

7.4

3 мая 2016 г.

1 Задача 4

```
In [127]: %matplotlib inline
import numpy as np
import math as mt
import matplotlib
import matplotlib.pyplot as plt
from pylab import *
from scipy.stats import *
```

```
In [128]: data = np.loadtxt('6.csv', delimiter=',', skiprows=3)
print(len(data))
```

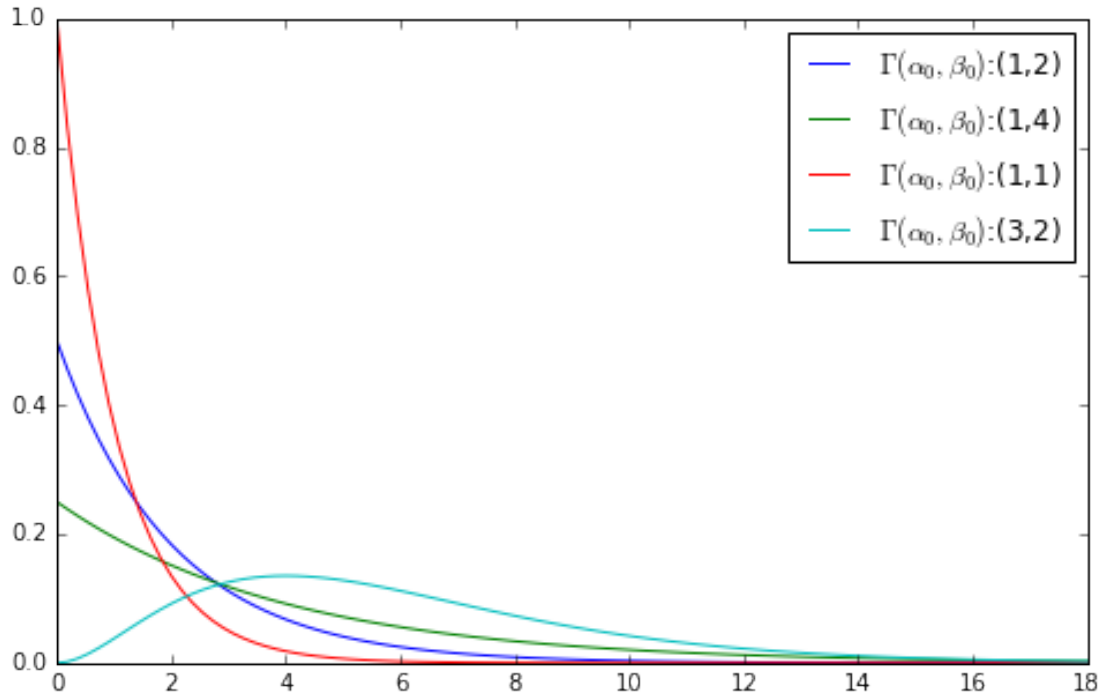
1000

Для $Exp(\theta)$ априорное распределение $\Gamma(\alpha_0, \beta_0)$ with mean = $\frac{\alpha_0}{\beta_0}$.
Байесовская оценка для параметра θ : $\theta^* = \frac{\alpha_0 + n}{\beta_0 + \sum_{i=1}^n x_i}$.

```
In [129]: def BayesEst(x, alpha, beta):
# Байесовская оценка для экспоненциального распределения
return (alpha + len(x))/(beta + sum(x))
```

Задача состоит в том, чтобы верно подобрать параметры α_0 β_0 .

```
In [130]: x = linspace(0,18,100)
params = [(1,2),(1,4),(1,1),(3,2)]
figure(figsize=(8,5))
for p in params:
    plot(x, gamma.pdf(x, p[0], scale=p[1]), \
         label='$\Gamma(\alpha_0, \beta_0)$'.format(p[0], p[1]))
legend()
show()
```



Поскольку в начальный момент времени мы ничего не знаем о “скорости выхода из строя” серверов, то нам не подходят параметры Гамма распределения, при которых образуется явный “горб” над каким-то конкретным числом. Нам следует этого избегать.

Значит, нужно брать параметр $\alpha_0 = 1$, а за параметр β_0 можно взять число 2, при котором график плотности выглядит не очень крутым.

In [131]: # Истинное значение theta

```
lmd = 1./66.
```

```
t_0 = 200
```

```
t = 70000
```

```
param = (1,1)
```

```
# Сюда запишу время между i-м моментом выхода из строя сервера и (i+1)-м,
```

```
# то есть delta - график "разности времен"
```

```
delta = []
```

```
for i in range(len(data)-1):
```

```
    delta.append(data[i+1] - data[i])
```

```
# Сюда запишу оценки для параметра theta
```

```
est = np.array([BayesEst(delta[:j+1]), param[0], param[1]) for j in range(len(data))])
```

```
# Строю график модуля разности истинного значения theta и байесовской оценки
```

```
figure(figsize=(12,5))
```

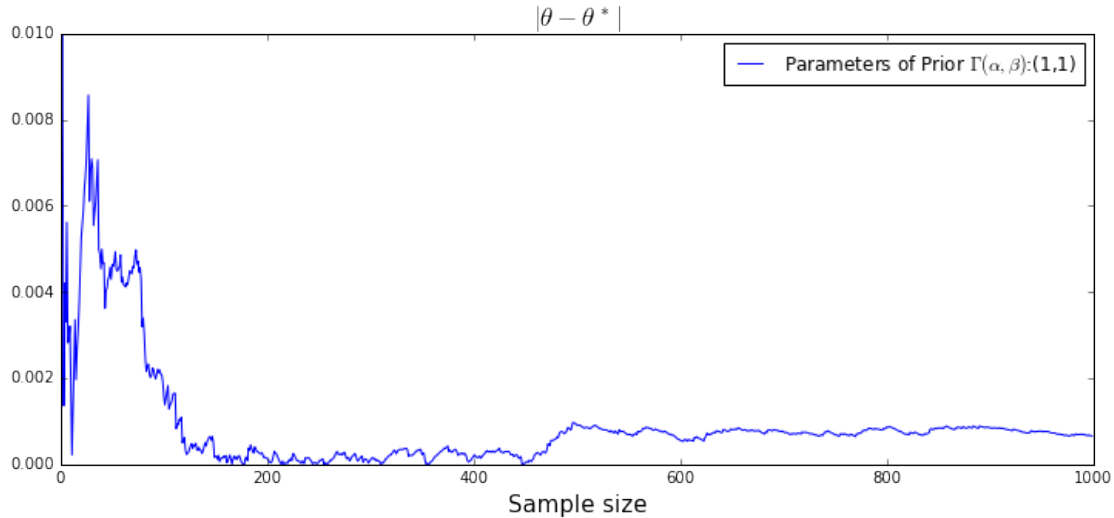
```
ylim((0,0.01))
```

```
xlabel("Sample size", fontsize=15)
```

```

plot(abs(lmd-est), label="Parameters of Prior $\Gamma(\alpha, \beta)$:({},{})"\
      .format(param[0], param[1]))
title("$\\theta - \\theta^*$", fontsize=17)
legend()
show()

```



Исправлена задача 6.1 под новые условия.

Теперь вместо λ будет стоять оценка θ^* в выражении для предсказанного числа серверов, которые необходимо заменить.

```

In [132]: condition_expect = [] # Здесь будут предсказанные значения количества серверов
time = [] # Сюда записываю время с шагом t_0
quant = [] # Количество серверов, вышедших из строя, на данный момент

plt.figure(figsize=(18,7))
for i in range(t//t_0):
    time.append(t_0*(i+1))
    quant.append(shape(np.where(data <= time[-1]))[1]) # Нахожу это количество подсчетом
                                                         # числа записей времени, меньших
                                                         # текущего времени
    condition_expect.append(((t-time[-1])*est[quant[-1]-1]) + quant[-1]) # Предсказанное значение

plt.xlabel('Time', fontsize=15)
plt.ylabel('Number of Servers to Buy', fontsize=15)
plt.plot(time,condition_expect)
plt.grid()
plt.show()

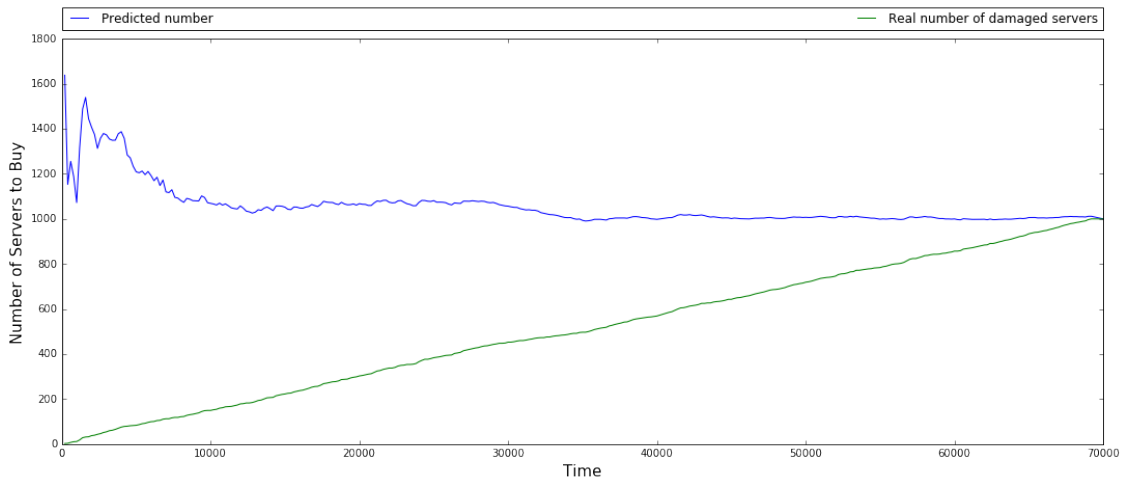
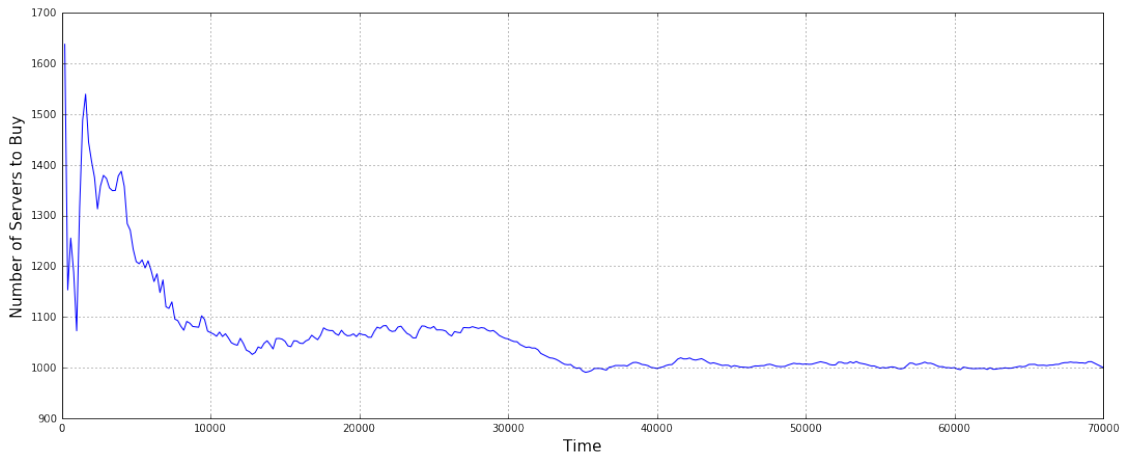
plt.figure(figsize=(18,7))

```

```

plt.xlabel('Time', fontsize=15)
plt.ylabel('Number of Servers to Buy', fontsize=15)
plt.plot(time, condition_expect, label='Predicted number')
plt.plot(time, quant, color='g', label='Real number of damaged servers')
plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3, ncol=2, \
           mode="expand", borderaxespad=0.)
plt.show()

```



```

In [133]: for i in range(len(est)):
           print("lambda = {:.4f}, {:.4f} - bayes estimate, {}/{}".format(lmd, est[i], i+1, len(est)))

```

lambda = 0.0152, 0.2476 - bayes estimate, 1/1000

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