Bias and variance of SVMs

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In this notebook, we will explore the bias and variance of SVM models, and see how we can tune this tradeoff.

```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd

from sklearn.svm import SVC
from sklearn.model_selection import GridSearchCV, train_test_split
from sklearn.metrics import accuracy_score

from tqdm import tqdm

from sklearn.datasets import make_blobs
```

Regularization level

Suppose we want to train a model to classify two "blobs" of data.

```
n_repeat = 100
n_test = 500
n_train = 100
sigma= 0.8
cluster_centers = np.array([[-1,1],[2,2]])
```

```
y_predict = np.zeros((n_test, n_repeat, 2))
```

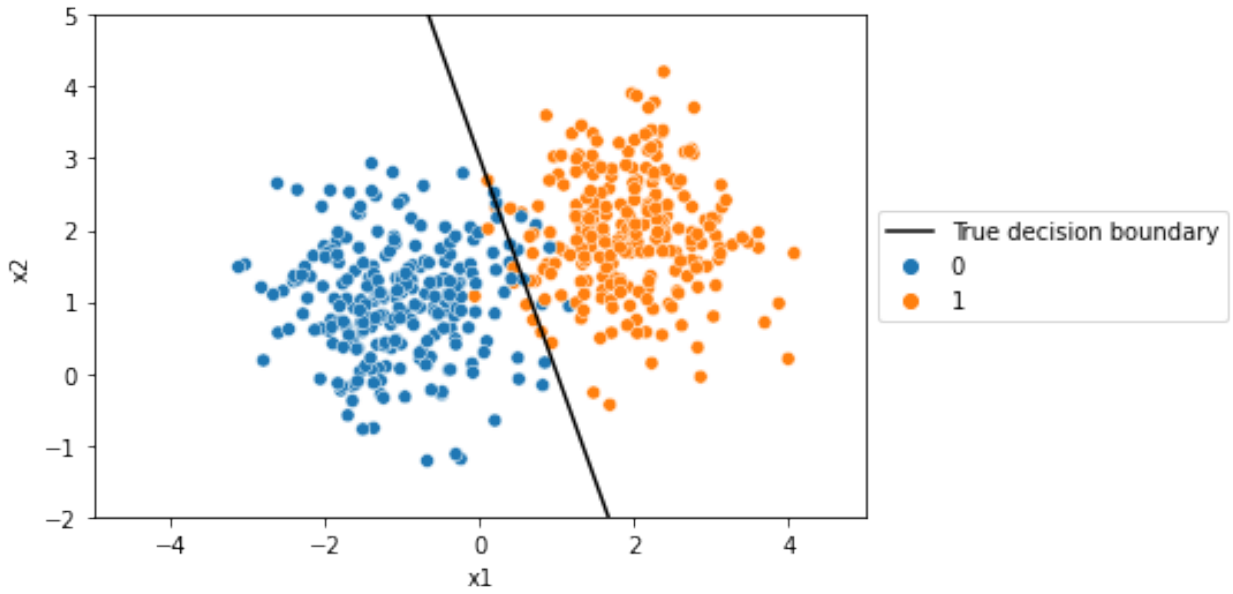
```
x_test, y_test = make_blobs(n_samples=n_test, centers=cluster_centers,
                  random_state=0, cluster_std=sigma)
```

```
sns.scatterplot(x=x_test[:,0], y=x_test[:,1], hue=y_test);

plt.xlabel("x1");
plt.ylabel("x2");
plt.xlim(-5,5);
plt.ylim(-2,5);

# get the true decision boundary
mid = [cluster_centers[:,0].mean(), cluster_centers[:,1].mean()]
slp =
    -1.0/((cluster_centers[1,1]-cluster_centers[0,1])/(cluster_centers[1,0]-cluster_centers[0,0]))
b = mid[1]-slp*mid[0]
x_true = np.arange(-5,5)
y_true = slp*x_true + b
sns.lineplot(x=x_true, y=y_true, color='black', label="True decision boundary")

plt.legend(loc='center left', bbox_to_anchor=(1, 0.5), ncol=1);
```



Which will have greater bias, and which will have greater variance?

- **Model A**: Linear SVM with $C = 0.01$
- **Model B**: Linear SVM with $C = 100$

Note: here is C in the SVM problem:

$$\begin{aligned} \underset{w,}{\text{minimize}} \quad & \frac{1}{2}\sum_{j=1}^{p} w_j^2 + C\sum_{i=1}^{n} \epsilon_i \\ \text{subject to} \quad & y_i(w_0 + \sum_{j=1}^{p} w_j x_{ij}) \geq 1 - \epsilon_i, \quad \forall i \\ & \epsilon_i \geq 0, \quad \forall i \end{aligned}$$

The greater the value of C, the more heavily the "margin violators" penalize the overall objective function. Therefore,

- If C is large, the margin must be narrow (with few "margin violators").
- If C is small, the margin may be wider (with more "margin violators").

```
Z_sim = np.zeros((40000, n_repeat, 2))

fig = plt.figure(figsize=(12,4))
ax_a, ax_b = fig.subplots(1, 2, sharex=True, sharey=True)

# now simulate training the model many times, on different training data every time
# and evaluate using the test data
for i in tqdm(range(n_repeat), total=n_repeat, desc="Simulation iteration"):

  # train both models on newly generated training data
  X, y = make_blobs(n_samples=n_test, centers=cluster_centers,
                    cluster_std=sigma)
```

```
  clf_a = SVC(kernel='linear', C=0.01).fit(X, y)
  clf_b = SVC(kernel='linear', C=100.0).fit(X, y)

  y_predict[:, i, 0] = clf_a.predict(x_test)
  y_predict[:, i, 1] = clf_b.predict(x_test)


  xx, yy = np.meshgrid(np.arange(-5, 5, .05),
                       np.arange(-5, 5, .05))

  Z = clf_a.decision_function(np.c_[xx.ravel(), yy.ravel()])
  Z_sim[:, i, 0] = Z
  Z = Z.reshape(xx.shape)
  ax_a.contour(xx, yy, Z, levels=[0.5], alpha=0.1, colors='plum');

  plt.xlim(-5,5);
  plt.ylim(-2,5);

  Z = clf_b.decision_function(np.c_[xx.ravel(), yy.ravel()])
  Z_sim[:, i, 1] = Z
  Z = Z.reshape(xx.shape)
  ax_b.contour(xx, yy, Z, levels=[0.5], alpha=0.1, colors='plum');

  plt.xlim(-5,5);
  plt.ylim(-2,5);


cs_a = ax_a.contour(xx, yy, Z_sim[:,:,0].mean(axis=1).reshape(200,200), levels=[0.5],
    colors='magenta', linewidths=2);
cs_b = ax_b.contour(xx, yy, Z_sim[:,:,1].mean(axis=1).reshape(200,200), levels=[0.5],
    colors='magenta', linewidths=2);

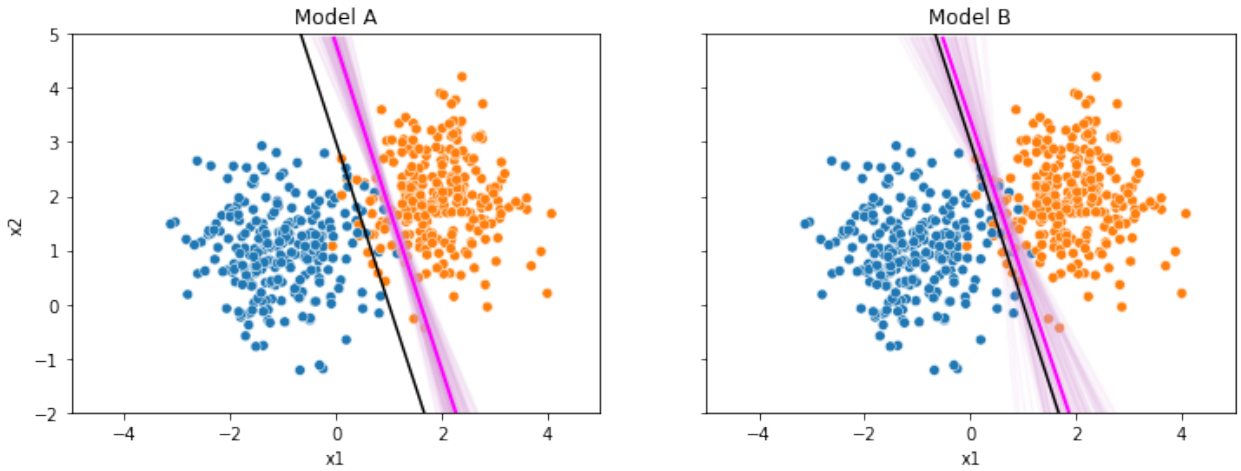
# plot data
sns.scatterplot(x=x_test[:,0], y=x_test[:,1], hue=y_test, ax=ax_a, legend=False);
sns.scatterplot(x=x_test[:,0], y=x_test[:,1], hue=y_test, ax=ax_b, legend=False);

sns.lineplot(x=x_true, y=y_true, color='black', ax=ax_a)
sns.lineplot(x=x_true, y=y_true, color='black', ax=ax_b)


ax_a.set_title("Model A");
ax_b.set_title("Model B");

ax_a.set_ylabel("x2");
ax_a.set_xlabel("x1");
ax_b.set_xlabel("x1");
```

```
Simulation iteration:       100%|| 100/100 [00:20<00:00,  4.96it/s]
```



Kernels

```
def generate_polynomial_classifier_data(n=100, xrange=[-1,1], coefs=[1,0.5,0,2], sigma=0.5):
  x = np.random.uniform(xrange[0], xrange[1], size=(n, 2))
  ysep = np.polynomial.polynomial.polyval(x[:,0],coefs)
  y = (x[:,1]>ysep).astype(int)
  x[:,0] = x[:,0] + sigma * np.random.randn(n)
  x[:,1] = x[:,1] + sigma * np.random.randn(n)
  return x, y
```

```
n_repeat = 100
n_test = 500
n_train = 1000
sigma= 0.3
coefs=np.array([0.3, 1, -1.5, -2])
xrange=[-1,1]
```

```
y_predict = np.zeros((n_test, n_repeat, 2))
```

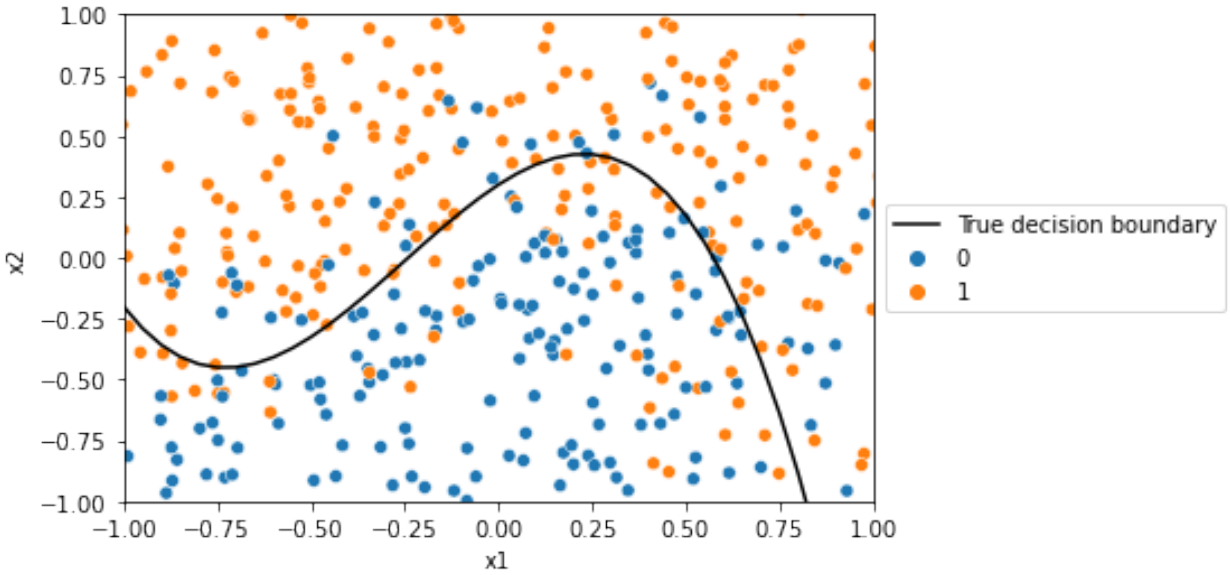
```
# generate test data once
x_test, y_test = generate_polynomial_classifier_data(n=n_test, xrange=xrange, coefs=coefs,
    sigma=sigma)
```

```
sns.scatterplot(x=x_test[:,0], y=x_test[:,1], hue=y_test);

plt.xlabel("x1");
plt.ylabel("x2");
plt.xlim((xrange[0], xrange[1]));
plt.ylim((xrange[0], xrange[1]));

# Plot true function
xtrue = np.arange(-1.5, 1.5, .05)
ytrue = np.polynomial.polynomial.polyval(xtrue,coefs)
sns.lineplot(x=xtrue, y=ytrue, color='black', label='True decision boundary');


plt.legend(loc='center left', bbox_to_anchor=(1, 0.5), ncol=1);
```



Suppose we want to train a model to classify data that is separated by a polynomial boundary.

Which will have greater bias, and which will have greater variance?

- **Model A**: SVM with linear kernel, $C = 1$
- **Model B**: SVM with RBF kernel, $C = 1$

```
Z_sim = np.zeros((3600, n_repeat, 2))

fig = plt.figure(figsize=(12,4))
ax_a, ax_b = fig.subplots(1, 2, sharex=True, sharey=True)

# now simulate training the model many times, on different training data every time
# and evaluate using the test data
for i in tqdm(range(n_repeat), total=n_repeat, desc="Simulation iteration"):

  # train both models on newly generated training data
  X, y = generate_polynomial_classifier_data(n=n_train, xrange=xrange, coefs=coefs,
      sigma=sigma)

  clf_a = SVC(kernel='linear', C=1).fit(X, y)
  clf_b = SVC(kernel='rbf', gamma=10,  C=1).fit(X, y)

  y_predict[:, i, 0] = clf_a.predict(x_test)
  y_predict[:, i, 1] = clf_b.predict(x_test)


  xx, yy = np.meshgrid(np.arange(-1.5, 1.5, .05),
                       np.arange(-1.5, 1.5, .05))

  Z = clf_a.decision_function(np.c_[xx.ravel(), yy.ravel()])
  Z_sim[:, i, 0] = Z
  Z = Z.reshape(xx.shape)
  ax_a.contour(xx, yy, Z, levels=[0.5], alpha=0.1, colors='plum');
  plt.xlim((xrange[0], xrange[1]));
  plt.ylim((xrange[0], xrange[1]));
```

```
  Z = clf_b.decision_function(np.c_[xx.ravel(), yy.ravel()])
  Z_sim[:, i, 1] = Z
  Z = Z.reshape(xx.shape)
  ax_b.contour(xx, yy, Z, levels=[0.5], alpha=0.1, colors='plum');
  plt.xlim((xrange[0], xrange[1]));
  plt.ylim((xrange[0], xrange[1]));


cs_a = ax_a.contour(xx, yy, Z_sim[:,:,0].mean(axis=1).reshape(60,60), levels=[0.5],
    colors='magenta', linewidths=2);
cs_b = ax_b.contour(xx, yy, Z_sim[:,:,1].mean(axis=1).reshape(60,60), levels=[0.5],
    colors='magenta', linewidths=2);

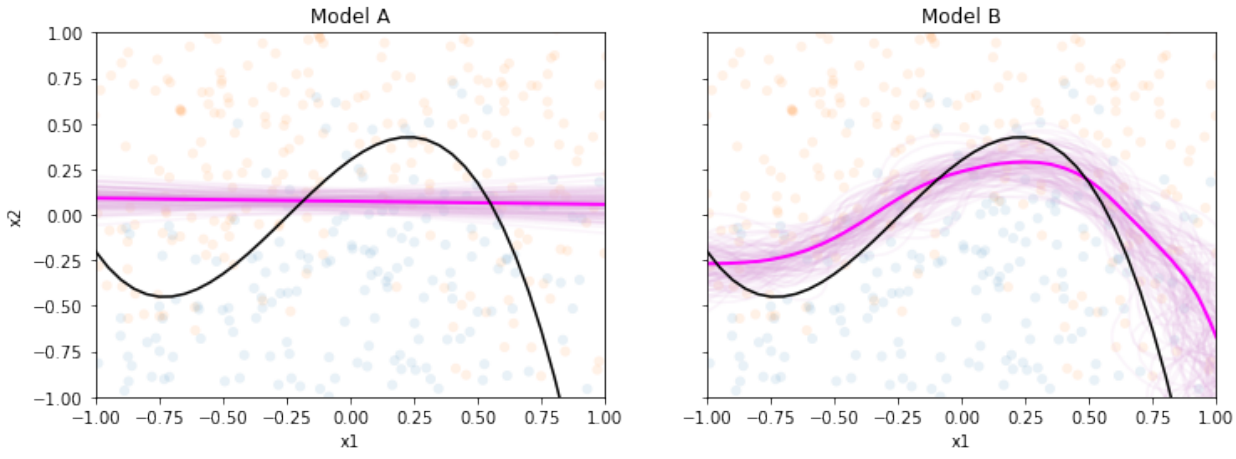

# Plot true function
xtrue = np.arange(-1.5, 1.5, .05)
ytrue = np.polynomial.polynomial.polyval(xtrue,coefs)
sns.lineplot(x=xtrue, y=ytrue, color='black', ax=ax_a);
sns.lineplot(x=xtrue, y=ytrue, color='black', ax=ax_b);

sns.scatterplot(x=x_test[:,0], y=x_test[:,1], hue=y_test, ax=ax_a, legend=False, alpha=0.1);
sns.scatterplot(x=x_test[:,0], y=x_test[:,1], hue=y_test, ax=ax_b, legend=False, alpha=0.1);

ax_a.set_title("Model A");
ax_b.set_title("Model B");

ax_a.set_ylabel("x2");
ax_a.set_xlabel("x1");
ax_b.set_xlabel("x1");
```

```
Simulation iteration:      100%|| 100/100 [00:22<00:00,  4.50it/s]
```



RBF parameter

Recall that the RBF kernel is defined as

$$K(x, z) = \exp(-\frac{\|x - z\|_2^2}{\sigma^2})$$

where σ is the bandwidth, or equivalently, using a γ parameter,

$$K(x, z) = \exp(-\gamma\|x - z\|_2^2)$$

For example, here is the RBF kernel centered on a single point, computed over the entire feature space, with two different values of γ:

```
from sklearn.metrics.pairwise import rbf_kernel

test_point = np.random.uniform(0,1,size=2)
xx, yy = np.meshgrid(np.arange(0 ,5, .05), np.arange(0, 5, .05))

gamma_a=0.05
gamma_b=5
Z_a = rbf_kernel(np.c_[xx.ravel(), yy.ravel()], test_point.reshape(1, -1), gamma=gamma_a)
Z_b = rbf_kernel(np.c_[xx.ravel(), yy.ravel()], test_point.reshape(1, -1), gamma=gamma_b)
```

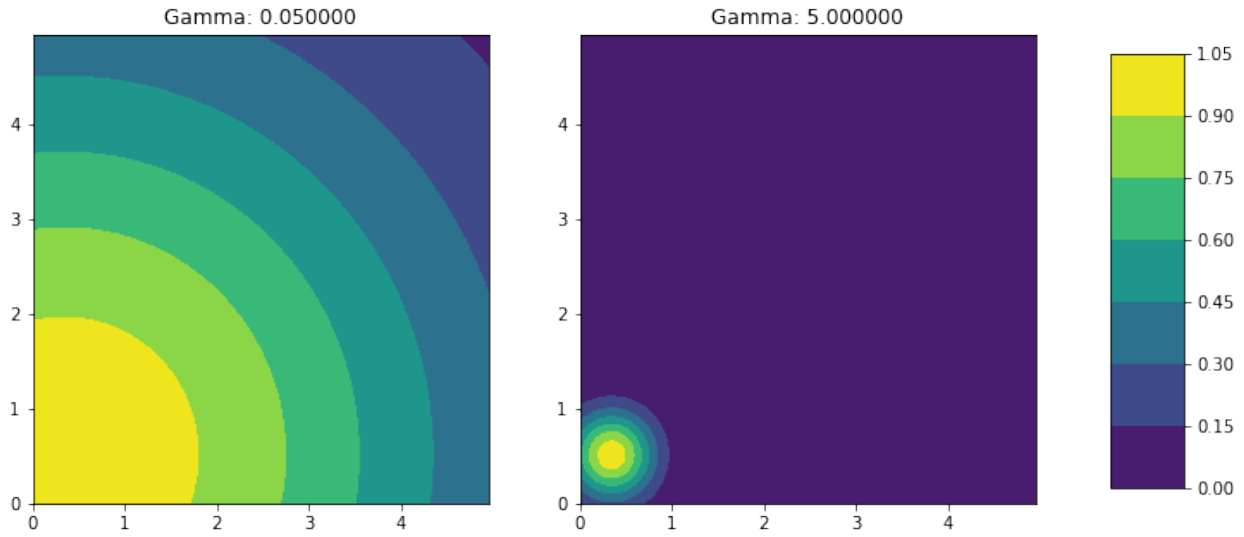
```
plt.figure(figsize=(12,5))
plt.subplot(1,2,1)

plt.scatter(x=test_point[0], y=test_point[1])
cs = plt.contourf(xx, yy, Z_a.reshape(xx.shape), vmin=0, vmax=1);
plt.title("Gamma: %f" % gamma_a);

plt.subplot(1,2,2)


plt.scatter(x=test_point[0], y=test_point[1])
cs = plt.contourf(xx, yy, Z_b.reshape(xx.shape), vmin=0, vmax=1);
plt.title("Gamma: %f" % gamma_b);

plt.subplots_adjust(right=0.8);
cbar_ax = plt.axes([0.85, 0.15, 0.05, 0.7]);
plt.colorbar(cax=cbar_ax);
```



We can see that when the kernel bandwidth is large (γ is small), the influence of each point extends much farther in the feature space than if the kernel bandwidth is small (γ is large).

Suppose we want to train a model to classify data that is separated by a polynomial boundary.

Which will have greater bias, and which will have greater variance?

- **Model A**: SVM with RBF kernel and $\gamma = 0.05$
- **Model B**: SVM with RBF kernel and $\gamma = 5$

```
n_repeat = 100
n_test = 500
n_train = 100
sigma= 0.3
coefs=np.array([0.3, 1, -1.5, -2])
xrange=[-1,1]
```

```
y_predict = np.zeros((n_test, n_repeat, 2))
```

```
# generate test data once
x_test, y_test = generate_polynomial_classifier_data(n=n_test, xrange=xrange, coefs=coefs,
    sigma=sigma)
```

```
Z_sim = np.zeros((3600, n_repeat, 2))

fig = plt.figure(figsize=(12,4))
ax_a, ax_b = fig.subplots(1, 2, sharex=True, sharey=True)

# now simulate training the model many times, on different training data every time
# and evaluate using the test data
for i in tqdm(range(n_repeat), total=n_repeat, desc="Simulation iteration"):

  # train both models on newly generated training data
  X, y = generate_polynomial_classifier_data(n=n_train, xrange=xrange, coefs=coefs,
      sigma=sigma)

  clf_a = SVC(kernel='rbf', gamma=0.05, C=10).fit(X, y)
```

```
  clf_b = SVC(kernel='rbf', gamma=5, C=10).fit(X, y)

  y_predict[:, i, 0] = clf_a.predict(x_test)
  y_predict[:, i, 1] = clf_b.predict(x_test)


  xx, yy = np.meshgrid(np.arange(-1.5, 1.5, .05),
                       np.arange(-1.5, 1.5, .05))

  Z = clf_a.decision_function(np.c_[xx.ravel(), yy.ravel()])
  Z_sim[:, i, 0] = Z
  Z = Z.reshape(xx.shape)
  ax_a.contour(xx, yy, Z, levels=[0.5], alpha=0.1, colors='plum');
  plt.xlim((xrange[0], xrange[1]));
  plt.ylim((xrange[0], xrange[1]));

  Z = clf_b.decision_function(np.c_[xx.ravel(), yy.ravel()])
  Z_sim[:, i, 1] = Z
  Z = Z.reshape(xx.shape)
  ax_b.contour(xx, yy, Z, levels=[0.5], alpha=0.1, colors='plum');
  plt.xlim((xrange[0], xrange[1]));
  plt.ylim((xrange[0], xrange[1]));


cs_a = ax_a.contour(xx, yy, Z_sim[:,:,0].mean(axis=1).reshape(60,60), levels=[0.5],
    colors='magenta', linewidths=2);
cs_b = ax_b.contour(xx, yy, Z_sim[:,:,1].mean(axis=1).reshape(60,60), levels=[0.5],
    colors='magenta', linewidths=2);

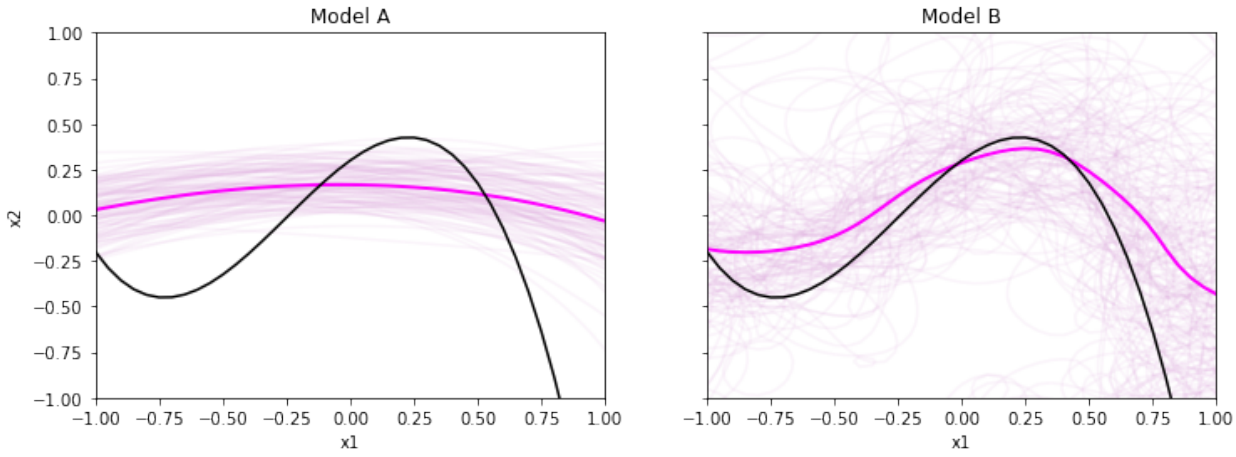

# Plot true function
xtrue = np.arange(-1.5, 1.5, .05)
ytrue = np.polynomial.polynomial.polyval(xtrue,coefs)
sns.lineplot(x=xtrue, y=ytrue, color='black', ax=ax_a);
sns.lineplot(x=xtrue, y=ytrue, color='black', ax=ax_b);

#sns.scatterplot(x_test[:,0], x_test[:,1], y_test, ax=ax_a, legend=False, alpha=0.1);
#sns.scatterplot(x_test[:,0], x_test[:,1], y_test, ax=ax_b, legend=False, alpha=0.1);

ax_a.set_title("Model A");
ax_b.set_title("Model B");

ax_a.set_ylabel("x2");
ax_a.set_xlabel("x1");
ax_b.set_xlabel("x1");
```

```
Simulation iteration:      100%|| 100/100 [00:04<00:00, 23.25it/s]
```



Hyperparameter search

For models with a single hyperparameter controlling bias-variance (for example: k in k nearest neighbors), we used sklearns's `KFoldCV` or `validation_curve` to test a range of values for the hyperparameter, and to select the best one.

When we have *multiple* hyperparameters to tune, we can use `GridSearchCV` to select the best *combination* of them.

For example, we just saw saw three ways to tune the bias-variance of an SVM classifier:

- Changing the kernel
- Changing C
- For an RBF kernel, changing γ

To get the best performance from an SVM classifier, we need to find the best *combination* of these hyperparameters. This notebook shows how to use `GridSearchCV` to tune an SVM classifier.

We will work with a subset of the MNIST handwritten digits data. First, we will get the data, and assign a small subset of samples to training and test sets.

```
from sklearn.datasets import fetch_openml
X, y = fetch_openml('mnist_784', version=1, return_X_y=True )
```

```
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size=1000, test_size=300)
```

Let's try this initial parameter "grid":

```
param_grid = [
  {'C': [0.1, 1000], 'kernel': ['linear']},
  {'C': [0.1, 1000], 'gamma': [0.01, 0.0001], 'kernel': ['rbf']},
 ]
param_grid
```

```
[{'C': [0.1, 1000], 'kernel': ['linear']},
 {'C': [0.1, 1000], 'gamma': [0.01, 0.0001], 'kernel': ['rbf']}]
```

Now we'll set up the grid search. We can use `fit` on it, just like any other `sklearn` model.

I added `return_train_score=True` to my `GridSearchSV` so that it will show me training scores as well:

```
clf = GridSearchCV(SVC(), param_grid, cv=3, refit=True, verbose=100, n_jobs=-1,
    return_train_score=True)
%time clf.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 6 candidates, totalling 18 fits
[Parallel(n_jobs=-1)]: Using backend LokyBackend with 8 concurrent workers.
Memmapping (shape=(1000, 784), dtype=float64) to new file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
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Memmapping (shape=(1000, 784), dtype=float64) to old file
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    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
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    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
```

```
Pickling array (shape=(1000,), dtype=object).
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Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   1 tasks      | elapsed:    5.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   2 tasks      | elapsed:    7.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   3 tasks      | elapsed:    8.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   4 out of  18 | elapsed:    9.3s remaining:   32.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_9247324698/23215-140178985755264-82a808de0b604527b147e35b39d61dac
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   5 out of  18 | elapsed:   10.1s remaining:   26.2s
[Parallel(n_jobs=-1)]: Done   6 out of  18 | elapsed:   10.2s remaining:   20.3s
[Parallel(n_jobs=-1)]: Done   7 out of  18 | elapsed:   10.5s remaining:   16.5s
[Parallel(n_jobs=-1)]: Done   8 out of  18 | elapsed:   12.3s remaining:   15.4s
[Parallel(n_jobs=-1)]: Done   9 out of  18 | elapsed:   13.9s remaining:   13.9s
[Parallel(n_jobs=-1)]: Done  10 out of  18 | elapsed:   15.9s remaining:   12.7s
[Parallel(n_jobs=-1)]: Done  11 out of  18 | elapsed:   16.1s remaining:   10.3s
```

```
[Parallel(n_jobs=-1)]: Done  12 out of  18 | elapsed:   16.4s remaining:    8.2s
[Parallel(n_jobs=-1)]: Done  13 out of  18 | elapsed:   16.5s remaining:    6.4s
[Parallel(n_jobs=-1)]: Done  14 out of  18 | elapsed:   17.7s remaining:    5.0s
[Parallel(n_jobs=-1)]: Done  15 out of  18 | elapsed:   17.9s remaining:    3.6s
[Parallel(n_jobs=-1)]: Done  16 out of  18 | elapsed:   19.3s remaining:    2.4s
[Parallel(n_jobs=-1)]: Done  18 out of  18 | elapsed:   21.5s remaining:    0.0s
[Parallel(n_jobs=-1)]: Done  18 out of  18 | elapsed:   21.5s finished
CPU times: user 1.49 s, sys: 217 ms, total: 1.71 s
Wall time: 22.8 s
```

```
GridSearchCV(cv=3, error_score=nan,
             estimator=SVC(C=1.0, break_ties=False, cache_size=200,
                           class_weight=None, coef0=0.0,
                           decision_function_shape='ovr', degree=3,
                           gamma='scale', kernel='rbf', max_iter=-1,
                           probability=False, random_state=None, shrinking=True,
                           tol=0.001, verbose=False),
             iid='deprecated', n_jobs=-1,
             param_grid=[{'C': [0.1, 1000], 'kernel': ['linear']},
                         {'C': [0.1, 1000], 'gamma': [0.01, 0.0001],
                          'kernel': ['rbf']}],
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring=None, verbose=100)
```

Here are the results:

```
pd.DataFrame(clf.cv_results_)
```

```
   mean_fit_time  std_fit_time  mean_score_time  std_score_time param_C  \
0       1.904941      0.121188         0.794550        0.254114     0.1
1       1.569635      0.375858         1.118351        0.107911    1000
2       4.341771      0.858102         0.822279        0.235311     0.1
3       3.922709      0.628115         1.035332        0.228026     0.1
4       4.308001      0.526039         1.049979        0.548820    1000
5       3.882048      0.349824         0.767956        0.091676    1000

  param_kernel param_gamma                                      params  \
0       linear         NaN               {'C': 0.1, 'kernel': 'linear'}
1       linear         NaN              {'C': 1000, 'kernel': 'linear'}
2          rbf        0.01     {'C': 0.1, 'gamma': 0.01, 'kernel': 'rbf'}
3          rbf      0.0001   {'C': 0.1, 'gamma': 0.0001, 'kernel': 'rbf'}
4          rbf        0.01    {'C': 1000, 'gamma': 0.01, 'kernel': 'rbf'}
5          rbf      0.0001  {'C': 1000, 'gamma': 0.0001, 'kernel': 'rbf'}

   split0_test_score  split1_test_score  split2_test_score  mean_test_score  \
0           0.832335           0.891892           0.873874         0.866034
1           0.832335           0.891892           0.873874         0.866034
2           0.116766           0.117117           0.114114         0.115999
3           0.116766           0.117117           0.114114         0.115999
4           0.116766           0.117117           0.114114         0.115999
5           0.116766           0.117117           0.114114         0.115999

   std_test_score  rank_test_score  split0_train_score  split1_train_score  \
0        0.024938                1            1.000000            1.000000
```

```
1        0.024938                1            1.000000           1.000000
2        0.001341                3            0.115616           0.115442
3        0.001341                3            0.115616           0.115442
4        0.001341                3            1.000000           1.000000
5        0.001341                3            1.000000           1.000000

   split2_train_score  mean_train_score  std_train_score
0            1.000000             1.000          0.00000
1            1.000000             1.000          0.00000
2            0.116942             0.116          0.00067
3            0.116942             0.116          0.00067
4            1.000000             1.000          0.00000
5            1.000000             1.000          0.00000
```

To inform our search, we will use our understanding of how SVMs work, and especially how the C and γ parameters control the bias and variance of the SVM.

Linear kernel

Let's tackle the linear SVM first, since it's faster to fit. We didn't see any change in the accuracy when we vary C. So, we should extend the range of C over which we search.

I'll try higher and lower values of C, to see what happens.

```
param_grid = [
  {'C': [1e-6, 1e-4, 1e-2, 1e2, 1e4, 1e6], 'kernel': ['linear']},
 ]
param_grid
```

```
[{'C': [1e-06, 0.0001, 0.01, 100.0, 10000.0, 1000000.0], 'kernel': ['linear']}]
```

```
clf = GridSearchCV(SVC(), param_grid, cv=3, refit=True, verbose=100, n_jobs=-1,
    return_train_score=True)
%time clf.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 6 candidates, totalling 18 fits
[Parallel(n_jobs=-1)]: Using backend LokyBackend with 8 concurrent workers.
Memmapping (shape=(1000, 784), dtype=float64) to new file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
```

```
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   1 tasks      | elapsed:    4.9s
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   2 tasks      | elapsed:    6.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).[Parallel(n_jobs=-1)]: Done   3 tasks      |
    elapsed:    6.2s

Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   4 out of  18 | elapsed:    6.2s remaining:   21.7s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_4776832271/23215-140178985785376-16c63d4e827f43dc953c4b2f6c933d0b
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   5 out of  18 | elapsed:    6.8s remaining:   17.6s
[Parallel(n_jobs=-1)]: Done   6 out of  18 | elapsed:    7.6s remaining:   15.3s
[Parallel(n_jobs=-1)]: Done   7 out of  18 | elapsed:    7.9s remaining:   12.4s
[Parallel(n_jobs=-1)]: Done   8 out of  18 | elapsed:    8.0s remaining:    9.9s
[Parallel(n_jobs=-1)]: Done   9 out of  18 | elapsed:    8.4s remaining:    8.4s
[Parallel(n_jobs=-1)]: Done  10 out of  18 | elapsed:    8.5s remaining:    6.8s
[Parallel(n_jobs=-1)]: Done  11 out of  18 | elapsed:    9.1s remaining:    5.8s
[Parallel(n_jobs=-1)]: Done  12 out of  18 | elapsed:    9.2s remaining:    4.6s
[Parallel(n_jobs=-1)]: Done  13 out of  18 | elapsed:    9.3s remaining:    3.6s
[Parallel(n_jobs=-1)]: Done  14 out of  18 | elapsed:   10.9s remaining:    3.1s
[Parallel(n_jobs=-1)]: Done  15 out of  18 | elapsed:   10.9s remaining:    2.2s
[Parallel(n_jobs=-1)]: Done  16 out of  18 | elapsed:   11.2s remaining:    1.4s
[Parallel(n_jobs=-1)]: Done  18 out of  18 | elapsed:   11.7s remaining:    0.0s
[Parallel(n_jobs=-1)]: Done  18 out of  18 | elapsed:   11.7s finished
CPU times: user 1.6 s, sys: 196 ms, total: 1.8 s
Wall time: 13.6 s
```

```
GridSearchCV(cv=3, error_score=nan,
             estimator=SVC(C=1.0, break_ties=False, cache_size=200,
                           class_weight=None, coef0=0.0,
                           decision_function_shape='ovr', degree=3,
                           gamma='scale', kernel='rbf', max_iter=-1,
                           probability=False, random_state=None, shrinking=True,
                           tol=0.001, verbose=False),
             iid='deprecated', n_jobs=-1,
             param_grid=[{'C': [1e-06, 0.0001, 0.01, 100.0, 10000.0, 1000000.0],
                          'kernel': ['linear']}],
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring=None, verbose=100)
```

```
pd.DataFrame(clf.cv_results_)
```

```
   mean_fit_time  std_fit_time  mean_score_time  std_score_time    param_C  \
0       1.382996      0.402043         0.732487        0.283969   0.000001
1       1.300493      0.355889         0.677238        0.146865     0.0001
2       1.383988      0.120403         0.759183        0.365900       0.01
3       1.089701      0.193386         0.491636        0.059341      100.0
4       1.119612      0.280808         0.526799        0.067583    10000.0
5       1.285502      0.184549         0.549808        0.048399  1000000.0

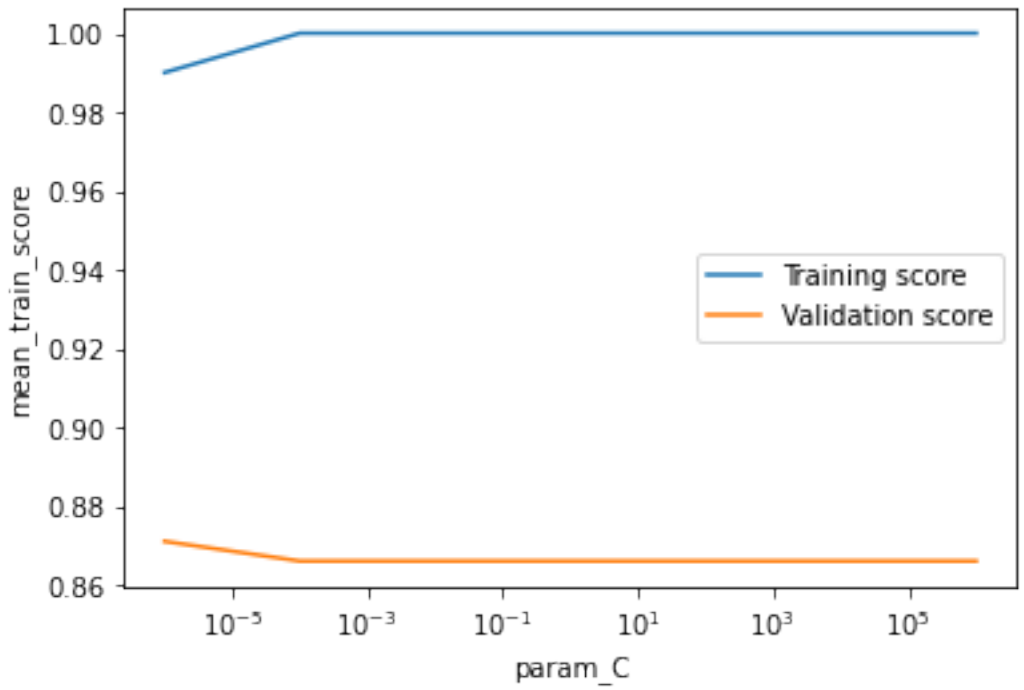
  param_kernel                                params  split0_test_score  \
0       linear      {'C': 1e-06, 'kernel': 'linear'}           0.841317
1       linear     {'C': 0.0001, 'kernel': 'linear'}           0.832335
2       linear       {'C': 0.01, 'kernel': 'linear'}           0.832335
3       linear      {'C': 100.0, 'kernel': 'linear'}           0.832335
4       linear    {'C': 10000.0, 'kernel': 'linear'}           0.832335
5       linear  {'C': 1000000.0, 'kernel': 'linear'}           0.832335

   split1_test_score  split2_test_score  mean_test_score  std_test_score  \
0           0.888889           0.882883         0.871030        0.021152
1           0.891892           0.873874         0.866034        0.024938
2           0.891892           0.873874         0.866034        0.024938
3           0.891892           0.873874         0.866034        0.024938
4           0.891892           0.873874         0.866034        0.024938
5           0.891892           0.873874         0.866034        0.024938

   rank_test_score  split0_train_score  split1_train_score  \
0                1            0.992492            0.988006
1                2            1.000000            1.000000
2                2            1.000000            1.000000
3                2            1.000000            1.000000
4                2            1.000000            1.000000
5                2            1.000000            1.000000

   split2_train_score  mean_train_score  std_train_score
0            0.989505          0.990001         0.001865
1            1.000000          1.000000         0.000000
2            1.000000          1.000000         0.000000
3            1.000000          1.000000         0.000000
4            1.000000          1.000000         0.000000
5            1.000000          1.000000         0.000000
```

```
sns.lineplot(data=pd.DataFrame(clf.cv_results_), x='param_C', y='mean_train_score',
    label="Training score");
sns.lineplot(data=pd.DataFrame(clf.cv_results_), x='param_C', y='mean_test_score',
    label="Validation score");
plt.xscale('log');
```



It looks like we get a slightly better validation score near the smaller values for C! What does this mean?

Let's try:

```
param_grid = [
  {'C': np.linspace(1e-5, 1e-7, num=10), 'kernel': ['linear']},
 ]
param_grid
```

```
[{'C': array([1.0e-05, 8.9e-06, 7.8e-06, 6.7e-06, 5.6e-06, 4.5e-06, 3.4e-06,
         2.3e-06, 1.2e-06, 1.0e-07]),
  'kernel': ['linear']}]
```

```
clf = GridSearchCV(SVC(), param_grid, cv=3, refit=True, verbose=100, n_jobs=-1,
    return_train_score=True)
%time clf.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 10 candidates, totalling 30 fits
[Parallel(n_jobs=-1)]: Using backend LokyBackend with 8 concurrent workers.
Memmapping (shape=(1000, 784), dtype=float64) to new file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
```

```
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
```

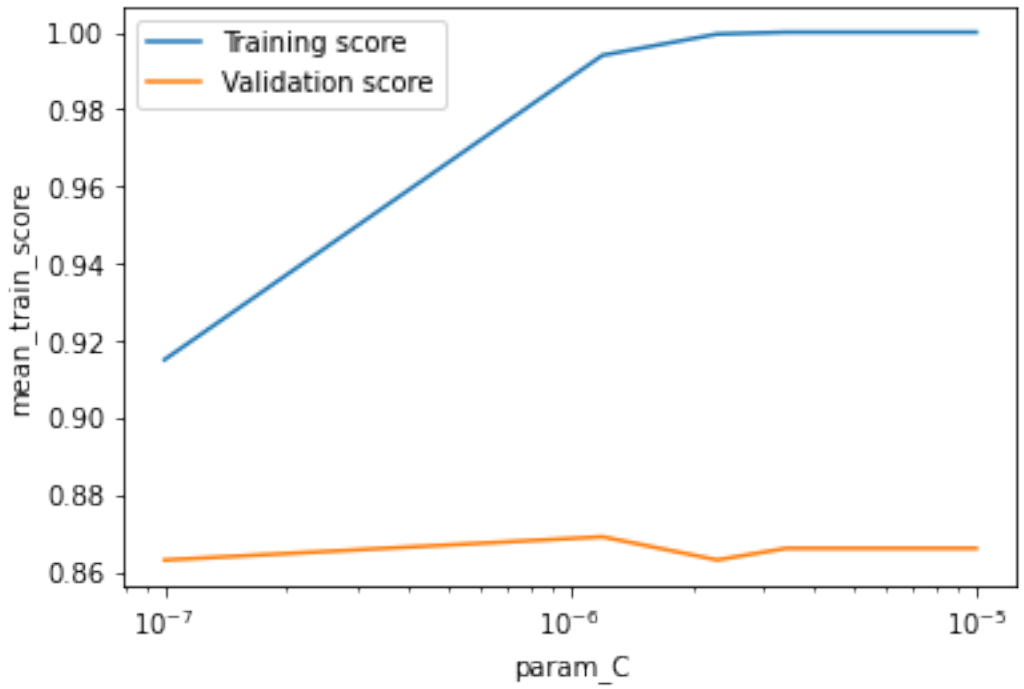
```
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   1 tasks      | elapsed:    5.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   2 tasks      | elapsed:    5.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   3 tasks      | elapsed:    6.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   4 tasks      | elapsed:    6.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   5 tasks      | elapsed:    7.1s
[Parallel(n_jobs=-1)]: Done   6 tasks      | elapsed:    7.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   7 tasks      | elapsed:    7.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   8 tasks      | elapsed:    7.8s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   9 tasks      | elapsed:    8.6s
Memmapping (shape=(1000, 784), dtype=float64) to old file
```

```
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  10 tasks      | elapsed:    9.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  11 tasks      | elapsed:    9.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  12 tasks      | elapsed:   10.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  13 tasks      | elapsed:   10.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  14 tasks      | elapsed:   10.8s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  15 tasks      | elapsed:   10.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  16 out of  30 | elapsed:   10.9s remaining:    9.6s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_1756250397/23215-140178923415056-53b15fd6d44346fb82f22baf14661723
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  17 out of  30 | elapsed:   11.2s remaining:    8.5s
[Parallel(n_jobs=-1)]: Done  18 out of  30 | elapsed:   11.8s remaining:    7.9s
[Parallel(n_jobs=-1)]: Done  19 out of  30 | elapsed:   12.8s remaining:    7.4s
[Parallel(n_jobs=-1)]: Done  20 out of  30 | elapsed:   13.3s remaining:    6.7s
[Parallel(n_jobs=-1)]: Done  21 out of  30 | elapsed:   13.9s remaining:    6.0s
[Parallel(n_jobs=-1)]: Done  22 out of  30 | elapsed:   14.2s remaining:    5.2s
[Parallel(n_jobs=-1)]: Done  23 out of  30 | elapsed:   14.5s remaining:    4.4s
[Parallel(n_jobs=-1)]: Done  24 out of  30 | elapsed:   14.7s remaining:    3.7s
```

```
[Parallel(n_jobs=-1)]: Done  25 out of  30 | elapsed:   14.9s remaining:    3.0s
[Parallel(n_jobs=-1)]: Done  26 out of  30 | elapsed:   15.6s remaining:    2.4s
[Parallel(n_jobs=-1)]: Done  27 out of  30 | elapsed:   16.0s remaining:    1.8s
[Parallel(n_jobs=-1)]: Done  28 out of  30 | elapsed:   17.1s remaining:    1.2s
[Parallel(n_jobs=-1)]: Done  30 out of  30 | elapsed:   17.5s remaining:    0.0s
[Parallel(n_jobs=-1)]: Done  30 out of  30 | elapsed:   17.5s finished
CPU times: user 1.66 s, sys: 184 ms, total: 1.84 s
Wall time: 19.5 s
```

```
GridSearchCV(cv=3, error_score=nan,
             estimator=SVC(C=1.0, break_ties=False, cache_size=200,
                           class_weight=None, coef0=0.0,
                           decision_function_shape='ovr', degree=3,
                           gamma='scale', kernel='rbf', max_iter=-1,
                           probability=False, random_state=None, shrinking=True,
                           tol=0.001, verbose=False),
             iid='deprecated', n_jobs=-1,
             param_grid=[{'C': array([1.0e-05, 8.9e-06, 7.8e-06, 6.7e-06, 5.6e-06, 4.5e-06,
                 3.4e-06,
       2.3e-06, 1.2e-06, 1.0e-07]),
                          'kernel': ['linear']}],
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring=None, verbose=100)
```

```
sns.lineplot(data=pd.DataFrame(clf.cv_results_), x='param_C', y='mean_train_score',
    label="Training score");
sns.lineplot(data=pd.DataFrame(clf.cv_results_), x='param_C', y='mean_test_score',
    label="Validation score");
plt.xscale('log');
```



We can be satisfied that we have found a good hyperparameter only when we see the high bias AND high variance side of the validation curve!

RBF kernel

Now, let's look at the RBF kernel.

In our first search, the accuracy of the RBF kernel is very poor. We may have high bias, high variance, (or both).

When $C = 0.1$ in our first search, both training and validation scores were low. This suggests high bias.

When $C = 1000$ in our first search, training scores were high and validation scores were low. This suggests high variance.

What next? We know from our discussion of bias and variance of SVMs that to combat overfitting, we can decrease γ and/or decrease C.

For now, let's keep the higher value of C, and try to reduce the overfitting by decreasing γ.

```
param_grid = [
  {'C': [1000], 'gamma': [1e-4, 1e-5, 1e-6, 1e-7, 1e-8, 1e-9, 1e-10, 1e-11], 'kernel':
      ['rbf']},
 ]
param_grid
```

```
[{'C': [1000],
  'gamma': [0.0001, 1e-05, 1e-06, 1e-07, 1e-08, 1e-09, 1e-10, 1e-11],
  'kernel': ['rbf']}]
```

```
clf = GridSearchCV(SVC(), param_grid, cv=2, refit=True, verbose=100, n_jobs=-1,
    return_train_score=True)
%time clf.fit(X_train, y_train)
```

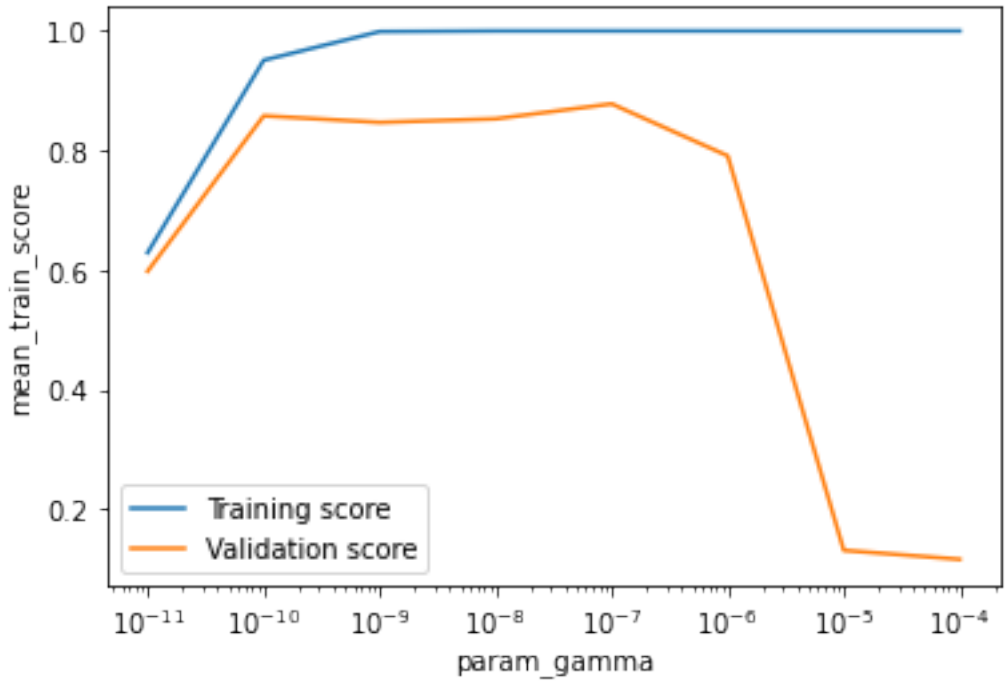
```
Fitting 2 folds for each of 8 candidates, totalling 16 fits
[Parallel(n_jobs=-1)]: Using backend LokyBackend with 8 concurrent workers.
Memmapping (shape=(1000, 784), dtype=float64) to new file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   1 tasks      | elapsed:    5.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
```

```
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   2 out of  16 | elapsed:    6.2s remaining:   43.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_3949321233/23215-140179135249088-7598e441d4b8486fa7b7ae3148fb3156
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(500,), dtype=int64).
Pickling array (shape=(500,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   3 out of  16 | elapsed:    7.1s remaining:   30.8s
[Parallel(n_jobs=-1)]: Done   4 out of  16 | elapsed:    7.2s remaining:   21.7s
[Parallel(n_jobs=-1)]: Done   5 out of  16 | elapsed:    7.6s remaining:   16.7s
[Parallel(n_jobs=-1)]: Done   6 out of  16 | elapsed:    7.7s remaining:   12.9s
[Parallel(n_jobs=-1)]: Done   7 out of  16 | elapsed:    8.1s remaining:   10.4s
[Parallel(n_jobs=-1)]: Done   8 out of  16 | elapsed:    8.1s remaining:    8.1s
[Parallel(n_jobs=-1)]: Done   9 out of  16 | elapsed:    8.3s remaining:    6.5s
[Parallel(n_jobs=-1)]: Done  10 out of  16 | elapsed:    8.5s remaining:    5.1s
[Parallel(n_jobs=-1)]: Done  11 out of  16 | elapsed:    9.1s remaining:    4.1s
[Parallel(n_jobs=-1)]: Done  12 out of  16 | elapsed:    9.8s remaining:    3.3s
[Parallel(n_jobs=-1)]: Done  13 out of  16 | elapsed:   10.0s remaining:    2.3s
[Parallel(n_jobs=-1)]: Done  14 out of  16 | elapsed:   10.3s remaining:    1.5s
[Parallel(n_jobs=-1)]: Done  16 out of  16 | elapsed:   11.8s remaining:    0.0s
[Parallel(n_jobs=-1)]: Done  16 out of  16 | elapsed:   11.8s finished
CPU times: user 1.9 s, sys: 164 ms, total: 2.07 s
Wall time: 13.9 s
```

```
GridSearchCV(cv=2, error_score=nan,
             estimator=SVC(C=1.0, break_ties=False, cache_size=200,
                           class_weight=None, coef0=0.0,
                           decision_function_shape='ovr', degree=3,
                           gamma='scale', kernel='rbf', max_iter=-1,
                           probability=False, random_state=None, shrinking=True,
                           tol=0.001, verbose=False),
             iid='deprecated', n_jobs=-1,
             param_grid=[{'C': [1000],
                          'gamma': [0.0001, 1e-05, 1e-06, 1e-07, 1e-08, 1e-09,
                                    1e-10, 1e-11],
                          'kernel': ['rbf']}],
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring=None, verbose=100)
```

```
sns.lineplot(data=pd.DataFrame(clf.cv_results_), x='param_gamma', y='mean_train_score',
    label="Training score")
sns.lineplot(data=pd.DataFrame(clf.cv_results_), x='param_gamma', y='mean_test_score',
    label="Validation score")
plt.xscale('log');
```



Here, we see that (at least for $C = 1000$), values of γ greater than `1e-5` seem to overfit, while decreasing γ lower than `1e-10` may underfit.

But we know that changing C also affects the bias variance tradeoff! For different values of C, the best value of γ will be different, and there may be a better *combination* of C and γ than any we have seen so far. We can try to increase and decrease C to see if that improves the validation score.

Now that we have a better idea of where to search, we can set up our "final" search grid.

We know that to find the best validation accuracy for the linear kernel, we should make sure our search space includes `1e-6` and `1e-7`. I chose to vary C from `1e-8` to `1e-4`. (I want to make sure the best value is not at the edge of the search space, so that we can be sure there isn't a better value if we go lower/higher.)

We know that to find the best validation accuracy for the RBF kernel, we should make sure our search space includes γ values around `1e-6` and `1e-7` when $C = 1000$. For larger values of C, we expect that we'll get better results with smaller values of γ. For smaller values of C, we expect that we'll get better results with larger values of γ. I chose to vary C from `1` to `1e6` and γ from `1e-4` to `1e-11`.

That's a big search grid, so this takes a long time to fit! (Try this at home with a larger training set to get an idea...)

```
param_grid = [
  {'C': [1e-8, 1e-7, 1e-6, 1e-5, 1e-4], 'kernel': ['linear']},
  {'C': [1, 1e2, 1e3, 1e4, 1e5, 1e6], 'gamma': [1e-4, 1e-5, 1e-6, 1e-7, 1e-8, 1e-9, 1e-10,
      1e-11], 'kernel': ['rbf']},
 ]
param_grid
```

```
[{'C': [1e-08, 1e-07, 1e-06, 1e-05, 0.0001], 'kernel': ['linear']},
 {'C': [1, 100.0, 1000.0, 10000.0, 100000.0, 1000000.0],
  'gamma': [0.0001, 1e-05, 1e-06, 1e-07, 1e-08, 1e-09, 1e-10, 1e-11],
  'kernel': ['rbf']}]
```

```
clf = GridSearchCV(SVC(), param_grid, cv=3, refit=True, verbose=100, n_jobs=-1,
    return_train_score=True)
clf.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 53 candidates, totalling 159 fits
[Parallel(n_jobs=-1)]: Using backend LokyBackend with 8 concurrent workers.
Memmapping (shape=(1000, 784), dtype=float64) to new file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e
```

```
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   1 tasks      | elapsed:    7.0s
[Parallel(n_jobs=-1)]: Done   2 tasks      | elapsed:    7.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   3 tasks      | elapsed:    7.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   4 tasks      | elapsed:    8.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   5 tasks      | elapsed:    8.6s
[Parallel(n_jobs=-1)]: Done   6 tasks      | elapsed:    8.6s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   7 tasks      | elapsed:    9.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   8 tasks      | elapsed:   10.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done   9 tasks      | elapsed:   10.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  10 tasks      | elapsed:   10.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  11 tasks      | elapsed:   10.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  12 tasks      | elapsed:   11.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  13 tasks      | elapsed:   11.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  14 tasks      | elapsed:   12.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  15 tasks      | elapsed:   13.9s
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  16 tasks      | elapsed:   17.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  17 tasks      | elapsed:   18.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  18 tasks      | elapsed:   18.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  19 tasks      | elapsed:   18.8s
[Parallel(n_jobs=-1)]: Done  20 tasks      | elapsed:   18.8s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  21 tasks      | elapsed:   19.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  22 tasks      | elapsed:   19.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  23 tasks      | elapsed:   20.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  24 tasks      | elapsed:   20.9s
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  25 tasks      | elapsed:   22.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  26 tasks      | elapsed:   23.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  27 tasks      | elapsed:   24.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  28 tasks      | elapsed:   25.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  29 tasks      | elapsed:   25.8s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  30 tasks      | elapsed:   26.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  31 tasks      | elapsed:   27.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  32 tasks      | elapsed:   28.3s
[Parallel(n_jobs=-1)]: Done  33 tasks      | elapsed:   28.8s
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  34 tasks      | elapsed:   30.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  35 tasks      | elapsed:   30.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  36 tasks      | elapsed:   32.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  37 tasks      | elapsed:   32.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  38 tasks      | elapsed:   33.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  39 tasks      | elapsed:   34.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  40 tasks      | elapsed:   35.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  41 tasks      | elapsed:   36.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  42 tasks      | elapsed:   37.4s
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  43 tasks      | elapsed:   37.9s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  44 tasks      | elapsed:   38.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  45 tasks      | elapsed:   39.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  46 tasks      | elapsed:   40.0s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  47 tasks      | elapsed:   40.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  48 tasks      | elapsed:   41.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  49 tasks      | elapsed:   41.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  50 tasks      | elapsed:   42.1s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  51 tasks      | elapsed:   42.2s
```

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Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  52 tasks      | elapsed:   42.4s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  53 tasks      | elapsed:   42.7s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  54 tasks      | elapsed:   43.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  55 tasks      | elapsed:   46.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  56 tasks      | elapsed:   46.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  57 tasks      | elapsed:   47.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  58 tasks      | elapsed:   47.8s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  59 tasks      | elapsed:   47.8s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  60 tasks      | elapsed:   48.7s
```

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Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  61 tasks      | elapsed:   52.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  62 tasks      | elapsed:   52.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  63 tasks      | elapsed:   52.6s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  64 tasks      | elapsed:   54.6s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  65 tasks      | elapsed:   54.7s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  66 tasks      | elapsed:   56.2s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  67 tasks      | elapsed:   56.5s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  68 tasks      | elapsed:   56.8s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  69 tasks      | elapsed:   57.8s
```

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Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  70 tasks      | elapsed:   59.3s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  71 tasks      | elapsed:   59.6s
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  72 tasks      | elapsed:  1.0min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  73 tasks      | elapsed:  1.0min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  74 tasks      | elapsed:  1.0min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  75 tasks      | elapsed:  1.0min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  76 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  77 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  78 tasks      | elapsed:  1.1min
```

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Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  79 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  80 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  81 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  82 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  83 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  84 tasks      | elapsed:  1.1min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  85 tasks      | elapsed:  1.2min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  86 tasks      | elapsed:  1.2min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  87 tasks      | elapsed:  1.2min
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  88 tasks      | elapsed:  1.2min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  89 tasks      | elapsed:  1.2min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  90 tasks      | elapsed:  1.2min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  91 tasks      | elapsed:  1.2min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  92 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  93 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  94 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  95 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  96 tasks      | elapsed:  1.3min
```

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Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  97 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  98 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done  99 tasks      | elapsed:  1.3min
[Parallel(n_jobs=-1)]: Done 100 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 101 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 102 tasks      | elapsed:  1.3min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 103 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 104 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 105 tasks      | elapsed:  1.4min
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 106 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 107 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 108 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 109 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 110 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 111 tasks      | elapsed:  1.4min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 112 tasks      | elapsed:  1.5min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 113 tasks      | elapsed:  1.5min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 114 tasks      | elapsed:  1.5min
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 115 tasks      | elapsed:  1.5min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 116 tasks      | elapsed:  1.5min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 117 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 118 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 119 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 120 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 121 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
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[Parallel(n_jobs=-1)]: Done 122 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 123 tasks      | elapsed:  1.6min
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 124 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 125 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 126 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 127 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 128 tasks      | elapsed:  1.6min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 129 tasks      | elapsed:  1.7min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 130 tasks      | elapsed:  1.7min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 131 tasks      | elapsed:  1.7min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 132 tasks      | elapsed:  1.7min
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 133 tasks      | elapsed:  1.7min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 134 tasks      | elapsed:  1.7min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 135 tasks      | elapsed:  1.7min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 136 tasks      | elapsed:  1.7min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 137 tasks      | elapsed:  1.8min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 138 tasks      | elapsed:  1.8min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 139 tasks      | elapsed:  1.8min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 140 tasks      | elapsed:  1.8min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 141 tasks      | elapsed:  1.8min
```

```
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 142 tasks      | elapsed:  1.8min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 143 tasks      | elapsed:  1.9min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(666,), dtype=int64).
Pickling array (shape=(334,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 144 tasks      | elapsed:  1.9min
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
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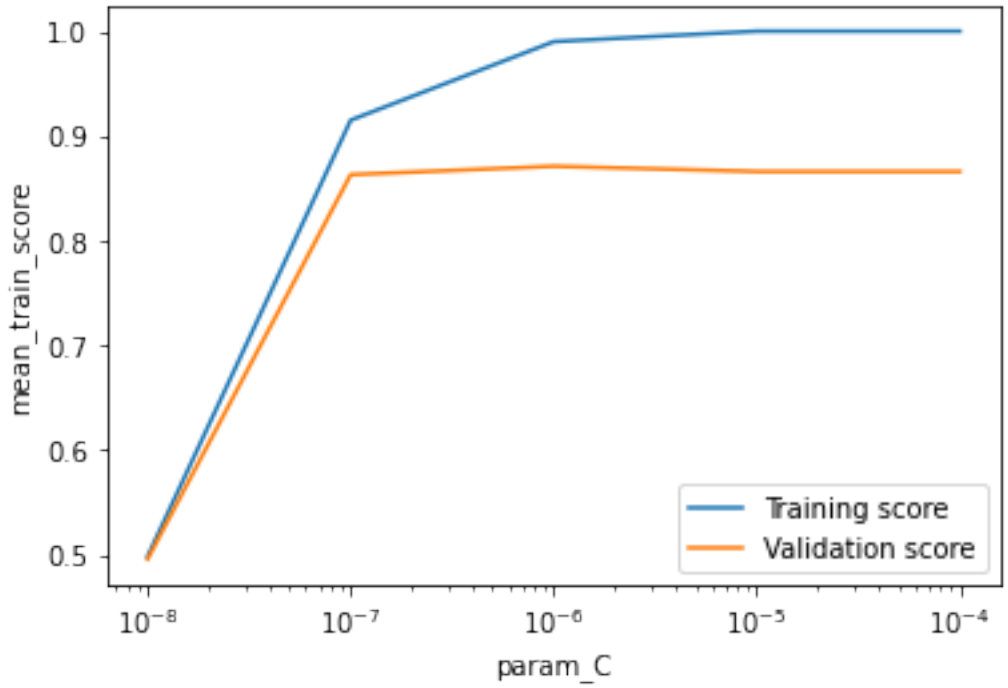
Pickling array (shape=(333,), dtype=int64).
Memmapping (shape=(1000, 784), dtype=float64) to old file
    /dev/shm/joblib_memmapping_folder_23215_5401181452/23215-140178923417856-9d61714b3a4a44ddab4a4d19313dd5e1
Pickling array (shape=(1000,), dtype=object).
Pickling array (shape=(667,), dtype=int64).
Pickling array (shape=(333,), dtype=int64).
[Parallel(n_jobs=-1)]: Done 146 out of 159 | elapsed:  1.9min remaining:    9.9s
[Parallel(n_jobs=-1)]: Done 148 out of 159 | elapsed:  1.9min remaining:    8.4s
[Parallel(n_jobs=-1)]: Done 150 out of 159 | elapsed:  1.9min remaining:    6.9s
[Parallel(n_jobs=-1)]: Done 152 out of 159 | elapsed:  1.9min remaining:    5.3s
[Parallel(n_jobs=-1)]: Done 154 out of 159 | elapsed:  1.9min remaining:    3.7s
[Parallel(n_jobs=-1)]: Done 156 out of 159 | elapsed:  1.9min remaining:    2.2s
[Parallel(n_jobs=-1)]: Done 159 out of 159 | elapsed:  2.0min finished
```

```
GridSearchCV(cv=3, error_score=nan,
             estimator=SVC(C=1.0, break_ties=False, cache_size=200,
                           class_weight=None, coef0=0.0,
                           decision_function_shape='ovr', degree=3,
                           gamma='scale', kernel='rbf', max_iter=-1,
                           probability=False, random_state=None, shrinking=True,
                           tol=0.001, verbose=False),
             iid='deprecated', n_jobs=-1,
             param_grid=[{'C': [1e-08, 1e-07, 1e-06, 1e-05, 0.0001],
                          'kernel': ['linear']},
                         {'C': [1, 100.0, 1000.0, 10000.0, 100000.0, 1000000.0],
                          'gamma': [0.0001, 1e-05, 1e-06, 1e-07, 1e-08, 1e-09,
                                    1e-10, 1e-11],
                          'kernel': ['rbf']}],
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring=None, verbose=100)
```

For the linear kernel, here's what we found:

```
df_cv   = pd.DataFrame(clf.cv_results_)
df_cv = df_cv[df_cv['param_kernel']=='linear']
```

```
sns.lineplot(data=df_cv, x='param_C', y='mean_train_score', label="Training score")
sns.lineplot(data=df_cv, x='param_C', y='mean_test_score', label="Validation score")
plt.xscale('log');
```



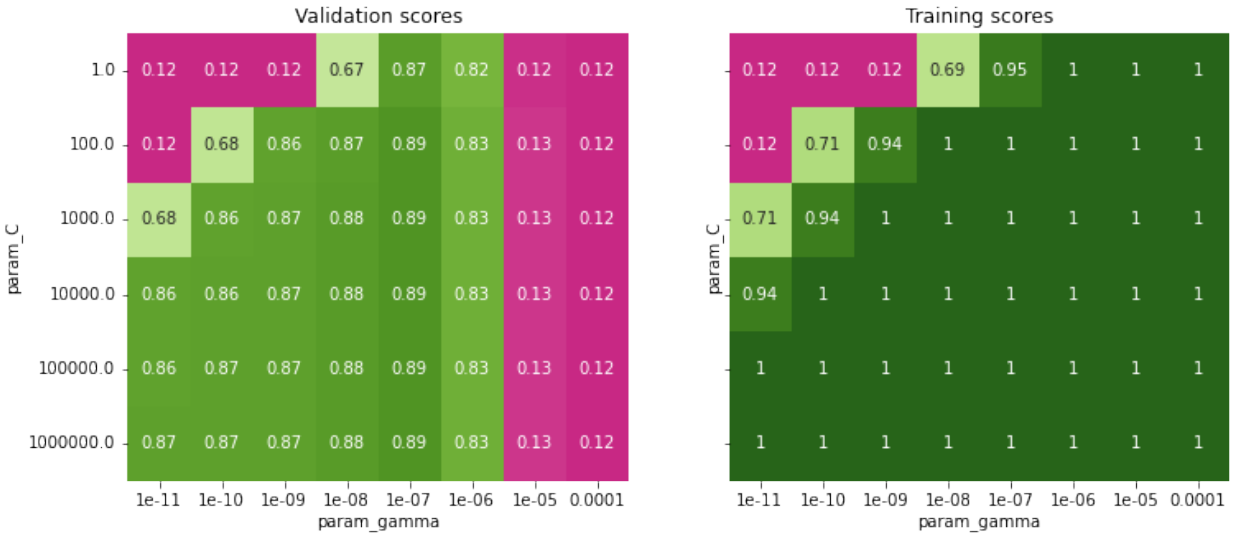
For the RBF kernel, here's what we found:

```
df_cv   = pd.DataFrame(clf.cv_results_)
df_cv = df_cv[df_cv['param_kernel']=='rbf']

plt.figure(figsize=(12,5))

ax1=plt.subplot(1,2,1)
pvt = pd.pivot_table(df_cv, values='mean_test_score', index='param_C', columns='param_gamma')
sns.heatmap(pvt, annot=True, cbar=False, vmin=0, vmax=1, cmap='PiYG');
plt.title("Validation scores");

ax2=plt.subplot(1,2,2, sharey=ax1)
plt.setp(ax2.get_yticklabels(), visible=False)
pvt = pd.pivot_table(df_cv, values='mean_train_score', index='param_C',
    columns='param_gamma')
sns.heatmap(pvt, annot=True, cbar=False, vmin=0, vmax=1, cmap='PiYG');
plt.title("Training scores");
```



We see that γ and C control the bias-variance tradeoff of the SVM model as follows.

- In the top left region, C is small (the margin is wider) and γ is small (the kernel bandwidth is large). In this region, the model has more bias (is prone to underfit). The validation scores and training scores are both low.
- On the right side (and we'd expect to see this on the bottom right if we extend the range of C even higher), C is large (the margin is narrower) and γ is large (the kernel bandwidth is small. In this region, the model has more variance (is likely to overfit). The validation scores are low, but the training scores are high.

In the middle, we have a region of good combinations of C and γ.

Since the parameter grid above shows us the validation accuracy decreasing both as we increase each parameter* and also as we decrease each parameter, we can be a bit more confident that we captured the point in the bias-variance surface where the error is smallest.

* C is different because increasing C even more may not actually change the margin.

We can see the "best" parameters, with which the model was re-fitted:

```
print(clf.best_params_)
```

```
{'C': 100.0, 'gamma': 1e-07, 'kernel': 'rbf'}
```

And we can evaluate the re-fitted model on the test set. (Note that the `GridSearchCV` only used the training set; we have not used the test set at all for model fitting.)

```
y_pred = clf.predict(X_test)
```

```
accuracy_score(y_pred, y_test)
```

```
0.9133333333333333
```