PH107 - Quantum physics.

Classical physics:- (traits)

1) All phenomenon - particle or wave
2) momentum & position exact. & are "objective" properties. (subjective in Q.phy)
3) wave exhibit features; not for particles; like interference, diffract^n

(slide)!

4) Doctrine of "determinism" → exact conditions gives exact results.

Quantum p. is bizarre:- (traits)

(slides)

Book: modern physics by serway, Moses and moyer 3rd Edition.

(seniors - buy & sell).

↓ {more, on slides}

× ———————— ×

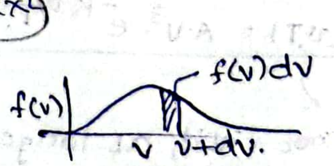
revise Faraday's law
Ampere's law
&
all E.M. laws.

→ degrees of freedom:-

for each trans. rot. $= \frac{1}{2}K_BT \longrightarrow KE_x = \frac{1}{2}mv_x^2$

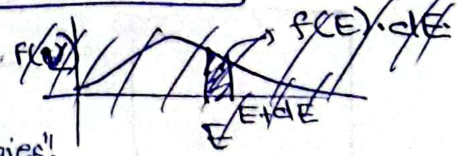
for each vib. $= K_BT \longrightarrow E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$

Supporting material



boltzmann distribution.

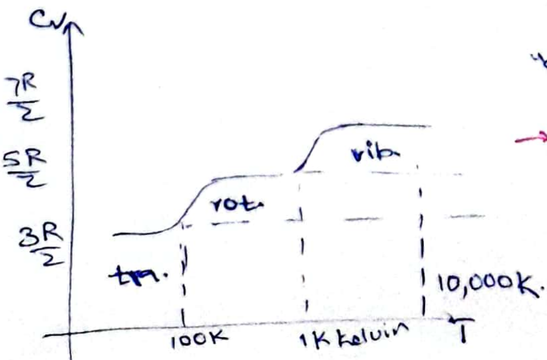
$$\boxed{f(E) = A \cdot e^{-E/K_BT}}$$



rigid diatomic $C_v = \frac{5R}{2}$.

non rigid diatomic $C_v = \frac{7}{2}R$.

* **Specific heat of $H_2(g)$**

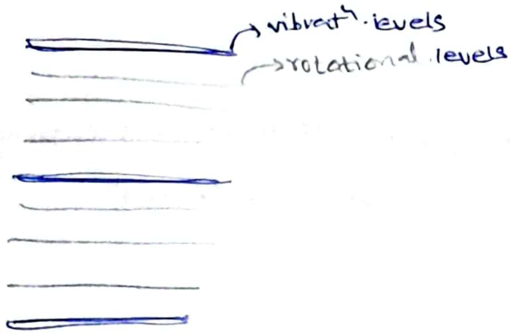


"quantisation of energies"!

→ no 'classical' explanation.

2-8/19

Quantisation of energy:- no quantisation for translational.



rotation frequency: $= \nu$

energy $= h\nu$.

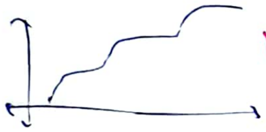
* thermal energy: $\frac{3}{2} K_B T$. → in 10^{-21} J order.

→ Triatomic Linear:-

CO_2 (case)

3 trans; 2 rot; 4 vib...

so; expected



but no!... all vibration are not same energy level.

Ozone (another example)

→ Blackbody radiation:- .

$$\boxed{J(\nu, T) = \frac{c}{4} \cdot u(\nu, T)}$$

J: power radiated per unit area

u: energy density per unit volume. just, on outside surface.

A particular vibration mode; will contribute; if $K_B T \simeq h\nu_{vib}$.

$K_B T$. → used as a scale factor for energy values in molecular scales. (& also as a unit of energy).

→ Wein's exponential law:-

$$u(\nu, T) = A \cdot \nu^3 \cdot e^{-K \cdot \nu / T}. \Rightarrow \boxed{\nu_{max} = (\alpha) \cdot T}$$

not valid; at longer λ.

waves in a box:-

Some derivation there!

we get.

$$\boxed{dN = \frac{8\pi\nu^2}{c^3} \cdot d\nu}$$

2/8/19

$$\boxed{u(\nu,T) = \frac{8\pi\nu^2}{c^3} K_B T}$$

Rayleigh-Jeans law.

give good results at high wavelengths.
low

Didnt understand anything!.

2/8/19

Class-2:-

1) Kirchoff theorem:-

$$e(\nu) = J(\nu,T)\cdot \underbrace{A(\nu)}_{\text{absorbance}}.$$

for blackbody: $A(\nu) = 1$.

$\therefore\ e(\nu) = J(\nu,T)$
↳ universal; for all bodies

2) Stephan's law:

$$\int^{\infty} e(\nu)d\nu \propto T^4 = \sigma T^4.$$

→ Energy density of blackbody:-

$J(\nu,T)$: power emitted per unit area; per ν interval;
$u(\nu,T)$: energy density per unit frequency

$$\boxed{J(\nu,T) = \frac{c}{4}\cdot u(\nu,T)}$$

↳ same variation ν T more ν

1) Weins law:-

$$\boxed{\frac{u(\nu,T)}{d\nu} = A\nu^3 e^{-\beta\nu/T} d\nu} \rightarrow h\nu \times \nu^2 \times \underbrace{e^{-\beta\nu/T} d\nu}.$$

(light as particles) classical! MB.

Should have been BE statistics.

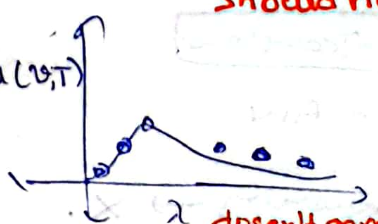
$u(\nu,T) = f(\nu)\cdot g(\nu/T)$ $\nu = \nu_0 \ldots$

$$\frac{du(\nu,T)}{d\nu} = 0. \quad \frac{du}{d\nu} = 0.$$

$T_0 \ldots$ $d\ (f(\nu)\cdot g(\nu/T))e^{-\nu/T_0}$

$$f'(\nu)\cdot e^{-\nu/T_0} + f(\nu)\cdot -\frac{1}{T_0}\cdot e^{-\nu/T_0} = 0.$$

$$\left(\frac{\nu}{T_0} = \text{constant}\right)$$



dosen't agree for longer wavelengths (i.e. above 6000nm).

2^{nd} idea: 2/8/19

→ Light standing in a box.

$n_x \left(\frac{\lambda_x}{2}\right) = L.$

$\lambda_x = \frac{2L}{n_x}$

$\therefore K_x = \frac{2\pi}{2L} \cdot n_x = \boxed{\frac{n_x \cdot \pi}{L}}$.

$A_x = A \sin\left(\frac{n_x \pi}{L} \cdot x\right)$

now! we need to know the no. of modes; per unit volume; in the frequency interval ν and $\nu + d\nu$.

cont!

$\boxed{\text{no. of modes per unit volume} = \frac{8\pi\nu^2}{c^3} d\nu.} = g(\nu)d\nu$

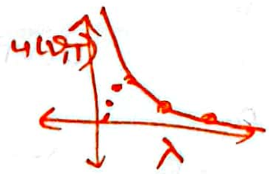
→ now; $u(\nu,T) = \frac{\text{no. of modes}}{\text{volume}} \times$ energy of each mode.
KT.

$\boxed{u(\nu,T) = \frac{8\pi\nu^2}{c^3} \cdot K_BT}$

Rayleigh-Jeans law.

for lower ν → fine

higher frequency → X.



ultraviolet catastrophe.

6/8/19

Special session:-

$E_y = (\cos k_z z \cdot \cos k_x x) \cdot e^{i\omega t}$

$k_z \cdot z = l\pi$

$k_x \cdot x = m\pi$

$\boxed{\frac{\omega}{k} = c}$

$\omega^2 = c^2(k_x^2 + k_z^2)$

$\boxed{\omega^2 = \pi^2 c^2 (l^2 + m^2)}$

→ **Planck hypothesis:-**

→ oscillators of walls of black body, emit or absorb radiation.

→ Oscillators of all frequencies are present.

→ energy of these oscillators is quantised. (ν is not!)

planck's equation:-

$$u(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

wein law:

$\boxed{\lambda_{max} \cdot T = 2.898 \times 10^{-3} m.k.}$

$\frac{du(\lambda T)}{d\lambda} = 0$ for the result

Total power:-

$J(\lambda T) = \frac{c}{4} \cdot u(\lambda, T)$

integrate & we get $= (\ ) \cdot T^4$

LASER:

(fluke formula) **Weins:** $h\nu \times \nu^2 \times \underbrace{e^{-h\nu/K_BT}}_{\text{mistake!}}$ ✓ ✓ ✓ ✓

Raileigh: $\boxed{\times \nu^2 \times \; K_BT}$ → mistake

Plancks: $h\nu \times \nu^2 \times \frac{1}{e^{h\nu/K_BT} - 1}$] correct. ✓ ✓ ✓

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Plank in:-

energy = no. of modes × avg. of each mode.

$\frac{8\pi\nu^2}{C^3} d\nu$ ✓

$\neq KT$.

but! energy is quantised.

$\boxed{E = nh\nu.}$

will learn ~~latery~~ below.

What is average.

as n is discreat.

this is the difference!

Quantum average :-

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu \, e^{-(nh\nu/K_BT)}}{\sum_{n=0}^{\infty} e^{-(nh\nu/K_BT)}}$$

$$= \frac{h\nu}{e^{\frac{h\nu}{K_BT}} - 1}$$

Classical average:-

taking continuous variation.

$$\langle E \rangle = \frac{\int_0^{\infty} E \cdot e^{-E/K_BT} \, d(E)}{\int_0^{\infty} e^{-E/K_BT} \, dE}$$

$$= K_BT$$

$\therefore$ Energy $u(\nu, T) = \frac{8\pi\nu^2}{C^3} \cdot \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1} = \frac{8\pi h\nu^3}{C^3} \cdot \frac{1}{\left\{e^{\frac{h\nu}{KT}} - 1\right\}}$

planck's formula.

if $h\nu \gg K_BT$

weins law

if $h\nu \ll K_BT$

rayleigh-jeans law.

NOW; trying to get weins displacement law:-

$$u(\nu, T) = \frac{\beta}{\lambda^5} \cdot \frac{1}{(e^{\alpha/\lambda T} - 1)} \quad (\text{use } \nu = \frac{c}{\lambda})$$

$$\frac{du(\lambda, T)}{d\lambda} = 0.$$

* we get (after solving) $\boxed{\lambda \cdot T = 2.898 \times 10^{-3} \, Km}$

Yeah!

try to get stephen's law:-

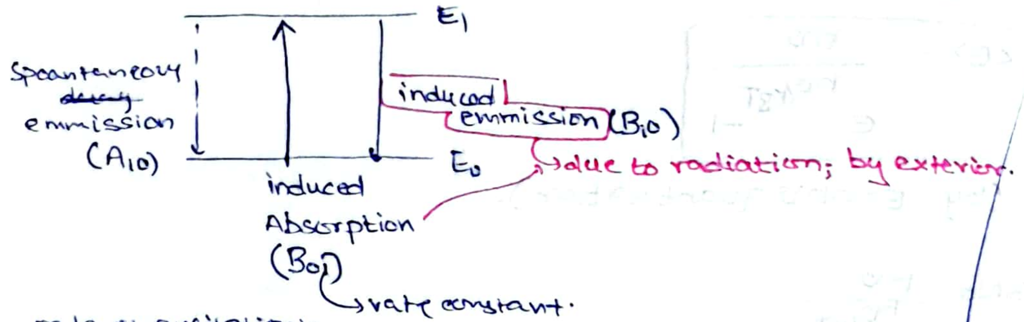
$$J(\nu, T) = \frac{c}{4} \cdot u(\nu, T)$$

$$\text{Net radiation} = \int_0^{\infty} J(\nu, T) \cdot d\nu = \left(2\pi^3 \ldots\right) T^4$$

Einstein's approach to planck's law:-

in atoms:-

A_{10}, B_{01}, B_{10} are rate constants.



rate of excitation:-

$$R_{01} = N_0 B_{01} \underbrace{u(\nu,T)}_{\text{radiation density.}}$$

Rate of emmission:-

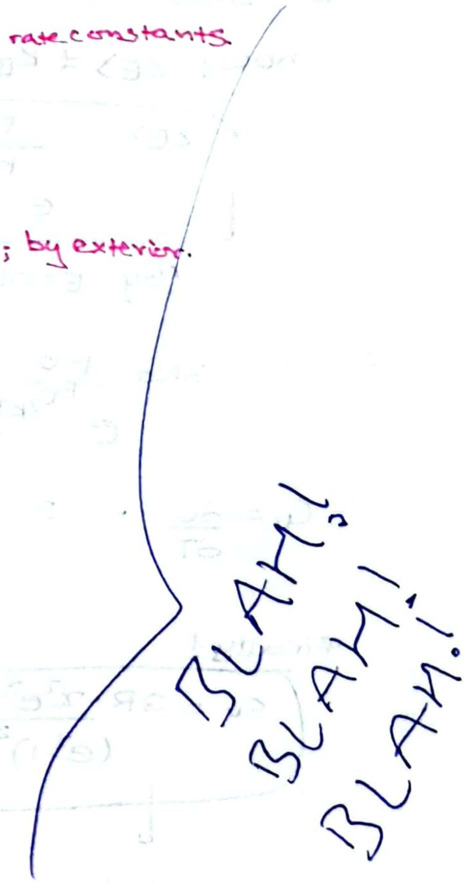
$$R_{10} = N_1\left(B_{10} \cdot u(\nu,T) + A_{10}\right)$$

∴ equilibrium:-

$$R_{10} = R_{01}$$

$$\therefore \frac{N_0}{N_1} = \frac{A_{10} + B_{10} \cdot u(\nu,T)}{B_{01} \cdot u(\nu,T)} = \frac{g_0}{g_1} e^{\left(\frac{E_1 - E_0}{KT}\right)}$$

degeneracies



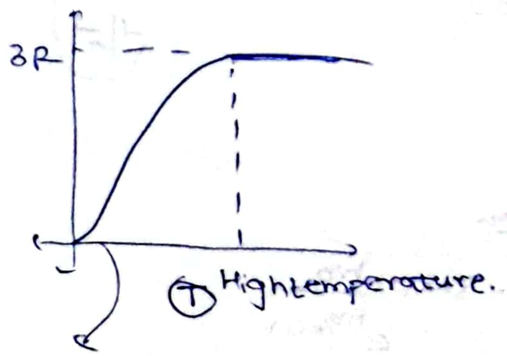
→ Specific heat of solids:-

1) Dulong-petit's law:-

$$\boxed{C_v = 3R}$$

but experimentally-

$C_v \to 0$ as $T \to 0$.



$C_v = aT^3$ model.

Einsteins model:-

At low temperatures, this formula; falls off rapidly as compared to expt. data.

$U = 3N_a \times \langle E \rangle$.

now! $\langle E \rangle \neq K_B T$.

$$\boxed{\langle E \rangle = \frac{h\nu}{e^{h\nu/K_B T} - 1}}$$

(by $E = nh\nu$ quantisation).

$\therefore \quad U = 3N_a \cdot \dfrac{h\nu}{e^{h\nu/KT} - 1}$

$\therefore C_v = \dfrac{\partial U}{\partial T} = 3$

explained! using quantisation.

finally!

$$\boxed{C_v = 3R \cdot \frac{x^2 e^x}{(e^x - 1)^2}} \; ; \quad x = \frac{h\nu}{K_B T} = \frac{T_E}{T}$$

$$\boxed{T_E = \frac{h\nu}{K_B} = \text{einstein temperature}}$$

For every metal!....

enough; if we know T_E....

* $T_E = \dfrac{h\nu}{K_B}$

const. for a given metal.

obtained experimentally.

lattice frequencies... $\boxed{\text{phonons}}$

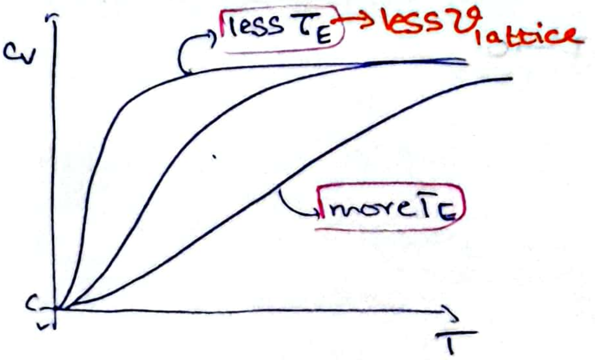
einstein; took same ν for all molecules

quanta of lattice vibrations are called phonons.

$3R \cdot \dfrac{x^2 \cdot e^x}{(e^x - 1)^2}$

$x = \dfrac{h\nu}{K_B T} = \dfrac{T_E}{T}$

* more T_E; less C_v at a T



$\boxed{1836}$

calcn:-

$U = \text{Energy} = 3N_a \cdot \dfrac{h\nu}{e^{h\nu/KT} - 1}$

$C_v = \left(\dfrac{dU}{dT}\right) = 3N_a h\nu \cdot \dfrac{+1}{(e^{h\nu/KT} - 1)^2} \cdot e^{\frac{h\nu}{KT}} \cdot \dfrac{h\nu}{K} \cdot \dfrac{1}{T^2}$

$$\boxed{3N_a K \cdot \left(\frac{h\nu}{KT}\right)^2 \cdot \frac{e^{h\nu/KT}}{(e^{h\nu/KT} - 1)^2}}$$

• frank-hertz experiment:-

X ———— X

photoelectric effect:-

Compton:-

particle nature of light!

so; does photon has momentum.

Bit of special Relativity:-

intrinsic rest mass $= m_0$.

$\therefore$ rest mass energy $= m_0 c^2$.

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma m_0.$$

$\therefore \boxed{\text{momentum} = \gamma m_0 \cdot v}$

$\therefore$ net energy $= \sqrt{p^2c^2 + m_0^2c^4} = \gamma m_0 c^2$

Compton scattering:- → Demonstrates momentum conservation involving photons.

classical : (thompson scattering);

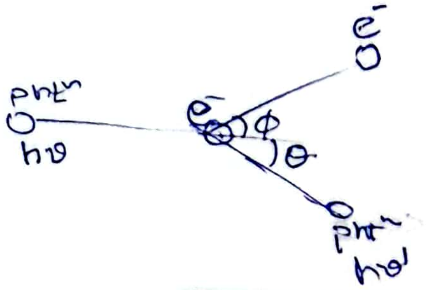
same λ; initially & finally. (as same ν) solve

compton:-

for light $E^2 = p^2c^2 + \cancel{m_0}c^4$

0 for photon

$\therefore E = pc = h\nu$



P cons. along y:-

$P_e \cdot \sin\phi = \frac{h\nu'}{c} \cdot \sin\theta.$

P cons along x:-

$\frac{h\nu}{c} = P_e \cdot \cos\phi + \frac{h\nu'}{c} \cdot \cos\theta.$

For e^-: $\boxed{E^2 = P_e^2c^2 + m_0^2c^4}$

Energy cons:

$h\nu + m_0c^2 = E_e + h\nu'$

finally $\boxed{\lambda' - \lambda = \lambda_c (1-\cos\theta)}$

$\boxed{\lambda_c = \frac{h}{m_0 c}}$ 2.43×10^{-12} m

so; to notice change; λ must be small. $\therefore$ high energy x-rays ➝ so.

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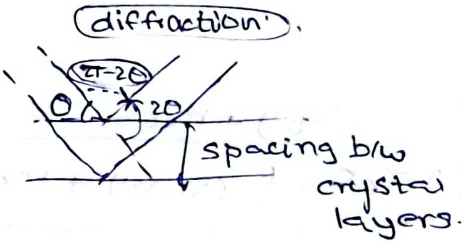
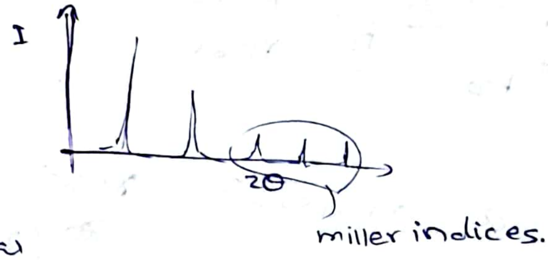
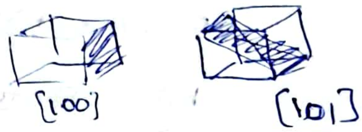
RANDOM:-

X-ray diffraction:-





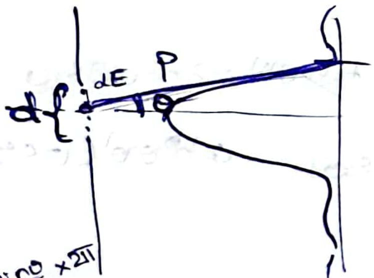


*photon duality:-

refraction:- can be explained; using only wave nature.

* photon can behave like a particle & a wave; depending upon experiment.

* Single slit electron diffraction:-



$$\boxed{\Delta P_y = p\sin\theta}$$ *

for first minima;

$$\boxed{\sin\theta = \frac{\lambda}{d}}$$

$$\Delta y \sim d \sim \frac{\lambda}{\sin\theta}$$

$$\therefore \Delta y \cdot \Delta P_y = \lambda p = \frac{h}{p} \cdot p = h$$

$$\therefore \boxed{\Delta y \cdot \Delta P_y = h}$$

$\frac{d\sin\theta}{n\lambda} \times 2\pi$

n

$\left(\frac{d}{n} \frac{\sin\theta}{\lambda} \cdot 2\pi\right)$

$$\boxed{\sin\theta = \frac{\lambda}{d}}$$

microscope:-

resolution $\approx$ wavelength of light used.

so; we can use the $\bar{e}$ beam; as we can change its wavelength.

Read textbook.

Group velocity & phase velocity:-

If a wave is given; how to give particle?

wave packet:-

- spatial beats by superposition of sinusoidal waves of nearby wavelengths

see slide images

2 waves: 

3 nearby wavelength: 

5 : 

many nearby wavelengths. 

wave packet.

(refers to particle position).

phase velocity, group velocity:-

phase position moves.

$\therefore$ phase velocity $= \frac{\omega}{k}$.

Group velocity:-

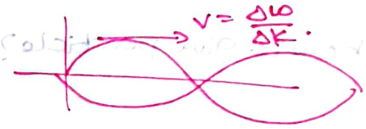
is the velocity; with which; the envelope moves.

Eg: $A\sin(kx - \omega t)$

$A\sin((k+\Delta k)x - (\omega+\Delta\omega)t)$

Sum. $= 2A\sin(\underbrace{kx-\omega t}) \cdot \cos(\underbrace{\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}})$

fast oscillating; Slow oscillating



$\therefore$ Group velocity $= \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} = V_{\text{Particle}}$ (here). $= V_g$

↳ velocity of wave packet.

So; develope a relation. b/w ω & k.

$\frac{\omega}{k} = c$.

$\therefore \omega = kc$.

$\frac{d\omega}{dk} = c$.

Eg: $v \propto \sqrt{\lambda}$. for a water ripple.

$\lambda \nu = c\sqrt{\lambda}$

$\nu = \frac{c}{\sqrt{\lambda}}$.

$\omega = \frac{2\pi c}{\sqrt{\lambda}}$.

$k = \frac{2\pi}{\lambda}$.

$\therefore \frac{d\omega}{dk} = \frac{1}{\lambda\sqrt{\lambda}} \cdot \lambda^2 = \sqrt{\lambda} \cdot k \cdot \checkmark = V_{group}$.

$\boxed{V_g = \frac{1}{2}V_p}$ *

relation b/w ω & k is $\boxed{\text{dispersion relationship.}}$

e.g: (i) $\frac{\omega}{k} = c$ for light in vaccum.

(ii) $\omega = A\sin\left(\frac{ka}{2}\right)$; for 1D-atom chain.

Particle to wave:-

particle $\rightarrow V$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

General true....

$$\boxed{\nu = \frac{E}{h}} = \frac{mc^2}{h}$$

$$\boxed{\text{Speed } (V_p) = \lambda\nu = \frac{c^2}{V}}$$

$\therefore V_p \neq V$.

$$\omega = 2\pi\nu = 2\pi\frac{mc^2}{h} = \frac{2\pi m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi m_0 v}{h\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{2\pi m_0 V}{h\sqrt{1-\frac{V^2}{c^2}}}$$

physics of a de-broglie wave.

$$V_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = V$$

(after differentiation)

$\therefore \boxed{V_g = V}$

→ Relationship b/w V_g & V_p:-

$$V_p = \frac{\omega}{k}.$$

$$V_g = \frac{d\omega}{dk} = \frac{d(V_p \cdot k)}{dk} = \left[V_p + k\cdot\frac{dV_p}{dk}\right] = \left[V_p - \lambda\cdot\frac{dV_p}{d\lambda}\right]$$

$$\boxed{V_g = V_p + k\cdot\frac{dV_p}{dk}} \boxed{= V_p - \lambda\cdot\frac{dV_p}{d\lambda}}$$

Non-dispersive medium:-

→ for which $\frac{dV_p}{dk} = 0$.

$\therefore \boxed{V_g = V_p}$

→ Dispersive medium:-

$$V_g \neq V_p.$$

Normal:- Anamolous dispersion:-

$V_g < V_p$. $V_g > V_p$.

★ electron has $\lambda_{deb.} = 2\times10^{-12}$ m...

Find KE, phase vel., V_g.

$h\vartheta = \gamma m_0 c^2$

$\frac{h}{\lambda} = \gamma m_0 v$

$V_p = \frac{c^2}{v}$

$v =$

$p^2c^2 =$

Sol) $E_{0\,rest} = m_0c^2$

$E = \sqrt{(pc)^2 + (m_0c^2)^2} = E_{0\,rest} + KE$

$\therefore \boxed{KE = \sqrt{E_0^2 + p^2c^2} - E_0}$

find v:-

$1 - \frac{v^2}{c^2} = \frac{E_0^2}{E^2}$

$\therefore \boxed{v = c\sqrt{1 - \left(\frac{E_0}{E}\right)^2}}$

$v = v_g \checkmark\ 0.77c$; $\bar{e}$; $V_g \cdot V_p = c^2 \rightarrow V_p = 1.3c.$

Dispersion relation for deBroglie wave:-

phase velocity:

$\boxed{V_p = \frac{\omega}{k}} = \frac{\vartheta}{\Delta}$

$\therefore V_p \cdot V_g = c^2$

$h\vartheta = E$

$\vartheta = \frac{E}{h}.$

→ Heisenberg's uncertainity relation:-

uncertainity in wavelength of component waves.

$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$ → hence; we can measure x, p_y, p_z preciesly. simultaneously.

$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

means:-

if a system is known to exist in energy E; over a limited period Δt. then this energy is uncertain by atleast an amount $\boxed{\frac{h}{4\pi\Delta t}}$.

→ to know energy of an exicited state with exact precision; it has to stay in that state as $\Delta t \to \infty$

* can $\bar{e}$ be found in nucleus?... cause; we have β decay na... physicists doubted.

* estimating atomic size; using uncertainity.

$\Delta x = \frac{radius}{2}$

$p = \Delta p = \frac{\hbar}{2\Delta x}$

$KE = \sqrt{(pc)^2 + (m_0c^2)^2} - m_0c^2 \simeq 19.75$ MeV.

β decay $\bar{e} \longrightarrow$ 1 MeV

experimental results show that no $\bar{e}$; particle in atom posses energy grater than 4 MeV

$p^+ \rightarrow$ potential Energy (-ve)

* Classical world

1) The observer is objective & passive

2) events happen, independent of whether observer is present or not.

3) this is known as Objective reality.

Quantum world.

1) the observer is not objective & passive.

2) the act of observation changes the physical system irrevocably

3) this is called subjective reality.

wherever 'h' is comparable to {(momentum scale) × (length dimension in experiment)} then wave nature is prominent.

Fourier transformations:-

any $f(x) = \sum a_n \sin nx + \sum b_n \cos nx$.

if $f(x) = \sum a_n \cdot \cos nx$. (take some n')

$$\int_{-\pi}^{\pi} f(x) \cdot \cos n'x \, dx = \sum a_n \cdot \int_{-\pi}^{\pi} \cos nx \cos n'x \, dx.$$

$$= \sum a_n \cdot \frac{1}{2} \cdot \int_{-\pi}^{\pi} \underbrace{\cos(n-n') \cdot x}_{\text{nonzero integral if } n=n'} + \underbrace{\cos(n+n')x}_{\text{its } \int_{-\pi}^{\pi} \text{ is always zero}} \, dx$$

$$= \frac{1}{2} \cdot a_{n'} \cdot 2\pi \qquad \boxed{n = n'}$$

$$= \pi a_{n'}$$

$$\therefore \boxed{a_{n'} = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \cos n'x \, dx}$$

in general: $f(t) = \sum_n a_n \sin nt + \sum_n b_n \cos nt$.

$$f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{i\omega t} \cdot d\omega$$

Here $g(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$.

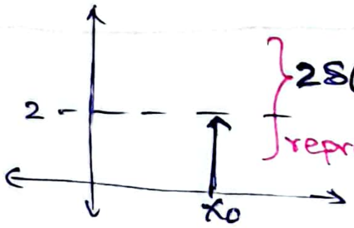
δ function:-

$$\delta(x) = 0 \quad \text{for} \quad x \neq 0$$
$$= \infty \quad \text{for} \quad x = 0$$

$$\delta(x - x_0) = \int_{t=-\infty}^{\infty} e^{i(t-x_0)}\,dt$$

$$\int_{-\infty}^{\infty} \delta(x)\,dx = \int_{-\varepsilon}^{\varepsilon} \delta(x)\,dx = 1.$$

Such that



$2\delta(x - x_0)$ representation.

* δ function makes sense; when it occurs as an integral.

$$\int_{-\infty}^{\infty} f(x)\cdot\delta(x - x_0)\,dx = \int_{x_0-\epsilon}^{x_0+\epsilon} f(x)\,\delta(x - x_0)\,dx$$

$$= f(x_0)$$

Examples

1) $f(t) = \alpha$.

$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i\omega t} dt$$

$$= \alpha \cdot \int_{-\infty}^{\infty} e^{-i\omega t} dt.$$

$$= \alpha \cdot \delta(0)$$

2) $f(t) = \cos\omega_0 t$.

$$f(t) = \frac{1}{2}(e^{i\omega_0 t} + e^{-i\omega_0 t})$$

$$g(\omega) = \int_{-\infty}^{\infty} \frac{1}{2} \cdot (e^{i\omega_0 t} + e^{-i\omega_0 t}) \cdot e^{-i\omega t} dt$$

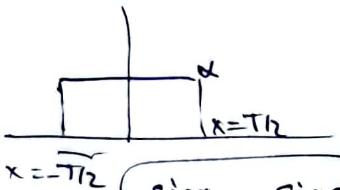
$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i\omega t} dt.$$

$$\frac{1}{2}(e^{i\omega_0 t} + e^{-i\omega_0 t})$$
$$e^{-i\omega t}$$

$$\frac{1}{2} e^{-it(\omega - \omega_0)} + \frac{1}{2} e^{-it(\omega + \omega_0)}$$

$$g(\omega) = \frac{1}{2}\delta(\omega - \omega_0) + \frac{1}{2}\delta(\omega + \omega_0)$$

3)

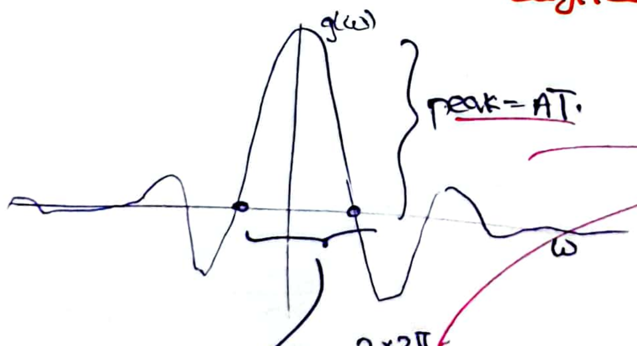


$$\boxed{\frac{\sin x}{x} = \text{sinc}(x)}$$

$$\boxed{g(\omega) = AT \cdot \text{sinc}\left(\frac{\omega T}{2}\right)}$$

used widely in digital communications.

here:



Width = $\frac{2 \times 2\pi}{T}$

both are inversely related.
hint at $\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$.

Tut 5:-

- electron interference.
  (YDSE)
- Davission Germer.

Q2) $\frac{1}{c^2}\cdot\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2}$

$\underbrace{e^{i(kx-\omega t)}}_{=0}$ is solⁿ.

$\sin(kz-\omega t)$

$\sin(k(x+y+z)-\omega t)$

$\boxed{\cos^2 k\frac{(x+y)}{2}}$

$\frac{1}{c^2}\cdot\frac{\partial^2 \psi}{\partial t^2} = \Sigma\frac{\partial^2 \psi}{\partial x^2}$

$\hookrightarrow \underbrace{e^{i(\vec{k}\cdot\vec{r}-\omega t)}}_{=0}$ is solⁿ.

$\therefore\ \vec{k_1} = k_0(\hat{x}+\hat{y}+\hat{z})$; $k^2 = k_0(\hat{z})$

$\therefore\ \boxed{\begin{aligned} y_1 &= A e^{i(\vec{k_1}\cdot\vec{r}-\omega t)} \\ y_2 &= A\cdot e^{i(\vec{k_2}\cdot\vec{r}-\omega t)}\end{aligned}}$

$y = A\cdot e^{i(k_0 z-\omega t)}\cdot\left[1+e^{ik_0(x+y)}\right]$

Intensity $= yy^* = |y|^2$

$\propto \cos^2\left(\frac{k}{2}(x+y)\right)$.

$I \neq |y_1|^2 + |y_2|^2 = |y_1+y_2|^2$

Gaussian function:-

* $$\boxed{f(x) = A\cdot e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}}$$

μ = Mean of x.

σ = standard deviation.
(measure of uncertainity).
1st time introduced.

Normalizat^n;
(imp. in Q.phy.)

i.e.
$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\boxed{\int_{-\infty}^{\infty} e^{-\alpha x^2} = \left(\frac{\pi}{\alpha}\right)^{1/2}}$$

$\therefore$ now
$$A\cdot\int_{-\infty}^{\infty} e^{-\frac{(x)^2}{2\sigma^2}} = 1$$
$$A = \frac{1}{\sqrt{2\pi}\,\sigma}$$
$$\therefore \boxed{f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

HWHM: half width half maxima.

HWHM

$$\therefore \boxed{HWHM = \sigma\cdot\sqrt{2\ln 2}}$$

$\therefore$ width $\propto \sigma$.

* $f(x) = A\cdot e^{-\frac{(x-2)^2}{\sigma^2}}$ sillie!

standard deviation $= \frac{\sigma}{\sqrt{2}}$.

F.T (furier transform^n) of Gauss fu^n:-

$$a(k) = \int_{-\infty}^{\infty} e^{\left(-\frac{x^2}{2\sigma^2}\right)}\cdot e^{-ikx}\,dx$$

(1 doubt in proof of slides).

* $$a(k) = \sqrt{2\pi}\,\sigma_x\cdot e^{-\left(\frac{k^2\sigma_x^2}{2}\right)}$$

$\therefore \sigma_k = \frac{1}{\sigma_x}$ $\therefore \boxed{\sigma_k\cdot\sigma_x = 1}$

the origin of uncertainity relation.

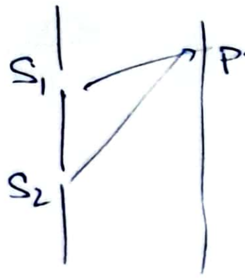
* for Gaussian fu^n:-

$\langle x\rangle = \mu$

$\langle x^2\rangle = \mu^2 + \sigma^2$.

$\therefore$ variance $(\sigma^2) = \langle x^2\rangle - (\langle x\rangle)^2 = \sigma^2$ satisfied!

→



$E_{net} = E_1 + E_2$
are imaginary

Intensity $= \underline{|E|^2}$. $\neq |E_1|^2 + |E_2|^2$

$= |E_1|^2 + |E_2|^2 + 2Re(E_1^* E_2)$

Similarly:- for $\bar{e}$ waves.

$\psi = \psi_1 + \psi_2$ (for matter waves).

$\therefore$ $P = \psi . \psi^* = |\psi|^2 =$ probability

* wave fun : $\psi(x,t)$

Average: $\langle O \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \cdot O \cdot \psi(x,t) dx.$

Superposition: $\boxed{\psi = c_1\psi_1 + c_2\psi_2}$

→ Wave packets:-

to form a true wave packet; that is zero everywhere outside Δx;

we need to add infinite harmonic waves;
with continuous variation
wave lengths (K).
& amplitudes.
a(K).

* Now; if $\psi(x)$ is given; we can find a(K).
position   momentum.
$\boxed{\hbar K = p}$.

Gaussian wavepacket:-

$\psi(x) = \int_{-\infty}^{\infty} a(K) \cdot e^{-iKx} \cdot dK.$

$a(K) = A \cdot e^{-\left(\frac{(K-K_0)^2}{2\sigma_K^2}\right)}$ → this K_0 will decide x_0??....

we know; if a(K) is gaussian; $\psi(x)$ is gaussian.

$\psi(x)$ will be

$\psi(x) = \underset{const}{(\ )} \cdot \underbrace{e^{iK_0x}}_{oscillatory} \cdot e^{-\frac{\sigma_K^2 x^2}{2}}$.

$\boxed{\sigma_x = \frac{1}{\sigma_K}}$

$\sigma_x = \frac{1}{\sigma_K}$

$\therefore \Delta k \cdot \Delta k = 1$

$\Delta x \cdot \Delta p = \hbar$.

the uncertainity is minimum for Gaussian wave packet.

$\therefore$ in general; $\boxed{\Delta x \cdot \Delta p \geq \hbar}$

this is the uncertainity relation for wave packet.

* Normalize $\psi(x) = c \cdot e^{ik_0 x} \cdot e^{\frac{-\sigma_k^2 x^2}{2}}$.

means $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$.. not $\int_{-\infty}^{\infty} \psi(x) dx = 1$

imaginary coming.

we get.

$$C = \frac{\sqrt{\sigma_k}}{\sqrt[4]{\pi}}$$

for contributions in Q. mechanics; we take $|\psi|^2$; not $\psi(x)$. So; we have a '2' factor.

* In quantum physics; average is

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} \psi^*(x) \cdot x \cdot \psi(x) dx}{\int_{-\infty}^{\infty} \psi^*(x) \cdot \psi(x) dx}$$

hence; we get $\frac{1}{\sqrt{2}}$ factors in standard deviations.

x ——— x

Basics:-

- wave function
- normalisation
- Operators ($\hat{O}$): turns functions into functions.

Eg: derivative operator: $O = \frac{d}{dx}$,

* $\boxed{\hat{A}\psi = \alpha \psi}$ α is scalar.

α is called eigen value.

$\psi(x)$ is eigen function of A.

Eg. in slides.

doubt:
if $\hat{A} = x$.
$\hat{A}(e^x) = \cancel{e^x}$ or xe^x ?

$[A,B] = AB - BA$ → commutator.

if $[A,B] = 0$; operators are commute.

Expectation value:-

$$\langle \hat{O} \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\, \hat{O} \psi(x,t) dx$$

quantum mechanical contribution

$\int_{-\infty}^{\infty} \psi^* \psi dx = 1$ here.

debroglie wave. $\psi(x,t) = A \cdot e^{-i(kx-\omega t)}$

we will write; for any general; as we have fourier with us.

1) $\frac{\partial}{\partial t}\psi(x,t) = A \cdot i\omega \cdot e^{+i(kx-\omega t)}$

2) $\frac{\partial}{\partial x}\psi(x,t) = +ik \cdot \psi(x,t)$.

& $\omega = \frac{E}{\hbar}$

$k = \frac{p_x}{\hbar}$

$\frac{\partial}{\partial t}\psi(x,t) = -\frac{A i E}{\hbar} \cdot \frac{\psi(x,t)}{A}$

$\hbar i \frac{\partial}{\partial x}\psi(x,t) = p_x \psi(x,t)$.

$\therefore +\hbar i \frac{\partial}{\partial t}\psi(x,t) = E \cdot \psi(x,t)$

$\therefore \boxed{\hat{p}_x = -\hbar i \frac{\partial}{\partial x}}$

$\therefore \boxed{\hat{E} = +\hbar i \frac{\partial}{\partial t}}$

only component!

1) Kinetic energy:- (consider a non-relativistic particle)

$K = \frac{p^2}{2m}$ } how much true? → true in non-relativistic domain.

$\therefore \hat{K} = \frac{1}{2m}\left(-\hbar i \frac{\partial}{\partial x}\right)^2$

$\boxed{\hat{K} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}}$ funny!

Facts:-

1) for every physical quantity (observable); there is an operator.

2) Heisenberg uncertainity involves non-commuting operators; these cannot be simultaneously measured preciesly.

$\hat{x}, \hat{p}$ are non commuting.

$x\frac{\partial}{\partial x}f(x) \neq \frac{\partial}{\partial x}(xf(x))$

→ checking commutability:-

1) $\hat{x}\hat{p}_x - \hat{p}_x\hat{x}$.

$= x \cdot -i\hbar\frac{\partial}{\partial x}(\psi(x)) + i\hbar\frac{\partial}{\partial x}(x \cdot \psi(x))$

$= i\hbar \cdot \psi(x)$ $\therefore [x, p_x] = i\hbar$

$\neq 0$

$\therefore$ noncommutable.

2) if $[\hat{K} \& \hat{p}]$ is done;

$[\hat{K}, \hat{p}] = 0$.

* let A, B, C be operators.

$[A,B] = C$.

commutator

then

general uncertainity relation

$\Delta A \cdot \Delta B \geq \frac{1}{2}|\langle C \rangle|$

expectation.

$\Delta x \cdot \Delta p_x \geq \frac{1}{2}|i\hbar| = \frac{\hbar}{2}$

$\boxed{\text{tut-6:}}$ tough guy!!

Uncertainity & Fouriers:-

P8) $\Delta x = 10^{-14}$ m

a) $\Delta p_x \simeq 10^{-20}$

$E = \sqrt{(pc)^2 + (m_e c^2)^2}$

$= 19.6$ MeV. High!

for a particle in nucleus; $E_{max} \approx 4$ MeV.

for proton; we also have Pot. Energy.

b)

ground state energy:-

$$= \frac{p^2}{2m} + \frac{1}{2}kx^2.$$

both ≈ 0.

$$\Delta p_x \cdot \Delta x = \frac{\hbar}{2}.$$

put $\Delta p_x = \frac{\hbar}{2(\Delta x)}$

⋮

Non commuting operators & uncertainity:-

$[A,B]=C.$

then $\underline{\Delta A \cdot \Delta B \geq \frac{1}{2}|\langle C\rangle|}$

x, P_x

we get

$[x,P_x]=i\hbar$

another operator.

$\therefore \quad \Delta x \cdot \Delta P_x \geq \frac{\hbar}{2}$

K, P:

commuting ✓.

$\therefore$ no uncertainity restriction.

- can measure both accurately.

* $\psi(x)=e^{-ix}$ is continuous, (how to say?) → draw $Re(\psi(x))$.

Finding $\psi(x)$:-

we will have $\psi(x,t)$; but we showed $\psi(x)$ also same eqn.
$\psi(x,t)=\psi(x)\cdot\phi(t)$.

$\hat{E}(\psi)=\hat{K}(\psi)+V(x)\psi(x)$

$$\boxed{i\hbar\frac{d}{dt}(\psi(x)) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)+V(x)\cdot\psi(x).}$$

$$(E)\psi(x)=\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}(\psi(x))+V(x)\psi(x)$$

eigen value.

} solve this to get $\psi(x)$. → ∞ soln
put boundary cond. to get limited solution.

now; **Hamiltonian (H) operator:**

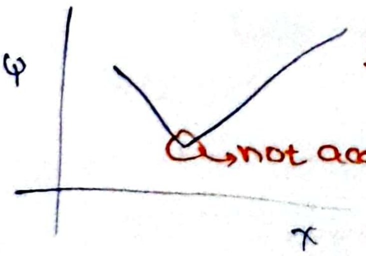
$$\hat{H}=\hat{K}+V(x) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)$$

Boundary conditions:-

1) finite, single valued, continuous.

2) diffrentiable. (so as to apply $\hat{P}_x$).

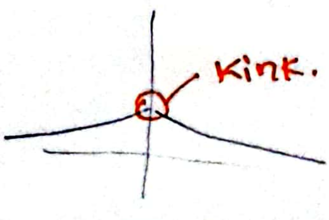
*



take care!

not acceptable! gotta be smooth.

$e^{-|x|}$ → unacceptable



kink.

* e^{x} → not acceptable.

* e^{-ix} → acceptable...

*Hermitian operator:-

an operator $\hat{O}$ is hermitian if

$$\boxed{\int_{-\infty}^{\infty} \psi_1^* \hat{O} \psi_2 \, dx = \int_{-\infty}^{\infty} (\hat{O}\psi_1)^* \psi_2 \, dx}$$

properties:

1) $\hat{O}$ expectation value is real.

$$\langle \hat{O} \rangle = \int_{-\infty}^{\infty} \psi^* \hat{O} \psi \, dx. \quad \left(\text{proof: show } \langle \hat{O} \rangle = \langle \hat{O} \rangle^* \right)$$

2) eigen values of $\hat{O}$ is real.

(proof; $\hat{O}\underset{\text{some eigenfu}^n.}{\psi} = \alpha \cdot \psi$.

$$\int_{-\infty}^{\infty} \psi^* \hat{O} \psi \, dx = \int_{-\infty}^{\infty} \alpha \cdot \psi \psi^* dx.$$

$= \alpha$ (eigen value is a no.; not a funct^n)

$$\int_{-\infty}^{\infty} (\hat{O}\psi)^* \psi \, dx = \int_{-\infty}^{\infty} \alpha^* \psi \psi^* dx.$$

$= \alpha^* \cancel{x}$ (eigen value is a number; not a function)

* $\hat{x}, \hat{P}_x, \hat{K}, \hat{E}$ are hermitian operators.

Hence, their eigen values (observables) are real quantities.

(what is eigen value?) $\rightarrow \delta(x - x_0)$

Solving ψ:

1) free particle:-

- no force is there
- $V = V_0$ (constant).

$$\therefore -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V_0 \psi = E\psi.$$

~~it happens that this is a eigen fu^n of $\hat{P}$~~

~~$\Delta P = 0$ for Ae^{ikx}.~~

~~$\Delta x \to \infty$~~

~~$\hat{P}\psi(x) = (\hbar k)\cdot \text{const} \cdot \psi(x)$.~~

$$\therefore \ddot{\psi} = -\frac{2m(E - V_0)}{\hbar^2} \psi$$

$\therefore$ let $\psi(x) = Ae^{\lambda x}$

$$\therefore e^{\lambda x}\lambda^2 = -\frac{2m(E-V_0)}{\hbar^2} e^{\lambda x}.$$

$$\boxed{K = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}}$$

$\rightarrow$ we will get $P_x = K\hbar = \sqrt{2m(KE)}$ obvi!

$E > V_0$:

$\lambda = \pm iK$

$$\therefore \boxed{\psi(x) = A \cdot e^{ikx} + B e^{-ikx}} \rightarrow \text{most general.}$$

as $\pm k = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$

here $\sigma_{P_x} = 0$

$\therefore \Delta x = \infty$.

- the $\psi(x) = A \cdot e^{ikx}$ cannot be normalized.

time dependent!

$\psi(x,t) = \psi(x) \cdot \phi(t)$

$\hookrightarrow e^{-i\omega t} = e^{-\frac{iEt}{\hbar}}$

$$\therefore \psi(x,t) = \underbrace{A \cdot e^{i(kx - \omega t)}}_{\text{wave moves } \rightarrow} + \underbrace{B \cdot e^{-i(kx + \omega t)}}_{\text{wave } \leftarrow}$$

$$\langle \hat{P}_x \rangle = \hbar k \cdot \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2}$$

case-ii: $E < V_0$ (tunneling effect). (we read; α-particles don't have needed energy; but still! they escape).

$$\lambda = \pm \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \pm K$$

real.

$$\psi(x) = C \cdot e^{Kx} + D \cdot e^{-Kx} \text{ general.}$$

as $x \to \infty$ $\quad e^{Kx} \to \infty$

$x \to -\infty$ $\quad e^{-Kx} \to \infty$

$\therefore \psi(x) = C \cdot e^{Kx}; \; x < 0$

$\quad\quad\quad\;\; D \cdot e^{-Kx}; \; x > 0.$

$C = D$ (for continuity).

$$\therefore \boxed{\psi(x) = C \cdot e^{-K|x|}}$$

not diff. at $x = 0$, if $C \neq 0$.

$\therefore C = 0$

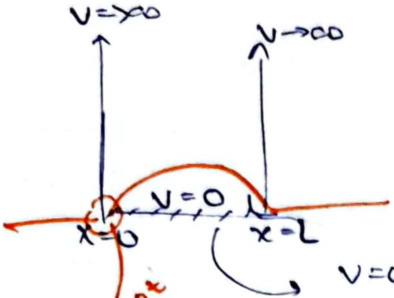
- no physical situation; for $E < V_0$ everywhere

But!; for some region; $E < V_0$ possible!

elsewhere ; $E > V_0$

24th sept

⇒ particle in a box:-



need not be diff. when '∞' potential spike.

$\psi(x)=0$ for $x\in\mathbb{R}-[0,L]$. (obvio).

$\psi(x) = A\sin(kx)+B\cos(kx)$.

equivalent.

$\therefore \frac{\partial^2\psi}{\partial x^2} = -k^2\psi$.

$\psi = A\cdot e^{+ikx}+B\cdot e^{-ikx}$.

$\psi(0)=0$.

$\Rightarrow B=0$

$\psi(L)=0$

$\Rightarrow \underbrace{A}_{\neq 0}\cdot\sin(kL)=0$

$$\boxed{k=\sqrt{\frac{2mE}{\hbar^2}}}$$

$kL = n\pi$

$$\boxed{L=\frac{n\pi}{k}}$$

$\therefore k^2=\frac{n^2\pi^2}{L^2}$

$$\boxed{E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}}$$

quantised!.

for one stationary state $\psi_n(x)$.

Normalize:

$|A|^2=\frac{2}{L}$.

$A=e^{i\theta}\sqrt{\frac{2}{L}}$ } let us take real.

* $\hat{H}\psi_n=E_n\psi_n$

E_n is a number.

but generally;

$\hat{H}\psi\neq E\psi$

if $\psi=c_1\psi_1+c_2\psi_2+\dots$

27.sept:

* Now; least energy $\neq E_0$

$= E_1$.

$\therefore$ Quantum systems posses "zero-point energy".

(in classical phy.; =0)

* one more approach:

$\frac{n\lambda_d}{2}=L$

$\lambda_d=\frac{2L}{n}=\frac{h}{p} \Rightarrow p=\frac{nh}{2L}$

$KE=E=\frac{p^2}{2m}=\frac{n^2\hbar^2\pi^2}{2mL^2}$

25th sept-2019.

Tut-7:

Operators.
wavefu^n.
Sch. equ^n for finding $\varphi(x)$.
particle in a box.

doubts:-

1) $\langle \hat{p} \rangle = \int_{-\infty}^{\infty} \varphi^* \hat{p} \varphi \, dx$

What if $\varphi(x)$ is not eigen function.

$\hat{p}\varphi = 'p'\varphi$

wont be same for all.

✓got it.

*

Q5)

(a) $\varphi(x) = A \cdot \left[\sin\left(\frac{\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right) \right]$

$= \sqrt{\frac{L}{2}} A \cdot \left[\varphi_1(x) + \varphi_2(x) \right]$

2) particle in box.

$\varphi(x) = A \cdot \left(\sin \frac{\pi}{L} x + \sin \frac{2\pi}{L} x \right)$

What about $\hat{E}, E$, $\hat{P}, P$ — need to collapse!.

(b) $\int_{-\infty}^{\infty} \varphi(x) \cdot \varphi^*(x) \, dx$

$= \frac{L}{2} A^2 \left(\int_{-\infty}^{\infty} \varphi_1^2(x) \, dx + \int_{-\infty}^{\infty} \varphi_2^2(x) \, dx \right.$

$+ \int_{-\infty}^{\infty} \varphi_1(x) \cdot \varphi_2(x) \, dx$.

both are orthogonal.

$\langle x \rangle = \int_{-\infty}^{\infty} \varphi x \varphi^* = 0$.

$\langle x^2 \rangle = \quad = ?$

wont be $p \cdot \varphi$. as not eigen.

$\langle p \rangle = \int_{-\infty}^{\infty} \varphi^* \left(-i\hbar \frac{\partial}{\partial x} \varphi \right) dx$.

$\langle p^2 \rangle = \int_{-\infty}^{\infty} \varphi^* \left(-\hbar^2 \frac{d^2}{dx^2} \varphi \right) dx$.

d)

energy mesurment is made $\longrightarrow \neq E$, $\hat{H}(E) \neq (\text{Energy}) \cdot (E)$

as $E \neq$ eigen.

$= \phi$

where $\hat{H}\phi = E\phi$

constant.

$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_1(x) + \varphi_2(x) \right)$

$\left| \int \phi^* \varphi \, dx \right|^2 = |C|^2 \ldots$

7) $\hat{H}\psi = -i\hbar\frac{\partial}{\partial t}\psi.$

$$\psi_n(x,t) = \psi_n(x)\cdot e^{-iE_nt/\hbar}$$

only for ψ_n, not for any ψ.

$\psi(x) = \frac{1}{\sqrt{2}}(\psi_1(x) + \psi_2(x)) \rightarrow$ not ~~a possible situation~~ stationary state. for one dim. box.

$$\psi(x,t) = \frac{1}{\sqrt{2}}\cdot\left(\psi_1(x)\cdot e^{-\frac{iE_1t}{\hbar}} + \psi_2(x)\cdot e^{-\frac{iE_2t}{\hbar}}\right)$$ (Batte knows).

Friday
27.sept.19

* zero point energy; by uncertainity.

$\Delta x_{max} = L$

$\Delta P_{min} = \frac{\hbar}{2L} \approx p$

$\therefore KE_{min} = \frac{p^2}{2m} = \frac{\hbar^2 \pi^2}{8mL^2}$?....

Quantum particle can't have zero energy.

where will the ψ go!

→ Orthogonality:-

ψ_1, ψ_2 are orthogonal if $\boxed{\int_{-\infty}^{\infty} \psi_1^* \cdot \psi_2 \, dx = 0}$

$\psi_n = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{n\pi}{L} \cdot x\right)$

* ψ_l & ψ_n are orthogonal for $l \neq n$.

for a Hermitian operator's domain;

eigen fun^{ns} of different eigen values are orthogonal.

$E\psi_{(x,t)} = i\hbar \frac{\partial}{\partial t} \psi(x,t)$

$E \cdot \phi(t) = i\hbar \frac{\partial}{\partial t} \cdot \phi(t)$.

$e^{\frac{-iE}{\hbar}t}$

→ time dependence:-

(write).

$\hat{H}\psi(x,t) = E\psi(x,t) = i\hbar \frac{\partial}{\partial t}\psi(x,t)$

$\psi(x,t) = \psi(x) \cdot \underset{e^{-i\omega t}}{\phi(t)}$.

now; dont think $\psi(x) \cdot \phi(t) = \psi(kx - \omega t)$

as $\boxed{\psi(x) \neq e^{ikx}}$ (aint true all the time).

$\therefore$

$\psi(x,t)$; if $= \psi(x) \cdot e^{-i\omega t}$.

then $|\psi(x)|^2$ dosen't change; because of $\phi(t)$ ....

take care.

hence "stationary state"!

uses!

* $\psi(x) \longrightarrow$ used to find P, E, KE

!!

$\psi(x)$ has all data in it!

$\longrightarrow$ used to find probability in $\overset{x_1}{B}, \overset{x_2}{B+b}$.

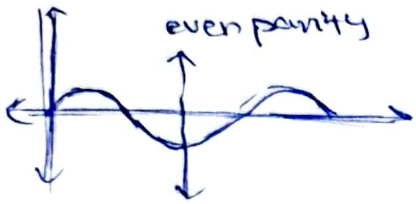
· uncertainity:-

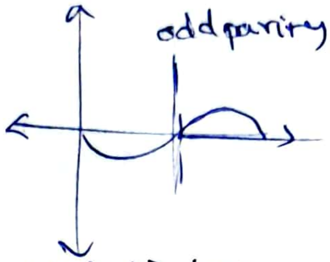
$\langle p \rangle = 0$. (as how much right; that much left).

$\langle p^2 \rangle \neq 0$.

$\langle x \rangle = \frac{L}{2}$? check!

symmetry:





* discussion: particle in 1D box.

* colours from Quantum dots:

Nanoscale semiconductor arrangement is called quantum dot.

" size dependent colour"

as $E_2 - E_1 = \frac{3\hbar^2\pi^2}{2m\underbrace{L^2}}$

this increases; colour becomes red.

Superposition:-

orthonormal eigen fn's.

$$* \quad \psi(x,t) = \sum C_n \psi_n(x) \cdot e^{-iE_nt/\hbar}$$

Now; $||\psi(x,t)||^2 = \psi_{x,t} \cdot \psi^*_{x,t}$

$=$ time dependent!

$\therefore$ this $\psi(x,t)$ is not a stationary state.

$* $ But $\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = |C_1|^2 + |C_2|^2 + \cdots$ (time independent!)

$\therefore \sum |C_i|^2 = 1.$

* see <slides>.

copenhagen → A place; where Quantum mechanics postulates were formulated.

---

* if a particle is in state ψ and an ideal measurment of variable A will yeild one of the eigen values 'α' (of $\hat{A}$) with probability $P(\alpha)$.

The state of system will change from ψ to ξ ($\hat{A}\xi = \alpha \cdot \xi$).

&

$$\underbrace{P(\alpha) = \left| \int_{-\infty}^{\infty} \xi^* \cdot \psi dx \right|^2}_{\text{yeilds the famous}} \rightarrow \text{copenhagen interpretation.}$$

$|\psi(x)|$ → to find particle.

→ Measurment:-

$$\psi(x,t) = \frac{1}{\sqrt{5}} \cdot \psi_1(x) \boxed{e^{-iE_1t/\hbar}} + \frac{2}{\sqrt{5}} \cdot \psi_2(x) \cdot \boxed{e^{-iE_2t/\hbar}}$$

only corresponds to phase; no change in $||\psi_1(x,t)||^2$ & $||\psi_2(x,t)||^2$.

if system is not disturbed;

$\psi(x,t)$ will "evolve" as such.

i.e. individual phase keep changing.

Ehrenfest theorem:- "differentiating expectation values!

$$\frac{d}{dt}\langle A\rangle = \frac{\langle [H,A]\rangle}{i\hbar} + \left\langle \frac{\partial A}{\partial t}\right\rangle$$

Mathematical ground:-

$[A,BC] = B[A,C] + [A,B]C$ (rel. order of B,C won't change).

$[\hat{A}B,C] = A[B,C] + [A,C]B$

$$[x^n,p] = x^n\left(-i\hbar\frac{\partial\psi}{\partial x}\right) - \left(-i\hbar\frac{\partial(x^n\psi)}{\partial x}\right)$$

$$\boxed{[x^n,p] = i\hbar\cdot n x^{n-1}\psi}$$

$[\hat{x},\hat{p}] = i\hbar$

to do $[x,p^2] = [x,pp] = p[x,p] + [x,p]p$
$= 2i\hbar\cdot\hat{p}$

→ equation of motion of "operator average":-

$$\boxed{\frac{d}{dt}\langle A\rangle = \left\langle\frac{\partial A}{\partial t}\right\rangle + \frac{i}{\hbar}\langle[H,A]\rangle}$$ (proof involved; $\hat{H}\psi = i\hbar\frac{\partial}{\partial t}\psi$)

& $\int\psi^* HA\psi = \int (H\psi)^* A\psi$
Hermitian

if A is not explicitly time dependent; $\hat{x}, \hat{p}$ ✓

$$\boxed{\frac{\partial\langle A\rangle}{\partial t} = \frac{i}{\hbar}\cdot\langle[H,A]\rangle}$$

if $[H,A]=0$;
A is constant of motion.

$$\frac{\partial\langle p\rangle}{\partial t} = -\langle V'(x)\rangle$$

$\langle F(x)\rangle$

Feel!

$$\boxed{\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m}}$$

Tut-8:

postulate is

$$\hat{H}\psi = i\hbar\frac{\partial}{\partial t}\psi \text{ (i guess)}.$$

this will give.

$$\hat{H}\psi_{(x)} = E\psi_{(x)}$$

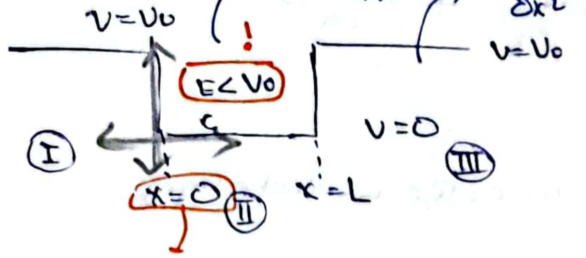
if $\psi(x,t) = \psi(x)\cdot\phi(t)$
then $\psi(x,t)$ is stationary State.

$\psi(x,t) = \sum c_i\psi_i(x,t)$
satisfies
$$\hat{H}\psi = i\hbar\frac{\partial}{\partial t}\psi$$

$\psi(x) = \sum c_i\psi_{i(x)}$
dosent satisfy.
$$\hat{H}\psi = E\psi.$$

* Body in a finite square well:-

$\frac{\partial^2}{\partial x^2}\psi = \frac{-2mE}{\hbar^2}\psi$. $\frac{\partial^2\psi}{\partial x^2} = \frac{-2m(E-V_0)}{\hbar^2}\psi$.



here; ψ gotta be differentiable....

→ ψ's differentibility is overlooked; only in case of $V \to \infty$.

Step 1: divide into region 1, 2, 3.

R I: $\frac{\partial^2\psi}{\partial x^2} = \frac{2m(V_0-E)}{\hbar^2}.\psi = \alpha^2\psi$.

& $\psi = A\cdot e^{\alpha x} + \underbrace{B\cdot e^{-\alpha x}}_{B=0;\text{ as boundary conditions.}}$

$\psi_1 = A\cdot e^{\alpha x}$

R II: $\frac{\partial^2\psi}{\partial x^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$.

$\psi_2 = \cancel{C\cdot e^{ikx} + D\cdot e^{-ikx}}$ → $C\sin kx + D\cos kx$.

now; we have α, k ... (interdependent).

R III: $\psi_3 = B\cdot e^{-\alpha x}$

on total; A, B, C, D, E are unknowns.

equns:
1) cont. at x = 0
2) diff. at x = 0
3). cont. at x = L
4). diff. at x = L
5) Normalisation!

Yippe! 5 var. — 5 equn.

A.1 = D.

$B\cdot e^{-\alpha L} = C\cdot \sin kL + D\cdot \cos kL$

⇒ $\cancel{\boxed{C=D}}$ as img. part is zero.

$A\cdot\alpha = Ck$.

$C = A\left(\frac{\alpha}{k}\right)$

⇒ $\boxed{\psi_2 = C\cos(kx)}$

$C\sin kL + D\cos kL = B\cdot e^{-\alpha L}$

$Ck\cdot\cos kL - Dk\sin kL = -\alpha B e^{-\alpha L}$

$\frac{D}{C} = \frac{\alpha}{k}$.

$\frac{k}{\alpha}(\tan kL - \frac{\alpha}{k})$

$\frac{k}{\alpha}$. $D\sin kL - C\cos kL$

$(\frac{\alpha}{k}\cdot \tan kL + 1)$

$= C\sin kL + D\cos kL$.

9th oct. 2019.

Tut 8:

Q1) Hermitian if $H = H^{\dagger}$ ↳ dagger. (we know; that integration definition.)

$\hat{C} = \hat{A}\hat{B}$; $A = A^{\dagger}$ as A is Hermitian.

$\boxed{\hat{C} = \hat{C}^{\dagger}} \Rightarrow \hat{A}\hat{B} = (\hat{A}\hat{B})^{\dagger} = (\hat{B}^{\dagger}.\hat{A}^{\dagger}) = B\cdot A$

$\therefore AB = BA$.

SYNTAX: product of operators $\hat{A}, \hat{B}$ is $\hat{A}\hat{B}$ (or) $\hat{B}\hat{A}$.
$\Downarrow \quad \hat{A}(\hat{B}\psi) \quad \hat{B}(\hat{A}\psi)$.

* $[A, B+C] = [A,B] + [A,C]$

$[A,BC] = \underbrace{B[A,C] + [A,B]C}_{\text{B, C order won't change.}}$

~~rough~~.

$\psi(x,y,z) = e^{-ikx} * e^{-iky} * e^{+ikz}$.

$$\frac{\tan kL - \frac{\alpha}{k}}{\frac{\alpha}{k}.\tan kL + 1} = \frac{\alpha}{k}.$$

Satisfying value is E.

$A^2 \tan kL - \tan kL + 2A$

$A^2 \tan kL + 2A - \tan kL = 0$

$\tan kL = \frac{2A}{1-AL}$.

$$\left.\frac{2\tan\frac{kL}{2}}{1-\tan^2\frac{kL}{2}} = \frac{2\cdot(\alpha/k)}{1-\left(\frac{\alpha}{k}\right)^2}\right\} \rightarrow$$ α, k; have only one variable ⇒ E (energy); solve numerically, or graphically to get "QUANTISED" values of E.

$\Updownarrow$

$\boxed{\tan\frac{kL}{2}} = \frac{\alpha}{k}$

(or)

$\boxed{\cot\frac{kL}{2}} = -\left(\frac{\alpha}{k}\right)$

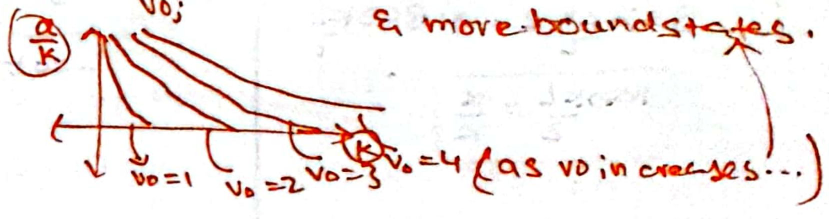
fixed graphs — vary with V_0;

BEWARE

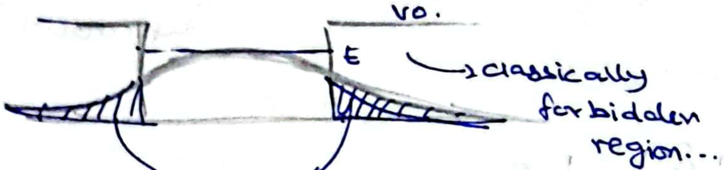
No. of xct points gives us no. of bound states.

* (not infinite here...).

→ as V_0 increases; it admits more & more bound states.



diagrams:-



* Particle is not "contained" by a finite well.

(think; what happens with $\bar{e}$ in a metal volume.)

→ Approximate energy expression:-

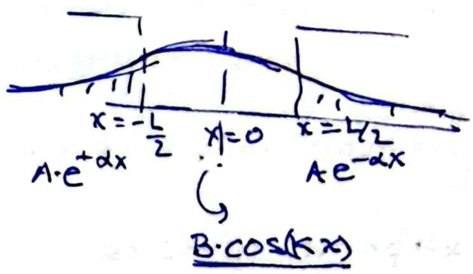
$$\boxed{\text{penetration depth } (\delta) = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(V_0-E)}}}$$

∴ effective dimension of well $= L+2\delta$.

$$\therefore \text{ Average energies} = \frac{n^2\hbar^2\pi^2}{2m(L+2\delta)^2}$$ we won't use, i.g.

*comparision of $\bar{e}$; in finite & infinite well:- see; lec 15, slide 18..

* even parity states:-



$$\therefore \quad B\cos\frac{kL}{2} = A\cdot e^{-\alpha\cdot\frac{L}{2}}$$

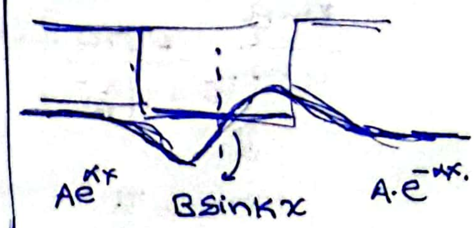
$$B(-\sin k\tfrac{L}{2})\cdot k = A(-\alpha)\cdot e^{-\alpha L/2}$$

$$\therefore \quad B\sin\frac{kL}{2}\cdot k = \alpha\cdot B\cos\frac{kL}{2}$$

$$\boxed{\tan\frac{kL}{2} = \frac{\alpha}{k}}$$

$n = 1, 3, 5, 7, \dots$

odd parity state:-



$$\therefore \quad B\sin\frac{kL}{2} = A\cdot e^{-\frac{\alpha L}{2}}$$

$$B\cos\frac{kL}{2}\cdot k = A(-\alpha)\cdot e^{-\frac{\alpha L}{2}}.$$

$$B\cos\frac{kL}{2}\cdot k = -\alpha\cdot B\sin\frac{kL}{2}$$

$$\boxed{-\cot\frac{kL}{2} = \frac{\alpha}{k}.}$$

$n = 2, 4, 6, \dots$

→ How many bound states; for a given value V_0, L:-

(semi infinite well case).

damn. you.

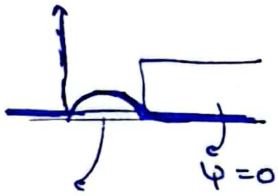
Solutions are

$$\cot\left(\frac{KL}{2}\right) = -\frac{\alpha}{K}.$$

* if $E = V_0$;

$$K = \frac{\sqrt{2mV_0}}{\hbar}$$

&



$\psi = 0$; as $E - V = 0$.

$\psi = \cancel{A\cos(Kx)} + A\sin KL$.

$\therefore$ $A\cos KL = 0$. & slope $= 0$

& $A\sin KL = 0$ too!; continuity of $\psi/_{x=L}$.

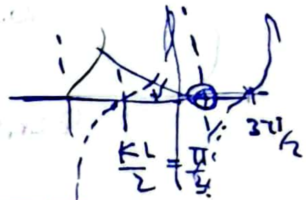
* for no. of solutions; $\frac{\alpha}{K} \to 0$; $\alpha \to 0$, $E \to V_0$

i.e. $K \to \left(K_0 = \frac{\sqrt{2mV_0}}{\hbar}\right)$

$\therefore$ if $K_0 \in \left[(2n_1-1)\frac{\pi}{L}, (2n_1+1)\frac{\pi}{L}\right)$

then n_1 bound states.

$$V_0 = \frac{\hbar^2 K_0^2}{2m} \ldots$$



for e.g. $K_0 (2n-1)\frac{\pi}{2}$.

$\left([2n_1-1)\frac{\pi}{L}, (2n_1+1)\frac{\pi}{L}\right)$

then; n_1 is our argum

Well's length is $\frac{L}{2}$!

See that length of well nicely ;)

we finally get $n_{max} = \{\text{something}\}$.

$$\boxed{n_{max} = \frac{1}{2} + \sqrt{\frac{V_0}{\left(\frac{\pi^2\hbar^2}{2mL^2}\right)}}}$$

L = length of semiinfinite well.

* Some example;- if we get $n_{max} = 3.76$;

we have 4 bound states.

* plane waves. $\varphi(x) = Ae^{ikx}$.

(cannot normalise.)

Quantum mechanics in 3D:-

* $\psi(\vec{r}) = A \cdot e^{i(\vec{k}\cdot\vec{r})}$

$= A \cdot e^{i(k_x \cdot x + k_y \cdot y + k_z \cdot z)}$

$= \psi(x)\,\psi(y)\,\psi(z).$

* for a cuboid:- (standard, $V=0$).

$\hat{H}\psi = E\psi.$

$\frac{-\hbar^2}{2m}\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)\psi = E\psi.$

$\therefore \ \Sigma(\hat{H}_x\psi) = E\psi.$

$\therefore$ see sol^n $H_x\psi = E_x\psi.$

we have $\psi_x = \sqrt{\frac{2}{L_x}} \cdot \sin\left(\frac{n_x \pi x}{L}\right)$

$\therefore \ \psi(x,y,z) = \sqrt{\frac{8}{V_{cuboid}}} \cdot \sin\frac{n_x\pi}{L}x \cdot \sin\frac{n_y\pi}{L}y \cdot \sin\left(\frac{n_z\pi z}{L}\right).$

& $E = E_x + E_y + E_z$

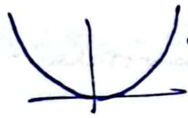
$= \frac{\hbar^2\pi^2}{2m}\left(\Sigma\frac{n_x^2}{L_x^2}\right)$

Degeneracy:-

if $g = n$; n-fold degenaracy.

degeneracy level.

Quantum Harmonic Oscillator:-



$*$ $F=-kx \Rightarrow \underbrace{V(x)=\frac{1}{2}kx^2}$.

in quantum mechanics; we rarely bring up "force".

Sch. equ^n:

$$\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{1}{2}kx^2\right)\psi = E\psi.$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar}\left(E-\frac{1}{2}kx^2\right)\psi=0.$$

$\boxed{\text{Hermite polynomials.}}$

what will be the solutions? tough! $\psi(x)=Ae^{-\alpha x^2}\left(\sum c_i x^i\right)$

put back in the; & solve...

Ground state:-

$$\psi(x)=A\cdot e^{-\alpha x^2}$$

put in sch. equ^n;

$$\left(4\alpha^2-\frac{mk}{\hbar^2}\right)x^2+\left(\frac{2mE_0}{\hbar^2}-2\alpha\right)=0.$$

$$\therefore \quad 4\alpha^2=\frac{mk}{\hbar^2}\;;\quad \frac{2mE_0}{\hbar^2}=2\alpha.$$

we know; $\frac{k}{m}=\omega^2$.

$$\therefore \quad 4\alpha^2=\frac{m^2\omega^2}{\hbar^2}$$

$$\boxed{\alpha=\frac{m\omega}{2\hbar}}$$

$$\Rightarrow \frac{2mE_0}{\hbar^2}=\frac{2m\omega}{2\hbar}$$

$$\boxed{E_0=\frac{1}{2}\hbar\omega}\ !!$$

Ground state energy.

$\omega=\sqrt{\frac{k}{m}}$ is a constant of an situation.

1^{st} e.s:-

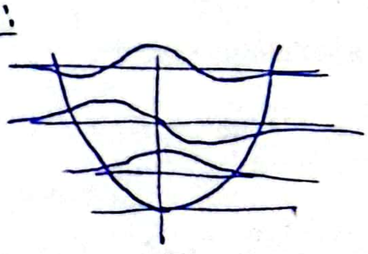
$$\psi=Ax\cdot e^{-\alpha x^2}.$$

we again get $\alpha=\frac{m\omega}{2\hbar}$ & $E_1=\frac{1}{2}\cdot(3)\hbar\omega$.

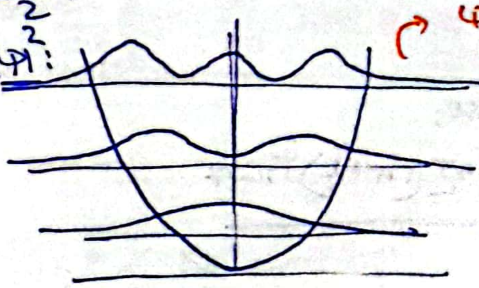
$\therefore$ Energies are:- $\frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \frac{7}{2}\hbar\omega, \ldots$

zero point energy $=E_0=\frac{\hbar\omega}{2}$.

ψ:



$|\psi|^2$:

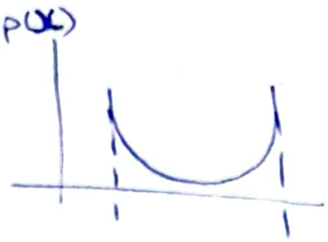


$\psi(x)$ extends beyond the classical turning points.

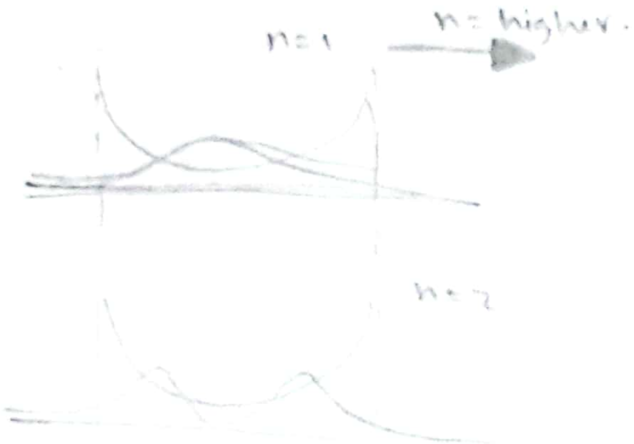
* classical probability of finding particle :-

$$P(x) \propto \frac{1}{v(x)}$$

$$P(x) = K \cdot \frac{1}{\sqrt{A^2 - x^2}}$$



* Quantum probability vs classical p.



* Uncertainity in G.S :-

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle = \frac{\hbar}{2m\omega}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle} = \sqrt{\frac{\hbar}{2m\omega}}$$

$$\langle p \rangle = 0$$

$$\langle p^2 \rangle = \left(\sqrt{\frac{\hbar m\omega}{2}}\right)^2$$

$$\therefore \Delta p = \sqrt{\frac{\hbar m\omega}{2}}$$

$$\therefore \Delta p \cdot \Delta x = \frac{\hbar}{2}$$

(i.e. G.S. is a minimum uncertainity state).

→ in 3D :-

$$\psi = \psi(x)\,\psi(y)\cdot\psi(z).$$

$$\therefore E_{net} = \left((n_x + n_y + n_z) + \frac{3}{2}\right)\hbar\omega$$ only when isotropic!!

i.e. only when
$\omega_x = \omega_y = \omega_z$.
i.e. $k_x = k_y = k_z$ ....
don't write blindly.

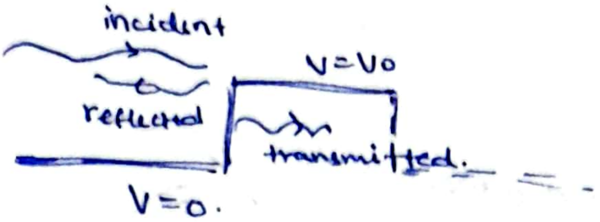
otherwise;

$$E = \sum \left(n_x + \frac{1}{2}\right)\hbar\omega_x$$

Free state problems:- Now; the particle is not bounded!.

- Scattering,
  tunneling.

* eg: A particle, coming from left; onto a step potential.



* if a particle is bounded-

; probability current is 0:

⇒ Probability current:-

$$i\hbar\frac{\partial}{\partial t}\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\cdot\psi. \quad (1)$$

$$-i\hbar\frac{\partial}{\partial t}\psi^* = \frac{-\hbar^2}{2m}\frac{\partial^2\psi^*}{\partial x^2} + V(x)\cdot\psi^* \quad (2)$$

$\psi^* \times (1) - \psi \times (2)$

$$\therefore \boxed{i\hbar\left(\frac{\partial}{\partial t}\psi\psi^*\right) = \frac{-\hbar^2}{2m}\left[\psi^*\frac{\partial^2\psi}{\partial x^2} - \psi\frac{\partial^2\psi^*}{\partial x^2}\right]}$$

wild thoughts:-

1) for free particle;
maybe
$\psi(x,t) \neq \psi(x)\cdot\psi(t)$
as it gotta be a travelling wave..

(but even then; $\hat{H}\psi = i\hbar\frac{\partial}{\partial t}\psi$ holds na!)

identity: $\frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right) = \psi^*\frac{\partial^2\psi}{\partial x^2} - \psi\frac{\partial^2\psi^*}{\partial x^2}$

$$\therefore i\hbar\cdot\frac{\partial}{\partial t}(\psi^*\psi) = \frac{-\hbar^2}{2m}\cdot\frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)$$

$$\therefore \frac{\partial}{\partial t}(\psi^*\psi) + \frac{\hbar}{2mi}\cdot\frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right) = 0.$$

if a particle is "moving"; then prob-current is finite.

$$\boxed{\frac{\partial\rho}{\partial t} + \frac{\partial J}{\partial x} = 0} \rightarrow \text{think like } \boxed{Jdt = \rho dx} ??....$$

J(x,t) is the "current" associated with charge density ρ.

ρ = probability

J =

$$\boxed{J = \frac{\hbar}{2im}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\cdot\frac{\partial\psi^*}{\partial x}\right)} \quad \boxed{J = \frac{\hbar}{m}\cdot Im\left(\psi^*\frac{\partial\psi}{\partial x}\right)}$$

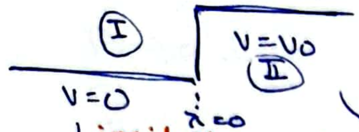
units: s^{-1}, $[Time]^{-1}$.

* if $\psi = A\cdot e^{ikx}$;

$$J = \frac{\hbar}{m}\cdot Im(A^2 e^{-ikx}\cdot ik\cdot e^{ikx}) = \frac{p}{m}|\psi|^2 = v\cdot|\psi|^2.$$

case-1) $E > V_0$.

$$\boxed{J = \frac{\hbar}{m} \cdot Im\left(\psi^* \frac{\partial \psi}{\partial x}\right)}$$



incident, reflected

$\psi_1 = A \cdot e^{ik_1x} + B \cdot e^{-ik_1x}$.

$k_1 = \frac{\sqrt{2mE}}{\hbar}$.

$\psi_2 = \underbrace{C \cdot e^{ik_2x}}_{\text{transmitted}} + D e^{-ik_2x}$.

no significance here

$\therefore D = 0$.

$\therefore \psi_1 = \underbrace{A \cdot e^{ik_1x}}_{P=\hbar k_1} + \underbrace{B \cdot e^{-ik_1x}}_{P=-\hbar k_1}$; $\psi_2 = \underbrace{C \cdot e^{ik_2x}}_{P=\hbar k_2}$.

continuity at 0.

$A + B = C$.

diff.

$(A-B) k_1 = C k_2$.

$A - B = C \cdot \frac{k_2}{k_1}$.

$2A = \frac{C(k_1+k_2)}{k_1}$

$C = \left(\frac{2k_1}{k_1+k_2}\right) A$. $\Rightarrow B = \left(\frac{k_1-k_2}{k_1+k_2}\right) A$.

$J = \frac{\hbar}{m} \cdot Im\left(\psi^* \frac{\partial \psi}{\partial x}\right)$

we cannot normalize the ψ_1 or ψ_2.

lets see probability current:-

Here $J = v \cdot |\psi(x)|^2$ (as $A \cdot e^{ikx}$ form).

$v = P/m$.

$J_1 = J_{incident} + J_{reflect}$

$= +\frac{\hbar k_1}{m} \cdot (|A|^2 - |B|^2)$

$J_2 = J_{trans} = \frac{\hbar k_2 |C|^2}{m}$.

$J_{incident} = |J_{reflection}| + J_{transmitted}$.

Reflection coefficient $(R) = \frac{J_R}{J_i} = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2$

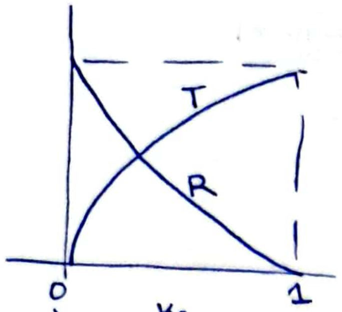
transmission coefficient $(T) = \frac{J_T}{J_i} = \frac{4k_1k_2}{(k_1+k_2)^2}$

$\therefore \boxed{R+T=1}$

* $K_1 = K\cdot\sqrt{E}$ (our case is $E > V_0$).

$K_2 = K\cdot\sqrt{E - V_0}$

$\therefore \frac{K_2}{K_1} = \sqrt{\left(1 - \frac{V_0}{E}\right)}$ & $R = f\left(\frac{K_2}{K_1}\right)$; $T = f\left(\frac{K_2}{K_1}\right)$



$T + R = 1$

0 meaning $E = V_0$ — body will probably be reflected

1 meaning $E >>> V_0$. — body will be transmitted.

conclusion:-

classically;

if $E > V_0$; the body "gotta" be transmitted;

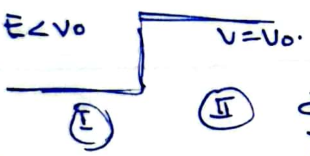
but

Quantumly;

the body has finite probability of reflecting.

---

case-II) $E < V_0$:-



$\frac{\partial^2\varphi}{\partial x} = -k_1^2\varphi.$

$\varphi_1 = A\cdot e^{+ik_1x}$ } right. $p = \hbar k_1$

$+ B\cdot e^{-ik_1x}$ left. $p = -\hbar k_1$.

$\frac{\partial^2\varphi}{\partial x} = k_2^2\cdot\varphi.$

$\varphi = C\cdot e^{k_2x} + D\cdot e^{-k_2x}$ } $\rightarrow \varphi_{11}(\infty) = 0$

$\therefore C = 0.$

But!

please understand.

e^{ikx} (imaginary function) $\rightarrow$ not normalizible

e^{kx} $\longrightarrow$ normalizable in $-\infty$ to 0.

$\angle$ $\boxed{\varphi_{11} = C\cdot e^{-k_2x}}$

real function; probability current $= 0$. !

solving:

continuity: $A + B = C$

slope : $Ak_1 - Bk_1 = C(-k_2)$

$A - B = -\frac{k_2}{k_1}\cdot C$

$\therefore B = \left(\frac{ik + \alpha}{ik - \alpha}\right)A$; $C = \frac{2ik}{ik - \alpha}\cdot A.$

note here: $\boxed{J_{reflected} = J_{incident}}$.

"try it out; $Im\left(\varphi^* \frac{\partial}{\partial x}\varphi\right) = 0$

$J_{transmission} = 0$.

∴ if $E < V_0$;

reflection coefficient $= 1$.

transmission coefficent $= 0$.

But note; $\psi \neq 0$ in the transmission region.

2. The particle can penetrate the barrier....

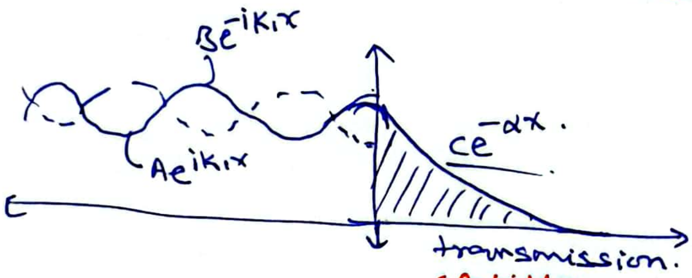
(existance is there; current isn't there).

↳ Like water in a pit; Of a flowing river.

* on right :

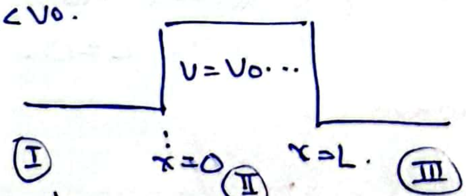
$$|\psi(x)|^2 \propto \left(\frac{4k^2}{k^2+\alpha^2}\right) e^{-2\alpha x}.$$

* Penetration depth $\delta = \frac{1}{\alpha} = \frac{\hbar^2}{\sqrt{2m(V_0-E)}}$



* STEP POTENTIAL:

$E < V_0$.



Solve to obtain reflection & transmission coefficients.

I: $Ae^{ik_1x} + Be^{-ik_1x}$

II: $Ce^{\alpha x} + De^{-\alpha x}$

no limiting $x \to \infty$ $x \to -\infty$; hence both terms; in general equation.

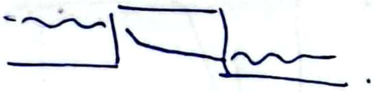
III: $\boxed{Fe^{ik_1x}}$

no e^{-ik_1x} term as; only transmitted wave is there. Note

Transmission coefficient; $= \frac{v|\psi(x)|}{v\text{-}|\psi(x)|}$

$= \left(\frac{F}{A}\right)^2$.

$\sinh(x) = \dfrac{e^{x}-e^{-x}}{2}$.

$$\boxed{\frac{1}{T} = 1 + \frac{1}{4}\left(\frac{\alpha}{k}+\frac{k}{\alpha}\right)^2 \sinh^2(\alpha L)}$$ — in  case.

$$\underbrace{k = \frac{\sqrt{2mE}}{\hbar}\;;\; \alpha = \frac{\sqrt{2m(V_0-E)}}{\hbar}}_{\text{obvious.}}$$

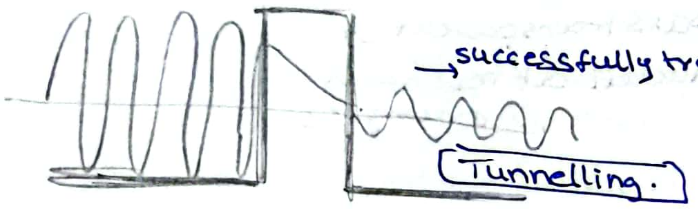
$$\therefore \boxed{T = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{E(V_0-E)}\right)\sinh^2(\alpha L)\right]^{-1}}$$

$$\left(1 + \frac{1}{4}\cdot\frac{V_0^2}{E(V_0-E)}\sinh^2(\alpha L)\right)^{-1}$$

two factors: V_0 (i.e. α) height
L. width.

(increase in any; decreases T).

*



Barrier penetration depth $= \dfrac{1}{\alpha} = \dfrac{\hbar}{\sqrt{2m(V_0-E)}}$

Limiting cases:-

1) $\alpha L >> 1$

2) $\alpha L << 1$

3) if $E > V_0$:

Simple modification; $\alpha = ik$; $k = \sqrt{\dfrac{2m(E-V_0)}{\hbar^2}}$

$$\therefore\; T = \left[1 + \frac{1}{4}\cdot\frac{V_0^2}{E(E-V_0)}\sin^2(kL)\right]^{-1}$$

→ Transmission Resonance:-

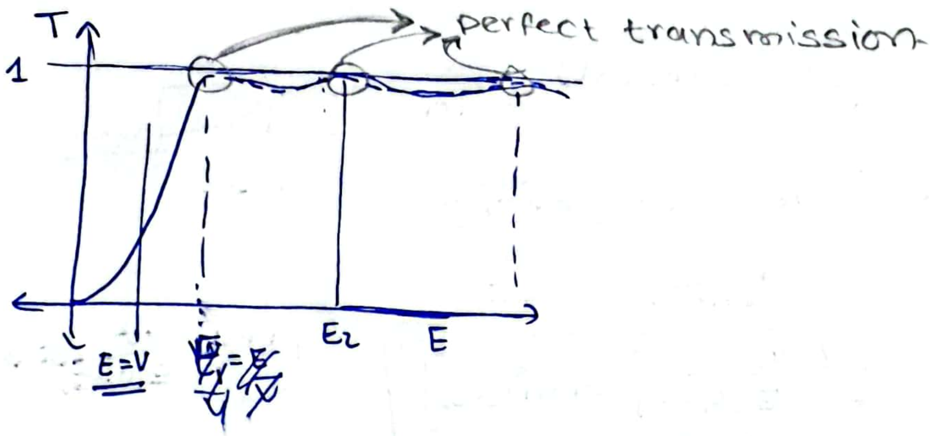
$\therefore$ for $kL = n\pi$; $T = 1$

transmission resonance as $T=1$.

$B=0$ in this case.?.

Transmission resonance:-

finally; for  thing;



* ramsaur effect: Nobel gases; transparent to e of certain energy.

Size resonance: neutrons pass transparently; through nuclei at resonant energies

* $KL = n\pi$

i) $\sqrt{2m(E-V_0)} = \dfrac{n\pi\hbar}{L}$

$$\boxed{E = V_0 + \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

– scanning tunneling microscope

for transmission resonance.

ii) or; $L = \dfrac{n\pi}{K} = \dfrac{n\pi}{2\pi}\cdot\lambda = n\left(\dfrac{\lambda}{2}\right)$

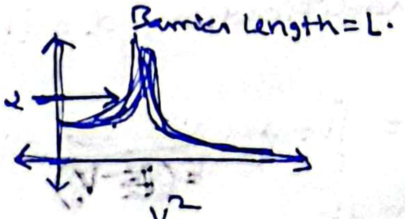
λ = wavelength of particle in barrier region.

∴ for transmsn resonance;

$$\text{Length}_{\text{barrier}} = n \times \frac{\lambda}{2}.$$

→ α-decay:-

λ = decay rate = $f \cdot T(E)$



$f = \dfrac{1}{\frac{2R}{v}}$ frequency.

$T(E) = \left(1 + \dfrac{V^2}{4E(V-E)}\cdot \sinh^2(\alpha L)\right)^{-1}$

V is appropiately taken.

$t_{1/2} = \dfrac{\ln 2}{\lambda}$

tut - last 2nd:

Statiscs:

MB: $\frac{N_i}{g_i} = e^{\alpha} \cdot e^{-\beta E_i}$

EB: $\frac{N_i}{g_i} = \frac{1}{e^{\alpha+\beta E_i} - 1}$

FD: $\frac{N_i}{g_i} = \frac{1}{e^{\alpha+\beta E_i} + 1}$

Q) even if $\bar{e}$ is kept in a box;
we will have spin degeneracy;
$s = 1/2$ (intrinsic to $\bar{e}$;
not to atom.)
$\therefore m_s = -\frac{1}{2}, \frac{1}{2}$.

Intro to quantum statistics:-

Founders of statistical physics:- L. Boltzman, J.W. Gibbs, J.C. Maxwell. - ignore particle-particle interaction.

* How many ways can the particles be distributed in energy levels; with constraints of fixed no. of particles & fixed total energy E.

Equilibrium distribution is the most probable way of distribution.

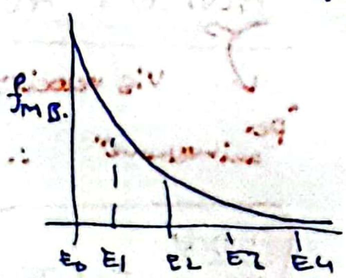
distinguishable particles
microstate can be more; for each macrostate.
called multiplicity.

indistinguishable particles
microstate = 1; for each macrostate.

1) Maxwell Boltzmann (MB):-

→ Assumes that each microstate is equiprobable.

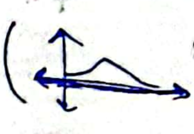
* if single degeneracy; there is observed; exponential decay in occurance of higher energy states.
(hence; not the real gases case)



$\therefore \boxed{f_{MB} = A \cdot e^{-\frac{E_i}{K_B T}}}$ for discrete energy levels.

proof is o.k.

$\boxed{\frac{n_i}{g_i} = A \cdot e^{-E_i/K_B T}}$ proof. ✓



(is when; density of states comes into picture).

* if energy levels are large; & are closely spaced;

$$f_{MB} = Ae^{-E_i/K_BT} \quad \text{(for discrete)}$$

i.e.

$$n_i = g_i \cdot A \cdot e^{-E_i/K_BT}$$

now: $\boxed{n(E)dE = g(E)\,dE \cdot A \cdot e^{-E/K_BT}}$

$n(E)dE \rightarrow$ no. of particles per unit volume with E to E+dE energy.

$g(E)dE \rightarrow$ density of states (no. of states in E & E+dE energy level). per unit volume

→ calculating density of states:-

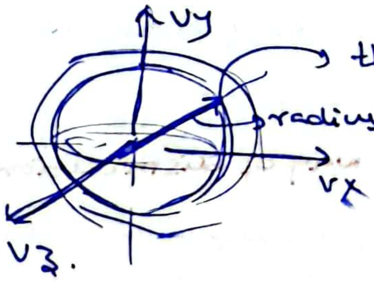
$E = \frac{1}{2}mv^2$ (for gas molecules).

So; now try to see possible values of $|\vec{v}|$.

∴ we see the different "directions" (v_x, v_y, v_z) to reflect upon the degeneracy of $|\vec{v}|$.

& this is now:-

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$



→ the volume in the shell "reflects" the degeneracy.

∴ $g(v)dv \propto 4\pi v^2 dv$

$g(v)\,dv = C \cdot 4\pi v^2 dv.$

v+dv	E+dE
v	E

now; $v = K \cdot \sqrt{E}$.

~~$g(E) \cdot \frac{dE}{2\sqrt{E}}$~~

$\boxed{g(v)dv = g(E)dE}$ feel karo.

∴ $g(E)dE = Av^2 dv$

$= AE \cdot \frac{1}{\sqrt{E}}$

$\boxed{g(E)dE = A\sqrt{E}}$

∴ now

$$n(v)\,dv = Av^2 e^{-\frac{1}{2}\frac{mv^2}{K_BT}} dv$$

"per unit volume"

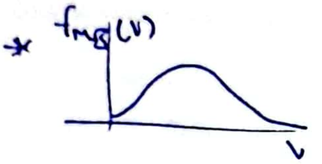
v is velocity

∴ $\int_{v=0}^{\infty} n(v)dv = \frac{N}{V}$ (! not N !)

$$\boxed{\frac{n(v)dv}{N/V} = 4\pi \left(\frac{m}{2\pi K_BT}\right)^{3/2} v^2 \exp(-mv^2/2K_BT) = f_{MB}(v)}$$

our chemistry formula.

fraction of molecules.

* 

Standard deviation $= \sqrt{\langle f_{MB}^2\rangle - \langle f_{MB}\rangle^2}$

$= \sqrt{\frac{3RT}{M} - \frac{8}{\pi}\frac{RT}{M}}$

$= \sqrt{3-\frac{8}{\pi}} \cdot \sqrt{\frac{RT}{M}}$ ✓

$\langle \text{Kinetic} \rangle_{\text{even molecule}} = \frac{3}{2} K_B T$

Statement of equipartition theorem..

→ **γ-function:-**

$$\gamma(a) = \int_0^\infty e^{-x} x^{a-1} dx$$

$$\gamma(a) = (a-1)\,\gamma(a-1)$$

$$\gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$\gamma\left(\frac{1}{2}\right) = \int_0^\infty e^{-x} \frac{1}{\sqrt{x}} dx$ $x = t^2$

$\gamma(1/2) = \int_0^\infty e^{-y} \sqrt{y}\, dy$ $\int_0^\infty e^{-t^2} \frac{1}{t} \cdot 2t\, dt$

$\gamma\left(\frac{1}{2}\right)\cdot\gamma\left(\frac{1}{2}\right) = \int_0^\infty\int_0^\infty e^{-(x+y)} \frac{\sqrt{y}}{\sqrt{x}}\, dx\, dy$

$x: r\cos\theta$, $y = r\sin\theta$

$\int\int e^{-r(\sin\theta+\cos\theta)}$

$\gamma\left(\frac{1}{2}\right) = 2\int_0^\infty e^{-t^2} dt$

$\gamma\left(\frac{1}{2}\right) = 2\int_0^\infty e^{-u^2} du$

$\gamma\left(\frac{1}{2}\right) = 2\sqrt{\int e^{-(x^2+y^2)} dx\, dy}$

$= 2\sqrt{\int e^{-r^2} r\, dr\, d\theta}$

$= 2\sqrt{\pi/2}\cdot\frac{1}{2}$

×———×

maxwel boltzman assumptions:-

1) distinguishable.
2) particles must be far apart.
3) no restriction on no. of particles in a level.
4) classical particles; no wave nature.

**** Validity:** (How to decide distinguishable or indistinguishable).
by 4th postulate;

$\boxed{\lambda_{\text{debroglie}} \lll \text{average distance}}$ b/w particles

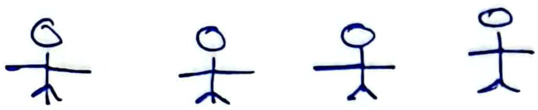
i.e. $\boxed{\frac{h}{\sqrt{2m\cdot\frac{3}{2}K_B T}} \ll \left(\frac{V}{N}\right)^{1/3}}$! dont forget $(\ )^{\frac{1}{3}}$.

∴ MB valid when; high T;
low particle density.
high mass.

PTO.

He at 273K ✓✓ Helium at 4.2K ××

when $\lambda_{db} << d$:

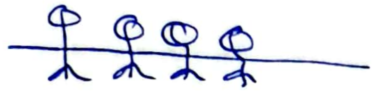


{identical particles}.

particles are distinguishable.

MB statistics applies.

if $d < \lambda_{db}$:



{identical}

particles are indistinguishable.

Bose-Einstein & FD statistics..

fermi dirac

* indistinguishable *.

$\psi(1,2) = \pm \psi(2,1)$

$\therefore$ two-particle wavefunction is either symmetric or anti symmetric. wrt interchange of particles.

$+$ } Bosons → integral spins. $h\nu(=1)$, $H(=1)$, He ....

photons — can be at the same place at same time.

$-$ } fermions → half integrals spins.

electrons, protons.

$e^-(=\frac{1}{2})$, $p^+(=\frac{1}{2})$, $n^0(=\frac{1}{2})$ ---

can't be at the same place at the same time.

unit of spin $= \hbar = 1.2\times10^{-()}$ Jouls second.

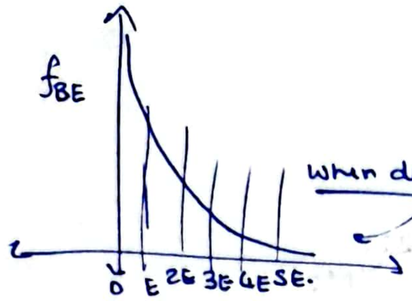
* Boson: more particles in same state is possible.

Fermions: if $\psi_a = \psi_b$; $\psi = \frac{1}{\sqrt{2}}\left(\psi_a(1)\psi_b(2) - \psi_b(1)\psi_a(2)\right) = 0$.

Fermions avoid each other.

→ Boson distribution:-

each macrostate; gives only one micro state.

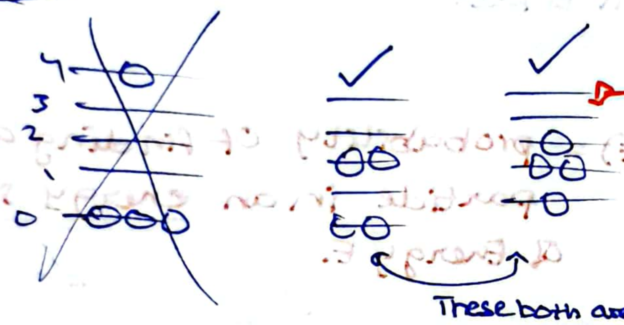


In bosonic systems; Lower energy levels; are more occupied than MB.

→ Fermion distribution:-

No more than two particles can be in a single level.
$(\uparrow, \downarrow)$

$E = 4E_0$



→ $4E_0$ not at all occupied.
! fermi distribution!

These both are equiprobable....
(as indistinguishable).

probability of occupation of energy levels:
sizable at lower energy levels.
sudden drop at higher energy levels.

→ Counting microstates: with degeneracy:-

1) MB distribution:-

$$n_i = N! \cdot \prod_{i=1}^{K} \frac{(g_i)^{N_i}}{N_i!}$$

3E — — — — g = 4
2E — — g = 2
E — — — g = 3
0 — g = 1

2) Bosonic distribution:-

— — — — g_i
N_i

no. of microstates: ${}^{N_i+g_i-1}C_{g_i-1}$

$$\therefore \text{count} = \prod_{i=1}^{K} {}^{N_i+g_i-1}C_{g_i-1}$$

Fermionic:

g_i

N_i

$g_i \geq N_i$ ways $= {}^{g_i}C_{N_i}$

$\therefore$ no. of microstates $= \prod_{i=1}^{n} {}^{g_i}C_{N_i}$

* In case of large no. of particles under the constraint of
(1) Fixed no. of particles.
(2) Fixed energy..
it can be; (rigorously) shown that:

B.E -

$$\boxed{f_{BE}(E) = \frac{1}{Be^{E/K_BT} - 1}}$$

$f(E)$ = probability of finding a particle in an energy state of Energy E.

FD:

$$\boxed{f_{FD}(E) = \frac{1}{He^{E/K_BT} + 1}}$$

if $g_i = 2$
then each state has $f(E)$
$\therefore$ net $= g_i \cdot f(E)$
all 'E' states.

against classical particle:-

$$\boxed{f_{MB}(E) = A \cdot e^{-E/K_BT}}$$

B,H ? by normalisation.

$\int f dE \neq 1$, as it is for each g_i of E_i...

$\int^{\infty} g_i(E) f dE = \frac{N}{V}$

to get that gas equation; we still need to take degeneracy (density of states) into account.

$$\boxed{f(E) = \frac{n_E dE}{g(E)dE}}$$ essentially.

after normalisation, we get:-

- $B = 1$, $\therefore$ $\boxed{f_{BE}(E) = \frac{1}{e^{E/K_BT} - 1}}$

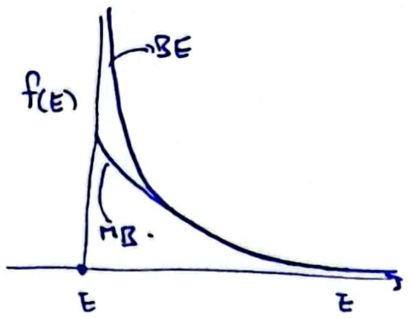
- $H = e^{-E_F/K_BT}$

$$\boxed{f_{FD}(E) = \frac{1}{e^{(E-E_F)/K_BT} + 1}}$$

E_F: fermi energy.

shouldn't be $-$.

Bose- E. statistics:-



as $E \to \infty$ $\quad$ $f_{BE} = f_{MB}$. (i.e. all classical particles behave as Bosons at low T.)

as $E \to 0$. $\quad$ f_{MB} = finite

$f_{BE} \to \infty$.

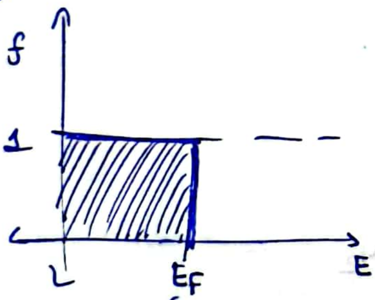
∴ at low temperatures, all particles drop to ground state

Hence;

Bose-einstein condensate

* Fermi-dirac statistics:-

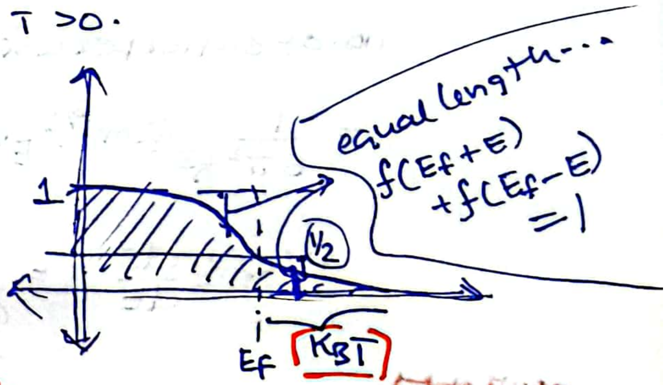
1) $T=0^+$



↳ fermi energy...

↳ defines highest energy at T=0K.

2) T > 0.



at T>0; the FD graph smears out.

* Fermi temperature:

$$\boxed{E_F = K_B \cdot T_F}$$

$$\boxed{T_F = \frac{E_F}{K_B}}$$

for $T > T_F$;

$f_{FD} \to f_{MB}$.

at high temperature:-

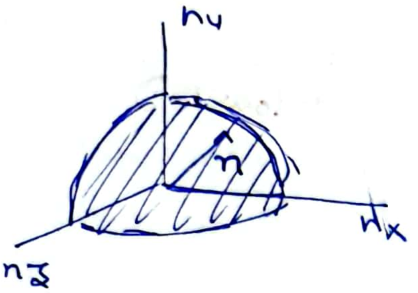
f_{BE}
f_{FD} $\to f_{MB}$.

Density of states; quantum picture:-

3D:

$$E_x = \frac{n_x^2 \hbar^2 \pi^2}{2mL^2}$$

$$E = \frac{\pi^2 \hbar^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2)$$



no. of states $\simeq$ volume of $\frac{1}{8}$th sphere

in that v_x, v_y, v_z it was proportional, here equal.

$$S = \frac{1}{8}\frac{4}{3}\pi n^3$$

$$n = \sqrt{2mE}\frac{L}{\pi\hbar}.$$

$$S = \frac{1}{6}\pi(2mE)^{3/2}\left(\frac{L}{\hbar\pi}\right)^3$$

$$V = L^3$$

no. of states per unit volume $= \frac{S}{V}$.

$$= \frac{1}{6\pi^2\hbar^3}(2m)^{3/2} \cdot E^{1/2}$$

$$g(E) = \frac{dS}{dE} = \frac{1}{4\pi^2\hbar^3}(2m)^{3/2}E^{1/2}$$

$$\hbar = \frac{h}{2\pi}$$

no. of states per unit energy per unit volume.

$$\therefore \boxed{g(E)_{3D} = \frac{2\pi}{h^3}(2m)^{3/2}E^{1/2}}$$

2D:

$$E = \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2)$$

$$S = \frac{1}{4}\pi n^2$$

$$\boxed{g(E)_{2D} = \frac{2\pi m}{h^2}}$$

1D:

$$S = n = \sqrt{2mE}\frac{L}{\pi\hbar}$$

$$\boxed{g(E) = \frac{(2m)^{1/2} \cdot E^{-1/2}}{h}}$$

3D $g(E) = \frac{2\pi}{h^3}(2m)^{3/2} E^{1/2}$

2D $g(E) = \frac{2\pi m}{h^2}$

1D $g(E) = \frac{(2m)^{1/2} E^{-1/2}}{h}$

→ without considering particle's spin.

if $\bar{e}$ are filled up;

$g(E) \times 2$ = density of states.

* if they just ask; "Number of states per unit energy"; $\boxed{g(E) \times \text{Volume}}$ is our answer.

* $g(p)dp = g(\nu)d\nu = g(E)dE = g(k)dk = \ldots$

Blah!

Phase space:-

system of n particles; defined by q_i, p_i

$q_i = (x_i, y_i, z_i)$

$p_i = p(x_i, p_{y_i}, p_{z_i})$

instantaneous state; defined by a point in phase space.

red box.

$$\boxed{g(p)dp = \frac{4\pi p^2 dp}{h^3}}$$

} B.E. statistics application.

→ Planks law for blackbody:-

$$g(p)dp = \frac{4\pi p^2}{h^3}dp.$$

to account for the two polarisations;

$$g(p)dp = 2 \times \frac{4\pi p^2}{h^3}dp.$$

ν = frequency.

$$\boxed{g(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu}\ !$$

no. of photons occupied $E = h\nu$ to $h(\nu + d\nu)$

$$\boxed{n(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu \; \frac{1}{e^{\frac{h\nu}{k_BT}} - 1}}$$

→ B.E statistics.

$$\therefore\ u(\nu)d\nu = n(\nu)d\nu(h\nu) = \boxed{\frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1}d\nu}$$

F.D. statistics applⁿ :-

Free é gas theory of metals :

{pending}.

* **Density of é :-**

$$n_e = Z_c \cdot N_A \cdot \frac{d_m}{A}$$

contribution from each atom

now; even for é; FD statistics applied :-

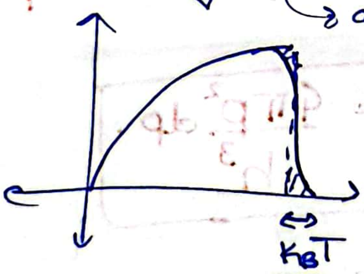
3D:

$$g(E) = \frac{4\pi}{h^3}\cdot\sqrt{2E}\cdot m_e^{3/2}\, dE$$

Spin has 2 orientations.

∴ $g(E)$ is $\frac{8\pi}{h^3}\cdot\sqrt{2E}\cdot m_e^{3/2}\, dE$

$$\therefore n_e = \frac{8\sqrt{2}\pi m_e^{3/2}}{h^3}\cdot \underbrace{\int_0^{\infty} E^{1/2}\,\frac{1}{e^{(E-E_F)/k_BT}+1}\, dE.}_{\text{calculate for arbitrary } T.}$$



say; at T=0...

now $n_e = (\quad)\cdot\int_0^{E_f}\sqrt{E}\cdot 1\, dE$:

we have..

$$\boxed{E_F = \frac{h^2}{2m_e}\left(\frac{3n_e}{8\pi}\right)^{2/3}}$$

E_F increase with n_e;
as é fill more states.

$$\therefore \frac{1}{2}m_e v_F^2 = E_F$$

fermi velocity

$$T_F = \frac{E_F}{K_B}$$

fermi temperature.

Tut-12:

1)

f.d....

$$n(E) = \frac{\int_{E-kT}^{E} g(E)\cdot f_{FD}(E)\,dE}{\int_{0}^{E} g(E) f_{FD}(E)\,dE}$$

$g(E)$ not mentioned!

use particle in 3D-box!

$g(E) = (\text{const})\cdot E^{1/2}$.

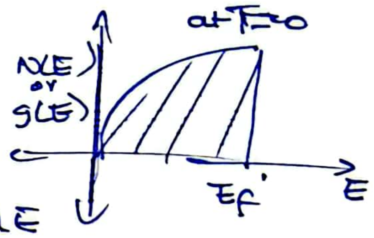
Sp. heat of Cu:- $\frac{d}{dT}\left(\frac{3}{2}kT \cdot \times \frac{3}{2}\frac{kT}{E_f}\right) N_a$.

contd:

Now; let us calculate E_{total}.

$E_{total} = \int_0^{E_f} E\,N(E)\,dE$ at $T=0$.

$N(E)\,dE = g(E)\,dE$



∴ finally

$$\boxed{E_{total} = N_E\cdot\left(\frac{3}{5}E_F\right)}$$

∴ Average $\bar{e}$ energy (in metal) at $T=0$ = $\frac{3}{5}E_F$

$$\boxed{\frac{3}{5}E_F} = \frac{\int_0^{E_f} E\cdot E^{1/2}\,dE}{\int_0^{E_f} E^{1/2}\,dE}$$

(This is 0; acc. to classical theory;

but quantum effects makes this impossible;

all $\bar{e}$ can't be in E=0 state).

Heat capacity due to $\bar{e}$:-

say excited no. of electrons = $n(E^+)\,dE$

= g