Assignment 1

(1) Test for consistency.

(i) $2x-3y+7z=5$ $\quad 3x+y-3z=13$ $\quad 2x+19y-47z-32$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - \frac{2}{3}R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \quad \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1/3 & -23/3 & \frac{29}{3} \\ 0 & 22 & -54 & 27 \end{bmatrix} \quad \begin{matrix} R_3 \to \\ R_3 - R_2 \text{ etc.} \end{matrix}$$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1/3 & -23/3 & \frac{29}{3} \\ 0 & 0 & 0 & 5 \end{bmatrix} \quad \rho(A) \neq \rho(AB) \longrightarrow \text{inconsistent.}$$

(ii) $2x-y+3z=8$ $\quad -x+2y+z=4$ $\quad 3x+y-4z=0$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + \frac{R_1}{2} \\ R_3 \to R_3 - \frac{3}{2}R_1 \end{matrix} \quad \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 5/2 & \frac{-15}{2} & 12 \end{bmatrix} \quad R_3 \to R_3 - \frac{5}{3}R_2$$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 0 & \frac{-10}{6} & -\frac{76}{3} \end{bmatrix} \quad \rho(A) = \rho(AB) = n = 3 \to \text{unique solution.}$$

(iii) $4x-y=12$ $\quad -x+5-2z=0$ $\quad -2x+4z=-8$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + \frac{R_1}{4} \\ R_3 \to R_3 - 2R_2 \end{matrix} \quad \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 18/4 & -2 & 3 \\ 0 & -10 & 8 & -8 \end{bmatrix} \quad \cancel{R_3 \to} R_3 + \frac{40}{19} R_2$$

(b) $x+y+z=6$, $\quad x+2y+3z=10$, $\quad x+2y+\lambda z=\mu$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_2 \end{matrix} \quad \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

(i) no soln $\Rightarrow \rho(A) \neq \rho(AB)$

$\lambda = 3, \ \mu \neq 10$

(ii) uni soln $= \rho(A) = \rho(AB) = n = 3$

$\boxed{\lambda \neq 3}$

(iii) infinite sol^n = $\rho(A) = \rho(AB) \neq 3$

$\lambda = 3 \quad \mu = 10$

(c) $x+y+z = 1 \qquad x+2y+4z = \lambda \qquad x+4y+10z = \lambda^2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - R_2 \\ R_2 \to R_2 - R_1 \end{matrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 2 & 6 & \lambda^2 - \lambda \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 0 & 0 & \lambda^2 - 3\lambda + 2 \end{bmatrix}$$

$\rho(A) = \rho(AB) \neq n \to \infty \; sol^n$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda - 2) + (\lambda - 2) = 0 \qquad \boxed{\lambda = 1} \; \boxed{\lambda = 2}$

for $\lambda = 1$

$x+y+z = 1$

$x + 3z = 0$

$z = k \quad y = -3k \quad x = 1+2k$

for $\lambda = 2$

$x+y+z = 1$

$y + 3z = 1$

$z = k \quad x = 2k$

$y = 1 - 3k$

(d) $x+3y-2z = 0 \qquad 2x - y + 4z = 0 \qquad x - 11y + 14z = 0$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \quad \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(AB) \neq n$

$\infty \; sol^n$

$x + 3y - 2z = 0 \qquad z = k$

$-2y + 8z = 0$

$x = \dfrac{10k}{7} \qquad y = \dfrac{8k}{7}$

(e) $3x + y - \lambda z = 0 \qquad 4x - 2y - 3z = 0 \qquad 2\lambda x + 4y + \lambda z = 0$

$$\begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 4 & -2 & -3 & 0 \\ 2\lambda & 4 & \lambda & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - \frac{4}{3}R_1 \\ R_3 \to R_3 - \frac{2\lambda}{3}R_1 \end{matrix} \quad \begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 0 & \frac{-10}{3} & \frac{-9-4\lambda}{3} & 0 \\ 0 & \frac{12-2\lambda}{3} & \frac{3\lambda - 2\lambda^2}{3} & \end{bmatrix}$$

$R_3 \rightarrow R_3 - \frac{(12-2\lambda)}{10}$ $\begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 0 & -10/3 & \frac{-9-4\lambda}{3} & 0 \\ 0 & 0 & x & 0 \end{bmatrix}$

For non-trivial soln

$\rho(A) \neq \rho(AB)$ 4n

$x = 0$

$-14\lambda^2 + 15\lambda + 69 = 0$

$14\lambda^2 - 15\lambda - 69 = 0$

$\lambda = \frac{15 \pm \sqrt{225 - (-3864)}}{28} = \frac{15 \pm 64}{28}$

$= 2.82, -1.75$

$\lambda = \frac{79}{28}$ $\lambda = \frac{-49}{28}$