Research and Simulation of Node Importance in Transport Network based on Weighted Model

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***Abstract*—In recent years, studies on network theory have attracted a growing attention, especially, it has become a hot issue to combine the theoretical approach of network with the practical problems. In this paper, we present a new weighted model for public transport network based on network efficiency. Distinguishing from traditional network analysis method, this model considers comprehensively the impact of spatial distance, traffic flow and congestion coefficient on transport network. Then, on the basis of this weighted model, evaluation of node importance is further introduced by method of removing node. Finally, we also carry out computer simulations on this model, an illustrative example is presented to demonstrate the validity of our models and methods. Simulation results prove that our methods are efficient and reliable for analyzing network characteristics and distinguishing the node importance.**

Keywords—Network theory; Weighted model; Node importance; Public transport network

I. INTRODUCTION

In china, as a result of the rapid development of economy, many metropolises have continually increased investments in urban public transport planning over the past few years, which resulting in complex public transport systems that possess numerous stations and intricate coupled relationships[1].In some theoretical studies[2-6],Urban public transport network are proved that it has the characteristic of small-world networks[7] and scale-free networks[8],which is a typical complex networks[9].Moreover, the scale-free networks is very sensitive to important nodes. Especially when a series of important nodes in public transport network go wrong, it will cause serious damage to the whole networks. In other words, the important nodes play a key role in the network function and dynamic process. Therefore, identifying important nodes in public transport network are meaningful for public transport planning.

In existing literature, Some works[2,3] explored related methods to calculate the node importance in complex network, and there are also a few literature about node importance calculation in the field of transportation[4,5,6].In the field of complex network, the node importance calculation mainly has the following three methods:(1)It is analyzed through the social economy, traffic and other specific evaluation indicators, which is widely used in the early days of the transportation network.(2)Judging by node attribute information. This method can identify topology network mainly through statistical indicators, and these statistical indicators are divided into local, global and position indicators, respectively reflecting the overall network of local attributes, global attributes and location information. Such as Ref.[10],it put forward a kind of method to determine node importance, which considering that node importance is related not only with itself, also with neighbor node degree. Ref.[11] provided a thorough introduction to some existing researches on the evaluation of node importance in public opinion, summarized the existing models and methods, and analyzed respective characteristics based on network topology and node attributes.(3)The system science method. This method is mainly implemented by removing the node of the network. The greater damage on the network after removing nodes, the greater importance is possessed by nodes, and vice versa. Ref.[12] introduced a method of node contraction. After node contraction, we can calculate agglomeration degree of node to determine node importance. For identifying node importance in public transport network, Ref.[13] has proposed one method to measure the node importance based on topology, the shortest path and the transportation capacity. Ref.[14] presented node shrinkage method in complex network, and the results show that the node importance is related to traffic flow and node position in the network. To the end, however, the impact of traffic flow and congestion on the evaluation is not taken into account in these existing methods. To address this problem, this paper considers comprehensively distance, traffic jams and traffic flow distribution as key factors, then proposed a more accurate public transport weighted-network model. The structure of this paper is organized as follows: in section 2, we present our weighted-network model to urban public transport system. In section 3, we propose a new evaluation method for the nodes of urban public network based on removing nodes. Then, for verifying the effectiveness of our method, we make simulation analysis on an illustrative example and explain experimental results of section 4 in detail. Finally, this paper is concluded in section 5.

II. A NEW WEIGHTED-NETWORK MODEL

For calculating node importance, the urban public transport system can be firstly modeled by a network model. In this paper, besides studying the characteristics of static topology, we also comprehensively consider the influence of dynamic factors, such as traffic jams, traffic passenger flow. We mainly propose a method named PTEW to evaluate the node importance comprehensively from more than one perspective. Next, we will introduce and propose definition of the model.

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A. The adjacency matrix of network

The adjacency matrix of public transport network is that: take stations as the nodes of network, when there is a route between stations i and station j, and no other stations between them, the value of matrix is Adj(i,j)=1,otherwise Adj(i,j)=0. Where 1<=i<=N, 1<=j<=N, N denotes the total number of stations in the planning area. The adjacency matrix is as following.

$$Adj = \begin{bmatrix} adj_{11} & adj_{12} & \cdots & adj_{1N} \\ adj_{21} & adj_{22} & \cdots & adj_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ adj_{N1} & adj_{N2} & \cdots & adj_{NN} \end{bmatrix} \quad (1)$$

B. The weighted method based on PTEW

(1) The PTEW method

Urban transport network is not only limited by road network, but also driven by transport efficiency. There are three existing methods to weight definition of urban public transport network [15-17]: (1) the distance between the stations; (2) the traffic between stations; (3) the number of lines between different stations. These three methods are defined with different focus, but all factors are not taken into account comprehensively. In order to fully consider the above factors, according to the definition of public transport efficiency, we proposed a method named Public Transport Efficiency Weighting method (PTEW), the corresponding weights are defined as followings:

$$w_{ij} = e_{ij} = \lambda \frac{x_{ij}}{T_{ij}} \quad (2)$$

Where x_{ij} denotes traffic flow between the station i and station j; T_{ij} denotes travel time the under the corresponding traffic; whereby e_{ij} denotes transport efficiency of arc section [i, j]. λ is correction coefficient, it is set to 1 in our experiment. The physical meaning of the public transport efficiency refers to the amount of passenger service under impedance per unit[20].In this paper, due to focusing on the research of the road traffic congestion for urban public transport network, so regardless of waiting time in station, we treat actual travel time as impedance time approximately.

(2) The weight calculation based on PTEW method

By weight definition of PTEW, for calculating the weight between stations, we need to know traffic flow between stations and actual travel time. Traffic flow can be provided by the Origin-Destination(OD) matrix, road impedance function also need to be taken into account when calculating the actual travel time.

(a) BPR function

Road impedance function can help us to calculate travel time, and reflect the relationship between the traffic time and road conditions. The U.S. highway Bureau of Public Roads (BPR) function is the most widely used road impedance function. The function reflects the relationship between travel time and traffic flow. Therefore, the BPR function can reflect the effects of traffic congestion when calculating the travel time. BPR function is as follows:

$$T_k = T_k^0 \left[1 + \alpha \left(\frac{x_k}{c_k} \right)^{\beta} \right] \quad (3)$$

Where T_k^0 denotes the impedance of road section k, when traffic equals to zero; α ,β indicate correction coefficient. x_k denotes traffic of road section, c_k is capacity of road section. Obviously, travel time is positive correlation with traffic, and negatively related to the road capacity.

(b) The improvement of BPR function

When the BPR function was proposed, it only considered the road traffic under normal conditions, regardless of serious traffic jams. However, traffic congestion is more and more serious in modern transportation, in order to accurately calculate travel time, it is very important to improve the BPR function.

In formula 3,we can see that the BPR function is monotonically increasing function of traffic, but in real-world transport system, because of traffic congestion, variation trend of traffic will reduce gradually, especially when traffic density increases to a certain threshold, traffic will begin to decrease gradually. Nonetheless, results of calculation using the BPR function will decrease, it is not identical to actual situation with traffic time increased. So you can't apply BPR function to calculation of traffic time under serious traffic congestion state[19].In order to accurately calculate travel time, this paper adopts the improved BPR function. Because of the increase of congestion, the travel time increases monotonously, while traffic density value also monotonously increasing. So the improved BPR function replaces traffic flow with traffic density to calculate travel time. The improved form of BPR function is shown in the following:

$$T_k = T_k^0 (1 + \alpha (1 - (1 - \frac{2\rho_k}{\rho_{k\max}})^z)^{\beta}) \quad (4)$$

Where ρ_k denotes road traffic density; $\rho_{\max}$ denotes the biggest traffic density for corresponding section. α , β , z is correction coefficient; in this study, the values of the three coefficient can be set to be $\alpha = 0.15$, $\beta = 4$,and $z = \frac{21}{11}$ [19].

(c) Definition of travel time matrix

As the above discussions, the travel time between different stations can be calculated by average travel time when it is non-crowded and traffic density, namely:

$$T_{ij} = \sum_{k \in s_{ij}} P_{ijk} T_k \quad (5)$$

Where s_{ij} stands for the set of road section, ρ_{ijk} is the ratio of distance which goes through road section k and the total length of road section k ; T_k is travel time of road, which is calculated by BPR function.

Combining with formula (4) and (5),we can get the final formula of travel time between adjacent stations:

$$T_{ij} = \sum_{k \in s_{ij}} P_{ijk} T_k^0 (1 + \alpha(1 - (1 - \frac{2\rho_k}{\rho_{k\max}})^z)^\beta) \quad (6)$$

(d) Definition of weighted matrix

Through OD matrix and travel time matrix, according to the formula (1), we will get the efficiency matrix of public transport by PTEW method, the matrix Weight(i,j) as follow:

$$Weight(i, j) = \frac{OD(i, j)}{T(i, j)} \quad (7)$$

Where OD(i,j) reflects the traffic data from station i to station j, T(i,i) denotes travel time from station i to station j. In this matrix, any element value represents the corresponding public transport efficiency between two stations.

III. EVALUATE NODE IMPORTANCE BASED ON REMOVING NODES

In order to evaluate how well a system works before and after a set of nodes being removing, we use the global efficiency of network, which is defined as the average of efficiency over all pairs of nodes.

$$imp(i) = \frac{E(G) - E(G - i)}{E(G)} \quad (8)$$

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} e_{ij} = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}} \quad (9)$$

Where d_{ij} is the length of the shortest path from node to node j. E(G) is defined as the global efficiency of network without removing nodes, E(G-i) denotes the global efficiency of network when node i is removed. However, in this paper, we need to replace $\frac{1}{d_{ij}}$ with Weight(i,j) to evaluate station importance, the formula as follows:

$$Eff(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} e_{ij} = \frac{1}{n(n-1)} \sum_{i \neq j \in G} Weight(i, j) \quad (10)$$

Also we obtain the formula:

$$\begin{aligned} IMP(i) &= \frac{Eff(G) - Eff(G - i)}{Eff(G)} \\ Eff(G) &= \frac{1}{n(n-1)} \sum_{i \neq j \in G} Weight(i, j) \\ Weight(i, j) &= \frac{OD(i, j)}{T(i, j)} \end{aligned} \quad (11)$$

Above all, here are steps of node importance calculation of urban public transport network:

Step1: modeling urban public transport network using L space, and establish its adjacency matrix.

Step2: calculate the travel time between adjacent stations, to form travel time matrix.

Step3: according to the adjacent travel time matrix, to calculate travel time between any stations through Floyd algorithm.

Step4: through OD data and travel time between any stations, calculate the final weight matrix.

Step5: according to the formula (10) to calculate the efficiency of network.

Step6: delete node i, and turn to Step1-Step5, then calculate efficiency of network which is deleted node.

Step7: according to the formula (11), calculate the node importance.

Step8: repeat Step6-Step7, until importance of all nodes are calculated.

IV. AN ILLUSTRATIVE EXAMPLE

As shown in Figure 1, the public transport network G contains 25 stations. For simplicity, without affecting the reliability of the experiment, we can make the following assumptions:(1)the transport network is an undirected network, the corresponding matrix is symmetric matrix;(2)we assume that the impedance of adjacent stations are identical.

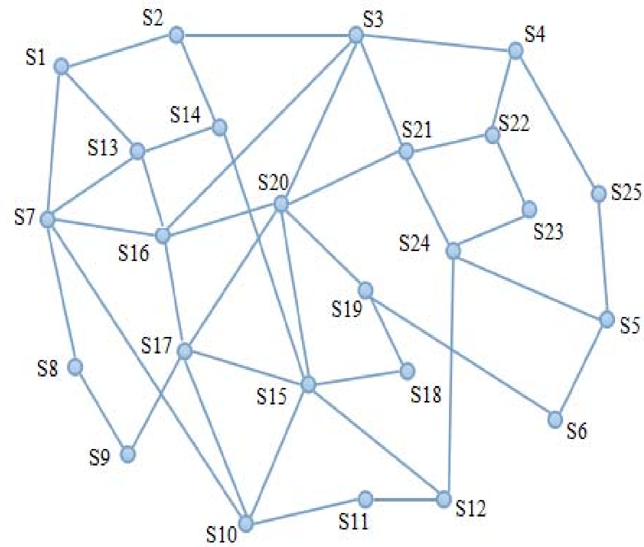

Figure 1 public transport system

To build weighted-network model of a corresponding public transport network, we need to know that: the OD matrix between any stations, free travel time matrix and traffic density saturation (i.e., the ratio of actual traffic density and maximum traffic density), in this paper, because the main work purposes to examine the changes of result with various parameters, we are more concerned about the relative size rather than the absolute size of the various parameters, so the setting of initial values will not cause substantial effects to our analysis. As stated in the literature [18][20],the values of initial parameters can be set within smaller specific random scope, the initial value of this example is as following:

$$
T0=\begin{bmatrix}
0&6&inf&inf&inf&inf&7&inf&inf&inf&inf&inf&6&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
6&0&5&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&6&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&5&0&6&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&5&inf&inf&inf&5&6&inf&inf&inf&inf\\
inf&inf&6&0&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&5&inf&inf&7\\
inf&inf&inf&inf&0&7&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&6&5\\
inf&inf&inf&inf&7&0&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&7&inf&inf&inf&inf&inf&inf\\
7&inf&inf&inf&inf&inf&0&6&inf&6&inf&inf&inf&inf&inf&7&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&6&0&6&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&6&0&inf&inf&inf&inf&inf&inf&inf&5&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&6&inf&inf&0&7&inf&inf&inf&5&inf&6&inf&inf&inf&inf&inf&inf&inf&inf\\
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6&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0&7&inf&7&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&6&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&7&0&7&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&inf&5&inf&5&inf&7&0&inf&5&6&inf&7&inf&inf&inf&inf&inf\\
inf&inf&5&inf&inf&inf&7&inf&inf&inf&inf&inf&7&inf&inf&0&6&inf&inf&7&inf&inf&inf&inf&inf\\
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inf&inf&inf&inf&inf&7&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&7&0&6&inf&inf&inf&inf&inf\\
inf&inf&5&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&7&7&5&inf&6&0&7&inf&inf&inf&inf\\
inf&inf&6&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&7&0&6&inf&5&inf\\
inf&inf&inf&5&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&6&0&5&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&5&0&6&inf\\
inf&inf&inf&inf&6&inf&inf&inf&inf&inf&inf&6&inf&inf&inf&inf&inf&inf&inf&inf&5&inf&6&0&inf\\
inf&inf&inf&7&5&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0
\end{bmatrix} \tag{12}
$$

$$
\rho=\begin{bmatrix}
0&0.1&inf&inf&inf&inf&0.3&inf&inf&inf&inf&inf&0.1&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
0.1&0&0.2&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.1&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&0.2&0&0.3&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.1&inf&inf&inf&0.1&0.2&inf&inf&inf&inf\\
inf&inf&0.3&0&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.1&inf&inf&0.1\\
inf&inf&inf&inf&0&0.3&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.2&0.2\\
inf&inf&inf&inf&0.3&0&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.2&inf&inf&inf&inf&inf&inf\\
0.3&inf&inf&inf&inf&inf&0&0.1&inf&0.2&inf&inf&inf&inf&inf&0.2&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&0.1&0&0.3&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&0.3&0&inf&inf&inf&inf&inf&inf&inf&0.3&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&0.2&inf&inf&0&0.3&inf&inf&inf&0.1&inf&0.3&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&inf&0.3&0&0.3&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.3&0&inf&inf&0.2&inf&inf&inf&inf&inf&inf&inf&inf&0.3&inf\\
0.1&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0&0.3&inf&0.2&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&0.1&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.3&0&0.1&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&inf&0.1&inf&0.2&inf&0.1&0&inf&0.3&0.2&inf&0.2&inf&inf&inf&inf&inf\\
inf&inf&0.1&inf&inf&inf&0.2&inf&inf&inf&inf&inf&0.2&inf&inf&0&0.3&inf&inf&0.2&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&0.3&0.3&inf&inf&inf&inf&0.3&0.3&0&inf&inf&0.1&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.2&inf&inf&0&0.1&inf&inf&inf&inf&inf&inf\\
inf&inf&inf&inf&inf&0.2&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.1&0&0.1&inf&inf&inf&inf&inf\\
inf&inf&0.1&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.2&0.2&0.1&inf&0.1&0&0.3&inf&inf&inf&inf\\
inf&inf&0.2&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.3&0&0.2&inf&0.3&inf\\
inf&inf&inf&0.1&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.2&0&0.1&inf&inf\\
inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0.1&0&0.1&inf\\
inf&inf&inf&inf&0.2&inf&inf&inf&inf&inf&inf&0.3&inf&inf&inf&inf&inf&inf&inf&inf&0.3&inf&0.1&0&inf\\
inf&inf&inf&0.1&0.2&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&inf&0
\end{bmatrix} \tag{13}
$$

$$
OD=\begin{bmatrix}
0&5&3&4&3&4&4&3&3&4&3&3&4&4&4&4&5&4&4&4&4&3&4&4&4\\
3&0&3&3&5&5&5&4&4&5&5&5&3&4&5&5&3&4&4&5&4&5&4&3&3\\
3&5&0&3&5&5&4&5&3&4&4&3&4&3&4&5&5&4&3&4&4&3&5&4&3\\
4&3&5&0&5&4&4&5&4&5&4&5&5&5&3&5&5&4&4&5&3&4&4&5&5\\
5&5&5&3&0&3&4&3&5&5&4&4&5&4&4&4&4&4&4&5&3&3&4&3&4\\
5&5&4&5&4&0&4&3&4&3&3&5&4&4&4&4&4&4&3&4&5&4&5&5&5\\
4&3&4&4&4&3&0&5&3&4&4&4&5&3&4&3&5&5&5&4&3&3&4&3&5\\
3&4&4&5&4&3&5&0&4&5&4&4&5&3&4&5&5&5&3&4&5&5&3&4&3\\
5&3&5&5&4&3&5&4&0&4&3&5&3&4&5&5&4&4&4&3&3&5&4&3&5\\
5&4&3&4&3&4&4&3&4&0&5&3&3&4&5&5&4&5&5&4&5&5&5&3&5\\
5&4&5&3&3&5&5&3&4&4&0&3&3&5&5&4&3&3&4&5&4&5&4&3&4\\
5&4&5&5&4&4&4&5&5&4&3&0&3&5&4&4&4&3&4&4&3&4&3&3&4\\
3&3&5&3&3&4&5&5&5&5&5&4&0&5&4&3&5&3&5&5&4&3&4&3&3\\
5&3&4&5&5&5&4&5&4&4&3&3&5&0&5&5&5&3&5&5&3&3&3&3&5\\
4&3&4&3&5&5&4&3&5&4&5&4&5&3&0&4&5&3&5&4&5&4&4&3&3\\
3&5&5&4&3&5&3&3&3&5&3&3&4&4&5&0&3&3&3&4&4&4&5&3&4\\
3&4&4&3&4&5&3&3&4&5&4&3&3&3&3&5&0&3&3&4&3&5&4&4&5\\
5&5&3&4&3&5&3&4&5&3&4&3&5&5&4&5&4&0&5&5&3&3&4&5&5\\
3&3&5&3&5&4&5&3&5&5&5&3&5&5&3&5&4&3&0&5&3&4&4&5&4\\
3&5&4&3&5&4&5&5&5&5&4&3&3&3&4&3&3&3&4&0&3&4&3&3&3\\
4&5&4&5&3&3&5&3&4&5&4&4&4&5&5&5&5&4&4&5&0&3&3&4&4\\
5&4&5&4&5&3&3&3&4&3&5&3&5&5&5&4&3&5&5&5&5&0&3&4&5\\
4&4&3&4&3&5&5&4&5&4&4&5&5&4&5&3&3&5&4&5&4&4&0&4&5\\
5&3&4&4&5&4&5&5&3&4&4&3&5&5&4&4&4&4&3&5&4&4&3&0&4\\
3&5&4&4&5&4&5&3&5&3&5&5&4&5&5&3&4&5&4&3&5&5&3&3&0
\end{bmatrix} \tag{14}
$$

The elements of the T_0 represent the free travel time between the adjacent stations, the elements of ρ represent road traffic density saturation of adjacent stations, the elements in the OD represent the number of passenger between any stations.

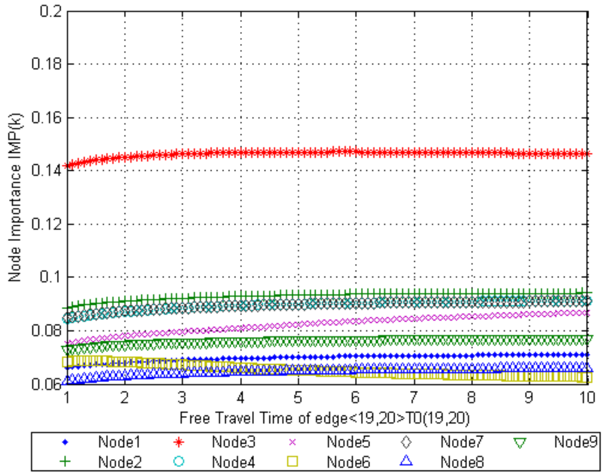
According to the steps of node importance calculation, the importance of each node can be obtained by applying the method of removing nodes. The results are shown in Table 1.

TABLE 1. NODE IMPORTANCE IN GRAPH G

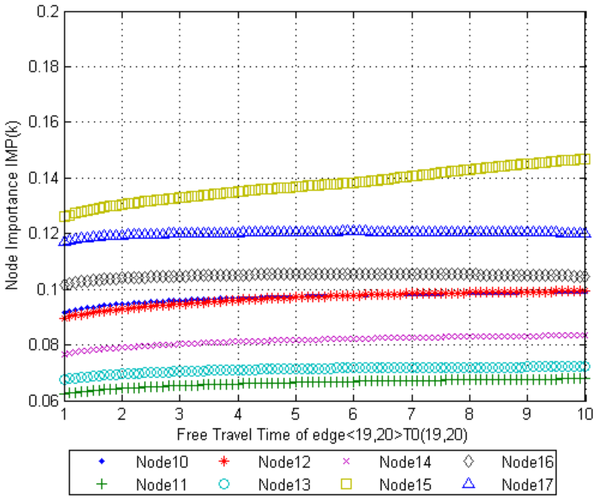
Node Number	Node Importance	Rankings
3	0.1470	1
15	0.1382	2
20	0.1282	3
17	0.1207	4
16	0.1056	5
24	0.1048	6
10	0.09783	7
12	0.09782	8
21	0.09650	9
2	0.09371	10
7	0.09041	11
4	0.09040	12
19	0.08638	13
5	0.08363	14
14	0.08222	15
22	0.08026	16
9	0.07654	17
13	0.07174	18
25	0.07134	19
1	0.07042	20
18	0.06885	21
11	0.06686	22
23	0.06664	23
8	0.06540	24
6	0.06446	25

As is shown in Table 1,node 3 and 15 is the most important node. Compared with the former, the importance of node 6, 8 is low. It is for the reason that the betweenness centrality of the node 3 and 15 is larger, after removing them, it will lead to a significant reduction in the efficiency of network; but the node 6, 8 is marginal in the network, it is of less significance for network after removing them. Because relative difference of the initial value setting of various parameters is small, importance of each node mainly relies on network topology and location of nodes.

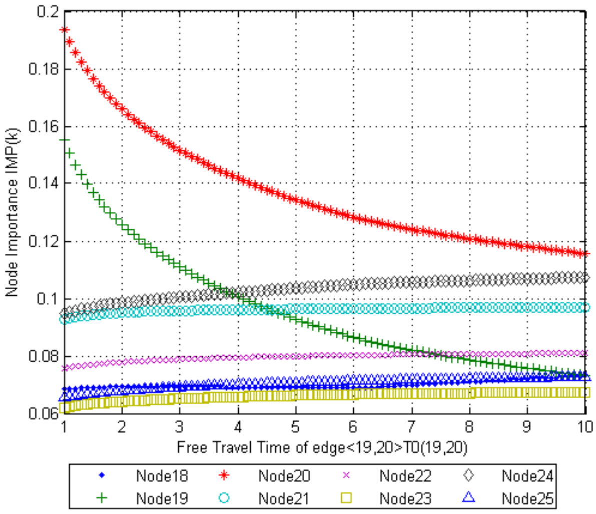
The parameters have important influence on the importance of nodes. Firstly, we focus on the relationship between node importance and distance. In our experiment, keeping other parameters unchanged, just changing the free travel time of edge <19, 20> randomly, the experimental results are as followings:



(2a)The change curves of the node 1-9 with the free travel time of the edge<19,20>



(2b)The change curves of the node 10-17 with the free travel time of the edge<19,20>

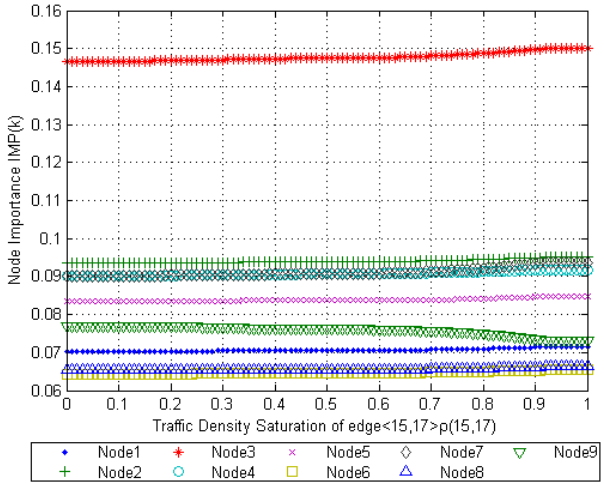


(2c)The change curves of the node 18-25 with the free travel time of the edge<19,20>

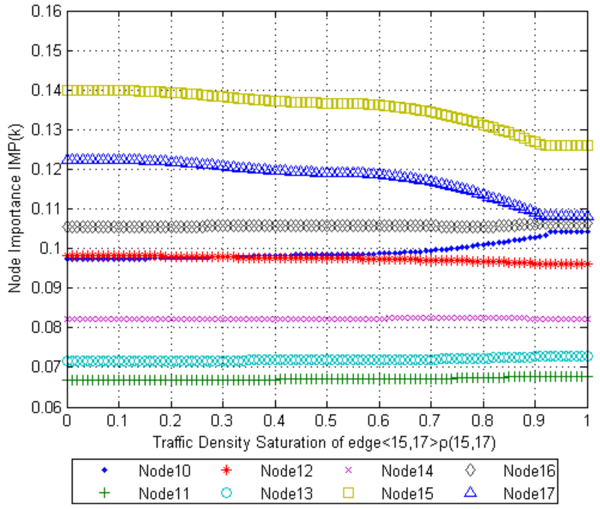
Figure 2 the changes of each node importance with free travel time of edge < 19, 20 >

Figure 2 shows the changes in the importance of each node when travel time of edge <19, 20> is changed. Moreover, the decrease of node importance falls very quickly, then turns gradually slow, because when free travel time of the edge < 19, 20 > is very small, there are multiple shortest paths through this path, but when the travel time increases to a certain threshold, the shortest paths through this edge decreases obviously, its impacts on network efficiency become small, so the node is decreased. The importance of other nodes will become larger conversely. It proves that the greater free travel time will lead to the less node importance.

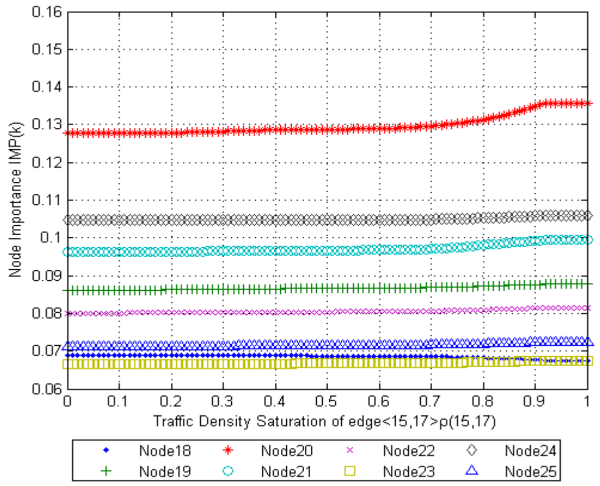
Then we will take edge <15, 17> randomly, to observe changes of each node importance with the change of traffic density saturation. The experimental results are as followings:



(3a)The change curves of the node 1-9 with the traffic density saturation of the edge<15,17>



(3b)The change curves of the node 10-17 with the traffic density saturation of the edge <15,17>

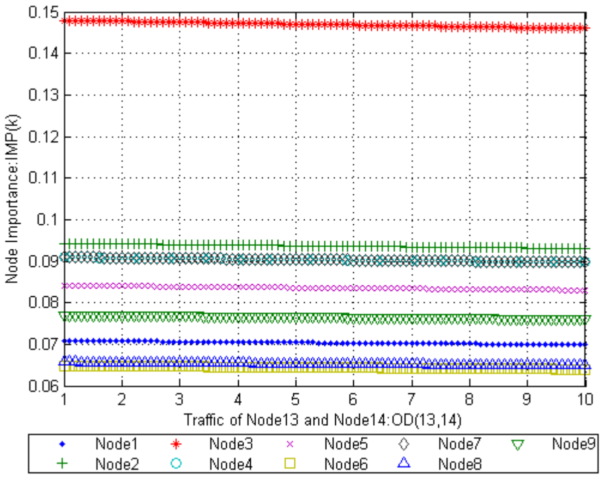


(3c)The change curves of the node 18-25 with the traffic density saturation of the edge<15,17>

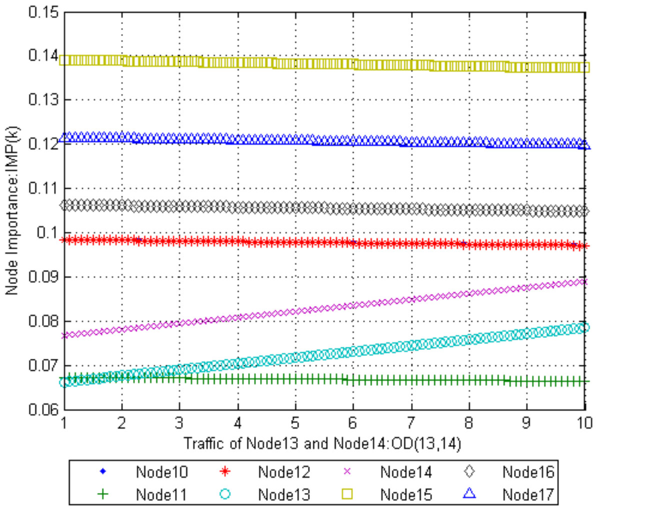
Figure 3 the changes of each node importance with traffic density saturation of edge <15,17>

As shown in Figure 3.It shows the node importance of node 15 and 17 is decreased when traffic density saturation of the edge <15, 17> become larger. It is for the reason that the increase of travel time is not obvious, when traffic density saturation is below a threshold. But when the road is in congestion, the travel time will increases obviously. It is noteworthy that when the traffic density saturation is around 0.9,each node importance keep unchangeable, it is for the reason that the road section is under serious congestion, the traffic time is very great, at this time, edge <15, 17> can be regarded as disconnected state. However, the node importance of node 10 and node 20 will increase significantly, because the shortest path through node 15 and node 17 is replaced with node 10 and node 20.

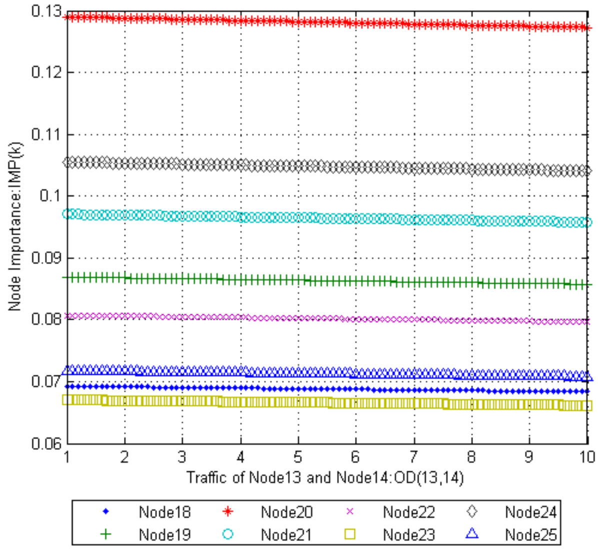
Finally, we can also observe the influences of traffic on each node importance. Taking a pair of nodes randomly, such as node 13 and node 14, and changing the traffic between them, the results are as followings:



(4a)The change curves of the node 1-9 with the traffic of the node 13 and node 14



(4b)The change curves of the node 10-17 with the traffic of the node 13 and node 14



(4c)The change curves of the node 18-25 with the traffic of the node 13 and node 14

Figure 4 the changes of each node importance with traffic of node 13 and node 14

From the Figure 4, the importance of node 13 and 14 is directly associated with traffic of the edge <13,14>, this is because public transport efficiency of the edge <13,14> increases with the increase of traffic. If removing node 13 or node 14, then the network efficiency will also decrease; On the other hand, some node importance decrease slowly, this is because the losses of network efficiency will not change after removing these nodes.

V. Conclusions

In this paper, we comprehensively consider multiple factors to evaluate the node importance of public transport network, and establish an urban public transport weighted-network model, in addition, a new evaluation method of removing node based on this model is put forward. The analysis results of the illustrative example prove that this method can more truly reflect the network topology, characteristic of distance, degree of the traffic jams and traffic flow.

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