HS402 - Saptarsh prosonno...

Midsem - 45%

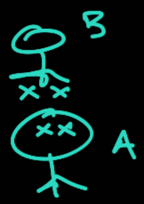
Endsem - 55%

very good!

1.08.22

- non-cooperative games.

<u>cheap talk</u>: free flow of information.



1. common language.
2. Same destination.

<u>Babbling</u>: zero information is passed.

coz 1. language barrier
2. conflict of interest.

"Private information"

* signaling (A's info to B)

A ⇄ B

screening

info flow from A to B.

A starts → <u>signalling</u>

B starts → <u>screening</u>
- exams
- audition
- interrogation.

* 2 kinds of games

  - simultaneous games.
  - dynamic games

→ <u>simultaneous games:-</u>

- Normal form representation - matrix kind of format

<u>Eg</u>: two drivers in a alley:

column player

row player

D_1 \ D_2	$wait_2$	go_2
$wait_1$	$-1,-1$	$-1,1$
go_1	$+1,-1$	$-2,-2$

<u>Payoffs</u>

solve by

"<u>Best response</u>"

- given other players action, what is best response.

game of chicken.

→ <u>Nash equilibrium:-</u>

* when all players are playing their best response outcome is nash equilibrium.

→ There is <u>no unilateral</u> profitable deviation for any players.

game of chicken

	$-1,-1$	$-1,+1$
	$+1,-1$	$-2,-2$

Both are equb. which are attained?

Beliefs

→ prisoner's dilemma:-

P_1 \ P_2	silent2	betray2
silent1	-1,-1	-3,0
betray1	0,-3	-2,-2

years in prison.

nash equilibrium..

Prisoner's dilemma

* note that (1,1) solution was better solution than nash equilibrium. But since non-coperative we have (2,2).

* Betray is dominant for both players. Hence, role of beliefs is muted.

Aug 02

→ Battle of the sexes:-

P_1 \ P_2	$movie_2$	$ballet_2$
$movie_1$	10, 7	0,0
$ballet_1$	0,0	7, 10

* Again, beliefs are important.

→ Matching the pennies :-

goal keeper / Striker	L_2	R_2
L_1	$-1, \underline{1}$	$\underline{1}, -1$
R_1	$\underline{1}, -1$	$-1, \underline{1}$

no equilibrium.

* nash equilibrium
- pure strategy NE — discussed till now.
- mixed strategy NE

* most random distribution – uniform distribution.

Mixed strategy NE:

Player 1 :- L_1 with prob. x
R_1 with prob. $1-x$

	S / G	y L_2	$1-y$ R_2
x	L_1	$-1, 1$	$1, -1$
$1-x$	R_1	$1, -1$	$-1, 1$

zero-sum game :-

in every cell, sum is 0.

→ no pure strategy NE exists in zero sum game.

* subpart of "constant sum games"

→ Best response correspondance :-

$EU_S(L_1) = y(-1) + 1(1-y)$

expected utility

$EU_S(R_1) = y(1) + (1-y)(-1)$

striker :-

In order for player 1 to play $x \in (0,1)$ it must be that

$EU_S(L_1) == EU_S(R_1)$

since

$$x = \begin{cases} 1 & \text{if } EU(L_1) > EU(R_1) \quad y < 1/2 \\ [0,1] & \text{if } EU(L_1) == EU(R_1) \\ 0 & \text{if } EU(L_1) < EU(R_1) \end{cases}$$

Best response correspondance

goal keeper :

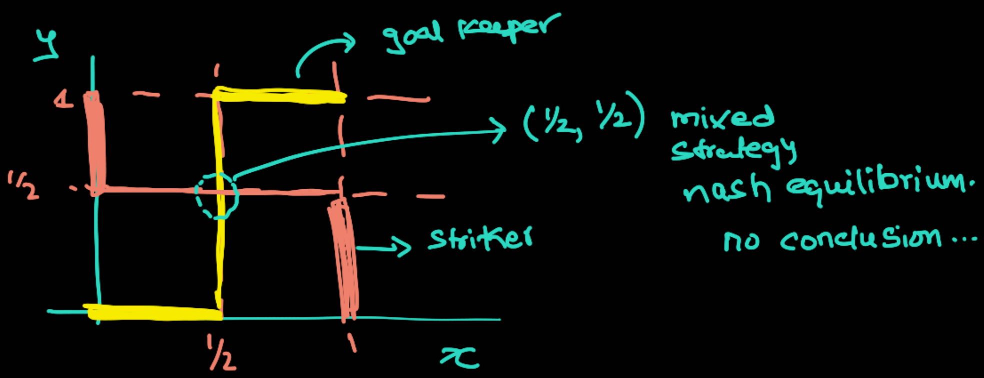
$EU_G(L_2) = x \cdot 1 + (1-x)(-1)$

$EU_G(R_2) = x(-1) + (1-x)(1)$

$$\therefore\ y = \begin{cases} 0 & \text{if } x < 1/2 \\ [0,1] & \text{if } x = 1/2 \\ 1 & \text{if } x > 1/2 \end{cases}$$

correspondance
↓
not a function;
is not well defined for
$y = 1/2$

Now, what is equilibrium ?



same analysis for prisoner's dilemma.

		y S_2	$1-y$ B_2
x	S_1	$-1,-1$	$-3,0$
$1-x$	B_1	$0,-3$	$-2,-2$

(P_1 rows, P_2 columns)

here, dominant strategy exists

for P_1:

$EU_{P_1}(S_1) = -y - 3(1-y)$
$= 2y - 3$

$EU_{P_1}(B_1) = 0 + 2y - 2$

$\therefore\ EU(B_1) > EU(S_1)$

$\therefore\ \underline{x = 0}\quad \forall y \in [0,1]$

similarly

$\underline{y = 0}$

Pure nash !

$B_1 \rightarrow$ dominant strategy

$S_1 \rightarrow$ dominated strategy.

Rule: never put +ve probability on dominated strategy.

Analysis for chicken game.

	$P_1 \backslash P_2$	y: G_2	$1-y$: W_2
x	G_1	$-2,-2$	$1,-1$
$1-x$	W_1	$-1,1$	$-1,-1$

$EU_2(G_2) = 1-3x$

$EU_2(W_2) = -1$

$$y = \begin{cases} 1 & x < 2/3 \\ [0,1] & x = 2/3 \\ 0 & x > 2/3 \end{cases}$$

$EU_1(G_1) = y(-2) + (1-y)(1)$

$EU_1(G_1) = 1-3y$

$EU_1(W_1) = -1$

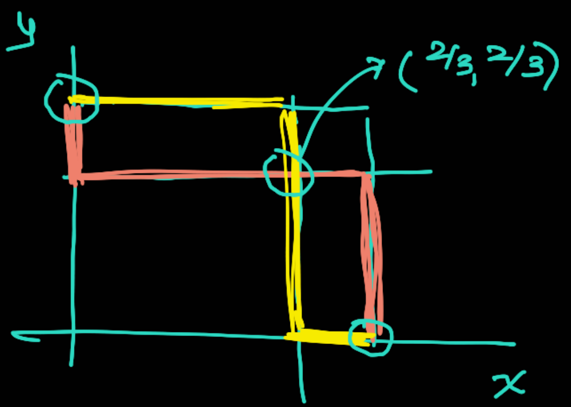
$EU_1 G_1 > EU_1 W_1$

$1-3y > -1$

$2 > 3y$

$y < 2/3.$

$$x = \begin{cases} 1 & y < \frac{2}{3} \\ [0,1] & y = 2/3 \\ 0 & y > \frac{2}{3} \end{cases}$$



~~missed one class on 8th Aug 2022.~~

but topic covered

Nash equb → formal definition.

"iterated elimination of strictly dominated strategies"
(less desired)

1 \ 2	L (3)	M	R (1)
U	1,0	1,2 NE	0,1
~~D~~ (2)	~~0,3~~	~~0,1~~	2,0

dominated strategies are eliminated.

→ prove that, after all iterative cancellations we arrive at Nash equilibrium...

Sol) n-players...

$(s_1^*, s_2^* \dots s_n^*)$ is remaining state.

suppose not!

$\exists i \in N$ & $s_i' \in S_i$ such that
(set of all actions)

$$u_i(s_1^*, s_2^* \dots s_i', s_{i+1}^* \dots s_n^*) > u_i(s_1^* \dots s_i^* \dots s_n^*)$$

but s_i' didn't survive the iterative cancellation

$\Rightarrow \exists\ \bar{s}_i \in S_i$ such that

$u_i(s_1^*, s_2^* \dots \bar{s}_i) > u_i(s_1^*, s_2^*, \dots s_i' \dots)$

& $\bar{s}_i$ also didn't survive the cancellation.

$\therefore$ ⋮

s_i^* is the survivor.

$\therefore\ u_i(s_1^*, s_2^* \dots s_i^*, \dots) > u_i(s_1^*, s_2^* \dots s_i', s_n^*)$

contradiction!

$\therefore$ we arrive at N.E.

(Funny proof
by contradiction)

* Dominance

strongly : $u_i(s_i^*) > u_i(s_i')$

weakly dominant : $u_i(s_i^*) \geq u_i(s_i')$

don't cancel weakly dominated strategies in iterated cancellation techniques.

→ Final offer arbitration model:-

↳ middle man

* firm $w_f = 100$

union $w_u = 110$

how much judge decides?

if he believes 104 → then w_f.

108 → then w_u.

(x) :

Judge ?...

x → judge's belief

(random variable)

say; $f(x)$, $F(x)$ are distributions. } these are fixed.

w_f, w_u are not.

firm union

then expected payoff

$$\circledast = w_f\left(F\left(\frac{w_f+w_u}{2}\right)\right) + w_u\left(1-F\left(\frac{w_f+w_u}{2}\right)\right)$$

$$w_f \cdot F\left(\frac{w_f+w_u}{2}\right) + w_u\left[1-F\left(\frac{w_f+w_u}{2}\right)\right.$$

now

firm → decrease $\circledast$

wage union → increase $\circledast$

} solve simultaneously to get w_f, w_u... for nash equilibrium.

Partial differentiation wrt w_f, w_u must be 0.

i) $\dfrac{\partial \circledast}{\partial w_f} = 0$

$$F\left(\frac{w_f+w_u}{2}\right) + w_f \cdot f\left(\frac{w_f+w_u}{2}\right)\cdot\frac{1}{2} + w_u\left(-f\left(\frac{w_f+w_u}{2}\right)\cdot\frac{1}{2}\right) = 0$$

$$F\left(\frac{w_f+w_u}{2}\right) + f\left(\frac{w_f+w_u}{2}\right)\left(\frac{w_f-w_u}{2}\right) = 0$$

ii) $\dfrac{\partial \circledast}{\partial w_u} = 0$

$$w_f \cdot \left(f\left(\frac{w_f+w_u}{2}\right)\right)\cdot\frac{1}{2} + \left(1-F\left(\frac{w_f+w_u}{2}\right)\right) + w_u\left(-f\left(\frac{w_f+w_u}{2}\right)\left(\frac{1}{2}\right)\right) = 0$$

$$\left(1-F\left(\frac{w_f+w_u}{2}\right)\right) + f\left(\frac{w_f+w_u}{2}\right)\cdot\left(\frac{w_f-w_u}{2}\right) = 0$$

$$\therefore \quad F\left(\frac{w_f+w_u}{2}\right) = \frac{1}{2}$$

median!

$$\therefore \quad \frac{w_f+w_u}{2} = \text{mean}$$

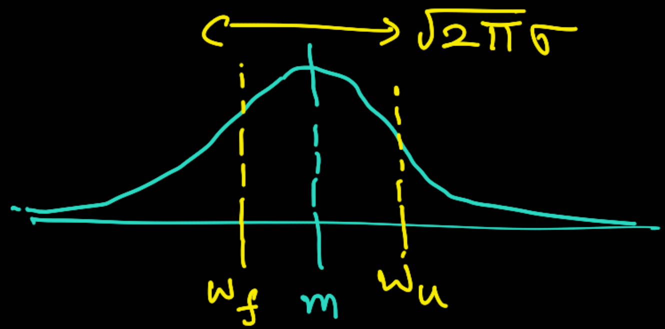
say $x \sim N(m,\sigma^2)$

$$\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-m)^2}{2\sigma^2}}$$

$$\& \quad f(\text{mean})\left(\frac{w_f-w_u}{2}\right) = \frac{1}{2}$$

$$\therefore \quad w_u - w_f = \sqrt{2\pi}\,\sigma$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-m)^2}{2\sigma^2}}$$



* the firm wants to go low, union wants high. Their mean is mean of x.

How much they can go? based on arbitration's (judge) variance.

→ The problem of commons :-

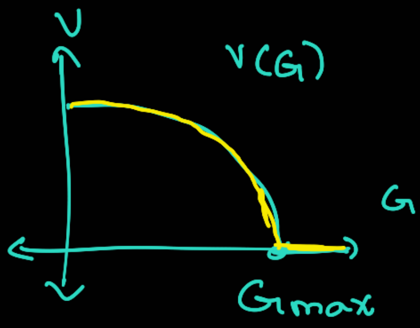
- n farmers in a village.
- i^{th} farmer → g_i number of goats.

$G = g_1 + g_2 + \cdots g_n$

- value of grazing per goat is $v(G)$.

$v' < 0, \; v'' < 0$

C → cost of grazing per goat.



$V(G) = 0$ if $G > G_{max}$.

Prove

if there were a central authority to maximize total value ; then $G_{central}$ would be less than when each farmer was playing his individual NE.

- zero-sum game has no pure nash equilibrium na?

central authority :-

say $g_1 \; g_2 \cdots g_n$

then

benefit $= G \cdot v(G)$

cost $= G \cdot c$

$\therefore$ total utility or total payoff $= G\,(v(G) - c)$

$\therefore$ for maximum utility

$$G\,(v'(G)) + (v(G) - c) = 0$$

$$V(G) + G v'(G) = C$$

- find $G_{central}$ from this.

individual :-

$G = g_1 + g_2 + \cdots g_n$

i^{th} farmer:

net payoff $= g_1 (v(G)) - g_1 C$

$\therefore$ for local maxima :

$$g_1\,(v'(G)) + (v(G) - c) = 0.$$

Similarly;

$$g_i\,(v'(G)) + (v(G) - c) = 0$$

sum all

$$G \cdot v'(G) + n\,(v(G)) - nc = 0$$

$$V(G_c) + G_c \cdot V'(G_c) = C$$

$$V(G) + \frac{G}{n} V'(G) = C$$

- G individual from this

$$V(G_i) + \frac{G_i}{n} V'(G_i) = C$$

write G^* → individual nash equbs. for all

$$V(G^*) + \frac{G^*}{n} \cdot V'(G^*) = C.$$

say $G^* < G_c$

then $V(G^*) > V(G_c)$

$$V'(G^*) > V'(G_c)$$

$$-V'(G^*) < -V'(G_c)$$

$\therefore$ $-G^* V'(G^*) < -G_c V'(G_c)$

$\therefore$ $-\frac{G^* V'(G^*)}{n} < -G_c \cdot V'(G_c)$

$\therefore$ $\frac{G^* V'(G^*)}{n} > G_c V'(G_c)$

$g_i(V(G))$

$- \ldots c_i$

$V(G) + g_i \, V'(G_i)$

$-C = 0.$

$\therefore$ $V(G^*) + \frac{1}{n} G^* V'(G^*) > V(G_c) + G_c V'(G_c)$

contradiction!

$\therefore$ $G^* > G_c$

$\therefore$ the net quantity utilized in individual nash equbs is more than the optimum quantity.

"The problem of COMMONS"

Nice illustration.

→ Second Price Auction (or) Vickrey Auction:-

* Here bidding is done simultaneously in confidential manner.
(rather than one after another)

* The highest bidder wins the auction, & pays the price of 2^{nd} highest bid.

→ n bidders; i^{th} bidder → valuation v_i & bidding b_i

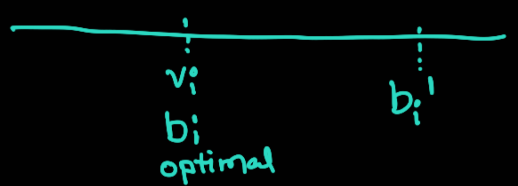
$$\text{utility } u_i = \begin{cases} v_i - \max\limits_{j \neq i} b_j \ ; & \text{if } b_i > \max\limits_{j \neq i} b_j \\ \dfrac{v_i - b_i}{m+1} \ ; & \text{if } b_i = \max\limits_{j \neq i} b_j \ , \ \{\# \ j \ \max\limits_{j \neq i} b_j\} = m \\ 0 & \text{otherwise} \end{cases}$$

winner is chosen at uniform random, in case of ties.

nash equb....

$b_i = v_i$ is player's weakly dominant strategy.

Lets see:

* 

say $b_i' > b_i^{optimal}$: then $u(b_i^{optimal}) \geq u(b_i')$

since we pay the second highest bid, & not our bidding.

if $v_i < \max_{j\neq i} b_j < b_i'$: $\underbrace{u(b_i)}_{\text{zero}} > \underbrace{u(b_i')}_{\text{negative}}$

if $v_i < b_i' < \max_{j\neq i} b_j$: $u(b_i) = u(b_i') = 0$

if $\max_{j\neq i} b_j < v_i < b_i'$: $u(b_i) = u(b_i') = v_i - \max_{j\neq i} b_j$

b_i is weakly dominant when $b_i' > b_i$

* $b_i > b_i'$

$u(b_i') = u(b_i) = v_i - \max_{j\neq i} b_j$

$\underbrace{u(b_i')}_{\text{zero}} < \underbrace{u(b_i)}_{\text{positive}}$

$u(b_i') = u(b_i) = 0$

$\therefore$ b_i is weakly dominant.

$\therefore$ bidding $b_i = v_i$ is the weakly dominant strategy.

→ Rationalizability :- } reducing choices, based on logic.

1 \ 2	C_1	C_2	C_3	C_4
R_1	0,7	2,5	7,0	0,1
R_2	5,2	3,3	5,2	0,1
R_3	7,0	2,5	0,7	0,1
~~R_4~~	~~0,0~~	~~0,-2~~	~~0,0~~	~~10,-1~~

1. C_4 is not rationalizable, since its never the best response of ②.
2. Then remove R_4
   ↳ can't remove it beforehand.

Dominance → binary concept

Rationalizable → not a binary concept

→ Cournot Game :-

firm 1 → X, marginal cost = 30

firm 2 → Y, marginal cost = 36

market quantity $Q = X + Y$

Price $P = 60 - Q$

what is the x^*, y^* nash equilibrium.

$$U_x = (60 - x - y)(x) - 30(x) = (30 - x - y)(x)$$

$$U_y = (24 - x - y)(y)$$

$$\frac{\partial U_x}{\partial x} = 0 \Rightarrow 30 - x - y - x = 0 ; \quad 2x + y = 30$$

$$\frac{\partial U_y}{\partial y} = 0 \rightarrow 24 - x - y - y = 0 ; \quad x + 2y = 24$$

$$x + y = 18$$

$$\therefore x = 12$$

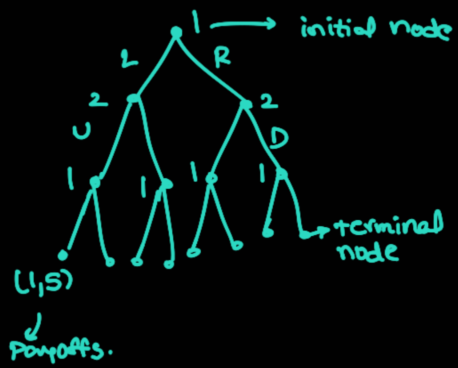
$$y = 6$$

we do iterative method by rationalizing $x, y \geq 0$ & then use diff. equns to decrease their window iteratively.

→ Dynamic Games. (till now one-shot games):-

Sequential Games

* depicted by game tree / extensive form representation.



→ Backward Induction :-

1st person "knows" that next person will go for profitable outcome.

three people need to pay for a park. Atleast 2 need to pay for park to be built.

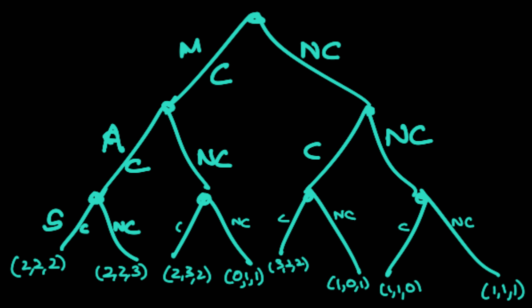
Game of perfect information. 3rd person knows decision of first two.

person — No cost / cost

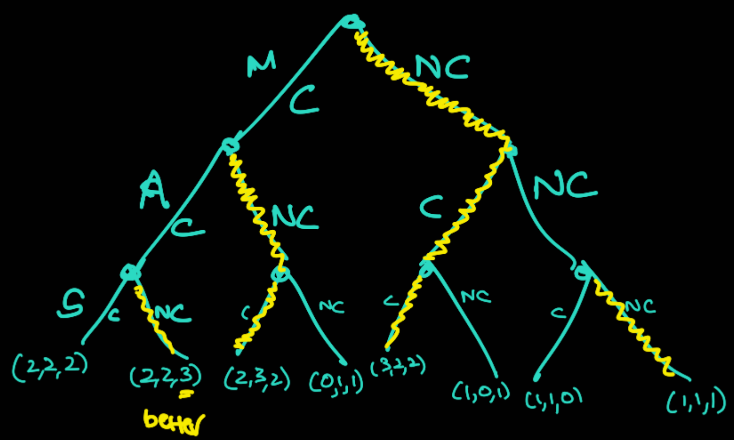
park — No park / park.

case	score/payoff
NC, P	3
C, P	2
NC, NP	1
C, NP	0

Morgan, Akash, Sanya



Back induction:



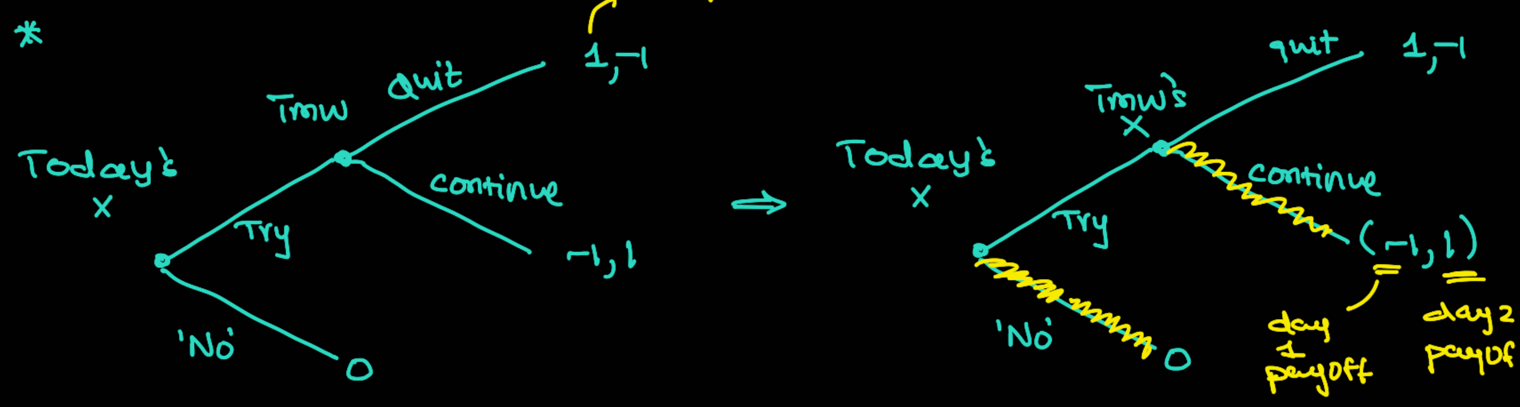
∴ First person has advantage.

& NOT THIRD PERSON !

→ Smoking Game :-

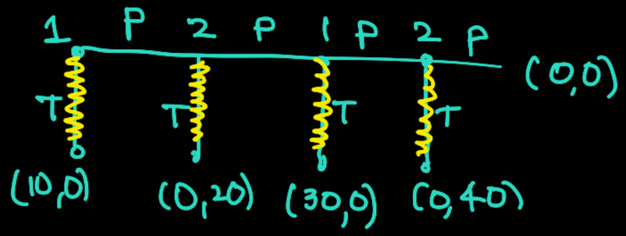
today's x; happy to quit tmw.

*



→ Centipede Game :-

pass to next round P
take money T (10 rupees will increase)

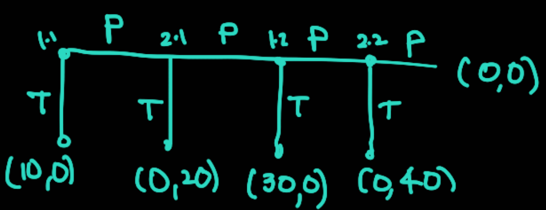


∴ immediately take!

* Information Set :-

set containing all the points at which a particular player has to take a decision.

* **Strategy** : Ascribes an action to every element in the information set.



no. of pure strategies $= 2^2$

$I_1 = \{1.1, 1.2\}$

$I_2 = \{2.1, 2.2\}$

$S_1 = \{(P,T)(P,P)(T,P)(T,T)\}$

probabilistic
0.5 P ...
0.5 T

we can play mixed strategies too!

→ splitting the pie :-

player-1 : Splits 100 to 2 parts

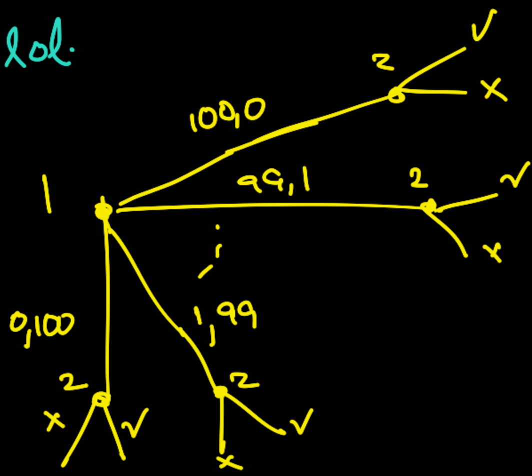
player-2 : decide whether this split is carried out or not.

- if not carried out; both get 0.

optimal: player-1 : 99

player-2 : 1

lol.



$IS_1 = \{ 1.1 \}$

$IS_2 = \{ 2.1, 2.2, \ldots 2.101 \}$

Strategy$_1$ example $= (10, 90)$

strategy 2 example $= (Y, N, N, Y \ldots\ldots)$ 101-tuple

num_strat$_1$ = 101

num_strat$_2$ = 2^{101} ✓✓

* **sequential games :-**
  - Game trees
  - Information Sets
  - Strategies
  - Backward Induction

* F_1 , F_2
  x y

KUNO Model?

simultaneous production.

$P = 60 - Q$

$Q = x + y$

$MC_1 = 30$

$MC_2 = 60$

→ Stackelberg Model :-

sequential decision of quantity of production.

leader 1. F_1 chooses x

follower 2. F_2 chooses y after F_1's decision

$Q = x + y$

$MC_1 = 30$

$MC_2 = 36$

$\Pi_1 = (60 - Q)x - 30x$ $\quad \Pi_2 = (60 - Q)y - 36y$

Backtracking :-

$$\frac{\partial \Pi_2}{\partial y} = 0 \Rightarrow (60 - Q) + (-y) - 36 = 0$$

$$\boxed{y^* = 12 - \frac{x}{2}}$$

use in Π_1 equation.

$$\pi_1 = \left(48 - \frac{3x}{2}\right)(x) - 30x$$

$$\frac{\partial \pi_1}{\partial x} = 0 \Rightarrow \quad 48 - \frac{1x}{2} - \frac{1}{2}x - 30 = 0$$

$$\Rightarrow \boxed{\begin{matrix} x^* = 18 & \pi_1^* = 162 \\ y^* = 3 & \pi_2^* = 9 \end{matrix}}$$

Stackleberg setup (sequential)

Payoff → π

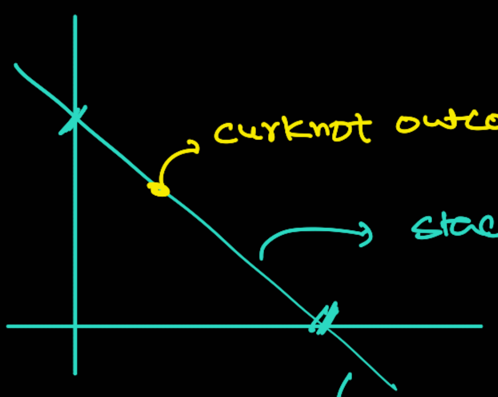
$$\boxed{\begin{matrix} x^* = 12 & \pi_1^* = 144 \\ y^* = 6 & \pi_2^* = 36 \end{matrix}}$$

solved previously... curknot setup (simultaneous)

∴ In sequential setup; there is <u>first movers advantage</u> only in this scenario!!

Eg: in a single-chance bidding; 2^{nd} player inevitably has the advantage; when he has the trueprice & the item & the 1^{st} player's bid.

* In this scenario <u>REVEALED PREFERANCE</u> of F_2 is preferred by F_1. Hence, F_1 wants Stackleberg setup.



curknot outcome

→ stackleberg outcome can choose from this range.

↳ player 2's response

$$* \quad \pi_{1\,\text{Stackleberg}} \geq \pi_{1\,\text{curknot}}$$

since F_1 can 'choose' curknot outcome itself if its the most optimal.

→ subgame :-

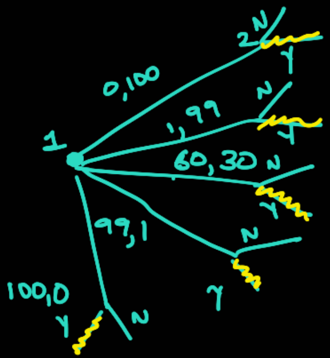
* 1. A subgame has a single node as the virtual node

2. The entire game henceforth has to be included in the subgame.

** subgame perfect Nash Equilibrium

* If the strategies are such that they constitute a N-E at every subgame.

Eg: Pie - cutting :-



$IS_1 = \{1\}$

$IS_2 = \{2\cdot 1, 2\cdot 2, \cdots 2\cdot 101\}$

$S_1 = (100, 0)$

$S_2 = (Y, Y, Y, \cdots$ 101 times$)$

} this is a subgame perfect NE

$S_1 = (60, 40)$

$S_2 =$ yes, only if pay ≥ 40

} this is also NE.

But not subgame perfect NE.

NE; bcause each player can't deviate unilaterally.

* NE has possibility of INCREDIBLE THREAT

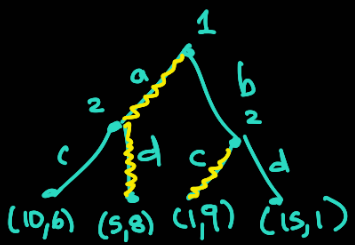
SPNE (subgame perfect) doesnot have threats.

* Backward Induction is SPNE.

* Always specify a NE by — Eq^nb Strategy of each agent

— Eq^nb payoffs.

SELF-NOTE:- what is nash equb. in a sequential game?



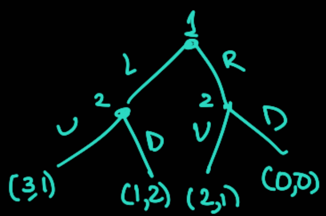
* Understand that we say 'A strategy is NE' and NOT 'A person is NE' or 'A node is NE'.
* Try unilateral deviation for proof.

$s_1 = (a)$

$s_2 = (d,c)$ — (c,d), (d,d) } both are unilateral devn. from one point in strategy space to another.

this is NE "strategy of all players"

→ Game trees to normal form :-



- Here, again INCREDIBLE THREAT by player 2.

⟹

we write "strategies" in Rows/columns.

1 \ 2	UU	UD	DU	DD
L	3,1	3,1	1,2	1,2
R	2,1	0,0	2,1	0,0

---> not a SPNE

Both are N.E.

Obtained via backward induction is SPNE

→ Repeated games:

- finite repetition
- infinite repetition

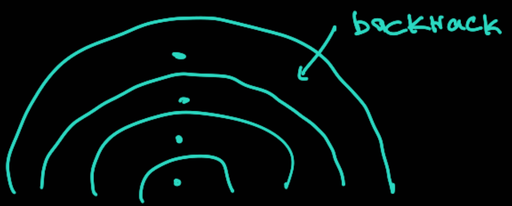
game of chickens

1 \ 2	L_2	R_2
L_1	2,2 NE	6,1
R_1	1,6	5,5

Result:

if $\exists$ a unique nash equilibrium then that will be played in ALL the rounds in finite repetition game.

* How will we solve finite repetition games? Game trees!



→ Discounting future payoffs:

* 100 rupees today is not equivalent to 100 rupees tomorrow.
* Agents prefer immediate payoffs.
* $\delta \rightarrow$ discounting factor;

$$\Pi_{net} = \Pi_1 + \delta \Pi_2 + \delta^2 \Pi_3$$

*

1 \ 2	L_2	M_2	R_2
L_1	1,1	5,0	0,0
M_1	0,5	4,4	0,0
R_1	0,0	0,0	3,3

$= G$.

$G(2)$; say $\delta = 1$

$$\Pi^1 = \Pi^1_1 + \Pi^1_2$$
$$\Pi^2 = \Pi^2_1 + \Pi^2_2$$

can we make them play $(4,4)$?

* Strategy of a player in a repeated game:

A complete plan of actions describing what a player plans to do

- For every round
- For every history in each round.

1 \ 2	L_2	M_2	R_2
L_1	1,1	5,0	0,0
M_1	0,5	4,4	0,0
R_1	0,0	0,0	3,3

observation-1:-

final round; both play NE. say (3,3)

strat. for P_1:

Round 1: play m_1

Round 2: If $h_1 = (m_1, m_2)$, play R_1

For all other history, play L_1

check if this is a NE.

strat. for P_2:

Rnd 1: play m_2

Round 2: if $h = (m_1, m_2)$ play R_2

for other history, play L_2.

payoffs $= 4+3 = 7$

* Now consider G(3) (3 rounds). & Give a strategy:

both players discount the future @ $0 < \delta < 1$.

Write down the pair of strategies & find the range of δ that sustains (4,4) in the first two rounds in G(3)

stick: punishment

1 \ 2	L_2	M_2	R_2
L_1	1,1	5,0	0,0
M_1	0,5	4,4	0,0
R_1	0,0	0,0	3,3

carrot

" simultaneous games; multiple rounds"

Sol)

if $\delta = 1$; all are NE strategies...

P1:

R1: M_1

R2: if $h_1 = (M_1, M_2)$ then M_1

else L_1 // punishment

to get better for both.

R3: if $h_2 = ((M_1, M_2), (M_1, M_2))$ play R_1

otherwise L_1

P2:-

R1: M_2

R2: if $h_1 = (M_1, M_2)$ then M_2

otherwise L_2

R3: if $h_2 = ((M_1, M_2)(M_1, M_2))$ play R_2

else L_2

* $4 + 4\delta + 3\delta^2 \geq 5 + \delta + \delta^2$.

lateral deviation...

$\delta \geq 0.28$

* $4 + 5\delta + \delta^2 < 4 + 4\delta + 3\delta^2$

$\delta \geq 0.5$

players should be scared of penalty; to play (4,4) in 1st round.

1 \ 2	L_2	M_2	R_2	P_2	Q_2
L_1	1,1	5,0	0,0	0,0	0,0
M_1	0,5	4,4	0,0	0,0	0,0
R_1	0,0	0,0	3,3	0,0	0,0
P_1	0,0	0,0	0,0	$\boxed{4, 1/2}$	0,0
Q_1	0,0	0,0	0,0	0,0	$\boxed{1/2, 4}$

punish point of Pl₁ (if Pl₂ betrays)

punish point of Pl₂

* Before, we needed a delegation.
when (1,1) was the stick...
since its penalty for Pl₂ also.

* unique nash eqlb

	L_2	R_2
L_1	1,1	5,0
R_1	0,5	4,4

$= G$

what will $G(T)$ be?

All will be $\underline{(1,1)}$

coz, unique NE.

& there's no mixed NE; since; we have a unique pure NE.

<u>Proof:</u> Back-tracking !

really?

*

	L_2	M_2	R_2
L_1	1,1	5,0	0,0
M_1	0,5	4,4	0,0
R_1	0,0	0,0	3,3

$= G$

$G(T) = ?$

* In the last round, both will play NE.

* Can we play other than NE in first few rounds?

Sol) Yes; if δ is large enough

"if players are enough concerned about future payoffs".

$G(2)$:-

$\pi^1 = \pi^1_1 + \delta \pi^1_2$

* there exists δ^* such that $\forall \delta > \delta^*$; (4,4) can be sustained.

- let $(\delta^*)_T$ be δ^* for $G(T)$.

Now; $(\delta^*)_2 \; \square \; (\delta^*)_3$

$<$
$>$
$=$
?

$(\delta^*)_2 \geq (\delta^*)_3$. !!

→ Reason:-

in $G(2)$ ___ ___

need to sustain (4,4)

in $G(3)$ ___ ___ ___

this will be easier to sustain in $G(3)$ than $G(2)$

as more penalty!

to sustain (4,4) here; same as $(\delta^*)_2$

$\therefore$ Actually $(\delta^*)_3 = (\delta^*_2)$

---

*

	L_2	R_2
L_1	1,1	5,0
R_1	0,5	4,4

Rough;

$\pi^1 = 4 + 100$

$\pi^2 = 4 +$

4 4

R_1 (0,5) L_2

L_1 (1/0) L_2

(1,1)

→ **Infinite games:-** $G(\infty)$

	L_2	R_2
L_1	1,1	5,0
R_1	0,5	4,4

Here, in infinite games; even with unique NE; we can have (4,4) since; there's no real backtracking

* **Again "punish policy":-**

$$4 + 4\delta + 4\delta^2 + \cdots = \frac{4}{1-\delta}$$

$$5 + \delta + \delta^2 + \cdots$$
$$5 + \frac{\delta}{1-\delta}$$

$$\frac{4}{1-\delta} \geq 5 + \frac{\delta}{1-\delta}$$

$$4 - \delta \geq 5 - 5\delta$$

$$\delta \geq 1/4$$

$$\delta \geq 0.25$$