) $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$

$Ax = 0$ $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ -2 & 1 & 3 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 5 & 9 \end{bmatrix}$

$R_3 = R_3 + R_2$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \rho(A) = 2$ linearly dependent , R3 has dimension 3.

S is linearly independent $\longrightarrow$ span R3

S is linearly independent and consist of vector in R3

(9) Explain one application of matrix operations in image processing with example.

One ~~example~~ application of matrix operation in image processing image transformation. It includes task such as scaling, translation and rotation.

Here are the steps.

(1) Rotation matrix :- To rotate an image, you can use a rot matrix. For a 2D image rotation, we ~~...~~ typically use =) 2

The form of the rotation matrix for 2D rotation by an angle θ.

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

(1) Represent pixel coordinates - each pixel in the original image can be represented as a vector containing its x and y coordinates.

(2) Apply rotation matrix - multiply each pixel vector by the rotation matix to get the new coordinates of the rotated pixel.

(3) Interpolation - After rotation, the new pixel positions might not align perfectly with the grid of pixels. Interpolation methods can be used to determine the intensity values of the pixels in the rotated image based on the intensity values of neighbouring pixels.

(4) Apply to entire page

for example - $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ Rotating img by 90° counter clock wise.

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ After performing matrix multiplication with each pixel coordinate, we will get our rotated image matx

$\begin{bmatrix} 3 & 6 & 9 \\ 2 & 5 & 8 \\ 1 & 4 & 7 \end{bmatrix}$ Then we will apply interpolation to get the final rotated image.

(10) Linear transformations play a crucial role in computer vision tasks, particularly in manipulation and analyzing ~~this operation efficiently~~ images. When it comes to rotating a 2D image, linear transformations are employed to achieve this operation efficiently.

In the context of rotating a 2D image, a linear transformation involves applying a transformation matrix to every pixel in image.

This transformation matrix represents the rotation operation. for 2D rotation, it looks like

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

where θ is the angle of rotation in radians.

When this transformation matrix is applied to the coordinates of each pixel in the original image. It effectively rotates the entire image around the origin.

Linear transformations are preferred for image rotation in computer vision because they preserve important properties such as lines remain straight and parallel lines remains parallel after transformation. Additionaly, linear transformations can be efficiently implemented using matrix multiplication, making them computationaly ~~tac~~ tractable.

In summary linear transformations are fundamental in rotation 2 D images in computer vision. They provide a mathematically and computationally efficient way to ~~preserve~~ manipulate images while preserving important geometric properties.