1) Machine Arithematic:-

- Introduction
- Finite digit Arithematic
- errors

2) Equations in one variable:-

- the bisection method
- fixed points
- fixed point iteration method
- Newton-Raphson method for root finding
- secant method for root finding
- Regula falsi, method of false position for root finding
- modified newton Ralphson for root multiplicity
- Aitken's Δ^2 method of accelerated convergance

3) Interpolation:-

- Lagrange interpolating polynomials.
- cumulative calculation of interpolating polynomials
- Neville's formula... $P = \frac{(x-x_j)P_j(x) - (x-x_i)P_i(x)}{x_i - x_k}$
- Neville's table.
- divided differences, its table
- Osculating polynomials.
- Hermite polynomials

-- Piecewise polynomial interpolation; Splining.

- cubic spline, quadratic spline

4) Numerical Differentiation and Integration:-

-- computing $f'(x)$.
(by interpolation the function with polynomials)

- (n+1) point formula
- Three point formula

-- Numerical Integration

- Numerical quadrature
- Trapezoidal & simpson's
- accuracy/precision
- Newton-cotes formulae. $(n=1,2,3,\ldots)$
  [open & close]
- Composite numerical integration - composite trapezoid, simpsons, midpoint.
- Adaptive quadrature method
- Gaussian Quadrature
- legendre polynomials → to compute Gaussian quadrature coefficients.

-- multiple dimensions

- multiple trapezoid, simpsons rule.

-- Improper integrals

- $$\int_a^b \frac{g(x)-P_4(x)}{(x-a)^p} + \int_a^b \frac{P_4(x)}{(x-a)^p}\, dx.$$

5.) Ordinary Differential Equations:-

- Lipschitz condition
- Euler's method
- local truncation error
- Taylor of order 'n'
- Runge-Kutta methods of order 2
- multi-order runge-kutta
- multi-step methods
- Adams-Bashforth, Adams-Moulton
- predictor-corrector model

6.) Numerical Linear Algebra:-

- GEM - operation count
- scaled partial pivoting
- LU decomposition
- PLU decomposition

-- iterative methods

- Norms on matrices

-- eigen values & vectors

- Jacobi method
- Gauss-Seidel method
- residual vector
- condition number
- Gerschgorin theorem
- Power method

round-off error.

truncation-error.

integrating → numerical quadrature

$$\int_a^b f(x)\,dx = \sum_{i=0}^{n} a_i f(x_i)$$

basically $\int_a^b L_i(x)$ $\boxed{n=1}$

lagrange.

$\frac{b-a}{n+2} = h$

$n=0$

error is

(1) $\frac{n+1}{2}$ f $\sum^{+1}$

$$\int_a^b (x-x_0)(x-x_1)\cdots(x-x_n)\cdot\frac{f^{n+1}(\xi(x))}{(n+1)!}\,dx$$

$\frac{h}{3}\left(f(x_0)+4f(x_1)+f(x_2)\right) - h^5$ h^3 2+2. $1+1\to 2$

$2\to 2$ $3+1$

4

1) normal integration
- trapezoid
- Simpson's rule
- Newton-cotes – open / close

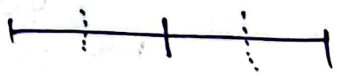
2) Composite integration

composite simpson

$n\to$ even.

* round-off error is same.

→ problem → Adaptive quadrature.



$|S(a,b) - S(a,\frac{a+b}{2}) - S(\frac{a+b}{2},b)| \le 15\epsilon$

then

$\left|\int_a^b f(x)dx - S(a,\frac{a+b}{2}) - S(\frac{a+b}{2},b)\right| < \epsilon$

$n=4$

$$\frac{h}{3}\left[f(a)+2\sum_{j=1}^{n/2-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right].$$

3) Gaussian quadrature :-

$$\int_a^b f(x)dx = \sum_{i=1}^{n} c_i f(x_i)$$

x_i in $[a,b]$ chosen

$$\int_{-1}^{1} P(x)dx = \sum_{i=1}^{n} c_i P(x_i)$$

tabulated

Standard: $[-1,1]$

$$c_i = \int_{-1}^{1}\prod_{j=1}^{n}\frac{x-x_j}{x_i-x_j}\,dx.$$

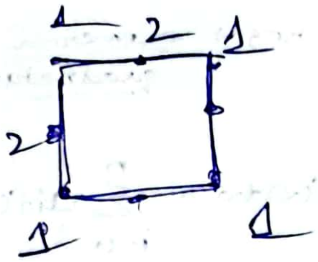
– legendre polynomials :-

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = x^2 - 1/3$
$P_3(x) = x^3 - \frac{3}{5}x$

roots, lie in $[-1,1]$

4) multiple Integrals

$\iint_R f(x,y)\,dA$



$S(a,b) = \frac{h}{3}(1+4+1)\ f(a)+f(\frac{a+b}{2})+f(b)$

$\frac{5}{6}$

5) improper integrals

$$\int \frac{g(x)}{(x-a)^p} \rightarrow \int \frac{g(x)-P_4(x)}{(x-a)^p} + \frac{P_4(x)}{(x-a)^p}$$

taylor

$$= g(a) + \frac{(x-a)}{1!} g'(a) + \frac{(x-a)^2}{2!} g''(a) + \cdots$$

composite Simpsons rule.

L29: Ordinary differential equations:-

* $f(t,y) \rightarrow$ Lipschitz Condition

$\rightarrow$ if $f(t,y)$ satisfies Lipschitz on y; initial-value prob

$y'(t) = f(t,y)$, $y(a) = \alpha$

has unique 'y'

$\rightarrow$ well-posed

universal:

$$y' = f(t,y),\ y(a) = \alpha$$

Methods:

1) Euler's method $\rightsquigarrow$ $h = \frac{b-a}{N}$; $t_0 = a$, $t_i = a + hi$

$$\boxed{w_{i+1} = w_i + h f(t_i, w_i)}$$

Error: $|y(t_i) - w_i| \le \frac{hM}{2L}\{e^{L(t_i - a)} - 1\}$

$L \rightarrow$ Lipschitz constant

$|y''| \le M$

Local truncation error:

say $w_{i+1} = w_i + h\phi(t_i, w_i)$

then take $w_{i+1} = y(t_i) + h\phi(t_i, y(t_i))$

then $\frac{y(t_{i+1}) - w_{i+1}}{h}$ = Local truncation error.

$$LTE = \frac{y_{i+1} - y_i}{h} - f(t_i, y_i)$$

$$= \frac{h \cdot f'(t_i, y_i)}{2} = \frac{h \cdot M}{2}$$

Euler: local truncation error $O(h)$

$h = \frac{1-0}{6}$

$\frac{h}{3}($

2) Higher order taylor method:-

$$y_{i+1} = y_i + \frac{f(t_i, y_i) \cdot h}{1} + \frac{h^2}{2} f'(t_i, y_i)$$

$$\therefore w_{i+1} = w_i + h \cdot T(t_i, w_i)$$

$$T(t_i, w_i) = f(t_i, w_i) + \frac{h}{2} f'(t_i, w_i) + \frac{h^2}{6} f''(t_i, w_i) + \dots$$

order 2

n^{th} order taylor: local truncation error $O(h^n)$

3) Runge-Kutta method:- (taylor's in 2-D)

- to attain same error as higher taylor, without f'', f'''...

$f(t, y) = P_n(t, y) + R_n(t, y)$

$f(t_0, y_0) + \left((t - t_0) \frac{\partial f}{\partial t}(t_0, y_0) + (y - y_0) \frac{\partial f}{\partial y}(t_0, y_0) \right)$

$+ \frac{1}{2!} [\dots]$

determine values a_1, α_1, β_1

so that $a_1 f(t + \alpha_1, y + \beta_1)$

equals $f(t + y) + \frac{h}{2} f'(t, y)$

$a_1 = 1$

$\alpha_1 = \frac{h}{2}$

$\beta_1 = \frac{h}{2} f(t, y)$

midpoint method!

$$\therefore w_{i+1} = w_i + f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)\right)$$

like some midpoint...

4) Modified Euler:

$$w_{i+1} = w_i + \frac{h}{2}\left[f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i))\right]$$

5) Higher order Runge-Kutta:-

$$T(t,y) = f(t,y) + \frac{h}{2} f'(t,y) + \frac{h^2}{6} f''(t,y)$$

↳ approximate this...

by

$$f(t+\alpha_1, y+\delta_1 f(t+\alpha_2, y+\delta_2 f(t,y)))$$ with $O(h^3)$ error

Heun's method.

Here

6) Order-4 Runge-Kutta:-

$w_0 = \alpha.$

writeing for convenience.

$$w_{i+1} = w_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$K_1 = hf(t_i, w_i)$$

$$K_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{K_1}{2}\right)$$

$$K_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}K_2\right)$$

local trunc. $O(h^4)$

$$K_4 = hf(t_{i+1}, w_i + K_3)$$

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multistep:

Adams-bashforth: explicit

$$w_{i+1} = w_i + \begin{matrix}(i-1)\\(i-2)\\(i-3)\end{matrix}$$

-moulton: implicit