

# ICPC TEAM REFERENCE DOCUMENT

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## 1 HeavyDS

### 1.1 Iterative

```
struct Data{
};

Data op(const Data &a, const Data &b){
}

struct segtree{
    vector<Data> d;
    int _n, log, size;
    unsigned int bit_ceil(unsigned int n) {
        unsigned int x = 1;
        while (x < (unsigned int)(n)) x *= 2;
        return x;
    }
    segtree(int n): _n(n) {
        size = (int)bit_ceil((unsigned int)(n));
        log = __builtin_ctz((unsigned int)size);
        d.resize(2*size, Data());
    }
    void update(int k) { d[k] = op(d[2*k], d[2*k+1]); }
    void set(int p, Data x) {
        p += size;
        d[p] = x;
        for (int i = 1; i <= log; i++) update(p >> i);
    }
    Data prod(int l, int r) { // !!! q(l,r) return l to r-1
        if (l == r) return Data();
        l += size;
        r += size;
        Data sml = Data(), smr = Data();
        while (l < r) {
            if (l&1) sml = op(sml, d[l++]);
            if (r&1) smr = op(d[--r], smr);
            l >>= 1;
            r >>= 1;
        }
        return op(sml, smr);
    }
};
```

### 1.2 Lazy

```
struct Min{
    int x;
    Min(int x = inf) : x(x) {}
};

Min operator+(const Min& a, const Min& b){
    return std::min(a.x, b.x);
}

void apply(int &a, int b){
    a += b;
}

void apply(Min& a, int b){
```

```
    a.x += b;
}

// Always make a class for them and define basic overloads
template<class Node, class Lazy, class Merge = std::plus<Node>>
struct LazySegTree {
    const int _n;
    const Merge _merge;
    std::vector<Node> _segT;
    std::vector<Lazy> _lazy;
    LazySegTree(int n) : _n(n), _merge(Merge()), _segT(4 << std::lg(n)), _lazy(4 << std::lg(n)) {}
    LazySegTree(std::vector<Node> init) :
        LazySegTree(init.size()) {
        build(init);
    }
    LazySegTree(int node, int st, int en) {
        if(en - st == 1) {
            _segT[node] = init[st];
            return;
        }
        int md = (st + en) >> 1;
        build(node << 1, st, md);
        build(node << 1 | 1, md, en);
        pull(node);
    }
    void build(int node, int st, int en) {
        if(en - st == 1) {
            _segT[node] = init[st];
            return;
        }
        int md = (st + en) >> 1;
        build(node << 1, st, md);
        build(node << 1 | 1, md, en);
        pull(node);
    }
    void pull(int node) {
        _segT[node] = _merge(_segT[node << 1], _segT[node << 1 | 1]);
    }
    void apply(int node, const Lazy& lazy) {
        if(node < _n) {
            apply(_segT[node], lazy);
            apply(_lazy[node], lazy);
        }
    }
    void push(int node) {
        if(node < _n) {
            apply(node << 1, _lazy[node]);
            apply(node << 1 | 1, _lazy[node]);
            _lazy[node] = Lazy();
        }
    }
    void update(int node, int st, int en, int idx, const Node& nodeVal) {
        if(en - st == 1) {
            _segT[node] = nodeVal;
            return;
        }
        push(node);
        int md = (st + en) >> 1;
        if(idx < md) {
            update(node << 1, st, md, idx, nodeVal);
        } else {
            update(node << 1 | 1, md, en, idx, nodeVal);
        }
        pull(node);
    }
    void update(int idx, const Node& nodeVal) {
        update(0, 0, _n, idx, nodeVal);
    }
    Node rangeQuery(int node, int st, int en, int ql, int qr) {
        if(st >= qr || en <= ql) {
            return Node();
        }
        if(ql < st || qr > en) {
            return _segT[node];
        }
        if(ql <= st && qr >= en) {
            return _segT[node];
        }
        int md = (st + en) >> 1;
        Node left = rangeQuery(node << 1, st, md, ql, qr);
        Node right = rangeQuery(node << 1 | 1, md, en, ql, qr);
        return _merge(left, right);
    }
};
```

```

        }
        if(st >= ql && en <= qr){
            return _segT[node];
        }
        push(node);
        int md = (st + en) >> 1;
        return _merge(rangeQuery(node << 1, st, md, ql, qr),
                      rangeQuery(node << 1 | 1, md, en, ql, qr));
    }
    Node rangeQuery(int ql, int qr){
        return rangeQuery(1, 0, _n, ql, qr);
    }
    void rangeApply(int node, int st, int en, int ql, int qr,
                    const Lazy& lazy){
        if(st >= qr || en <= ql){
            return;
        }
        if(st >= ql && en <= qr){
            apply(node, lazy);
            return;
        }
        push(node);
        int md = (st + en) >> 1;
        rangeApply(node << 1, st, md, ql, qr, lazy);
        rangeApply(node << 1 | 1, md, en, ql, qr, lazy);
        pull(node);
    }
    void rangeApply(int ql, int qr, const Lazy& lazy){
        rangeApply(1, 0, _n, ql, qr, lazy);
    }
}

// LazySegTree<Min, int> seg(a);
// For range [i, j] use i, j + 1

```

### 1.3 PersistentSegmentTree

```

struct PST{
    struct Node{
        int sum = 0;
        int l = 0, r = 0;
    };
    const int n;
    vector<Node> tree;
    int timer = 1;
    PST(int n, int mx_nodes) : n(n), tree(mx_nodes) {}
    Node merge(int l, int r){
        return Node({tree[l].sum + tree[r].sum, l, r});
    }
    int build(int st, int en, const vector<int> &init){
        if(st == en){
            tree[timer] = Node({init[st], 0, 0});
            return timer++;
        }
        int md = (st + en) >> 1;
        tree[timer] = merge(build(st, md, init), build(md + 1,
                                                       en, init));
        return timer++;
    }
    int update(int root, int idx, int val, int st, int en){
        if(st == en){
            tree[timer] = Node({val, 0, 0});
            return timer++;
        }
        int md = (st + en) >> 1;
        if(idx <= md){
            tree[timer] = merge(update(tree[root].l, idx, val,
                                       st, md), tree[root].r);
        }
        else{
            tree[timer] = merge(tree[root].l,
                               update(tree[root].r, idx, val, md + 1, en));
        }
        return timer++;
    }
    int query(int root, int l, int r, int st, int en){
        if(r < st || en < l) return 0;

```

```

        if(l <= st && en <= r) return tree[root].sum;
        int md = (st + en) >> 1;
        return query(tree[root].l, l, r, st, md) +
               query(tree[root].r, l, r, md + 1, en);
    }
    int build(const vector<int> &init){
        return build(0, n - 1, init);
    }
    int update(int root, int idx, int val){
        return update(root, idx, val, 0, n - 1);
    }
    int query(int root, int l, int r){
        return query(root, l, r, 0, n - 1);
    }
};

```

### 1.4 Fenwick\_sum

```

struct FenwickTree {
    vector<int> bit; // binary indexed tree
    int n;
    FenwickTree(int n) {
        this->n = n;
        bit.assign(n, 0);
    }
    FenwickTree(vector<int> const &a) : FenwickTree(a.size()) {
        for (size_t i = 0; i < a.size(); i++)
            add(i, a[i]);
    }
    int sum(int r) {
        int ret = 0;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret += bit[r];
        return ret;
    }
    int sum(int l, int r) {
        return sum(r) - sum(l - 1);
    }
    void add(int idx, int delta) {
        for (; idx < n; idx = idx | (idx + 1))
            bit[idx] += delta;
    }
};

```

### 1.5 SparseTable

```

int log_floor(int x){
    return x ? __builtin_clzll(1) - __builtin_clzll(x) : -1;
}
template<typename T, class F = function<T(const T&, const T&)>>
struct SPARSE_TABLE{
    int n;
    vector<vector<T>> st;
    F fun;
    SPARSE_TABLE(const vector<T> &v, const F &f) : fun(f){
        n = static_cast<int>(v.size());
        int maxN = log_floor(n) + 1;

        st.resize(maxN);
        st[0] = v;

        for(int i = 1; i < maxN; i++){
            st[i].resize(n - (1 << i) + 1);

            for(int j = 0; j + (1 << i) <= n; j++){
                st[i][j] = fun(st[i - 1][j], st[i - 1][j + (1
                                                               << (i - 1))]);
            }
        }
    }
    T get(int l, int r){
        int h = log_floor(r - l + 1);
        // works for idempotent functions only ykw
        return fun(st[h][l], st[h][r - (1 << h) + 1]);
    }
};

```

```
}
```

## 1.6 MergeSortTree

```
template <typename T>
struct MergeSortTree {
    int n;
    vector<vector<T>> tree;

    // Constructor: builds the tree from input vector a
    MergeSortTree(const vector<T>& a) {
        n = a.size();
        tree.resize(4 * n);
        build(1, 0, n - 1, a);
    }

    void build(int node, int start, int end, const vector<T>&
    → a) {
        if (start == end) {
            tree[node] = {a[start]};
            return;
        }
        int mid = (start + end) / 2;
        build(2 * node, start, mid, a);
        build(2 * node + 1, mid + 1, end, a);

        // Merge two sorted vectors
        merge(tree[2 * node].begin(), tree[2 * node].end(),
              tree[2 * node + 1].begin(), tree[2 * node +
              → 1].end(),
              back_inserter(tree[node]));
    }

    // Internal query function for Lower Bound (smallest value
    → >= x)
    T query_lb(int node, int start, int end, int l, int r, T x)
    → {
        if (start > end || start > r || end < l)
            return numeric_limits<T>::max(); // Return INF

        if (start >= l && end <= r) {
            auto it = lower_bound(tree[node].begin(),
            → tree[node].end(), x);
            if (it != tree[node].end())
                return *it;
            return numeric_limits<T>::max();
        }

        int mid = (start + end) / 2;
        return min(query_lb(2 * node, start, mid, l, r, x),
                  query_lb(2 * node + 1, mid + 1, end, l, r,
                  → x));
    }

    // Internal query function for Count (number of elements <=
    → x)
    int query_count(int node, int start, int end, int l, int r,
    → T x) {
        if (start > end || start > r || end < l)
            return 0;

        if (start >= l && end <= r) {
            // upper_bound gives first element > x, so index
            → gives count of elements <= x
            return upper_bound(tree[node].begin(),
            → tree[node].end(), x) - tree[node].begin();
        }

        int mid = (start + end) / 2;
        return query_count(2 * node, start, mid, l, r, x) +
            query_count(2 * node + 1, mid + 1, end, l, r,
            → x);
    }

    // Public Interface: Find smallest number >= x in range [l,
    → r]
}
```

```
T lower_bound_val(int l, int r, T x) {
    return query_lb(1, 0, n - 1, l, r, x);
}

// Public Interface: Count numbers <= x in range [l, r]
int count_less_equal(int l, int r, T x) {
    return query_count(1, 0, n - 1, l, r, x);
}
};
```

## 2 Graphs

### 2.1 Dinic

```
template<class T>
struct Flow {
    const int n;
    struct Edge {
        int to;
        T cap;
        Edge(int to, T cap) : to(to), cap(cap) {}
    };
    std::vector<Edge> e;
    std::vector<std::vector<int>> g;
    std::vector<int> cur, h;
    Flow(int n) : n(n), g(n) {}

    bool bfs(int s, int t) {
        h.assign(n, -1);
        std::queue<int> que;
        h[s] = 0;
        que.push(s);
        while (!que.empty()) {
            const int u = que.front();
            que.pop();
            for (int i : g[u]) {
                auto [v, c] = e[i];
                if (c > 0 && h[v] == -1) {
                    h[v] = h[u] + 1;
                    if (v == t) {
                        return true;
                    }
                    que.push(v);
                }
            }
        }
        return false;
    }

    T dfs(int u, int t, T f) {
        if (u == t) {
            return f;
        }
        auto r = f;
        for (int &i = cur[u]; i < (int)(g[u].size()); ++i) {
            const int j = g[u][i];
            auto [v, c] = e[j];
            if (c > 0 && h[v] == h[u] + 1) {
                auto a = dfs(v, t, std::min(r, c));
                e[j].cap -= a;
                e[j ^ 1].cap += a;
                r -= a;
                if (r == 0) {
                    return f;
                }
            }
        }
        return r;
    }

    void addEdge(int u, int v, T c) {
        g[u].push_back(e.size());
        e.emplace_back(v, c);
        g[v].push_back(e.size());
        e.emplace_back(u, 0);
    }
};
```

```

T maxFlow(int s, int t) {
    T ans = 0;
    while (bfs(s, t)) {
        cur.assign(n, 0);
        ans += dfs(s, t, std::numeric_limits<T>::max());
    }
    return ans;
}
void get(int end){
    vector<int> path;
    cur.assign(n, 0);
    int u = 1;
    while(u != end){
        path.push_back(u);
        dbg(path);
        for (int &i = cur[u]; i < (int)(g[u].size()); ++i)
        {
            const int j = g[u][i];
            auto [v, c] = e[j];
            dbg(u, v, c, j);
            if (j % 2 == 0 && c == 0) {
                e[j].cap = 1;
                u = v;
                break;
            }
        }
        path.push_back(end);
        cout << path.size() << endl;
        for(int i : path) cout << i << " ";
        cout << endl;
    }
}

queue<int> q;
vector<bool> vis(n + 1, false);
q.push(1);
vis[1] = true;
while(!q.empty()){
    int u = q.front(); q.pop();
    for(auto i: mf.g[u]){
        auto [v, c] = mf.e[i];
        if(c > 0 && !vis[v]){
            vis[v] = true;
            q.push(v);
        }
    }
}
for(int i = 1; i <= n; i ++){
    for(auto id : mf.g[i]){
        auto [v, c] = mf.e[id];
        if(v != s and c == 0){
            cout << i << " " << v - n << endl;
        }
    }
}
}

```

## 2.2 LCA

```

// LCA and Binary Lifting
void dfs(int v, int p){
    up[0][v] = p;
    for(int i = 1; i < 20; i ++){
        up[i][v] = up[i - 1][up[i - 1][v]];
    }
    for(auto x : g[v]){
        if(x == p) continue;
        dst[x] = dst[v] + 1;
        dfs(x, v);
    }
}
auto lca = [&](int x, int y){
    if(dst[x] < dst[y]) swap(x, y);
    for(int i = 19; i >= 0; i --){
        if(dst[up[i][x]] >= dst[y]){
            x = up[i][x];
        }
    }
}

```

```

if(x == y) return x;
for(int i = 19; i >= 0; i --){
    if(up[i][x] != up[i][y]){
        x = up[i][x];
        y = up[i][y];
    }
}
return up[0][x];
}

```

## 2.3 CutPoint

```

vll g[NUM];
bool vis[NUM];
ll in[NUM];
ll low[NUM];
int timer = 0;

void CutPoints(ll src, ll parent = -1)
{
    vis[src] = true;
    in[src] = low[src] = timer;
    timer++;
    int children = 0;
    for (auto child : g[src])
    {
        if (child == parent)
            continue;
        if (vis[child])
        {
            low[src] = min(low[src], in[child]);
        }
        else
        {
            CutPoints(child, src);
            low[src] = min(low[src], low[child]);
            if (low[child] >= in[src] && parent != -1)
            {
                // Src is a cut point!
                // CAUTION: Might get called multiple times due
                // to satisfiability of multiple children
            }
            children++;
        }
    }
    if (parent == -1 && children > 1){
        // Root is a cut point!
        // used children instead of size of adj bcoz we dont
        // use back edges
    }
}

```

## 2.4 TreeIsomorphism

```

class Tree {
public:
    vector<vector<int>> adj;
    vector<int> centroid;
    vector<int> sub;
    vector<int> id;
    vector<int64_t> powr;

    void dfs(int curNode, int prevNode) {
        sub[curNode] = 1;
        bool is_centroid = true;
        vector<pair<int, int>> nodes;
        for (int v : adj[curNode]) {
            if (v != prevNode) {
                dfs(v, curNode);
                sub[curNode] += sub[v];
                if (sub[v] > (int)adj.size() / 2) {
                    is_centroid = false;
                }
            }
        }
    }
}

```

```

        nodes.emplace_back(id[v], v);
    }
    // Canonical ordering: sort children by their hash
    sort(nodes.begin(), nodes.end());

    id[curNode] = 1;
    for (auto& p : nodes) {
        // Polynomial hash combination
        id[curNode] = ((powr[sub[p.second] + 1] *
                        id[curNode]) % MOD + id[p.second]) % MOD;
    }
    id[curNode] *= 2;
    id[curNode] %= MOD;

    // Check "parent" component size for centroid
    // definition
    if ((int)adj.size() - sub[curNode] > (int)adj.size() / 2) {
        is_centroid = false;
    }
    if (is_centroid) {
        centroid.push_back(curNode);
    }
}

vector<int> getCentroids() {
    // Clear previous runs if any
    centroid.clear();
    dfs(0, -1);
    return centroid;
}

// Computes hash for a specific rooting
int getHash(int root) {
    dfs(root, -1);
    return id[root];
}

void add_edge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

Tree(int n) {
    adj.resize(n);
    sub.resize(n);
    id.resize(n);
    powr.resize(n + 1);
    powr[0] = 1;
    for (int i = 1; i <= n; i++) {
        powr[i] = (2 * powr[i - 1]) % MOD;
    }
}
};

int main() {
    // Example: Two trees that look different but are
    // structurally the same
    // Tree 1: 0-1, 1-2
    Tree t1(3);
    t1.add_edge(0, 1);
    t1.add_edge(1, 2);

    // Tree 2: 1-0, 0-2 (Same line graph, just labeled
    // differently)
    Tree t2(3);
    t2.add_edge(1, 0);
    t2.add_edge(0, 2);

    // Step 1: Find centroids for both trees
    // This allows us to find a "Canonical Root"
    vector<int> c1 = t1.getCentroids();
    vector<int> c2 = t2.getCentroids();

    bool isomorphic = false;

    // Step 2: Compare hashes rooted at centroids
    // A tree can have up to 2 centroids. We must check all
    // combinations.
}

```

```

for (int root1 : c1) {
    for (int root2 : c2) {
        if (t1.getHash(root1) == t2.getHash(root2)) {
            isomorphic = true;
        }
    }
}

if (isomorphic)
    cout << "The trees are Isomorphic!" << endl;
else
    cout << "The trees are Different." << endl;

return 0;
}

```

## 2.5 CentroidDecomp

```

int nn;
void dfs1(int u, int p)
{
    sub[u] = 1;
    nn++;
    for (auto it = g[u].begin(); it != g[u].end(); it++)
        if (*it != p)
    {
        dfs1(*it, u);
        sub[u] += sub[*it];
    }
}
int dfs2(int u, int p)
{
    for (auto it = g[u].begin(); it != g[u].end(); it++)
        if (*it != p && sub[*it] > nn / 2)
            return dfs2(*it, u);
    return u;
}
void decompose(int root, int p)
{
    nn = 0;
    dfs1(root, root);
    int centroid = dfs2(root, root);
    if (p == -1)
        p = centroid;
    par[centroid] = p;
    for (auto it = g[centroid].begin(); it != g[centroid].end(); it++)
    {
        g[*it].erase(centroid);
        decompose(*it, centroid);
    }
    g[centroid].clear();
}

```

## 2.6 VirtualTree

```

const int MAXN = 200005;

// --- PREREQUISITES (LCA & DFS Times) ---
vector<int> adj[MAXN];
int tin[MAXN], tout[MAXN], timer;
int up[MAXN][20], depth[MAXN];

void dfs_lca(int u, int p, int d) {
    tin[u] = ++timer;
    depth[u] = d;
    up[u][0] = p;
    for (int i = 1; i < 20; i++)
        up[u][i] = up[up[u][i-1]][i-1];

    for (int v : adj[u]) {
        if (v != p) dfs_lca(v, u, d + 1);
    }
    tout[u] = timer;
}

```

```

}

bool is_ancestor(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v) {
    if (is_ancestor(u, v)) return u;
    if (is_ancestor(v, u)) return v;
    for (int i = 19; i >= 0; i--) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

// --- VIRTUAL TREE TEMPLATE ---
vector<int> vt_adj[MAXN]; // Adjacency list for Virtual Tree

// Sorts nodes by DFS entry time (Critical for construction)
bool compare_tin(int a, int b) {
    return tin[a] < tin[b];
}

void build_virtual_tree(vector<int>& nodes) {
    // 1. Sort by DFS entry time
    sort(nodes.begin(), nodes.end(), compare_tin);

    // 2. Add LCAs of adjacent sorted nodes to the set
    // This guarantees all necessary connecting nodes are
    // present
    int k = nodes.size();
    for (int i = 0; i < k - 1; i++) {
        nodes.push_back(lca(nodes[i], nodes[i+1]));
    }

    // 3. Sort again and remove duplicates
    sort(nodes.begin(), nodes.end(), compare_tin);
    nodes.erase(unique(nodes.begin(), nodes.end()),
    // nodes.end());

    // 4. Build edges using a stack
    // Clear previous virtual tree edges for these specific
    // nodes
    for (int u : nodes) vt_adj[u].clear();

    vector<int> stack;
    stack.push_back(nodes[0]);

    for (int i = 1; i < nodes.size(); i++) {
        int u = nodes[i];
        // Pop stack while the top is NOT an ancestor of u
        // (This means we finished the subtree of stack.back())
        while (stack.size() > 1 && !is_ancestor(stack.back(),
        // u)) {
            stack.pop_back();
        }
        // Add edge from the current direct ancestor to u
        // Note: The graph is directed downwards usually, or
        // undirected depending on need
        vt_adj[stack.back()].push_back(u);
        vt_adj[u].push_back(stack.back()); // Add this if
        // undirected

        stack.push_back(u);
    }
}
}

```

## 2.7 Centroid

```

function<void(int, int)> dfs2 = [&](int v, int p){
    int mx = 0;
    for(auto u: g[v]){
        if(u == p || vis[u]) continue;
        mx = max(mx, sbt[u]);
    }
    mx = max(mx, n - sbt[v]);
    if(mx <= n / 2){

```

```

        centroid = v;
    }
    for(auto u: g[v]){
        if(u == p || vis[u]) continue;
        dfs2(u, v);
    }
};

```

## 2.8 Mcmf

```

using i64 = long long;
template<class T>
struct MinCostFlow {
    struct _Edge {
        int to;
        T cap;
        T cost;
        _Edge(int to_, T cap_, T cost_) : to(to_), cap(cap_), cost(cost_) {}
    };
    int n;
    std::vector<_Edge> e;
    std::vector<std::vector<int>> g;
    std::vector<T> h, dis;
    std::vector<int> pre;
    bool dijkstra(int s, int t) {
        dis.assign(n, std::numeric_limits<T>::max());
        pre.assign(n, -1);
        std::priority_queue<std::pair<T, int>, std::vector<std::pair<T, int>>, std::greater<std::pair<T, int>>> que;
        dis[s] = 0;
        que.emplace(0, s);
        while (!que.empty()) {
            T d = que.top().first;
            int u = que.top().second;
            que.pop();
            if (dis[u] != d) {
                continue;
            }
            for (int i : g[u]) {
                int v = e[i].to;
                T cap = e[i].cap;
                T cost = e[i].cost;
                if (cap > 0 && dis[v] > d + h[u] - h[v] +
                cost) {
                    dis[v] = d + h[u] - h[v] + cost;
                    pre[v] = i;
                    que.emplace(dis[v], v);
                }
            }
        }
        return dis[t] != std::numeric_limits<T>::max();
    }
    MinCostFlow() {}
    MinCostFlow(int n_) {
        init(n_);
    }
    void init(int n_) {
        n = n_;
        e.clear();
        g.assign(n, {});
    }
    void addEdge(int u, int v, T cap, T cost) {
        g[u].push_back(e.size());
        e.emplace_back(v, cap, cost);
        g[v].push_back(e.size());
        e.emplace_back(u, 0, -cost);
    }
    std::pair<T, T> flow(int s, int t) {
        T flow = 0;
        T cost = 0;
        h.assign(n, 0);
        while (dijkstra(s, t)) {
            for (int i = 0; i < n; ++i) {
                h[i] += dis[i];
            }

```

```

    }
    T aug = std::numeric_limits<int>::max();
    for (int i = t; i != s; i = e[pre[i] ^ 1].to) {
        aug = std::min(aug, e[pre[i]].cap);
    }
    for (int i = t; i != s; i = e[pre[i] ^ 1].to) {
        e[pre[i]].cap -= aug;
        e[pre[i] ^ 1].cap += aug;
    }
    flow += aug;
    cost += aug * h[t];
}
return std::make_pair(flow, cost);
}

struct Edge {
    int from;
    int to;
    T cap;
    T cost;
    T flow;
};

std::vector<Edge> edges() {
    std::vector<Edge> a;
    for (int i = 0; i < e.size(); i += 2) {
        Edge x;
        x.from = e[i + 1].to;
        x.to = e[i].to;
        x.cap = e[i].cap + e[i + 1].cap;
        x.cost = e[i].cost;
        x.flow = e[i + 1].cap;
        a.push_back(x);
    }
    return a;
}
};

```

```

        mx_sz = c_sz;
        heavy[v] = x;
    }
    return subt[v] = sz;
}

void dfs_hld(int v, int h){
    root[v] = h;
    pos[v] = cur_pos, inv[cur_pos] = v;
    cur_pos++;
    if(heavy[v] != -1) dfs_hld(heavy[v], h);
    for(auto x : g[v]){
        if(x == par[v] || x == heavy[v]) continue;
        dfs_hld(x, x);
    }
}

bool is_anc(int a, int b) {
    return pos[a] <= pos[b] && pos[b] < pos[a] + subt[a];
}

int kth(int v, int k) {
    for (int i = 0; i < 20; i++) {
        if (k & (1 << i)) {
            v = up[v][i];
        }
    }
    return v;
}

int hehe(int v, int u) {
    // u is ancestor of v
    int dif = dpth[v] - dpth[u] - 1;
    return kth(v, dif);
}

mint qry(int a, int b) {
    mint res = 0;
    for (; root[a] != root[b]; b = par[root[b]]) {
        if (dpth[root[a]] > dpth[root[b]])
            swap(a, b);
        mint cur_heavy_path_max = seg.prod(pos[root[b]],
        ↪ pos[b] + 1).x;
        res += cur_heavy_path_max;
    }
    if (dpth[a] > dpth[b])
        swap(a, b);
    mint last_heavy_path_max = seg.prod(pos[a], pos[b] +
    ↪ 1).x;
    res += last_heavy_path_max;
    return res;
}
};


```

## 2.9 Hld

```

struct hld {
    vector<vector<int>> g;
    vector<int> par, dpth, heavy, root, pos, subt, inv;
    vector<vector<int>> up;
    segtree seg;
    int cur_pos = 0;
    hld(int n){
        g.resize(n + 1), par.resize(n + 1), dpth.resize(n +
        ↪ 1);
        heavy.resize(n + 1, -1), root.resize(n + 1),
        ↪ pos.resize(n + 1), inv.resize(n + 1);
        subt.resize(n + 1, 1);
        up.resize(n + 1);
        for(int i = 0; i <= n; i ++){
            up[i].resize(20, 0);
        }
        seg = segtree(n + 1);
        cur_pos = 0;
    }
    void add_edge(int u, int v){
        g[u].push_back(v);
        g[v].push_back(u);
    }
    void run(int rt){
        dfs_sz(rt, 0);
        dfs_hld(rt, rt);
    }
    int dfs_sz(int v, int p){
        int sz = 1, mx_sz = 0;
        par[v] = p;
        up[v][0] = p;
        for(int i = 1; i < 20; i ++){
            up[v][i] = up[up[v][i - 1]][i - 1];
        }
        for(auto x : g[v]){
            if(x == p) continue;
            dpth[x] = dpth[v] + 1;
            int c_sz = dfs_sz(x, v);
            sz += c_sz;
            if(c_sz > mx_sz){

```

## 2.10 Euler

```

// Hierholzer's Algorithm undirected
// check remove the seen for directed
void dfs(int node) {
    while (!g[node].empty()) {
        auto [son, idx] = g[node].back();
        g[node].pop_back();
        if (seen[idx]) { continue; }
        seen[idx] = true;
        dfs(son);
    }
    path.push_back(node);
}
for (int node = 0; node < n; node++) {
    if (degree[node] % 2) {
        cout << "IMPOSSIBLE" << endl;
        return 0;
    }
}
dfs(0);
if (path.size() != m + 1) {
    cout << "IMPOSSIBLE";
}

```

## 2.11 Kosaraju

```

vector<int> g[n + 1], rg[n + 1];
for(auto [x, y, w] : edgs){
    g[x].push_back(y);
    rg[y].push_back(x);
}
vector<int> order;
vector<int> vis(n + 1, -1);
function<void(int)> dfs = [&](int v){
    vis[v] = 1;
    for(auto x : g[v]){
        if(vis[x] == -1) dfs(x);
    }
    order.push_back(v);
};
for(int i = 1; i <= n; i++) if(vis[i] == -1) dfs(i);
reverse(al(order));
int idx = 1;
for(int i = 1; i <= n; i++) vis[i] = -1;
function<void(int, vector<int>&)> dfs2 = [&](int v,
→ vector<int>& cmp){
    vis[v] = 1;
    for(auto x : rg[v]){
        if(vis[x] == -1) dfs2(x, cmp);
    }
    cmp.push_back(v);
};
vector<int> dag(n + 1, -1);
for(auto x : order){
    if(vis[x] == -1){
        vector<int> cmp;
        dfs2(x, cmp);
        for(auto y : cmp) dag[y] = idx;
        idx++;
    }
}
set<int> ng[idx], idg(idx, 0);
for(int i = 1; i <= n; i++){
    for(auto x : g[i]){
        if(dag[x] != dag[i]){
            ng[dag[i]].insert(dag[x]);
        }
    }
}
for(int i = 1; i < idx; i++){
    for(auto x : ng[i]) idg[x]++;
}
queue<int> q;
vector<int> dp(idx, 0);
for(int i = 1; i < idx; i++){
    if(idg[i] == 0){
        q.push(i);
        dp[i] = 1;
    }
}
while(!q.empty()){
    auto v = q.front(); q.pop();
    for(auto x : ng[v]){
        idg[x]--;
        dp[x] = max(dp[x], dp[v] + 1);
        if(!idg[x]) q.push(x);
    }
}

```

```

void fill_post(int at) {
    visited[at] = true;
    for (int n : rev_adj[at]) {
        if (!visited[n]) { fill_post(n); }
    }
    post[at] = timer++;
}

void find_comp(int at) {
    visited[at] = true;
    comp[at] = id;
    for (int n : adj[at]) {
        if (!visited[n]) { find_comp(n); }
    }
}

public:
    const vector<vector<int>> &adj;
    TarjanSolver(const vector<vector<int>> &adj)
        : adj(adj), rev_adj(adj.size(), post(adj.size())),
        → comp(adj.size()),
        visited(adj.size()) {
        vector<int> nodes(adj.size());
        for (int n = 0; n < adj.size(); n++) {
            nodes[n] = n;
            for (int next : adj[n]) { rev_adj[next].push_back(n); }
        }
        for (int n = 0; n < adj.size(); n++) {
            if (!visited[n]) { fill_post(n); }
        }
        std::sort(nodes.begin(), nodes.end(),
            [&](int n1, int n2) { return post[n1] > post[n2];
            → });
        visited.assign(adj.size(), false);
        for (int n : nodes) {
            if (!visited[n]) {
                find_comp(n);
                id++;
            }
        }
    }
    int comp_num() const { return id; }
    int get_comp(int n) const { return comp[n]; }
};

void tarjan(int u) {
    dfn[u] = low[u] = ++ clk;
    s[++ p] = u;
    ins[u] = 1;
    for(int i = head[u] ; i ; i = rest[i]) {
        int v = to[i];
        if(wid[i] > mid) continue;
        if(!dfn[v]) {
            tarjan(v);
            low[u] = min(low[u], low[v]);
        } else if(ins[v]){
            low[u] = min(low[u], dfn[v]);
        }
    }
    if(dfn[u] == low[u]) {
        ++ cnt;
        int v;
        do {
            ins[v = s[p --]] = 0;
            id[v] = cnt;
        } while(u != v);
    }
}

```

## 2.12 Tarjan

```

class TarjanSolver {
private:
    vector<vector<int>> rev_adj;
    vector<int> post;
    vector<int> comp;

    vector<bool> visited;
    int timer = 0;
    int id = 0;

```

## 2.13 Bridges

```

// Bridges
vector<int> tin(n + 1, -1), low(n + 1, -1), vis(n + 1, 0);
int clk = 0;
vector<pair<int, int>> ans;
function<void(int, int)> dfs = [&](int v, int p){
    vis[v] = 1;
    tin[v] = low[v] = clk++;
    for(auto x : g[v]){

```

```

        if(x == p) continue;
        if(!vis[x])
            dfs(x, v);
        low[v] = min(low[v], low[x]);
        if(low[x] > tin[v]){
            ans.push_back({v, x});
        }
    }

for(int i = 1; i <= n; i ++){
    if(!vis[i]){
        dfs(i, -1);
    }
}

// Articulation Points
function<void(int, int)> dfs = [&](int v, int p){
    vis[v] = 1;
    tin[v] = low[v] = clk++;
    int c = 0;
    for(auto x : g[v]){
        if(x == p) continue;
        if(vis[x]){
            low[v] = min(low[v], tin[x]);
        }
        else{
            dfs(x, v);
            low[v] = min(low[v], low[x]);
            if(low[x] >= tin[v] && p != -1){
                ans.insert(v);
            }
            c++;
        }
    }
    if(c > 1 && p == -1){
        ans.insert(v);
    }
};

for(int i = 1; i <= n; i ++){
    if(!vis[i]){
        dfs(i, -1);
    }
}

```

## 2.14 Toposort

```

vector<int> topoSort(const vector<vector<int>>& gr) {
vector<int> indeg(n), q;
for (auto& li : gr) for (int x : li) indeg[x]++;
rep(i, 0, n) if (indeg[i] == 0) q.push_back(i);
rep(j, 0, sz(q)) for (int x : gr[q[j]])
if (--indeg[x] == 0) q.push_back(x);
return q;
}

```

## 2.15 Distances

```

// Bellman Ford
vector<int> d(n, INF);
d[v] = 0;
for (int i = 0; i < n - 1; ++i)
    for (Edge e : edges)
        if (d[e.a] < INF)
            d[e.b] = min(d[e.a], d[e.a] + e.cost);

// Dijkstra Algorithm
void dijkstra(int s, vector<int> & d, vector<int> & p) {
    int n = adj.size();
    d.assign(n, INF);
    p.assign(n, -1);

    d[s] = 0;

```

```

        using pii = pair<int, int>;
        priority_queue<pii, vector<pii>, greater<pii>> q;
        q.push({0, s});
        while (!q.empty()) {
            int v = q.top().second;
            int d_v = q.top().first;
            q.pop();
            if (d_v != d[v])
                continue;

            for (auto edge : adj[v]) {
                int to = edge.first;
                int len = edge.second;

                if (d[v] + len < d[to]) {
                    d[to] = d[v] + len;
                    p[to] = v;
                    q.push({d[to], to});
                }
            }
        }

// Floyd Warshall
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (d[i][k] < INF && d[k][j] < INF)
                d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
        }
    }
}

```

## 3 Misc

### 3.1 Dsu\_dynconn

```

struct dsu_state{
    int v, u, rnk_v, rnk_u;
    dsu_state() {}
    dsu_state(int v, int u, int rnk_v, int rnk_u)
        : v(v), u(u), rnk_v(rnk_v), rnk_u(rnk_u) {}
};

struct dsuRoll{
    vector<int> par, rnk;
    stack<dsu_state> st;
    int comps;
    dsuRoll() {}
    dsuRoll(int n){
        par.resize(n + 1), rnk.resize(n + 1, 0);
        for(int i = 1; i <= n; i ++){
            par[i] = i;
            rnk[i] = 0;
        }
        comps = n;
    }
    int find(int v){
        return (v == par[v]) ? v : find(par[v]);
    }
    bool unite(int x, int y){
        x = find(x), y = find(y);
        if(x == y) return false;
        if(rnk[x] > rnk[y]) swap(x, y);
        st.push(dsu_state(x, y, rnk[x], rnk[y]));
        par[x] = y;
        if(rnk[x] == rnk[y]) rnk[y]++;
        comps--;
        return true;
    }
    void rollback(){
        if(st.empty()) return;
        auto cur = st.top(); st.pop();
        comps++;
        par[cur.v] = cur.v;
        par[cur.u] = cur.u;
    }
}

```

```

        rnk[cur.v] = cur.rnk_v;
        rnk[cur.u] = cur.rnk_u;
    }

};

struct qry{
    int v, u;
    bool f;
    qry() {}
    qry(int v, int u) : v(v), u(u) {}

};

struct dynCon{
    int n, sz;
    vector<vector<qry>> seg;
    dsuRoll dsu;
    dynCon(int _n, int _q) : n(_n), sz(_q) {
        dsu = dsuRoll(_n);
        seg.resize(4 * _q + 5);
    }
    void add(int idx, int st, int en, int ql, int qr, qry& q){
        if(st > qr || en < ql) return;
        if(st >= ql && en <= qr){
            seg[idx].push_back(q);
            return;
        }
        int md = (st + en) >> 1;
        add(idx << 1, st, md, ql, min(qr, md), q);
        add(idx << 1 | 1, md + 1, en, max(md + 1, ql), qr, q);
    }
    void add(int l, int r, qry q){
        add(l, 0, sz, l, r, q);
    }
    void run(int v, int l, int r, vector<int>& ans){
        for(auto &q : seg[v]) q.f = dsu.unite(q.v, q.u);
        if(l == r) { /* do something */ }
        else{
            int md = (l + r) >> 1;
            run(v << 1, l, md, ans);
            run(v << 1 | 1, md + 1, r, ans);
        }
        for(auto &q : seg[v]) if(q.f) dsu.rollback();
    }
    vector<int> solve(){
        vector<int> ans(sz + 1);
        run(1, 0, sz, ans);
        return ans;
    }
};

int n, q;
fscanf(in, "%d %d", &n, &q);
map<pair<int, int>, int> mp;
vector<int> ans(q + 1);
dynCon dc(n, q);
for(int i = 0; i < q; i ++){
    char c; fscanf(in, " %c", &c);
    if(c == '?'){
        ans[i] = 1;
    }
    else{
        int v, u;
        fscanf(in, "%d %d", &v, &u);
        if(v > u) swap(v, u);
        if(c == '+'){
            mp[{v, u}] = i;
        }
        else{
            dc.add(mp[{v, u}], i - 1, qry(v, u));
            mp.erase({v, u});
        }
    }
}
for(auto [p, start] : mp){
    dc.add(start, q, qry(p.first, p.second));
}

auto f = dc.solve();

```

## 3.2 IntervalContainer

```

/* Description: Add and remove intervals from a set of disjoint
   intervals.
Will merge the added interval with any overlapping intervals in
the set when
adding. Intervals are [inclusive, exclusive). */
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it);
    }
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    }
    return is.insert(before, {L,R});
}
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
    else (*int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
}

```

## 3.3 2SAT\_Shrey

```

struct two_sat
{
    int n;
    vector<vector<int>> g, gr; // gr is the
                                // reversed graph
    vector<int> comp, topological_order, answer; // comp[v]: ID
                                                // of the SCC containing node v
    vector<bool> vis;
    two_sat() {}

    two_sat(int _n) { init(_n); }

    void init(int _n)
    {
        n = _n;
        g.assign(2 * n, vector<int>());
        gr.assign(2 * n, vector<int>());
        comp.resize(2 * n);
        vis.resize(2 * n);
        answer.resize(2 * n);
    }

    void add_edge(int u, int v)
    {
        g[u].push_back(v);
        gr[v].push_back(u);
    }

    // For the following three functions
    // int x, bool val: if 'val' is true, we take the variable
    // to be x. Otherwise we take it to be x's complement.

    // At least one of them is true
    void add_clause_or(int i, bool f, int j, bool g)
    {
        add_edge(i + (f ? n : 0), j + (g ? 0 : n));
        add_edge(j + (g ? n : 0), i + (f ? 0 : n));
    }

    // Only one of them is true
    void add_clause_xor(int i, bool f, int j, bool g)
    {

```

```

{
    add_clause_or(i, f, j, g);
    add_clause_or(i, !f, j, !g);
}

// Both of them have the same value
void add_clause_and(int i, bool f, int j, bool g)
{
    add_clause_xor(i, !f, j, g);
}

// Topological sort
void dfs(int u)
{
    vis[u] = true;

    for (const auto &v : g[u])
        if (!vis[v])
            dfs(v);

    topological_order.push_back(u);
}

// Extracting strongly connected components
void scc(int u, int id)
{
    vis[u] = true;
    comp[u] = id;

    for (const auto &v : gr[u])
        if (!vis[v])
            scc(v, id);
}

// Returns true if the given proposition is satisfiable and
↳ constructs a valid assignment
bool satisfiable()
{
    fill(vis.begin(), vis.end(), false);

    for (int i = 0; i < 2 * n; i++)
        if (!vis[i])
            dfs(i);

    fill(vis.begin(), vis.end(), false);
    reverse(topological_order.begin(),
    ↳ topological_order.end());

    int id = 0;
    for (const auto &v : topological_order)
        if (!vis[v])
            scc(v, id++);

// Constructing the answer
for (int i = 0; i < n; i++)
{
    if (comp[i] == comp[i + n])
        return false;
    answer[i] = (comp[i] > comp[i + n] ? 1 : 0);
}

return true;
}

```

```

typedef tree<ll, null_type, less<ll>, rb_tree_tag,
    tree_order_statistics_node_update> indexed_set;
typedef tree<ll, null_type, less_equal<ll>, rb_tree_tag,
    tree_order_statistics_node_update> indexed_multiset;
#pragma GCC optimize("O3")
#pragma GCC target("avx2,bmi,bmi2,popcnt,lzcnt")
mt19937
    rng(chrono::steady_clock::now().time_since_epoch().count());

void __print(int x) {cerr << x;}
void __print(long x) {cerr << x;}
void __print(long long x) {cerr << x;}
void __print(unsigned x) {cerr << x;}
void __print(unsigned long x) {cerr << x;}
void __print(unsigned long long x) {cerr << x;}
void __print(float x) {cerr << x;}
void __print(double x) {cerr << x;}
void __print(long double x) {cerr << x;}
void __print(char x) {cerr << '\'' << x << '\'';}
void __print(const char *x) {cerr << '\"' << x << '\"';}
void __print(const string &x) {cerr << '\"' << x << '\"';}
void __print(bool x) {cerr << (x ? "true" : "false");}

template<typename T, typename V>
void __print(const pair<T, V> &x) {cerr << '{';
    __print(x.first); cerr << ','; __print(x.second); cerr << '}';
template<typename T>
void __print(const T &x) {int f = 0; cerr << '{'; for (auto &i:
    x) cerr << (f++ ? "," : ""), __print(i); cerr << "}";
void __print() {cerr << "]\\n";}
template <typename T, typename... V>
void __print(T t, V... v) {__print(t); if (sizeof...(v)) cerr <<
    ", "; __print(v...);}

// the debug works with collections types which you can iterate
// by for (auto i: a)
#ifndef ONLINE_JUDGE
#define debug(x...) cerr << "[" << #x << "] = ["; __print(x)
#else
#define debug(x...)
#endif

#define pb push_back
#define sz(x) static_cast<ll>(x.size())
#define pyes cout << "Yes\\n"
#define pno cout << "No\\n"
#define ce cout << '\\n'
#define endl '\\n'
#define fi first
#define se second
#define rev(v) reverse(v.begin(), v.end())
#define srt(v) sort(v.begin(), v.end())
#define all(v) v.begin(), v.end()
#define mnv(v) *min_element(v.begin(), v.end())
#define mxv(v) *max_element(v.begin(), v.end())
#define ull vector<ll>
#define up vector<pair<long long, long long>>
#define rep(i, n) for (ll i = 0; i < n; i++)
#define forf(i, a, b) for (ll i = a; i < b; i++)

const ll mod7 = 1e9 + 7;
const ll mod9 = 998244353;

void vin(vector<ll> &a, int n)
{
    for (int i = 0; i < n; i++)
    {
        ll x;
        cin >> x;
        a.push_back(x);
    }
}

template <typename T>
void pin(vector<T> a)
{
    for (int i = 0; i < (int)a.size(); i++)
    {
        cout << a[i] << " ";
    }
}

```

### 3.4 BoilerPlate Shrey

```
// Jaane wo kaise, log the jinke, pyaar ko pyaar milaaaaa
// Hamne to jab kaliyan maangi, kaaton ka haaar milaaaaa

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
typedef long long ll;
typedef long double ld;
using namespace std;
using namespace __gnu_pbds;
```

```

        cout << a[i] << " ";
    }
    ce;
}

ll power(ll a, ll b)
{
    ll res = 1;
    while (b > 0)
    {
        if (b & 1)
            res = (res * a);
        a = (a * a);
        b >>= 1;
    }
    return res;
}

ll exp(ll x, ll y, ll p)
{
    ll res = 1;
    x %= p;
    while (y)
    {
        if (y & 1)
        {
            res *= x;
            res %= p;
        }
        x *= x;
        x %= p;
        y >>= 1;
    }
    return res;
}

const int NUM = 1e6 + 7;
const ll INF = 1e18 + 5;
vp moves = {{0, 1}, {1, 0}, {-1, 0}, {0, -1}};

void solve()
{
}

// clear all data structures between test cases
// handle all corner cases
// confusing between n and m , i and j?
// uninitialized variables?

int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    cout.tie(0);
    // freopen("input.txt", "r", stdin);
    // freopen("output.txt", "w", stdout);
    // memset(dp, -1, sizeof(dp));
    ll t = 1;
    cin >> t;
    while (t--)
    {
        solve();
    }
    return 0;
}

```

### 3.5 Hungarian

```

// ans[i] = j means ith worker is assigned jth task
// O(n^3) time complexity
// It finds the assignment where the sum of costs is the lowest
// possible value.

void hungarian(vector<vector<long double>> &a, vll &ans)
{
    ll n = a.size();
    vector<ld> u(n + 1, 0), v(n + 1, 0);
    vll p(n + 1, 0), way(n + 1, 0);

```

```

for (ll i = 1; i <= n; ++i)
{
    vector<ld> minv(n + 1, INF);
    vector<bool> used(n + 1, false);
    ll j0 = 0;
    p[0] = i;
    do
    {
        used[j0] = true;
        ll i0 = p[j0], j1;
        long double delta = LDBL_MAX;
        for (ll j = 1; j <= n; ++j)
        {
            if (!used[j])
            {
                long double cur = a[i0 - 1][j - 1] - u[i0]
                ↪ - v[j];
                if (cur < minv[j])
                {
                    minv[j] = cur;
                    way[j] = j0;
                }
                if (minv[j] < delta)
                {
                    delta = minv[j];
                    j1 = j;
                }
            }
        }
        for (ll j = 0; j <= n; ++j)
        {
            if (used[j])
            {
                u[p[j]] += delta;
                v[j] -= delta;
            }
            else
            {
                minv[j] -= delta;
            }
        }
        j0 = j1;
    } while (p[j0] != 0);
    do
    {
        ll j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}

for (ll j = 1; j <= n; ++j)
{
    ans[p[j] - 1] = j - 1;
}
}

```

### 3.6 MEX\_DS

```

template <class T>
int calcMex(vector<T> v)
{
    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());
    int n = int(v.size());
    for (int i = 0; i < n; ++i)
        if (v[i] != i)
            return i;
    return n;
}

class Mex {
private:
    map<int, int> frequency;
}

```

```

set<int> missing_numbers;
vector<int> A;

public:
    Mex(vector<int> const& A) : A(A) {
        for (int i = 0; i <= A.size(); i++)
            missing_numbers.insert(i);

        for (int x : A) {
            ++frequency[x];
            missing_numbers.erase(x);
        }
    }

    int mex() {
        return *missing_numbers.begin();
    }

    void update(int idx, int new_value) {
        if (--frequency[A[idx]] == 0)
            missing_numbers.insert(A[idx]);
        A[idx] = new_value;
        ++frequency[new_value];
        missing_numbers.erase(new_value);
    }
};

```

```

        data.push_back(pii(A[i], i));
    }
    sort(data.begin(), data.end());
}

bool inter(int ql, int qr) {
    return min(R, qr) >= max(L, ql);
}

bool covered(int ql, int qr) {
    return ql <= L && R <= qr;
}

void makeAdd(int ql, int qr, long long delta) {
    if (!inter(ql, qr))
        return;
    if (covered(ql, qr)) {
        add += delta;
        return;
    }
    for (int i = 0; i < data.size(); i++)
        if (data[i].second >= ql && data[i].second <= qr)
            {
                data[i].first += delta;
            }
    sort(data.begin(), data.end());
}

int getFirst(long long x) {
    x -= add;
    pii p = pii(x, 0);
    auto it = lower_bound(data.begin(), data.end(), p);
    if (it == data.end() || it->first != x)
        return inf;
    return it->second;
}

int getLast(long long x) {
    x -= add;
    pii p = pii(x + 1, 0);
    auto it = lower_bound(data.begin(), data.end(), p);
    if (it == data.begin())
        return -inf;
    it--;
    if (it->first == x)
        return it->second;
    return -inf;
}
};
```

## 3.7 VeniceSet

```

// This can be applied to all Data Structures that support (add,
// remove and query) and only need the operation (updateAll).

class VeniceSet
{
public:
    multiset<ll> ice;
    ll level = 0;
    void add(ll x)
    {
        ice.insert(x + level);
    }
    void remove(ll x)
    {
        ice.erase(ice.find(x + level));
    }
    void decrement_all(ll x) // update all
    {
        level += x;
    }
    ll get_min() // query
    {
        return *ice.begin() - level;
    }
    ll get_size()
    {
        return ice.size();
    }
};

```

## 3.8 SortedArraysMO

```

int A[MAXN];
typedef pair<long long, int> pii;

struct Block {
    vector<pii> data;
    int L;
    int R;
    long long add;

    Block(int l, int r) {
        L = l;
        R = r;
        add = 0;
        for (int i = l; i <= r; i++) {

```

## 3.9 OrderedSet

```

// Remember to change define int long long
#include "ext/pb_ds/assoc_container.hpp"
#include "ext/pb_ds/tree_policy.hpp"
using namespace __gnu_pbds;
template<class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
template<class key, class value, class cmp = std::less<key>>
using ordered_map = tree<key, value, cmp, rb_tree_tag,
    tree_order_statistics_node_update>;
// find_by_order(k) returns iterator to kth element starting
// from 0;
// order_of_key(k) returns count of elements strictly smaller
// than k;
template<class T>
using min_heap = priority_queue<T, vector<T>, greater<T>>;

```

## 3.10 Twosat

```

struct twoSAT{
    int n, m;
    vector<vector<int>> adj, adjT;
    vector<int> val, order, comp, vis;

```

```

twoSAT(int n) : n(n), m(2 * n), adj(2 * n), adjT(2 * n),
    val(n), comp(2 * n, -1), vis(2 * n) {
    order.reserve(2 * n);
}
void dfs1(int v){
    vis[v] = 1;
    for(auto u : adj[v]){
        if(!vis[u]) dfs1(u);
    }
    order.push_back(v);
}
void dfs2(int v, int cl){
    comp[v] = cl;
    for(auto u : adjT[v]){
        if(comp[u] == -1) dfs2(u, cl);
    }
}
void add_f(int a, bool na, int b, bool nb){
    a = (2 * a) ^ na;
    b = (2 * b) ^ nb;
    adj[a ^ 1].push_back(b);
    adj[b ^ 1].push_back(a);
    adjT[b].push_back(a ^ 1);
    adjT[a].push_back(b ^ 1);
}
bool solver(){
    order.clear();
    fill(vis.begin(), vis.end(), 0);
    for(int i = 0; i < m; i++){
        if(!vis[i]) dfs1(i);
    }
    comp.assign(m, -1);
    for(int i = 0, j = 0; i < m; i++){
        int v = order[m - i - 1];
        if(comp[v] == -1) dfs2(v, j++);
    }
    val.assign(n, 0);
    for(int i = 0; i < m; i += 2){
        if(comp[i] == comp[i + 1]) return false;
        val[i >> 1] = comp[i] > comp[i + 1];
    }
    return true;
}
};
```

## 3.11 Fast

```

namespace fast {
    char B[1 << 18], *S = B, *T = B;
#define getc() (S == T ? (T = (S = B) + fread(B, 1, 1 << 18, stdin), S == T) ? 0 : *S++)
long long read() {
    long long ret = 0; char c;
    int f = 0;
    while (c = getc(), (c != '-' || (c < '0' || c > '9')));
    for (; (c >= '0' && c <= '9') || c == '-'; c = getc()){
        if(c == '-') f = 1;
        else ret = ret * 10 + c - '0';
    }
    return f ? -ret : ret;
}
```

## 3.12 Template

```

#include<bits/stdc++.h>
#ifndef ONLINE_JUDGE
#include "template.cpp"
#else
#define dbg(...)
#define debugArr(...)
#endif
using namespace std;
#define int long long
#define endl "\n"
```

```

#define al(v) v.begin(), v.end()
#define set_bits __builtin_popcountll
const int N = 1e9 + 7;
//const int N = 998244353;
const int inf = 0x3f3f3f3f3f3f3f3f;
const int MAXN = 2e5 + 7;
mt19937_64 rng(chrono::steady_clock::now());
// .time_since_epoch().count());
void solve(){
}
int t = 1;
int32_t main(){
    ios::sync_with_stdio(0);
    cin.tie(0);
    int t; cin >> t;
    while(t --){
        solve();
    }
}

#pragma GCC optimize ("Ofast")
#pragma GCC target ("avx2")
```

## 3.13 Basis

```

struct basis{
    array<int, 20> b;
    int sz = 0;
    basis(){
        b.fill(0);
        sz = 0;
    }
    void insert(int x){
        for(int i = 19; i >= 0; i--){
            if((x >> i) & 1){
                if(!b[i]){
                    b[i] = x;
                    sz++;
                    return;
                }
                x ^= b[i];
            }
        }
    }
    bool query(int x){
        for(int i = 19; i >= 0; i--){
            if(x == 0) return true;
            if((x >> i) & 1){
                if(!b[i]) return false;
                x ^= b[i];
            }
        }
        return x == 0;
    }
};
```

## 3.14 ConvexHullTrick

```

template<class T>
using min_heap = priority_queue<T, vector<T>, greater<T>>;
// When lines are added in increasing order of slopes
// Queries minimum
struct CHT {
    struct Line {
        int m, c;
        Line () {}
        Line (int _m, int _c) : m(_m), c(_c) {}

        double intersect(const Line &other) const {
            return (double)(other.c - c) / (m - other.m);
        }

        int operator()(int x) const {
            return m * x + c;
        }
    };
};
```

```

vector<double> points;
vector<Line> lines;
void init(Line l){
    points.push_back(-inf);
    lines.push_back(l);
}
void add_line(Line l){
    while(lines.size() >= 2 &&
        l.intersect(lines[lines.size() - 2]) <=
        points.back())){
        points.pop_back();
        lines.pop_back();
    }
    if(!lines.empty()){
        points.push_back(l.intersect( lines.back())));
    }
    if(!lines.empty() && lines.back().m == l.m){
        if(lines.back().c >= l.c) return;
        lines.pop_back();
        points.pop_back();
    }
    lines.push_back(l);
}
int query(int x){
    int idx = upper_bound(al(points), x) - points.begin() -
        1;
    return lines[idx](x);
}
void solve(){
    int n, c; cin>>n>>c;
    vector<int> a(n + 1), dp(n + 1);
    for(int i = 1; i <= n; i ++){
        cin>>a[i];
    }
    CHT cht;
    cht.init(CHT::Line(-2 * a[1], a[1] * a[1]));
    for(int i = 2; i <= n; i ++){
        dp[i] = cht.query(a[i]) + a[i] * a[i] + c;
        cht.add_line(CHT::Line(-2 * a[i], dp[i] + a[i] *
            a[i]));
    }
    cout<<dp[n]<<endl;
}

// Anyhow works, Queries Maximum
struct Line {
    mutable int k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(int x) const { return p < x; }
};

struct CHT : multiset<Line, less<> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    int div(int a, int b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(int k, int m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    int query(int x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};

```

## 4 Strings

### 4.1 Hash

```

void go_hsh(string s){
    hsh[0] = s[0];
    ppow[0] = 1;
    for(int i = 1; i < s.size(); i ++){
        hsh[i] = (((hsh[i - 1] * p) % mod) + s[i]) % mod;
        ppow[i] = (ppow[i - 1] * p) % mod;
    }
}
int get(int l, int r){
    if(!l) return hsh[r];
    return (hsh[r] - (hsh[l - 1] * ppow[r - l + 1])) % mod +
        mod) % mod;
}

```

### 4.2 Manacher

```

// Finds all palindromic substrings in O(n) time
// Returns a vector where the ith element represents the length
// of the longest palindromic substring
// centered at index i (for odd-length palindromes) or between
// indices i and i+1 (for even-length palindromes)
vector<int> manacher_odd(string s){
    int n = s.size();
    s = ('$' + s + '^');
    vector<int> ans(n + 2);
    int l = 1, r = 1;
    for(int i = 1; i <= n; i ++){
        ans[i] = max(0LL, min(r - i, ans[l + (r - i)]));
        while(s[i + ans[i]] == s[i - ans[i]]){
            ans[i]++;
        }
        if(i + ans[i] > r){
            l = i - ans[i];
            r = i + ans[i];
        }
    }
    vector<int> res;
    // we get answer as
    // even lengthed palindrome between i & i+1 as
    // (ans[i+1]-1)/2
    // odd centered at i as ans[i]/2
    return vector<int>(ans.begin() + 1, ans.end() - 1);
}
vector<int> manacher(string s){
    string t = "";
    for(auto x : s){
        t += "#";
        t += x;
    }
    t += "#";
    // cout<<t<<endl;
    auto res = manacher_odd(t);
    return vector<int>(res.begin() + 1, res.end() - 1);
}

Manacher.h
Description: For each position in a string, computes p[0][i] =
    half length
of longest even palindrome around pos i, p[1][i] = longest odd
    (half rounded
down).
Time: O (N)
e7ad79, 13 lines
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi,2> p = {vi(n+1), vi(n)};
    rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
        int t = r-i+z;
        if (i<r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-!z;
        while (L>=1 && R+1<n && s[L-1] == s[R+1])

```

```

p[z][i]++;
L--, R++;
if (R>r) l=L, r=R;
}
return p;
}

```

### 4.3 SuffixArray

```

struct SuffixArray {
    vector<int> sa, lcp;

    // Append 0 to s in case s is a vector
    SuffixArray(string &s, int lim = 256){
        int n = s.size() + 1, k = 0, a, b;
        vector<int> x(n), y(n), ws(max(n, lim)), rank(n);
        for(int i = 0; i < n - 1; i++) x[i] = s[i];

        sa = lcp = y, iota(sa.begin(), sa.end(), 0);
        for(int j = 0, p = 0; p < n; j = max(1LL, j * 2), lim =
        ↪ p){
            p = j, iota(y.begin(), y.end(), n - j);
            for(int i = 0; i < n; i++) if (sa[i] >= j) y[p++] =
            ↪ = sa[i] - j;
            fill(ws.begin(), ws.end(), 0);
            for(int i = 0; i < n; i++) ws[x[i]]++;
            for(int i = 1; i < lim; i++) ws[i] += ws[i - 1];
            for(int i = n; i --;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            for(int i = 1; i < n; i++) a = sa[i - 1], b =
            ↪ = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j])
                ↪ ? p - 1 : p++;
        }
        for(int i = 1; i < n; i++) rank[sa[i]] = i;
        for(int i = 0, j; i < n - 1; lcp[rank[i++]] = k) {
            for(k && k--, j = sa[rank[i] - 1]; s[i + k] == s[j]
            ↪ + k];
            k++;
        }
    }
};

```

### 4.4 Trie

```

const int MAXB = 30;
struct node{
    int cnt;
    int t[2];
    set<int> id;
    node (){
        cnt = 0;
        t[0] = t[1] = -1;
    }
};
struct btrie{
    vector<node> trie;
    btrie() {
        trie.resize(1);
    }
    int new_node(){
        trie.push_back(node());
        return (int)trie.size() - 1;
    }
    void insert(int x, int id){
        int cur = 0;
        trie[cur].cnt++;
        for(int i = MAXB; i >= 0; i --){
            int b = (x >> i) & 1;
            if(trie[cur].t[b] == -1){
                trie[cur].t[b] = new_node();
            }
            cur = trie[cur].t[b];
            trie[cur].cnt++;
        }
        trie[cur].id.insert(id);
    }
};

```

```

}
void remove(int x, int id){
    int cur = 0;
    trie[cur].cnt--;
    for(int i = MAXB; i >= 0; i --){
        int b = (x >> i) & 1;
        cur = trie[cur].t[b];
        trie[cur].cnt--;
    }
    assert(trie[cur].id.count(id));
    trie[cur].id.erase(id);
}

pair<int, int> min_xor(int x){
    int cur = 0, res = 0, idx = -1;
    for(int i = MAXB; i >= 0; i --){
        int b = ((x >> i) & 1);
        if(trie[cur].t[b] != -1 &&
        ↪ trie[trie[cur].t[b]].cnt > 0){
            cur = trie[cur].t[b];
        }
        else{
            res |= (1LL << i);
            cur = trie[cur].t[b ^ 1];
            if(cur == -1) return {res, -1};
        }
    }
    assert(!trie[cur].id.empty());
    idx = *trie[cur].id.begin();
    return {res, idx};
};

auto get = [&](int x){
    // I need y such that x ^ y is maximum;
    // if x has 0 then find 1 then 0;
    // if x has 1 then find 0 then 1;
    int curr = 0;
    int ans = 0;
    for(int i = 29; i >= 0; i --){
        int bit = (x >> i) & 1;
        if(bit){
            if(trie[curr][0]){
                ans += (1 << i);
                curr = trie[curr][0];
            } else {
                curr = trie[curr][1];
            }
        } else {
            if(trie[curr][1]){
                ans += (1 << i);
                curr = trie[curr][1];
            } else {
                curr = trie[curr][0];
            }
        }
    }
    return ans;
};

```

### 4.5 Kmp

```

// p[i] = length of the longest proper prefix of s[0..i]
// which is also a suffix of s[0..i]
vector<int> prefix_function(string s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}

```

## 5 Mathematics

### 5.1 Sieve

```
int low[MAXN]; // smallest prime factor
int phi[MAXN]; // Euler Totient Function
int mu[MAXN]; // Möbius function
vector<int> primes;
void sieve() {
    for (int i = 1; i < MAXN; i++) {
        phi[i] = i;
        mu[i] = 1;
    }
    for (int i = 2; i < MAXN; i++) {
        if (low[i] == 0) { // i is prime
            low[i] = i;
            primes.push_back(i);
            phi[i] = i - 1;
            mu[i] = -1;

            for (int j = i; j < MAXN; j += i) {
                if (low[j] == 0) low[j] = i;
                phi[j] -= phi[j] / i;
                if ((j / i) % i == 0)
                    mu[j] = 0;
                else
                    mu[j] *= -1;
            }
        }
    }
}
```

### 5.2 Vector

```
struct vec
{
    vector<ld> x;

    vec(ll n)
    {
        x.resize(n, 0);
    }
    vec operator+(const vec &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] + other.x[i];
        }
        return res;
    }
    vec operator-(const vec &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] - other.x[i];
        }
        return res;
    }
    vec operator*(const ld &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] * other;
        }
        return res;
    }
    vec operator/(const ld &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] / other;
        }
        return res;
    }
}
```

```
ld dot(const vec &other) const
{
    ld ans = 0;
    rep(i, x.size())
    {
        ans += x[i] * other.x[i];
    }
    return ans;
}
ld norm_sq() const
{
    ld ans = 0;
    rep(i, x.size())
    {
        ans += x[i] * x[i];
    }
    return ans;
}
ld norm() const
{
    return sqrt(norm_sq());
}
};

ld dist(vec &p, vec &a, vec &b)
{
    vec ab = b - a;
    vec ap = p - a;
    ld seglensq = ab.norm_sq();
    if (seglensq < 1e-12)
    {
        return ap.norm();
    }
    ld t = ap.dot(ab) / seglensq;
    if (t < 0)
        t = 0;
    if (t > 1)
        t = 1;
    vec q = a + (ab * t);
    return (q - p).norm();
}
```

### 5.3 SOSDp

```
int dpOr[MAXN], dpAnd[MAXN], A[MAXN];
for(int i = 0; i < n; i ++){
    dpOr[A[i]]++, dpAnd[A[i]]++;
}
for(int bit = 0; bit < 20; bit ++){
    for(int msk = 0; msk < MAXN; msk ++){
        if((msk >> bit) & 1){
            dpOr[msk] += dpOr[msk ^ (1 << bit)];
        }
        else{
            dpAnd[msk] += dpAnd[msk | (1 << bit)];
        }
    }
}
```

### 5.4 PollardRho

```
// https://judge.yosupo.jp/problem/factorize
#include<bits/stdc++.h>
using namespace std;

using ll = long long;
namespace PollardRho {
    mt19937
    ↵ rnd(chrono::steady_clock::now().time_since_epoch().count());
    const int P = 1e6 + 9;
    ll seq[P];
    int primes[P], spf[P];
    inline ll add_mod(ll x, ll y, ll m) {
        return (x += y) < m ? x : x - m;
    }
}
```

```

}

inline ll mul_mod(ll x, ll y, ll m) {
    ll res = __int128(x) * y % m;
    return res;
    // ll res = x * y - (ll)((long double)x * y / m + 0.5) * m;
    // return res < 0 ? res + m : res;
}

inline ll pow_mod(ll x, ll n, ll m) {
    ll res = 1 % m;
    for (; n; n >>= 1) {
        if (n & 1) res = mul_mod(res, x, m);
        x = mul_mod(x, x, m);
    }
    return res;
}

// O(it * (log n)^3), it = number of rounds performed
inline bool miller_rabin(ll n) {
    if (n <= 2 || (n & 1)) return (n == 2);
    if (n < P) return spf[n] == n;
    ll c, d, s = 0, r = n - 1;
    for (; !(r & 1); r >>= 1, s++) {}
    // each iteration is a round
    for (int i = 0; primes[i] < n && primes[i] < 32; i++) {
        c = pow_mod(primes[i], r, n);
        for (int j = 0; j < s; j++) {
            d = mul_mod(c, c, n);
            if (d == 1 && c != 1 && c != (n - 1)) return false;
            c = d;
        }
        if (c != 1) return false;
    }
    return true;
}

void init() {
    int cnt = 0;
    for (int i = 2; i < P; i++) {
        if (!spf[i]) primes[cnt++] = spf[i] = i;
        for (int j = 0, k; (k = i * primes[j]) < P; j++) {
            spf[k] = primes[j];
            if (spf[i] == spf[k]) break;
        }
    }
}

// returns O(n^(1/4))
ll pollard_rho(ll n) {
    while (1) {
        ll x = rnd() % n, y = x, c = rnd() % n, u = 1, v, t = 0;
        ll *px = seq, *py = seq;
        while (1) {
            *py++ = y = add_mod(mul_mod(y, y, n), c, n);
            *py++ = y = add_mod(mul_mod(y, y, n), c, n);
            if ((x = *px++) == y) break;
            v = u;
            u = mul_mod(u, abs(y - x), n);
            if (!u) return __gcd(v, n);
            if (++t == 32) {
                t = 0;
                if ((u = __gcd(u, n)) > 1 && u < n) return u;
            }
            if (t && (u = __gcd(u, n)) > 1 && u < n) return u;
        }
    }
}

vector<ll> factorize(ll n) {
    if (n == 1) return vector<ll>();
    if (!miller_rabin(n)) return vector<ll>{n};
    vector<ll> v, w;
    while (n > 1 && n < P) {
        v.push_back(spf[n]);
        n /= spf[n];
    }
    if (n >= P) {
        ll x = pollard_rho(n);
        v = factorize(x);
        w = factorize(n / x);
        v.insert(v.end(), w.begin(), w.end());
    }
    return v;
}

```

```

}

void solve(){
    ll m, n, k; cin>>m>>n>>k;
    set<ll> st;
    for(ll i = 0; i < n; i ++){
        ll x; cin>>x;
        auto f = PollardRho::factorize(x);
        for(auto y : f) st.insert(y);
    }
    for(ll i = 0; i < k; i ++){
        ll a, b, c; cin>>a>>b>>c;
    }
    for(auto x : st) cout<<x<<" ";
    cout<<endl;
}

int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    PollardRho::init();
    int t = 1;
    while (t--) {
        solve();
    }
    return 0;
}

```

## 5.5 Gaussian Elimination

```

void solve()
{
    ll p, f;
    cin >> p >> f;
    vector<vll> adj(p);
    vll deg(p, 0);
    rep(i, f)
    {
        ll a, b;
        cin >> a >> b;
        --a;
        --b;
        adj[a].pb(b);
        adj[b].pb(a);
        deg[a]++;
        deg[b]++;
    }

    vector<bitset<NUM + 1>> mat(p);
    rep(v, p)
    {
        for (auto u : adj[v])
            mat[v].set(u);
        if (deg[v] & 1)
            mat[v].set(v);
        else
            mat[v].set(p);
    }

    // The below code is taken from internet (Gaussian
    // Elimination in GF(2))
    ll row = 0;
    for (ll col = 0; col < p && row < p; ++col)
    {
        ll sel = -1;
        for (ll r = row; r < p; ++r)
        {
            if (mat[r].test(col))
            {
                sel = r;
                break;
            }
        }
        if (sel == -1)
            continue;
        if (sel != row)
            swap(mat[sel], mat[row]);
        for (ll r = 0; r < p; ++r)

```

```

{
    if (r != row && mat[r].test(col))
    {
        mat[r] ^= mat[row];
    }
    ++row;
}

for (ll r = 0; r < p; ++r)
{
    bool allzero = true;
    for (ll c = 0; c < p; ++c)
        if (!mat[r].test(c))
        {
            allzero = false;
            break;
        }
    if (allzero && mat[r].test(p))
    {
        cout << 'N' << endl;
        return;
    }
}

cout << 'Y' << endl;
}

```

## 5.6 Bits

```

mask &= ~(1LL << i); // Clear i-th bit
mask &= (mask - 1); // Turn off lowest set bit
bool is_pow2 = (mask > 0 && (mask & (mask - 1)) == 0); // Is
// power of two
int leading_zeros = __builtin_clzll(mask); // Count leading
// zeros
int trailing_zeros = __builtin_ctzll(mask); // Count trailing
// zeros
int lsb = mask & (-mask); // Isolate least significant bit
mask &= (mask - 1); // Turn off least significant bit

// Iterate over all subsets of a bitmask
for(int sub = mask; sub; sub = (sub - 1) & mask){
    // sub is a non-empty subset
}

```

## 5.7 Euler\_Totient

```

int phi(int n) { // O(sqrt(n))
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1)
        result -= result / n;
    return result;
}

void phi_1_to_n(int n) { // O(nloglogn)
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)
        phi[i] = i;

    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}

```

## 5.8 Comb\_mint

```

struct mint {
    int val;
    mint(long long v = 0) {
        if (v < 0) {
            v = v % N + N;
        }
        if (v >= N) {
            v %= N;
        }
        val = v;
    }
    static int mod_inv(int a, int m = N) {
        int g = m, r = a, x = 0, y = 1;
        while (r != 0) {
            int q = g / r;
            g %= r; swap(g, r);
            x -= q * y; swap(x, y);
        }
        return x < 0 ? x + m : x;
    }
    explicit operator int() const {
        return val;
    }
    mint& operator+=(const mint &other) {
        val += other.val;
        if (val >= N) val -= N;
        return *this;
    }
    mint& operator-=(const mint &other) {
        val -= other.val;
        if (val < 0) val += N;
        return *this;
    }
    typedef unsigned long long ull;
    ull fast_N(ull a, ull b, ull M = N) {
        long long ret = a * b - M * ull(1.L / M * a * b);
        return ret + M * (ret < 0) - M * (ret >= (long long)M);
    }
    mint& operator*=(const mint &other) {
        val = fast_N((ull) val, other.val);
        return *this;
    }
    mint& operator/=(const mint &other) {
        return *this *= other.inv();
    }
    friend mint operator+(const mint &a, const mint &b) {
        return mint(a) += b;
    }
    friend mint operator-(const mint &a, const mint &b) {
        return mint(a) -= b;
    }
    friend mint operator*(const mint &a, const mint &b) {
        return mint(a) *= b;
    }
    friend mint operator/(const mint &a, const mint &b) {
        return mint(a) /= b;
    }
    mint& operator++() {
        val = val == N - 1 ? 0 : val + 1;
        return *this;
    }
    mint& operator--() {
        val = val == 0 ? N - 1 : val - 1;
        return *this;
    }
    mint operator++(int32_t) { mint before = *this; ++*this;
        return before; }
    mint operator--(int32_t) { mint before = *this; --*this;
        return before; }
    mint operator-() const {
        return val == 0 ? 0 : N - val;
    }
    bool operator==(const mint &other) const { return val ==
        other.val; }
    bool operator!=(const mint &other) const { return val !=
        other.val; }
    mint inv() const {
        return mod_inv(val);
    }
}

```

```

mint pow(long long p) const {
    assert(p >= 0);
    mint a = *this, result = 1;

    while (p > 0) {
        if (p & 1)
            result *= a;

        a *= a;
        p >>= 1;
    }
    return result;
}

friend ostream& operator<<(ostream &stream, const mint &m)
→ {
    return stream << m.val;
}

friend istream& operator>>(istream &stream, mint &m) {
    return stream >> m.val;
}
};

template<typename T, const int P>
class comb {
public:
    int n;
    vector<T> fac, ifac, inv, pow;
    comb(int n) : n(n), fac(n), ifac(n), inv(n), pow(n) {
        fac[0] = ifac[0] = inv[0] = pow[0] = T(1);
        for(int i = 1; i <= n; i++) {
            inv[i] = T(1) / T(i);
            fac[i] = fac[i - 1] * T(i);
            ifac[i] = ifac[i - 1] * inv[i];
            pow[i] = pow[i - 1] * T(P);
        }
    }

    T ncr(int n, int r) {
        if(n < r) return T(0);
        if(n < 0 || r < 0) return T(0);
        return fac[n] * ifac[r] * ifac[n - r];
    }
};

comb<mint, 2> com(MAXN);

```

## 5.9 NCr\_Shrey

```

const ll MAXN = 1e7;
const ll MOD = 1e9 + 7;

vector<ll> fac(MAXN + 1);
vector<ll> inv(MAXN + 1);

/** Computes x^y modulo p in O(log p) time. */
ll exp(ll x, ll y, ll p)
{
    ll res = 1;
    x %= p;
    while (y) {
        if (y & 1)
        {
            res *= x;
            res %= p;
        }
        x *= x;
        x %= p;
        y >>= 1;
    }
    return res;
}

void factorial() {
    fac[0] = 1;
    for (ll i = 1; i <= MAXN; i++) {
        fac[i] = (fac[i - 1] * i) % MOD;
    }
}

```

```

void inverses() {
    inv[MAXN] = exp(fac[MAXN], MOD - 2, MOD);
    for (ll i = MAXN; i >= 1; i--) {
        inv[i - 1] = (inv[i] * i) % MOD;
    }
}

ll choose(int n, int r) {
    return fac[n] * inv[r] % MOD * inv[n - r] % MOD;
}

```

## 5.10 Matrix

```

template<typename T>
struct Matrix{
    vector<vector<T>> mat;
    int n, m;
    Matrix() {}
    Matrix(int _n, int _m, bool ident = false) {
        n = _n; m = _m;
        mat.assign(n, vector<T>(m, 0));
        if (ident) {
            for (int i = 0; i < n; i++)
                mat[i][i] = 1;
        }
    }
    Matrix(const vector<vector<T>> &v) {
        n = v.size();
        m = v[0].size();
        mat = v;
    }
    Matrix operator*(const Matrix &b) const {
        Matrix res(n, b.m);
        for (int i = 0; i < n; i++) {
            for (int k = 0; k < m; k++) {
                if (mat[i][k].val == 0) continue;
                for (int j = 0; j < b.m; j++) {
                    res.mat[i][j] += mat[i][k] * b.mat[k][j];
                }
            }
        }
        return res;
    }
};

template<typename T>
Matrix<T> mat_pow(Matrix<T> a, long long p) {
    Matrix<T> res(a.n, a.n, true);
    while (p) {
        if (p & 1) res = res * a;
        a = a * a;
        p >>= 1;
    }
    return res;
}

```

## 5.11 Extended\_GCD

```

ll extended_gcd(ll a, ll b, ll &x, ll &y)
{
    if (b == 0)
    {
        x = 1;
        y = 0;
        return a;
    }
    ll x1, y1;
    ll d = extended_gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

```

## 5.12 SegmentedSeive

```

// 1. Standard Sieve to generate primes up to sqrt(MaxR)
// MaxR = 10^12 usually, so limit = 10^6
void simpleSieve(int limit, vector<int> &primes)
{
    vector<bool> is_prime(limit + 1, true);
    is_prime[0] = is_prime[1] = false;

    for (int p = 2; p * p <= limit; p++)
    {
        if (is_prime[p])
        {
            for (int i = p * p; i <= limit; i += p)
                is_prime[i] = false;
        }
    }

    for (int p = 2; p <= limit; p++)
    {
        if (is_prime[p])
        {
            primes.push_back(p);
        }
    }
}

// 2. Segmented Sieve for range [L, R]
vector<long long> segmentedSieve(long long L, long long R)
{
    // A. Generate small primes up to sqrt(R)
    int lim = sqrt(R);
    vector<int> small_primes;
    simpleSieve(lim, small_primes);

    // B. Create a boolean array for the range [L, R]
    // is_prime[i] corresponds to number (L + i)
    vector<bool> is_prime(R - L + 1, true);

    // C. Mark multiples of small primes
    for (int p : small_primes)
    {
        // Find the first multiple of p that is >= L
        // Formula: (L + p - 1) / p * p gives ceil(L/p) * p
        long long start = (L + p - 1) / p * p;

        // Corner case: If start is the prime itself, move to
        // next multiple
        // (We don't want to mark the prime itself as
        // composite)
        if (start == p)
            start += p;

        // Mark all multiples in range
        // Index conversion: number x maps to index (x - L)
        for (long long i = start; i <= R; i += p)
        {
            // Check bounds just in case (start could be > R if
            // p > R)
            if (i >= L)
            {
                is_prime[i - L] = false;
            }
        }
    }

    // D. Collect primes
}
```

```

vector<long long> primes;
for (long long i = 0; i <= R - L; i++)
{
    if (is_prime[i])
    {
        long long num = L + i;
        if (num > 1)
        {
            // 1 is not prime
            primes.push_back(num);
        }
    }
}
return primes;
}
```

## 5.13 MillerRabin

```

using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};

bool MillerRabin(u64 n) { // returns true if n is prime, else
    // returns false.
    if (n < 2)
        return false;

    int r = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        r++;
    }

    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37})
    {
        if (n == a)
            return true;
        if (check_composite(n, a, d, r))
            return false;
    }
    return true;
}
```

## 1 Combinatorial

## 2 Permutations

### 2.1 Cycles

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

### 2.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

### 2.3 Burnside's lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

## 3 Partitions and subsets

### 3.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$$\begin{array}{c|ccccccccccccc} n & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 20 & 50 & 100 \\ \hline p(n) & 1 & 1 & 2 & 3 & 5 & 7 & 11 & 15 & 22 & 30 & 627 & \sim 2e5 & \sim 2e8 \end{array}$$

### 3.2 Lucas' Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

## 4 General purpose numbers

### 4.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).  $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$   
Sums of powers:

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^{\infty} f(i) &= \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

## 4.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$c(n, k) = c(n - 1, k - 1) + (n - 1)c(n - 1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k)x^k = x(x + 1) \dots (x + n - 1)$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$

$$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$$

## 4.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j + 1)$ ,  $k + 1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n - k)E(n - 1, k - 1) + (k + 1)E(n - 1, k)$$

$$E(n, 0) = E(n, n - 1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 4.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

## 4.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ . For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$$

## 4.6 Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$

# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$

# with degrees  $d_i$ :  $(n - 2)! / ((d_1 - 1)! \dots (d_n - 1)!)$

## 4.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with  $n + 1$  leaves (0 or 2 children).
- ordered trees with  $n + 1$  vertices.
- ways a convex polygon with  $n + 2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.