7.1

25 апреля 2016 г.

1 Задача 1

In [274]:
```
%matplotlib inline
import numpy as np
import math as mt
import matplotlib
import matplotlib.pyplot as plt
from pylab import *
from scipy.stats import *
```

In [275]:
```
sz = 100  # Размер выборки
x = norm.rvs(size=sz)
```

2 Модель $N(\theta, 1)$

Априорное распределение - $N(\mu_0, \sigma_0^2)$ with mean= μ_0, $\mu = \frac{\sum_{i=1}^{n} X_i + \frac{\mu_0}{\sigma_0^2}}{\frac{1}{\sigma_0^2} + n}$. Следовательно, $\theta^* = \frac{\sum_{i=1}^{n} X_i + \frac{\mu_0}{\sigma_0^2}}{\frac{1}{\sigma_0^2} + n}$.

In [276]:
```
means = np.zeros(sz)  # Здесь будут храниться средние
for i in range(sz):
    means[i] = x[:i].mean()
```

In [277]:
```
def BayesEst1(x, a, sigma):
    return float((sum(x) + float(a/sigma**2))/(len(x) + (1./sigma**2)))
```

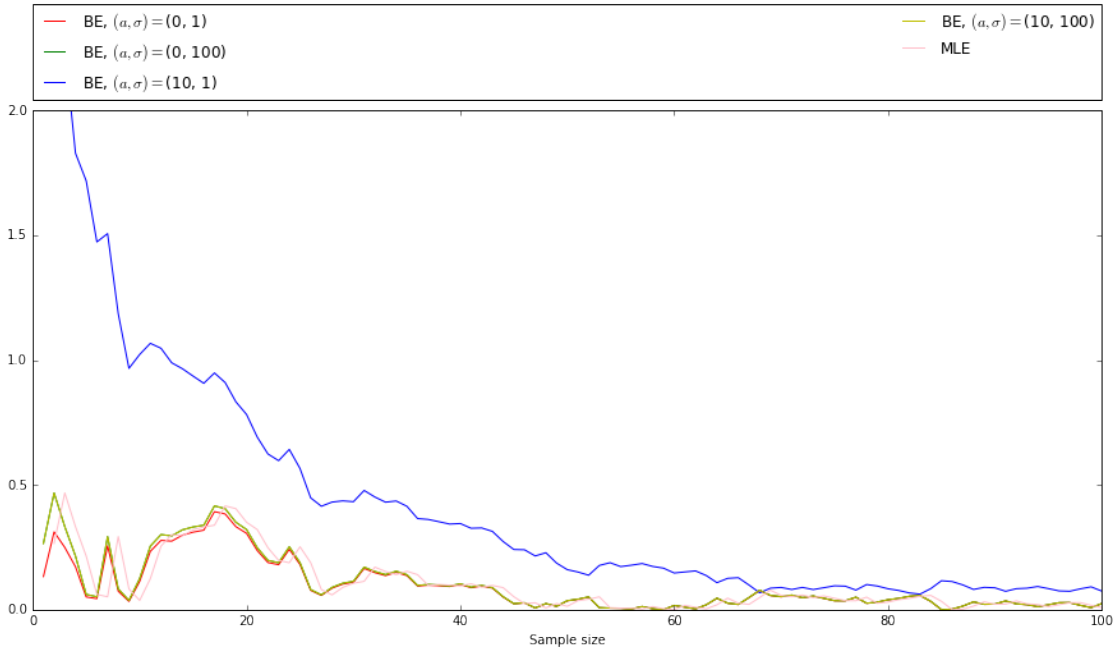
In [278]:
```
colors = ['r','g','b','y','pink']
params = np.array([(0,1), (0,100), (10,1), (10,100)])

plt.figure(figsize=(15,7))
col = 0
for par in params:
    tmp = np.array([BayesEst1(x[:(i+1)],par[0],par[1]) for i in range(sz)])
    plt.xlabel('Sample size')
    # BE stands for 'Bayes estimator'
    plt.plot(np.arange(1,sz+1), abs(tmp), color=colors[col], \
```

```
            label='BE, $(a, \sigma)=$({}, {})'.format(par[0], par[1]))
        col += 1

    plt.plot(np.arange(1,sz+1), abs(means), color=colors[col], label='MLE')
    plt.ylim((0,2))
    plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3, ncol=2, \
               mode="expand", borderaxespad=0.)

    plt.show()
```



3 Модель $N(0, \theta)$

Априорное распределение - Inverse-gamma Distribution $\Gamma_{inv}(\alpha_0, \beta_0)$ with mean= $\frac{\beta_0}{\alpha_0 - 1}$, where α_0- параметр сдвига, β_0- параметр масштаба, $\alpha = \alpha_0 + \frac{n}{2}, \beta = \beta_0 + \frac{\sum_{i=1}^{n} X_i^2}{2}$. Следовательно, $\theta^* = \frac{\beta}{\alpha - 1} = \frac{2\beta_0 + \sum_{i=1}^{n} X_i^2}{2\alpha_0 + n - 2}$.

In [283]:
```
sigmas = np.zeros(sz)  # Здесь будут храниться выборочные дисперсии
for i in range(sz):
    sigmas[i] = var(x[:(i+1)])
```

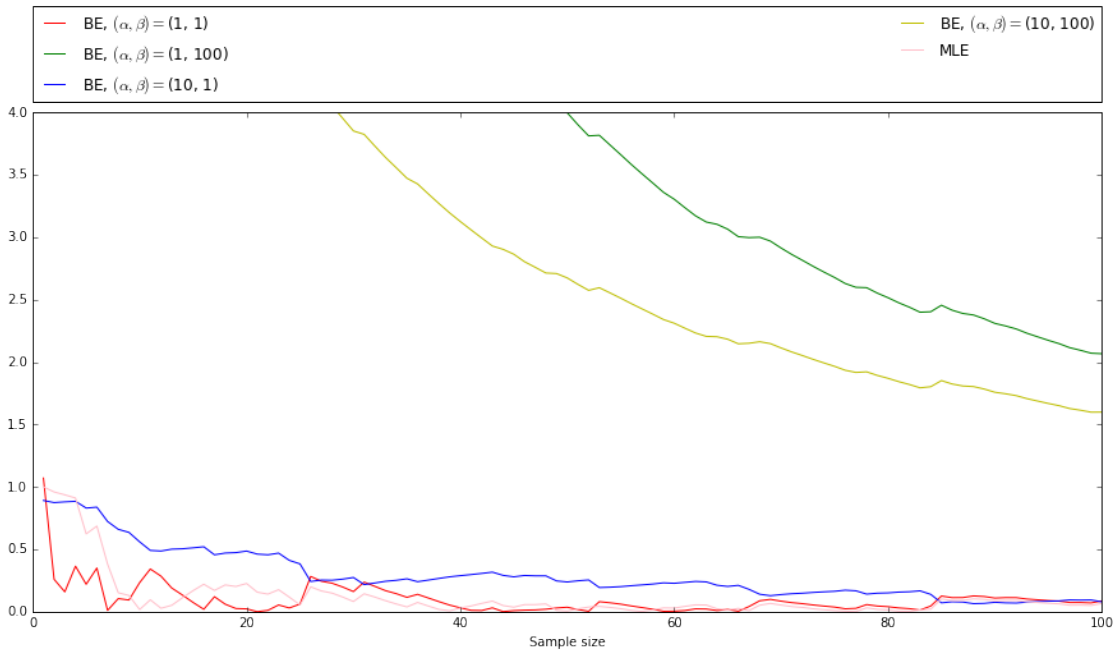
In [284]:
```
def BayesEst2(x, alpha, beta):
#     return float(((2. * lambd) + alpha)/((2. * alpha) + sum([i**2 for i in x])))
    return float(((2. * beta) + sum([i**2 for i in x]))/((2. * alpha) + len(x) - 2.))
```

In [285]:
```
colors = ['r','g','b','y','pink']
params = np.array([(1,1), (1,100), (10,1), (10,100)])
```

```
plt.figure(figsize=(15,7))
col = 0
for par in params:
    tmp = np.array([BayesEst2(x[:(i+1)],par[0],par[1]) for i in range(sz)])
    plt.xlabel('Sample size')
    # BE stands for 'Bayes estimator'
    plt.plot(np.arange(1,sz+1), abs(tmp-1), color=colors[col], \
             label='BE, $(\\alpha, \\beta)=$({}, {})'.format(par[0], par[1]))
    col += 1

plt.plot(np.arange(1,sz+1), abs(sigmas-1), color=colors[col], label='MLE')
plt.ylim((0,4))
plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3, ncol=2, \
           mode="expand", borderaxespad=0.)

plt.show()
```



In [ ]: