1 HeavyDS

1.1 Iterative

```
struct Data{
};

Data op(const Data &a, const Data &b){
}
struct segtree{
    vector<Data> d;
    int _n, log, size;
    unsigned int bit_ceil(unsigned int n) {
        unsigned int x = 1;
        while (x < (unsigned int)(n)) x <<= 1;
        return x;
    }
    segtree(int n): _n(n) {
        size = (int)bit_ceil((unsigned int)(n));
        log = __builtin_ctz((unsigned int)size);
        d.resize(2*size, Data());
    }
    void update(int k) { d[k] = op(d[2*k],
    ↪  d[2*k+1]); }
    void set(int p, Data x) {
        p += size;
        d[p] = x;
        for (int i = 1; i <= log; i++) update(p >>
        ↪  i);
    }
    Data prod(int l, int r) { // !!! q(l,r) return
    ↪  l to r-1
        if (l == r) return Data();
        l += size;
        r += size;
        Data sml = Data(), smr = Data();
        while (l < r) {
            if (l&1) sml = op(sml, d[l++]);
            if (r&1) smr = op(d[--r], smr);
            l >>= 1;
            r >>= 1;
        }
        return op(sml, smr);
    }
};
```

1.2 Lazy

```
struct Min{
    int x;
    Min(int x = inf) : x(x) {}
};

Min operator+(const Min& a, const Min& b){
    return std::min(a.x, b.x);
}

void apply(int &a, int b){
    a += b;
}

void apply(Min& a, int b){
    a.x += b;
}

// Always make a class for them and define basic
↪  overloads
template<class Node, class Lazy, class Merge =
↪  std::plus<Node>>
struct LazySegTree {
    const int _n;
    const Merge _merge;
    std::vector<Node> _segT;
    std::vector<Lazy> _lazy;
    LazySegTree(int n) : _n(n), _merge(Merge()),
    ↪  _segT(4 << std::__lg(n)), _lazy(4 <<
    ↪  std::__lg(n)) {}
    LazySegTree(std::vector<Node> init) :
    ↪  LazySegTree(init.size()) {
        std::function<void(int, int, int)> build =
        ↪  [&](int node, int st, int en){
            if(en - st == 1){
                _segT[node] = init[st];
                return;
            }
            int md = (st + en) >> 1;
            build(node << 1, st, md);
            build(node << 1 | 1, md, en);
            pull(node);
        };
        build(1, 0, _n);
    }
    void pull(int node){
        _segT[node] = _merge(_segT[node << 1],
        ↪  _segT[node << 1 | 1]);
    }
    void apply(int node, const Lazy& lazy){
        ::apply(_segT[node], lazy);
        ::apply(_lazy[node], lazy);
    }
    void push(int node){
        apply(node << 1, _lazy[node]);
        apply(node << 1 | 1, _lazy[node]);
        _lazy[node] = Lazy();
    }
    void update(int node, int st, int en, int idx,
    ↪  const Node& nodeVal){
        if(en - st == 1){
            _segT[node] = nodeVal;
            return;
        }
        push(node);
        int md = (st + en) >> 1;
        if(idx < md){
            update(node << 1, st, md, idx,
            ↪  nodeVal);
        }
        else{
            update(node << 1 | 1, md, en, idx,
            ↪  nodeVal);
        }
        pull(node);
    }
    void update(int idx, const Node& nodeVal){
        update(1, 0, _n, idx, nodeVal);
    }
    Node rangeQuery(int node, int st, int en, int
    ↪  ql, int qr){
        if(st >= qr || en <= ql){
            return Node();
        }
        if(st >= ql && en <= qr){
            return _segT[node];
        }
        push(node);
        int md = (st + en) >> 1;
        return _merge(rangeQuery(node << 1, st, md,
        ↪  ql, qr), rangeQuery(node << 1 | 1, md,
        ↪  en, ql, qr));
    }
    Node rangeQuery(int ql, int qr){
        return rangeQuery(1, 0, _n, ql, qr);
    }
    void rangeApply(int node, int st, int en, int
    ↪  ql, int qr, const Lazy& lazy){
        if(st >= qr || en <= ql){
            return;
        }
        if(st >= ql && en <= qr){
            apply(node, lazy);
            return;
        }
        push(node);
        int md = (st + en) >> 1;
        rangeApply(node << 1, st, md, ql, qr,
        ↪  lazy);
        rangeApply(node << 1 | 1, md, en, ql, qr,
        ↪  lazy);
        pull(node);
    }
    void rangeApply(int ql, int qr, const Lazy&
    ↪  lazy){
        rangeApply(1, 0, _n, ql, qr, lazy);
    }
};

// LazySegTree<Min, int> seg(a);
// For range [i, j] use i, j + 1
```

1.3 PersistentSegmentTree

```
struct PST{
    struct Node{
        int sum = 0;
        int l = 0, r = 0;
    };
    const int n;
    vector<Node> tree;
    int timer = 1;
    PST(int n, int mx_nodes) : n(n), tree(mx_nodes)
    ↪  {}
    Node merge(int l, int r){
        return Node({tree[l].sum + tree[r].sum, l,
        ↪  r});
    }
    int build(int st, int en, const vector<int>
    ↪  &init){
        if(st == en){
            tree[timer] = Node({init[st], 0, 0});
            return timer ++;
        }
        int md = (st + en) >> 1;
        tree[timer] = merge(build(st, md, init),
        ↪  build(md + 1, en, init));
        return timer ++;
    }
    int update(int root, int idx, int val, int st,
    ↪  int en){
        if(st == en){
            tree[timer] = Node({val, 0, 0});
            return timer ++;
        }
        int md = (st + en) >> 1;
        if(idx <= md){
            tree[timer] =
            ↪  merge(update(tree[root].l, idx,
            ↪  val, st, md), tree[root].r);
        }
        else{
            tree[timer] = merge(tree[root].l,
            ↪  update(tree[root].r, idx, val, md +
            ↪  1, en));
        }
        return timer ++;
    }
    int query(int root, int l, int r, int st, int
    ↪  en){
        if(r < st || en < l) return 0;
        if(l <= st && en <= r) return
        ↪  tree[root].sum;
        int md = (st + en) >> 1;
        return query(tree[root].l, l, r, st, md) +
        ↪  query(tree[root].r, l, r, md + 1, en);
    }
    int build(const vector<int> &init){
        return build(0, n - 1, init);
    }
    int update(int root, int idx, int val){
        return update(root, idx, val, 0, n - 1);
    }
    int query(int root, int l, int r){
        return query(root, l, r, 0, n - 1);
    }
};
```

1.4 Fenwick_sum

```
struct FenwickTree {
    vector<int> bit;  // binary indexed tree
    int n;
    FenwickTree(int n) {
        this->n = n;
        bit.assign(n, 0);
    }
    FenwickTree(vector<int> const &a) :
    ↪  FenwickTree(a.size()) {
        for (size_t i = 0; i < a.size(); i++)
            add(i, a[i]);
    }
    int sum(int r) {
        int ret = 0;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret += bit[r];
        return ret;
    }
    int sum(int l, int r) {
        return sum(r) - sum(l - 1);
    }
    void add(int idx, int delta) {
        for (; idx < n; idx = idx | (idx + 1))
            bit[idx] += delta;
    }
};
```

1.5 SparseTable

```
int log_floor(int x){
    return x ? __builtin_clzll(1) -
    ↪  __builtin_clzll(x)  : -1 ;
}
template<typename T , class F = function<T(const
↪  T&,const T&)>>
struct SPARSE_TABLE{
    int n;
    vector<vector<T>> st;
    F fun;
    SPARSE_TABLE(const vector<T> &v , const F &f) :
    ↪  fun(f){
        n = static_cast<int>(v.size());
        int maxN = log_floor(n) + 1;

        st.resize(maxN);
        st[0] = v;

        for(int i = 1; i < maxN; i++){
            st[i].resize(n - (1 << i) + 1);

            for(int j = 0; j + (1 << i) <= n;
            ↪  j++){
                st[i][j] = fun(st[i - 1][j] , st[i
                ↪  - 1][j + (1 << (i - 1))]);
            }
        }
    }
    T get(int l, int r){
        int h = log_floor(r - l + 1);
        // works for idempotent functions only ykw
        return fun(st[h][l] , st[h][r - (1 << h) +
        ↪  1]);
    }
};
```

1.6 MergeSortTree

```
template <typename T>
struct MergeSortTree {
    int n;
    vector<vector<T>> tree;

    // Constructor: builds the tree from input
    ↪  vector a
    MergeSortTree(const vector<T>& a) {
        n = a.size();
        tree.resize(4 * n);
        build(1, 0, n - 1, a);
    }

    void build(int node, int start, int end, const
    ↪  vector<T>& a) {
        if (start == end) {
            tree[node] = {a[start]};
            return;
        }
        int mid = (start + end) / 2;
        build(2 * node, start, mid, a);
        build(2 * node + 1, mid + 1, end, a);

        // Merge two sorted vectors
        merge(tree[2 * node].begin(), tree[2 *
        ↪  node].end(),
              tree[2 * node + 1].begin(), tree[2 *
              ↪  node + 1].end(),
              back_inserter(tree[node]));
    }

    // Internal query function for Lower Bound
    ↪  (smallest value >= x)
    T query_lb(int node, int start, int end, int l,
    ↪  int r, T x) {
        if (start > end || start > r || end < l)
            return numeric_limits<T>::max(); //
            ↪  Return INF

        if (start >= l && end <= r) {
            auto it =
            ↪  lower_bound(tree[node].begin(),
            ↪  tree[node].end(), x);
            if (it != tree[node].end())
                return *it;
            return numeric_limits<T>::max();
        }

        int mid = (start + end) / 2;
        return min(query_lb(2 * node, start, mid,
        ↪  l, r, x),
                   query_lb(2 * node + 1, mid + 1,
                   ↪  end, l, r, x));
    }

    // Internal query function for Count (number
    ↪  of elements <= x)
    int query_count(int node, int start, int end,
    ↪  int l, int r, T x) {
        if (start > end || start > r || end < l)
            return 0;

        if (start >= l && end <= r) {
            // upper_bound gives first element > x,
            ↪  so index gives count of elements
            ↪  <= x
            return upper_bound(tree[node].begin(),
            ↪  tree[node].end(), x) -
            ↪  tree[node].begin();
        }

        int mid = (start + end) / 2;
        return query_count(2 * node, start, mid, l,
        ↪  r, x) +
               query_count(2 * node + 1, mid + 1,
               ↪  end, l, r, x);
    }

    // Public Interface: Find smallest number >= x
    ↪  in range [l, r]
    T lower_bound_val(int l, int r, T x) {
        return query_lb(1, 0, n - 1, l, r, x);
    }

    // Public Interface: Count numbers <= x in
    ↪  range [l, r]
    int count_less_equal(int l, int r, T x) {
        return query_count(1, 0, n - 1, l, r, x);
    }
};
```

1.7 PolyMul

```
namespace GetPrimitve{
    vi Divisors;
    vi Divisor(lli x){vi ans;
        for(lli i=2;i*i<=x;i++){

            ↪  if(x%i==0){ans.EB(i);while(x%i==0)x/=i;}
        }
        if(x>1)ans.EB(x);return ans;
    }
    bool check(int prim,int p,vi &divs){
        for(auto v:divs){
            if(binpow(prim,(p-1)/v,p)==1)return 0;
        }return 1;
    }
    int getRoot(int p){
        int ans=2;vi divs=Divisor(p-1);
        while(!check(ans,p,divs))ans++;
        return ans;
    }
};
//7340033       5
//998244353     3
const lli MAXB = 1 << 21;
using polybase = lli;
polybase A[MAXB], B[MAXB], C[MAXB];
template<lli NTTMOD,lli PRIMITIVE_ROOT> struct
↪  POLYMUL{
    lli modInv(lli a) { return a <= 1 ? a : (long
    ↪  long) (NTTMOD - NTTMOD / a) *
    ↪  modInv(NTTMOD % a) % NTTMOD; }
    void NTT(polybase P[], lli n, lli oper) {
      for(int i=1,j=0;i<n-1;i++){
        for(int s=n;j^=s>>=1,~j&s;);
        if(i<j)swap(P[i],P[j]);
      }
      for(int d=0;(1<<d)<n;d++){
        int m=1<<d,m2=m*2;
        lli
        ↪  unit_p0=::binpow(PRIMITIVE_ROOT,(NTTMOD-1)
        if(oper<0)unit_p0=modInv(unit_p0);
        for(int i=0;i<n;i+= m2) {
          polybase unit = 1;
          for (int j = 0; j < m; j++) {
            polybase &P1 = P[i + j + m], &P2 = P[i
            ↪  + j];
            polybase t = unit * P1 % NTTMOD;
            P1 = (P2 - t + NTTMOD) % NTTMOD; P2 =
            ↪  (P2 + t) % NTTMOD;
            unit = unit * unit_p0 % NTTMOD;
          }
        }
      }
    }
    vector<polybase> mul(const vector<polybase> &a,
    ↪  const vector<polybase> &b) {
        vector<polybase> ret(max(0LL, (lli)
        ↪  a.size() + (lli) b.size() - 1), 0);
        int len = 1; while (len < (lli)ret.size())
        ↪  len <<= 1;
        for (int i = 0; i < len; i++) A[i] = i <
        ↪  (int)a.size() ? a[i] : 0;
        for (int i = 0; i < len; i++) B[i] = i <
        ↪  (int)b.size() ? b[i] : 0;
        NTT(A, len, 1); NTT(B, len, 1);
        for (lli i = 0; i < len; i++) C[i] =
        ↪  (polybase) A[i] * B[i] % NTTMOD;
        NTT(C, len, -1); for (lli i = 0, inv =
        ↪  modInv(len); i < (lli)ret.size(); i++)
        ↪  ret[i] = (long long) C[i] * inv %
        ↪  NTTMOD;
        return ret;
    }
    vi binpow(vi b,lli p){
        vi ans=vi(1,1);
        for(;p;p>>=1){
            if(p&1)ans=mul(ans,b);
            b=mul(b,b);
        }
        return ans;
    }
    vi calc(vi &arr,int l,int r){
        if(l==r){
            //base case;
            return {};
        }
        int mid=(l+r)>>1;
        vi x=calc(arr,l,mid),y=calc(arr,mid+1,r);
        return mul(x,y);
    }
};
POLYMUL<998244353,3> ntt;


lli phi(lli x){// will be a power of 2 mostly.
    if(x==1)return 1;
    else return x/2;
}
```

```
vi arr[20];
void solve(){
    lli n,k,q;
    arr[1].assign(100001,0);
    cin>>n>>k;
    vi divs;
    lli cur=1;
    while(n%cur==0){
        divs.EB(cur);
        cur*=2;
    }
    fr(i,k){
        lli x;cin>>x;arr[1][x]++;
    }
    rep(i,2,19){
        arr[i]=ntt.mul(arr[i-1],arr[i-1]);

        ↪  if(arr[i].size()>100001)arr[i].resize(100001);
    }
    sort(all(divs));
    cin>>q;
    fr(_i,q){
        lli S;cin>>S;
        lli ans=0;
        for(int i=0;i<divs.size();i++){
            if(S%(n/divs[i])!=0)continue;

            ↪  ans=(ans+arr[i+1][S/(n/divs[i])]*phi(n/divs[i]))%MOD;
        }
        ans=ans*binpow(n,MOD-2,MOD)%MOD;
        cout<<ans<<endl;
    }
}

// youKnowWho's FFT
using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd> & a, bool invert) {
    int n = a.size();
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1) j ^= bit;
        j ^= bit;
        if (i < j) swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <<= 1) {
        double ang = 2 * PI / len * (invert ? -1 :
        ↪  1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] *
                ↪  w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }
    if (invert) {
        for (cd & x : a) x /= n;
    }
}
vector<int> multiply(vector<int> const& a, int n)
{
    vector<cd> fa(n);
    for (int i = 0; i < (int)a.size(); i++) {
        fa[i] = a[i];
    }
    for(int i = a.size(); i < n; i ++){
        fa[i] = 0;
    }
    fft(fa, false);
    for(int i = 0; i < n; i ++){
        fa[i] *= fa[i];
    }
    fft(fa, true);

    vector<int> res(n);
    for(int i = 0; i < n; i ++){
        res[i] = round(fa[i].real());
    }
    return res;
}

vector<int> p(MAXN);
void solve(){
    int n; cin>>n;
    int mx = 1;
    vector<int> s(n + 1);
    for(int i = 1; i <= n; i ++){
        cin>>s[i];
        mx = max(mx, s[i]);
    }
    mx += 1000;
    for(int i = 1; i <= n; i ++){
        p[s[i]] = 1;
    }

    n = 1;
    while(n <= MAXN){
        n <<= 1;
    }
    vector<int> res = multiply(p, n);
    // dbg(res);

    int c = 0;
    for(int i = 2; i <= 2 * mx; i += 2){
        int md = i / 2;
        if(p[md]){
            c += (res[i] - 1) / 2;
        }
    }
    cout<<c<<endl;
}
```

1.8 FWT

```
Given two arrays $A$ and $B$ of size $n = 2^k$,
↪  compute an array $C$ such that
\[
C[x] = \sum_{\substack{i \oplus j = x}} A[i] \cdot
↪  B[j].
\]

template <typename T>
struct FWT {

    // ===== XOR =====
    void fwt_xor(T a[], int n) {
        for (int d = 1; d < n; d <<= 1) {
            for (int i = 0; i < n; i += (d << 1))
            ↪ {
                for (int j = 0; j < d; j++) {
                    T x = a[i + j];
                    T y = a[i + j + d];
                    a[i + j]     = x + y;
                    a[i + j + d] = x - y;
                }
            }
        }
    }

    void ufwt_xor(T a[], int n) {
        for (int d = 1; d < n; d <<= 1) {
            for (int i = 0; i < n; i += (d << 1))
            ↪ {
                for (int j = 0; j < d; j++) {
                    T x = a[i + j];
                    T y = a[i + j + d];
                    a[i + j]     = (x + y) >> 1;
                    a[i + j + d] = (x - y) >> 1;
                }
            }
        }
    }

    // ===== AND =====
    void fwt_and(T a[], int n) {
        for (int d = 1; d < n; d <<= 1) {
            for (int i = 0; i < n; i += (d << 1))
            ↪ {
                for (int j = 0; j < d; j++) {
                    a[i + j] += a[i + j + d];
                }
            }
        }
    }

    void ufwt_and(T a[], int n) {
        for (int d = 1; d < n; d <<= 1) {
            for (int i = 0; i < n; i += (d << 1))
            ↪ {
                for (int j = 0; j < d; j++) {
                    a[i + j] -= a[i + j + d];
                }
            }
        }
    }

    // ===== OR =====
    void fwt_or(T a[], int n) {
        for (int d = 1; d < n; d <<= 1) {
            for (int i = 0; i < n; i += (d << 1))
            ↪ {
                for (int j = 0; j < d; j++) {
                    a[i + j + d] += a[i + j];
                }
            }
        }
    }

    void ufwt_or(T a[], int n) {
        for (int d = 1; d < n; d <<= 1) {
            for (int i = 0; i < n; i += (d << 1))
            ↪ {
                for (int j = 0; j < d; j++) {
                    a[i + j + d] -= a[i + j];
                }
            }
        }
    }

    // ===== CONVOLUTIONS =====
    void xor_convolution(T a[], T b[], int n) {
        fwt_xor(a, n);
        fwt_xor(b, n);
        for (int i = 0; i < n; i++) a[i] *= b[i];
        ufwt_xor(a, n);
    }

    void and_convolution(T a[], T b[], int n) {
        fwt_and(a, n);
        fwt_and(b, n);
        for (int i = 0; i < n; i++) a[i] *= b[i];
        ufwt_and(a, n);
    }

    void or_convolution(T a[], T b[], int n) {
        fwt_or(a, n);
        fwt_or(b, n);
        for (int i = 0; i < n; i++) a[i] *= b[i];
        ufwt_or(a, n);
    }
};
```

2 Graphs

2.1 Dinic

```
template<class T>
struct Flow {
    const int n;
    struct Edge {
        int to;
        T cap;
        Edge(int to, T cap) : to(to), cap(cap) {}
    };
    std::vector<Edge> e;
    std::vector<std::vector<int>> g;
    std::vector<int> cur, h;
    Flow(int n) : n(n), g(n) {}

    bool bfs(int s, int t) {
        h.assign(n, -1);
        std::queue<int> que;
        h[s] = 0;
        que.push(s);
        while (!que.empty()) {
            const int u = que.front();
            que.pop();
            for (int i : g[u]) {
                auto [v, c] = e[i];
                if (c > 0 && h[v] == -1) {
                    h[v] = h[u] + 1;
                    if (v == t) {
                        return true;
                    }
                    que.push(v);
                }
            }
        }
        return false;
    }

    T dfs(int u, int t, T f) {
        if (u == t) {
            return f;
        }
        auto r = f;
        for (int &i = cur[u]; i <
        ↪  (int)(g[u].size()); ++i) {
            const int j = g[u][i];
            auto [v, c] = e[j];
            if (c > 0 && h[v] == h[u] + 1) {
                auto a = dfs(v, t, std::min(r,
                ↪  c));
                e[j].cap -= a;
                e[j ^ 1].cap += a;
                r -= a;
                if (r == 0) {
                    return f;
                }
            }
        }
        return f - r;
    }
    void addEdge(int u, int v, T c) {
        g[u].push_back(e.size());
        e.emplace_back(v, c);
        g[v].push_back(e.size());
        e.emplace_back(u, 0);
    }
    T maxFlow(int s, int t) {
        T ans = 0;
        while (bfs(s, t)) {
            cur.assign(n, 0);
            ans += dfs(s, t,
            ↪  std::numeric_limits<T>::max());
        }
        return ans;
    }
    void get(int end){
        vector<int> path;
        cur.assign(n, 0);
        int u = 1;
        while(u != end){
            path.push_back(u);
            dbg(path)
            for (int &i = cur[u]; i <
            ↪  (int)(g[u].size()); ++i) {
                const int j = g[u][i];
                auto [v, c] = e[j];
                dbg(u, v, c, j)
                if (j % 2 == 0 && c == 0) {
                    e[j].cap = 1;
                    u = v;
                    break;
                }
            }
        }
        path.push_back(end);
        cout<<path.size()<<endl;
        for(int i : path) cout<<i<<" ";
        cout<<endl;
    }
};

queue<int> q;
vector<bool> vis(n + 1, false);
q.push(1);
vis[1] = true;
while(!q.empty()){
    int u = q.front(); q.pop();
    for(auto i: mf.g[u]){
        auto [v, c] = mf.e[i];
        if(c > 0 && !vis[v]){
            vis[v] = true;
            q.push(v);
        }
    }
}
for(int i = 1; i <= n; i ++){
    for(auto id : mf.g[i]){
        auto [v, c] = mf.e[id];
        if(v != s and c == 0){
            cout<<i<<" "<<v - n<<endl;
        }
    }
}
```

2.2 LCA

```
// LCA and Binary Lifting
void dfs(int v, int p){
    up[0][v] = p;
    for(int i = 1; i < 20; i ++){
        up[i][v] = up[i - 1][up[i - 1][v]];
    }
    for(auto x : g[v]){
        if(x == p) continue;
        dst[x] = dst[v] + 1;
        dfs(x, v);
    }
}
auto lca = [&](int x, int y){
        if(dst[x] < dst[y]) swap(x, y);
        for(int i = 19; i >= 0; i --){
            if(dst[up[i][x]] >= dst[y]){
                x = up[i][x];
            }
        }
        if(x == y) return x;
        for(int i = 19; i >= 0; i --){
            if(up[i][x] != up[i][y]){
                x = up[i][x];
                y = up[i][y];
            }
        }
        return up[0][x];
    };
```

2.3 CutPoint

```
vll g[NUM];
bool vis[NUM];
ll in[NUM];
ll low[NUM];
int timer = 0;

void CutPoints(ll src, ll parent = -1)
{
    vis[src] = true;
    in[src] = low[src] = timer;
    timer++;
    int children = 0;
    for (auto child : g[src])
    {
        if (child == parent)
            continue;
        if (vis[child])
        {
            low[src] = min(low[src], in[child]);
        }
        else
        {
            CutPoints(child, src);
            low[src] = min(low[src], low[child]);
            if (low[child] >= in[src] &&
            ↪  parent!=-1)
            {
                // Src is a cut point!
                // CAUTION: Might get called
                ↪  multiple times due to
                ↪  satisfiablity of multiple
                ↪  children
            }
            children++;
        }
    }
    if (parent==-1 && children>1){
        // Root is a cut point!
        // used children instead of size of adj
        ↪  bcoz we dont use back edges
    }
}
```

2.4 CentroidDecomp

```
int nn;
void dfs1(int u, int p)
{
    sub[u] = 1;
    nn++;
    for (auto it = g[u].begin(); it != g[u].end();
    ↪  it++)
        if (*it != p)
        {
            dfs1(*it, u);
            sub[u] += sub[*it];
        }
}
int dfs2(int u, int p)
{
    for (auto it = g[u].begin(); it != g[u].end();
    ↪  it++)
        if (*it != p && sub[*it] > nn / 2)
            return dfs2(*it, u);
    return u;
}
void decompose(int root, int p)
{
    nn = 0;
    dfs1(root, root);
    int centroid = dfs2(root, root);
    if (p == -1)
        p = centroid;
    par[centroid] = p;
    for (auto it = g[centroid].begin(); it !=
    ↪  g[centroid].end(); it++)
    {
        g[*it].erase(centroid);
        decompose(*it, centroid);
    }
    g[centroid].clear();
}
```

2.5 VirtualTree

```
const int MAXN = 200005;

// --- PREREQUISITES (LCA & DFS Times) ---
vector<int> adj[MAXN];
int tin[MAXN], tout[MAXN], timer;
int up[MAXN][20], depth[MAXN];

void dfs_lca(int u, int p, int d) {
    tin[u] = ++timer;
    depth[u] = d;
    up[u][0] = p;
    for (int i = 1; i < 20; i++)
        up[u][i] = up[up[u][i-1]][i-1];

    for (int v : adj[u]) {
        if (v != p) dfs_lca(v, u, d + 1);
    }
    tout[u] = timer;
}
```

```
bool is_ancestor(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v) {
    if (is_ancestor(u, v)) return u;
    if (is_ancestor(v, u)) return v;
    for (int i = 19; i >= 0; i--) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

// --- VIRTUAL TREE TEMPLATE ---
vector<int> vt_adj[MAXN]; // Adjacency list for
↪  Virtual Tree

// Sorts nodes by DFS entry time (Critical for
↪  construction)
bool compare_tin(int a, int b) {
    return tin[a] < tin[b];
}

void build_virtual_tree(vector<int>& nodes) {
    // 1. Sort by DFS entry time
    sort(nodes.begin(), nodes.end(), compare_tin);

    // 2. Add LCAs of adjacent sorted nodes to the
    ↪  set
    // This guarantees all necessary connecting
    ↪  nodes are present
    int k = nodes.size();
    for (int i = 0; i < k - 1; i++) {
        nodes.push_back(lca(nodes[i],
        ↪  nodes[i+1]));
    }

    // 3. Sort again and remove duplicates
    sort(nodes.begin(), nodes.end(), compare_tin);
    nodes.erase(unique(nodes.begin(), nodes.end()),
    ↪  nodes.end());

    // 4. Build edges using a stack
    // Clear previous virtual tree edges for these
    ↪  specific nodes
    for (int u : nodes) vt_adj[u].clear();

    vector<int> stack;
    stack.push_back(nodes[0]);

    for (int i = 1; i < nodes.size(); i++) {
        int u = nodes[i];
        // Pop stack while the top is NOT an
        ↪  ancestor of u
        // (This means we finished the subtree of
        ↪  stack.back())
        while (stack.size() > 1 &&
        ↪  !is_ancestor(stack.back(), u)) {
            stack.pop_back();
        }
        // Add edge from the current direct
        ↪  ancestor to u
        // Note: The graph is directed downwards
        ↪  usually, or undirected depending on
        ↪  need
        vt_adj[stack.back()].push_back(u);
        vt_adj[u].push_back(stack.back()); // Add
        ↪  this if undirected

        stack.push_back(u);
    }
}
```

2.6 Centroid

```
function<void(int, int)> dfs2 = [&](int v, int p){
        int mx = 0;
        for(auto u: g[v]){
            if(u == p || vis[u]) continue;
            mx = max(mx, sbt[u]);
        }
        mx = max(mx, n - sbt[v]);
        if(mx <= n / 2){
            centroid = v;
        }
        for(auto u: g[v]){
            if(u == p || vis[u]) continue;
            dfs2(u, v);
        }
    };
```

2.7 Mcmf

```
using i64 = long long;
template<class T>
struct MinCostFlow {
    struct _Edge {
        int to;
        T cap;
        T cost;
        _Edge(int to_, T cap_, T cost_) : to(to_),
        ↪  cap(cap_), cost(cost_) {}
    };
    int n;
    std::vector<_Edge> e;
    std::vector<std::vector<int>> g;
    std::vector<T> h, dis;
    std::vector<int> pre;
    bool dijkstra(int s, int t) {
        dis.assign(n,
        ↪  std::numeric_limits<T>::max());
        pre.assign(n, -1);
        std::priority_queue<std::pair<T, int>,
        ↪  std::vector<std::pair<T, int>>,
        ↪  std::greater<std::pair<T, int>>> que;
        dis[s] = 0;
        que.emplace(0, s);
        while (!que.empty()) {
            T d = que.top().first;
            int u = que.top().second;
            que.pop();
            if (dis[u] != d) {
                continue;
            }
            for (int i : g[u]) {
                int v = e[i].to;
                T cap = e[i].cap;
                T cost = e[i].cost;
                if (cap > 0 && dis[v] > d + h[u] -
                ↪  h[v] + cost) {
                    dis[v] = d + h[u] - h[v] +
                    ↪  cost;
                    pre[v] = i;
                    que.emplace(dis[v], v);
                }
            }
        }
        return dis[t] !=
        ↪  std::numeric_limits<T>::max();
    }
    MinCostFlow() {}
    MinCostFlow(int n_) {
        init(n_);
    }
    void init(int n_) {
        n = n_;
        e.clear();
        g.assign(n, {});
    }
    void addEdge(int u, int v, T cap, T cost) {
        g[u].push_back(e.size());
        e.emplace_back(v, cap, cost);
        g[v].push_back(e.size());
        e.emplace_back(u, 0, -cost);
    }
    std::pair<T, T> flow(int s, int t) {
        T flow = 0;
        T cost = 0;
        h.assign(n, 0);
        while (dijkstra(s, t)) {
            for (int i = 0; i < n; ++i) {
                h[i] += dis[i];
            }
            T aug =
            ↪  std::numeric_limits<int>::max();
            for (int i = t; i != s; i = e[pre[i] ^
            ↪  1].to) {
                aug = std::min(aug,
                ↪  e[pre[i]].cap);
            }
            for (int i = t; i != s; i = e[pre[i] ^
            ↪  1].to) {
                e[pre[i]].cap -= aug;
                e[pre[i] ^ 1].cap += aug;
            }
            flow += aug;
            cost += aug * h[t];
        }
        return std::make_pair(flow, cost);
    }
    struct Edge {
        int from;
        int to;
        T cap;
        T cost;
        T flow;
    };
    std::vector<Edge> edges() {
        std::vector<Edge> a;
        for (int i = 0; i < e.size(); i += 2) {
            Edge x;
            x.from = e[i + 1].to;
            x.to = e[i].to;
            x.cap = e[i].cap + e[i + 1].cap;
            x.cost = e[i].cost;
            x.flow = e[i + 1].cap;
            a.push_back(x);
        }
        return a;
    }
};
```

2.8 Hld

```
struct hld {
    vector<vector<int>> g;
    vector<int> par, dpth, heavy, root, pos, subt,
    ↪  inv;
    vector<vector<int>> up;
    segtree seg;
    int cur_pos = 0;
    hld(int n){
        g.resize(n + 1), par.resize(n + 1),
        ↪  dpth.resize(n + 1);
        heavy.resize(n + 1, -1), root.resize(n +
        ↪  1), pos.resize(n + 1), inv.resize(n +
        ↪  1);
        subt.resize(n + 1, 1);
        up.resize(n + 1);
        for(int i = 0; i <= n; i ++){
            up[i].resize(20, 0);
        }
        seg = segtree(n + 1);
        cur_pos = 0;
    }
    void add_edge(int u, int v){
        g[u].push_back(v);
        g[v].push_back(u);
    }
    void run(int rt){
        dfs_sz(rt, 0);
        dfs_hld(rt, rt);
    }
    int dfs_sz(int v, int p){
        int sz = 1, mx_sz = 0;
        par[v] = p;
        up[v][0] = p;
        for(int i = 1; i < 20; i ++){
            up[v][i] = up[up[v][i - 1]][i - 1];
        }
        for(auto x : g[v]){
            if(x == p) continue;
            dpth[x] = dpth[v] + 1;
            int c_sz = dfs_sz(x, v);
            sz += c_sz;
            if(c_sz > mx_sz){
                mx_sz = c_sz;
                heavy[v] = x;
            }
        }
        return subt[v] = sz;
    }
    void dfs_hld(int v, int h){
        root[v] = h;
        pos[v] = cur_pos, inv[cur_pos] = v;
        cur_pos ++;
        if(heavy[v] != -1) dfs_hld(heavy[v], h);
        for(auto x : g[v]){
            if(x == par[v] || x == heavy[v])
            ↪  continue;
            dfs_hld(x, x);
        }
    }
    bool is_anc(int a, int b) {
        return pos[a] <= pos[b] && pos[b] < pos[a]
        ↪  + subt[a];
    }
    int kth(int v, int k) {
        for (int i = 0; i < 20; i++) {
            if (k & (1 << i)) {
                v = up[v][i];
            }
        }
        return v;
    }
    int hehe(int v, int u) {
        // u is ancestor of v
        int dif = dpth[v] - dpth[u] - 1;
        return kth(v, dif);
    }
    mint qry(int a, int b) {
        mint res = 0;
        for (; root[a] != root[b]; b =
        ↪  par[root[b]]) {
            if (dpth[root[a]] > dpth[root[b]])
                swap(a, b);
            mint cur_heavy_path_max =
            ↪  seg.prod(pos[root[b]], pos[b] +
            ↪  1).x;
            res += cur_heavy_path_max;
        }
        if (dpth[a] > dpth[b])
            swap(a, b);
        mint last_heavy_path_max = seg.prod(pos[a],
        ↪  pos[b] + 1).x;
        res += last_heavy_path_max;
        return res;
    }
};
```

2.9 Euler

```
// Hierholzer's Algorithm undirected
// check remove the seen for directed
void dfs(int node) {
  while (!g[node].empty()) {
    auto [son, idx] = g[node].back();
    g[node].pop_back();
    if (seen[idx]) { continue; }
    seen[idx] = true;
    dfs(son);
  }
  path.push_back(node);
}
for (int node = 0; node < n; node++) {
    if (degree[node] % 2) {
        cout << "IMPOSSIBLE" << endl;
        return 0;
    }
}
dfs(0);
if (path.size() != m + 1) {
    cout << "IMPOSSIBLE";
}
```

2.10 Kosaraju

```
vector<int> g[n + 1], rg[n + 1];
for(auto [x, y, w] : edgs){
    g[x].push_back(y);
    rg[y].push_back(x);
}
vector<int> order;
vector<int> vis(n + 1, -1);
function<void(int)> dfs = [&](int v){
    vis[v] = 1;
    for(auto x : g[v]){
        if(vis[x] == -1) dfs(x);
    }
    order.push_back(v);
};
for(int i = 1; i <= n; i ++) if(vis[i] == -1)
↪  dfs(i);
reverse(al(order));
int idx = 1;
for(int i = 1; i <= n; i ++) vis[i] = -1;
function<void(int, vector<int>&)> dfs2 = [&](int v,
↪  vector<int>& cmp){
    vis[v] = 1;
    for(auto x : rg[v]){
        if(vis[x] == -1) dfs2(x, cmp);
    }
    cmp.push_back(v);
};
vector<int> dag(n + 1, -1);
for(auto x : order){
    if(vis[x] == -1){
        vector<int> cmp;
        dfs2(x, cmp);
        for(auto y : cmp) dag[y] = idx;
        idx ++;
    }
}
set<int> ng[idx], idg(idx, 0);
for(int i = 1; i <= n; i ++){
    for(auto x : g[i]){
        if(dag[x] != dag[i]){
            ng[dag[i]].insert(dag[x]);
        }
    }
}
for(int i = 1; i < idx; i ++){
    for(auto x : ng[i]) idg[x] ++;
}
queue<int> q;
vector<int> dp(idx, 0);
for(int i = 1; i < idx; i ++){
    if(idg[i] == 0){
        q.push(i);
        dp[i] = 1;
    }
}
while(!q.empty()){
    auto v = q.front(); q.pop();
    for(auto x : ng[v]){
        idg[x] --;
        dp[x] = max(dp[x], dp[v] + 1);
        if(!idg[x]) q.push(x);
    }
}
```

2.11 Bridges

```
// Bridges
vector<int> tin(n + 1, -1), low(n + 1, -1), vis(n +
↪  1, 0);
int clk = 0;
vector<pair<int, int>> ans;
function<void(int, int)> dfs = [&](int v, int p){
    vis[v] = 1;
    tin[v] = low[v] = clk ++;
    for(auto x : g[v]){
        if(x == p) continue;
        if(!vis[x])
            dfs(x, v);
        low[v] = min(low[v], low[x]);
        if(low[x] > tin[v]){
            ans.push_back({v, x});
        }
    }
};

for(int i = 1; i <= n; i ++){
    if(!vis[i]){
        dfs(i, -1);
    }
}

// Articulation Points
function<void(int, int)> dfs = [&](int v, int p){
    vis[v] = 1;
    tin[v] = low[v] = clk ++;
    int c = 0;
    for(auto x : g[v]){
        if(x == p) continue;
        if(vis[x]){
            low[v] = min(low[v], tin[x]);
        }
        else{
            dfs(x, v);
            low[v] = min(low[v], low[x]);
            if(low[x] >= tin[v] && p != -1){
                ans.insert(v);
            }
            c ++;
        }
    }
    if(c > 1 && p == -1){
        ans.insert(v);
    }
};

for(int i = 1; i <= n; i ++){
    if(!vis[i]){
        dfs(i, -1);
    }
}
```

2.12 Toposort

```
vector<int> topoSort(const vector<vector<int>>& gr)
↪  {
vector<int> indeg(n), q;
for (auto& li : gr) for (int x : li) indeg[x]++;
rep(i,0,n) if (indeg[i] == 0) q.push_back(i);
rep(j,0,sz(q)) for (int x : gr[q[j]])
if (--indeg[x] == 0) q.push_back(x);
return q;
}
```

2.13 Distances

```
// Bellman Ford
vector<int> d(n, INF);
    d[v] = 0;
    for (int i = 0; i < n - 1; ++i)
        for (Edge e : edges)
            if (d[e.a] < INF)
                d[e.b] = min(d[e.b], d[e.a] +
                ↪  e.cost);



// Dijkstra Algorithm
void dijkstra(int s, vector<int> & d, vector<int>
↪  & p) {
    int n = adj.size();
    d.assign(n, INF);
    p.assign(n, -1);

    d[s] = 0;
    using pii = pair<int, int>;
    priority_queue<pii, vector<pii>, greater<pii>>
    ↪  q;
    q.push({0, s});
    while (!q.empty()) {
        int v = q.top().second;
        int d_v = q.top().first;
        q.pop();
        if (d_v != d[v])
            continue;

        for (auto edge : adj[v]) {
            int to = edge.first;
            int len = edge.second;

            if (d[v] + len < d[to]) {
                d[to] = d[v] + len;
                p[to] = v;
                q.push({d[to], to});
            }
        }
    }
}

// Floyd Warshall
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (d[i][k] < INF && d[k][j] < INF)
                d[i][j] = min(d[i][j], d[i][k] +
                ↪  d[k][j]);
        }
    }
}
```

3 Misc

3.1 Dsu_dynconn

```
struct dsu_state{
    int v, u, rnk_v, rnk_u;
    dsu_state() {}
    dsu_state(int v, int u, int rnk_v, int rnk_u)
        : v(v), u(u), rnk_v(rnk_v), rnk_u(rnk_u)
        ↪  {}
};
struct dsuRoll{
    vector<int> par, rnk;
    stack<dsu_state> st;
    int comps;
    dsuRoll() {}
    dsuRoll(int n){
        par.resize(n + 1), rnk.resize(n + 1, 0);
        ↪
        for(int i = 1; i <= n; i ++){
            par[i] = i;
            rnk[i] = 0;
        }
        comps = n;
    }
    int find(int v){
        return (v == par[v]) ? v : find(par[v]);
    }
    bool unite(int x, int y){
        x = find(x), y = find(y);
        if(x == y) return false;
        if(rnk[x] > rnk[y]) swap(x, y);
        st.push(dsu_state(x, y, rnk[x], rnk[y]));
        par[x] = y;
        if(rnk[x] == rnk[y]) rnk[y] ++;
        comps --;
        return true;
    }
    void rollback(){
        if(st.empty()) return;
        auto cur = st.top(); st.pop();
        comps ++;
        par[cur.v] = cur.v;
        par[cur.u] = cur.u;
        rnk[cur.v] = cur.rnk_v;
        rnk[cur.u] = cur.rnk_u;
    }
};

struct qry{
    int v, u;
    bool f;
    qry() {}
    qry(int v, int u) : v(v), u(u) {}
};

struct dynCon{
    int n, sz;
    vector<vector<qry>> seg;
    dsuRoll dsu;
    dynCon(int _n, int _q) : n(_n), sz(_q) {
        dsu = dsuRoll(_n);
        seg.resize(4 * _q + 5);
    }
    void add(int idx, int st, int en, int ql, int
    ↪  qr, qry& q){
        if(st > qr || en < ql) return;
        if(st >= ql && en <= qr){
            seg[idx].push_back(q);
            return;
        }
        int md = (st + en) >> 1;
        add(idx << 1, st, md, ql, min(qr, md), q);
        add(idx << 1 | 1, md + 1, en, max(md + 1,
        ↪  ql), qr, q);
    }
    void add(int l, int r, qry q){
        add(1, 0, sz, l, r, q);
    }
    void run(int v, int l, int r, vector<int>&
    ↪  ans){
        for(auto &q : seg[v]) q.f = dsu.unite(q.v,
        ↪  q.u);
        if(l == r) { /* do something */ }
        else{
            int md = (l + r) >> 1;
            run(v << 1, l, md, ans);
            run(v << 1 | 1, md + 1, r, ans);
        }
        for(auto &q : seg[v]) if(q.f)
        ↪  dsu.rollback();
    }
    vector<int> solve(){
        vector<int> ans(sz + 1);
        run(1, 0, sz, ans);
        return ans;
    }
};

int n, q;
```

```
fscanf(in, "%d %d", &n, &q);
map<pair<int, int>, int> mp;
vector<int> ans(q + 1);
dynCon dc(n, q);
for(int i = 0; i < q; i ++){
    char c; fscanf(in, " %c", &c);
    if(c == '?'){
        ans[i] = 1;
    }
    else{
        int v, u;
        fscanf(in, "%d %d", &v, &u);
        if(v > u) swap(v, u);
        if(c == '+'){
            mp[{v, u}] = i;
        }
        else{
            dc.add(mp[{v, u}], i - 1, qry(v, u));
            mp.erase({v, u});
        }
    }
}

for(auto [p, start] : mp){
    dc.add(start, q, qry(p.first, p.second));
}

auto f = dc.solve();
```

3.2 IntervalContainer

```
/* Description: Add and remove intervals from a
↪  set of disjoint intervals.
Will merge the added interval with any overlapping
↪  intervals in the set when
adding. Intervals are [inclusive, exclusive). */
set<pii>::iterator addInterval(set<pii>& is, int L,
↪  int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
    }
    if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
    }
    return is.insert(before, {L,R});
    }
    void removeInterval(set<pii>& is, int L, int R)
    ↪  {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
}
```

3.3 2SAT_Shrey

```
struct two_sat
{
    int n;
    vector<vector<int>> g, gr;                    //
    ↪  gr is the reversed graph
    vector<int> comp, topological_order, answer; //
    ↪  comp[v]: ID of the SCC containing node v
    vector<bool> vis;

    two_sat() {}

    two_sat(int _n) { init(_n); }

    void init(int _n)
    {
        n = _n;
        g.assign(2 * n, vector<int>());
        gr.assign(2 * n, vector<int>());
        comp.resize(2 * n);
        vis.resize(2 * n);
        answer.resize(2 * n);
    }

    void add_edge(int u, int v)
    {
        g[u].push_back(v);
        gr[v].push_back(u);
    }

    // For the following three functions
    // int x, bool val: if 'val' is true, we take
    ↪  the variable to be x. Otherwise we take it
    ↪  to be x's complement.

    // At least one of them is true
    void add_clause_or(int i, bool f, int j, bool
    ↪  g)
    {
        add_edge(i + (f ? n : 0), j + (g ? 0 :
        ↪  n));
        add_edge(j + (g ? n : 0), i + (f ? 0 :
        ↪  n));
    }

    // Only one of them is true
    void add_clause_xor(int i, bool f, int j, bool
    ↪  g)
    {
        add_clause_or(i, f, j, g);
        add_clause_or(i, !f, j, !g);
    }

    // Both of them have the same value
    void add_clause_and(int i, bool f, int j, bool
    ↪  g)
    {
        add_clause_xor(i, !f, j, g);
    }

    // Topological sort
    void dfs(int u)
    {
        vis[u] = true;

        for (const auto &v : g[u])
            if (!vis[v])
                dfs(v);

        topological_order.push_back(u);
    }

    // Extracting strongly connected components
    void scc(int u, int id)
    {
        vis[u] = true;
        comp[u] = id;

        for (const auto &v : gr[u])
            if (!vis[v])
                scc(v, id);
    }

    // Returns true if the given proposition is
    ↪  satisfiable and constructs a valid
    ↪  assignment
    bool satisfiable()
    {
        fill(vis.begin(), vis.end(), false);

        for (int i = 0; i < 2 * n; i++)
            if (!vis[i])
                dfs(i);

        fill(vis.begin(), vis.end(), false);
        reverse(topological_order.begin(),
        ↪  topological_order.end());

        int id = 0;
        for (const auto &v : topological_order)
            if (!vis[v])
                scc(v, id++);

        // Constructing the answer
        for (int i = 0; i < n; i++)
        {
            if (comp[i] == comp[i + n])
                return false;
            answer[i] = (comp[i] > comp[i + n] ? 1
            ↪  : 0);
        }

        return true;
    }
};
```

3.4 Hungarian

```
// ans[i] = j means ith worker is assigned jth
↪  task
// O(n^3) time complexity
// It finds the assignment where the sum of costs
↪  is the lowest possible value.
```

```
void hungarian(vector<vector<long double>> &a, vll
↪  &ans)
{
    ll n = a.size();
    vector<ld> u(n + 1, 0), v(n + 1, 0);
    vll p(n + 1, 0), way(n + 1, 0);

    for (ll i = 1; i <= n; ++i)
    {
        vector<ld> minv(n + 1, INF);
        vector<bool> used(n + 1, false);
        ll j0 = 0;
        p[0] = i;
        do
        {
            used[j0] = true;
            ll i0 = p[j0], j1;
            long double delta = LDBL_MAX;
            for (ll j = 1; j <= n; ++j)
            {
                if (!used[j])
                {
                    long double cur = a[i0 - 1][j -
                    ↪  1] - u[i0] - v[j];
                    if (cur < minv[j])
                    {
                        minv[j] = cur;
                        way[j] = j0;
                    }
                    if (minv[j] < delta)
                    {
                        delta = minv[j];
                        j1 = j;
                    }
                }
            }
            for (ll j = 0; j <= n; ++j)
            {
                if (used[j])
                {
                    u[p[j]] += delta;
                    v[j] -= delta;
                }
                else
                {
                    minv[j] -= delta;
                }
            }
            j0 = j1;
        } while (p[j0] != 0);
        do
        {
            ll j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
    }

    for (ll j = 1; j <= n; ++j)
    {
        ans[p[j] - 1] = j - 1;
    }
}
```

3.5 OrderedSet

```
// Remember to change define int long long
#include "ext/pb_ds/assoc_container.hpp"
#include "ext/pb_ds/tree_policy.hpp"
using namespace __gnu_pbds;
template<class T>
using ordered_set = tree<T, null_type,less<T>,
↪  rb_tree_tag, tree_order_statistics_node_update>
↪  ;
template<class key, class value, class cmp =
↪  std::less<key>>
using ordered_map = tree<key, value, cmp,
↪  rb_tree_tag,
↪  tree_order_statistics_node_update>;
// find_by_order(k)  returns iterator to kth
↪  element starting from 0;
// order_of_key(k) returns count of elements
↪  strictly smaller than k;
template<class T>
using min_heap =
↪  priority_queue<T,vector<T>,greater<T> >;
```

3.6 Fast

```
namespace fast {
    char B[1 << 18], *S = B,*T = B;
    #define getc() (S == T && (T = (S = B) +
    ↪  fread(B, 1, 1 << 18, stdin), S == T) ? 0 :
    ↪  *S++)
    long long read() {
        long long ret = 0; char c;
        int f = 0;
        while (c = getc(), (c != '-') && (c < '0'
        ↪  || c > '9'));
        for (; (c >= '0' && c <= '9') || c == '-';
        ↪  c = getc()){
            if(c == '-') f = 1;
            else ret = ret * 10 + c - '0';
        }
        return f ? -ret : ret;
    }
}
```

3.7 Template

```
#include<bits/stdc++.h>
using namespace std;
#define int long long
#define endl "\n"
#define al(v) v.begin(), v.end()
#define set_bits __builtin_popcountll
const int N = 1e9 + 7;
//const int N = 998244353;
const int inf = 0x3f3f3f3f3f3f3f3f;
const int MAXN = 2e5 + 7;
mt19937_64 rng(chrono::steady_clock::now()
↪  .time_since_epoch().count());
void solve(){
}
int t = 1;
int32_t main(){
    ios::sync_with_stdio(0);
    cin.tie(0);
    int t; cin>>t;
    while(t --){
        solve();
    }
}
#pragma GCC optimize ("Ofast")
#pragma GCC target ("avx2")
```

3.8 Basis

```
struct basis{
    array<int, 20> b;
    int sz = 0;
    basis(){
        b.fill(0);
        sz = 0;
    }
    void insert(int x){
        for(int i = 19; i >= 0; i --){
            if((x >> i) & 1){
                if(!b[i]){
                    b[i] = x;
                    sz ++;
                    return;
                }
                x ^= b[i];
            }
        }
    }
    bool query(int x){
        for(int i = 19; i >= 0; i --){
            if(x == 0) return true;
            if((x >> i) & 1){
                if(!b[i]) return false;
                x ^= b[i];
            }
        }
        return x == 0;
    }
};
```

3.9 ConvexHullTrick

```
template<class T>
using min_heap =
↪  priority_queue<T,vector<T>,greater<T>>;
// When lines are added in increasing order of
↪  slopes
// Queries minimum
struct CHT {
    struct Line {
        int m, c;
        Line () {}
        Line (int _m, int _c) : m(_m), c(_c) {}
        double intersect(const Line &other) const
        ↪ {
            return (double)(other.c - c) / (m -
            ↪  other.m);
        }

        int operator()(int x) const {
            return m * x + c;
        }
    };
    vector<double> points;
    vector<Line> lines;
    void init(Line l){
        points.push_back(-inf);
        lines.push_back(l);
    }
    void add_line(Line l){
        while(lines.size() >= 2 &&
        ↪  l.intersect(lines[lines.size() - 2])
        ↪ <= points.back()){
            points.pop_back();
            lines.pop_back();
        }
        if(!lines.empty()){
            points.push_back(l.intersect(
            ↪  lines.back()));
        }
        if(!lines.empty() && lines.back().m ==
        ↪  l.m){
            if(lines.back().c >= l.c) return;
            lines.pop_back();
            points.pop_back();
        }
        lines.push_back(l);
    }
    int query(int x){
        int idx = upper_bound(al(points), x) -
        ↪  points.begin() - 1;
        return lines[idx](x);
    }
};
void solve(){
    int n, c; cin>>n>>c;
    vector<int> a(n + 1), dp(n + 1);
    for(int i = 1; i <= n; i ++){
        cin>>a[i];
    }

    CHT cht;
    cht.init(CHT::Line(-2 * a[1], a[1] * a[1]));
    for(int i = 2; i <= n; i ++){
        dp[i] = cht.query(a[i]) + a[i] * a[i] + c;
        cht.add_line(CHT::Line(-2 * a[i], dp[i] +
        ↪  a[i] * a[i]));
    }
    cout<<dp[n]<<endl;
}

// Anyhow works, Queries Maximum
struct Line {
    mutable int k, m, p;
    bool operator<(const Line& o) const { return k
    ↪  < o.k; }
    bool operator<(int x) const { return p < x; }
};

struct CHT : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) =
    ↪  a/b)
    int div(int a, int b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf
        ↪  : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(int k, int m) {
        auto z = insert({k, m, 0}), y = z++, x =
        ↪  y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x,
        ↪  y = erase(y));
        while ((y = x) != begin() && (--x)->p >=
        ↪  y->p)
            isect(x, erase(y));
    }
    int query(int x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};
```

4 Strings

4.1 Hash

```
void go_hsh(string s){
    hsh[0] = s[0];
    ppow[0] = 1;
    for(int i = 1; i < s.size(); i ++){
        hsh[i] = (((hsh[i - 1] * p) % mod) + s[i])
        ↪  % mod;
        ppow[i] = (ppow[i - 1] * p) % mod;
    }
}
int get(int l, int r){
    if(!l) return hsh[r];
    return (hsh[r] - (hsh[l - 1] * ppow[r - l + 1])
    ↪  % mod + mod) % mod;
}
```

4.2 Manacher

```
// Finds all palindromic substrings in O(n) time
// Returns a vector where the ith element
↪  represents the length of the longest
↪  palindromic substring
```

```
// centered at index i (for odd-length palindromes)
↪  or between indices i and i+1 (for even-length
↪  palindromes)
vector<int> manacher_odd(string s){
    int n = s.size();
    s=('$' + s + '^');
    vector<int> ans(n + 2);
    int l = 1, r = 1;
    for(int i = 1; i <= n; i ++){
        ans[i] = max(0LL, min(r - i, ans[l + (r -
        ↪  i)]));
        while(s[i + ans[i]] == s[i - ans[i]]){
            ans[i] ++;
        }
        if(i + ans[i] > r){
            l = i - ans[i];
            r = i + ans[i];
        }
    }
    vector<int> res;
    // we get answer as
    // even lengthed palindrome between i & i+1 as
    ↪  (ans[i+1]-1)/2
    // odd centered at i as ans[i]/2
    return vector<int>(ans.begin()+1,ans.end()-1);
}
vector<int> manacher(string s){
    string t = "";
    for(auto x : s){
        t += "#";
        t += x;
    }
    t += "#";
    // cout<<t<<endl;
    auto res = manacher_odd(t);
    return vector<int>(res.begin() + 1, res.end() -
    ↪  1);
}

Manacher.h
Description: For each position in a string,
↪  computes p[0][i] = half length
of longest even palindrome around pos i, p[1][i] =
↪  longest odd (half rounded
down).
Time: O (N)
e7ad79, 13 lines
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi,2> p = {vi(n+1), vi(n)};
    rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
    p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
    }
    return p;
}
```

4.3 SuffixArray

```
struct SuffixArray {
    vector<int> sa, lcp;

    // Append 0 to s in case s is a vector
    SuffixArray(string &s, int lim = 256){
        int n = s.size() + 1, k = 0, a, b;
        vector<int> x(n), y(n), ws(max(n, lim)),
        ↪  rank(n);
        for(int i = 0; i < n - 1; i ++) x[i] =
        ↪  s[i];

        sa = lcp = y, iota(sa.begin(), sa.end(),
        ↪  0);
        for(int j = 0, p = 0; p < n; j = max(1LL,
        ↪  j * 2), lim = p){
            p = j, iota(y.begin(), y.end(), n -
            ↪  j);
            for(int i = 0; i < n; i ++) if (sa[i]
            ↪  >= j) y[p++] = sa[i] - j;
            fill(ws.begin(), ws.end(), 0);
            for(int i = 0; i < n; i ++)
            ↪  ws[x[i]]++;
            for(int i = 1; i < lim; i ++) ws[i] +=
            ↪  ws[i - 1];
            for(int i = n; i --;)
            ↪  sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            for(int i = 1; i < n; i ++) a = sa[i -
            ↪  1], b = sa[i], x[b] =
                        (y[a] == y[b] && y[a + j]
                        ↪  == y[b + j]) ? p - 1 :
                        ↪  p++;
        }
        for(int i = 1; i < n; i ++) rank[sa[i]] =
        ↪  i;
        for(int i = 0, j; i < n - 1;
        ↪  lcp[rank[i++]] = k) {
            for(k && k--, j = sa[rank[i] - 1]; s[i
            ↪  + k] == s[j + k];
            k++);
        }
    }
};
```

4.4 Trie

```
const int MAXB = 30;
struct node{
    int cnt;
    int t[2];
    set<int> id;
    node (){
        cnt = 0;
        t[0] = t[1] = -1;
    }
};
struct btrie{
    vector<node> trie;
    btrie() {
        trie.resize(1);
    }
    int new_node(){
        trie.push_back(node());
        return (int)trie.size() - 1;
    }
    void insert(int x, int id){
        int cur = 0;
        trie[cur].cnt ++;
        for(int i = MAXB; i >= 0; i --){
            int b = (x >> i) & 1;
            if(trie[cur].t[b] == -1){
                trie[cur].t[b] = new_node();
            }
            cur = trie[cur].t[b];
            trie[cur].cnt ++;
        }
        trie[cur].id.insert(id);
    }
    void remove(int x, int id){
        int cur = 0;
        trie[cur].cnt --;
        for(int i = MAXB; i >= 0; i --){
            int b = (x >> i) & 1;
            cur = trie[cur].t[b];
            trie[cur].cnt --;
        }
        assert(trie[cur].id.count(id));
        trie[cur].id.erase(id);
    }
    pair<int, int> min_xor(int x){
        int cur = 0, res = 0, idx = -1;
        for(int i = MAXB; i >= 0; i --){
            int b = ((x >> i) & 1);
            if(trie[cur].t[b] != -1 &&
            ↪ trie[trie[cur].t[b]].cnt > 0){
                cur = trie[cur].t[b];
            }
            else{
                res |= (1LL << i);
                cur = trie[cur].t[b ^ 1];
                if(cur == -1) return {res, -1};
            }
        }
        assert(!trie[cur].id.empty());
        idx = *trie[cur].id.begin();
        return {res, idx};
    }
};
auto get = [&](int x){
    // I need y such that x ^ y is maximum;
    // if x has 0 then find 1 then 0;
    // if x has 1 then find 0 then 1;
    int curr = 0;
    int ans = 0;
    for(int i = 29; i >= 0; i --){
        int bit = (x >> i) & 1;
        if(bit){
            if(trie[curr][0]){
                ans += (1 << i);
                curr = trie[curr][0];
            } else {
                curr = trie[curr][1];
            }
        } else {
            if(trie[curr][1]){
                ans += (1 << i);
                curr = trie[curr][1];
            } else {
                curr = trie[curr][0];
            }
        }
    }
    return ans;
};
```

4.5 Kmp

```
// p[i] = length of the longest proper prefix of
↪  s[0..i]
// which is also a suffix of s[0..i]
vector<int> prefix_function(string s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
```

5 Mathematics

5.1 Sieve

```
int low[MAXN];      // smallest prime factor
int phi[MAXN];      // Euler Totient Function
int mu[MAXN];       // Mobius function
vector<int> primes;
void sieve() {
    for (int i = 1; i < MAXN; i++) {
        phi[i] = i;
        mu[i] = 1;
    }
    for (int i = 2; i < MAXN; i++) {
        if (low[i] == 0) {                 // i is
        ↪  prime
            low[i] = i;
            primes.push_back(i);
            phi[i] = i - 1;
            mu[i] = -1;

            for (int j = i; j < MAXN; j += i) {
                if (low[j] == 0) low[j] = i;
                phi[j] -= phi[j] / i;
                if ((j / i) % i == 0)
                    mu[j] = 0;
                else
                    mu[j] *= -1;
            }
        }
    }
}
```

5.2 Vector

```
struct vec
{
    vector<ld> x;

    vec(ll n)
    {
        x.resize(n, 0);
    }
    vec operator+(const vec &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] + other.x[i];
        }
        return res;
    }
    vec operator-(const vec &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] - other.x[i];
        }
        return res;
    }
    vec operator*(const ld &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] * other;
        }
        return res;
    }
    vec operator/(const ld &other) const
    {
        vec res(x.size());
        rep(i, x.size())
        {
            res.x[i] = x[i] / other;
        }
        return res;
    }
    ld dot(const vec &other) const
    {
        ld ans = 0;
        rep(i, x.size())
        {
            ans += x[i] * other.x[i];
        }
        return ans;
    }
    ld norm_sq() const
    {
        ld ans = 0;
        rep(i, x.size())
        {
            ans += x[i] * x[i];
        }
        return ans;
    }
    ld norm() const
    {
        return sqrt(norm_sq());
    }
};

ld dist(vec &p, vec &a, vec &b)
{
    vec ab = b - a;
    vec ap = p - a;
    ld seglensq = ab.norm_sq();
    if (seglensq < 1e-12)
    {
        return ap.norm();
    }
    ld t = ap.dot(ab) / seglensq;
    if (t < 0)
        t = 0;
    if (t > 1)
        t = 1;
    vec q = a + (ab * t);
    return (q - p).norm();
}
```

5.3 SOSDp

```
int dpOr[MAXN], dpAnd[MAXN], A[MAXN];
for(int i = 0; i < n; i ++){
    dpOr[A[i]] ++, dpAnd[A[i]] ++;
}
for(int bit = 0; bit < 20; bit ++){
    for(int msk = 0; msk < MAXN; msk ++){
        if((msk >> bit) & 1){
            dpOr[msk] += dpOr[msk ^ (1 << bit)];
        }
        else{
            dpAnd[msk] += dpAnd[msk | (1 << bit)];
        }
    }
}
```

5.4 PollardRho

```
// https://judge.yosupo.jp/problem/factorize
#include<bits/stdc++.h>
using namespace std;

using ll = long long;
namespace PollardRho {
  mt19937
  ↪  rnd(chrono::steady_clock::now().time_since_epoch
  const int P = 1e6 + 9;
  ll seq[P];
  int primes[P], spf[P];
inline ll add_mod(ll x, ll y, ll m) {
  return (x += y) < m ? x : x - m;
}
inline ll mul_mod(ll x, ll y, ll m) {
  ll res = __int128(x) * y % m;
  return res;
  // ll res = x * y - (ll)((long double)x * y /
  ↪  m + 0.5) * m;
  // return res < 0 ? res + m : res;
}
inline ll pow_mod(ll x, ll n, ll m) {
  ll res = 1 % m;
  for (; n; n >>= 1) {
    if (n & 1) res = mul_mod(res, x, m);
    x = mul_mod(x, x, m);
  }
  return res;
}
// O(it * (logn)^3), it = number of rounds
↪  performed
inline bool miller_rabin(ll n) {
  if (n <= 2 || (n & 1 ^ 1)) return (n == 2);
  if (n < P) return spf[n] == n;
  ll c, d, s = 0, r = n - 1;
  for (; !(r & 1); r >>= 1, s++) {}
  // each iteration is a round
  for (int i = 0; primes[i] < n && primes[i] <
  ↪  32; i++) {
    c = pow_mod(primes[i], r, n);
    for (int j = 0; j < s; j++) {
      d = mul_mod(c, c, n);
      if (d == 1 && c != 1 && c != (n - 1))
      ↪  return false;
      c = d;
    }
    if (c != 1) return false;
  }
  return true;
}
void init() {
  int cnt = 0;
  for (int i = 2; i < P; i++) {
    if (!spf[i]) primes[cnt++] = spf[i] = i;
    for (int j = 0, k; (k = i * primes[j]) < P;
    ↪  j++) {
      spf[k] = primes[j];
      if (spf[i] == spf[k]) break;
    }
  }
}
// returns O(n^(1/4))
ll pollard_rho(ll n) {
  while (1) {
    ll x = rnd() % n, y = x, c = rnd() % n, u =
    ↪  1, v, t = 0;
    ll *px = seq, *py = seq;
    while (1) {
      *py++ = y = add_mod(mul_mod(y, y, n), c,
      ↪  n);
      *py++ = y = add_mod(mul_mod(y, y, n), c,
      ↪  n);
      if ((x = *px++) == y) break;
      v = u;
      u = mul_mod(u, abs(y - x), n);
      if (!u) return __gcd(v, n);
      if (++t == 32) {
        t = 0;
        if ((u = __gcd(u, n)) > 1 && u < n)
        ↪  return u;
      }
    }
    if (t && (u = __gcd(u, n)) > 1 && u < n)
    ↪  return u;
  }
}
vector<ll> factorize(ll n) {
  if (n == 1) return vector <ll>();
  if (miller_rabin(n)) return vector<ll> {n};
  vector <ll> v, w;
  while (n > 1 && n < P) {
    v.push_back(spf[n]);
    n /= spf[n];
  }
  if (n >= P) {
    ll x = pollard_rho(n);
    v = factorize(x);
    w = factorize(n / x);
    v.insert(v.end(), w.begin(), w.end());
  }
  return v;
}
}

void solve(){
    ll m, n, k; cin>>m>>n>>k;
    set<ll> st;
    for(ll i = 0; i < n; i ++){
        ll x; cin>>x;
        auto f = PollardRho::factorize(x);
        for(auto y : f) st.insert(y);
    }
    for(ll i = 0; i < k; i ++){
        ll a, b, c; cin>>a>>b>>c;
    }
    for(auto x : st) cout<<x<<" ";
    cout<<endl;
}
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  PollardRho::init();
  int t = 1;
  while (t--) {
    solve();
  }
  return 0;
}
```

5.5 GaussianElimination

```
void solve()
{
    ll p, f;
    cin >> p >> f;
    vector<vll> adj(p);
    vll deg(p, 0);
    rep(i, f)
    {
        ll a, b;
        cin >> a >> b;
        --a;
        --b;
        adj[a].pb(b);
        adj[b].pb(a);
        deg[a]++;
        deg[b]++;
    }

    vector<bitset<NUM + 1>> mat(p);
    rep(v, p)
    {
        for (auto u : adj[v])
            mat[v].set(u);
        if (deg[v] & 1)
            mat[v].set(v);
        else
            mat[v].set(p);
    }

    // The below code is taken from internet
    ↪  (Gaussian Elimination in GF(2))
    ll row = 0;
    for (ll col = 0; col < p && row < p; ++col)
    {
        ll sel = -1;
        for (ll r = row; r < p; ++r)
        {
            if (mat[r].test(col))
            {
                sel = r;
                break;
            }
        }
        if (sel == -1)
            continue;
        if (sel != row)
            swap(mat[sel], mat[row]);
        for (ll r = 0; r < p; ++r)
        {
            if (r != row && mat[r].test(col))
            {
                mat[r] ^= mat[row];
            }
        }
        ++row;
    }

    for (ll r = 0; r < p; ++r)
    {
        bool allzero = true;
        for (ll c = 0; c < p; ++c)
            if (mat[r].test(c))
            {
                allzero = false;
                break;
            }
        if (allzero && mat[r].test(p))
        {
            cout << 'N' << endl;
            return;
        }
    }

    cout << 'Y' << endl;
}
```

5.6 Crt

```
int extGcd(int a, int b, int &x, int &y){
    if(b == 0){
        x = 1; y = 0;
        return a;
    }
    int x1, y1;
    int d = extGcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - (a / b) * y1;
    return d;
}

int modinv(int a, int m){
    int x, y;
    int g = extGcd(a, m, x, y);
    int inv = (x % m + m) % m;
    return inv;
}
// solves the two congruences and returns the
↪  minimum non-negative solution
int get(int a, int b, int c, int d, int n){
    // ax = -b mod n -> ax - kn = -b -> gcd(a, n)
    ↪  / b
    // cx = -d mod n

    int g1 = gcd(a, n);
    if((-b) % g1 != 0) return -1;
    int m1 = n / g1;
    int r1 = ((-b / g1) % m1 + m1) % m1;
    int x1 = (modinv(a / g1, m1) * r1) % m1;

    int g2 = gcd(c, n);
    if((-d) % g2 != 0) return -1;
    int m2 = n / g2;
    int r2 = ((-d / g2) % m2 + m2) % m2;
    int x2 = (modinv(c / g2, m2) * r2) % m2;

    // x = x1 mod m1 and x = x2 mod m2
    int g = gcd(m1, m2);
    if((x2 - x1) % g != 0) return -1;
    int M = (m1 / g) * m2;
    int inv = modinv(m1 / g, m2 / g);
    int ans = (x1 + (((x2 - x1) / g * inv) % (m2 /
    ↪  g)) * m1) % M;
    ans = (ans + M) % M;
    return ans;
}
```

5.7 Bits

```
mask &= ~(1LL << i); // Clear i-th bit
mask &= (mask - 1); // Turn off lowest set bit
bool is_pow2 = (mask > 0 && (mask & (mask - 1)) ==
↪  0); // Is power of two
int leading_zeros = __builtin_clzll(mask); //
↪  Count leading zeros
int trailing_zeros = __builtin_ctzll(mask); //
↪  Count trailing zeros
int lsb = mask & (-mask); // Isolate least
↪  significant bit
mask &= (mask - 1); // Turn off least significant
↪  bit

// Iterate over all subsets of a bitmask
for(int sub = mask; sub; sub = (sub - 1) & mask){
    // sub is a non-empty subset
}
```

5.8 Euler_Totient

```
int phi(int n) { // O(sqrt(n))
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1)
        result -= result / n;
    return result;
}

void phi_1_to_n(int n) { // O(nloglogn)
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)
        phi[i] = i;

    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}
```

5.9 NCr_Shrey

```
const ll MAXN = 1e7;
const ll MOD = 1e9 + 7;

vector<ll> fac(MAXN + 1);
vector<ll> inv(MAXN + 1);

/** Computes x^y modulo p in O(log p) time. */
ll exp(ll x, ll y, ll p)
{
    ll res = 1;
    x %= p;
    while (y)
    {
        if (y & 1)
        {
            res *= x;
            res %= p;
        }
        x *= x;
        x %= p;
        y >>= 1;
    }
    return res;
}

void factorial() {
    fac[0] = 1;
    for (ll i = 1; i <= MAXN; i++) {
        fac[i] = ( fac[i - 1] * i ) % MOD;
    }
}

void inverses() {
    inv[MAXN] = exp(fac[MAXN], MOD - 2, MOD);
    for (ll i = MAXN; i >= 1; i--) {
        inv[i - 1] = (inv[i] * i) % MOD;
    }
}

ll choose(int n, int r) {
    return fac[n] * inv[r] % MOD * inv[n - r] %
    ↪  MOD;
}
```

5.10 Matrix

```
template<typename T>
struct Matrix{
    vector<vector<T>> mat;
    int n, m;
    Matrix() {}
    Matrix(int _n, int _m, bool ident = false) {
        n = _n; m = _m;
        mat.assign(n, vector<T>(m, 0));
        if (ident) {
            for (int i = 0; i < n; i++)
                mat[i][i] = 1;
        }
    }
    Matrix(const vector<vector<T>> &v) {
        n = v.size();
        m = v[0].size();
        mat = v;
    }
    Matrix operator*(const Matrix &b) const {
        Matrix res(n, b.m);
        for (int i = 0; i < n; i++) {
            for (int k = 0; k < m; k++) {
                if (mat[i][k].val == 0) continue;
                for (int j = 0; j < b.m; j++) {
                    res.mat[i][j] += mat[i][k] *
                    ↪  b.mat[k][j];
                }
            }
        }
        return res;
    }
};
template<typename T>
Matrix<T> mat_pow(Matrix<T> a, long long p) {
    Matrix<T> res(a.n, a.n, true);
    while (p) {
        if (p & 1) res = res * a;
        a = a * a;
        p >>= 1;
    }
    return res;
}
```

5.11 MillerRabin

```
using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};

bool MillerRabin(u64 n) { // returns true if n is
↪   prime, else returns false.
    if (n < 2)
        return false;

    int r = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        r++;
    }

    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23,
    ↪   29, 31, 37}) {
        if (n == a)
            return true;
        if (check_composite(n, a, d, r))
            return false;
    }
    return true;
}
```