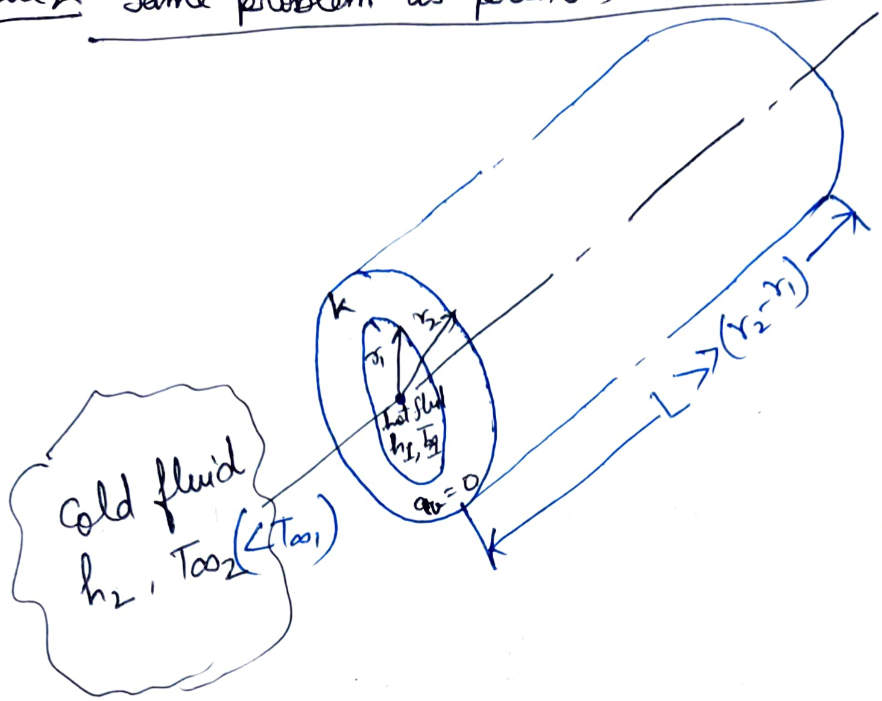
**Que 2**: Same problem as previous but with Newtonian/convection BC's




$$T_r = C \ln r + D \quad \text{—— (I)}$$

BC's @ $r = r_1$

$$h_1 \cancel{(2\pi r_1 l)} \left[ T_{\infty 1} - T_r \right]_{r=r_1} = -k \cancel{(2\pi r_1 l)} \left\{ \frac{dT_r}{dr} \right\}_{r=r_1}$$

$$h_1 (T_{\infty 1} - T_r) = -k \left\{ \frac{dT}{dr} \right\} \quad \text{—— (II)}$$

@ $r = r_2$

$$-k \cancel{(2\pi r_2 L)} \cdot \frac{dT_r}{dr} = h_2 \cancel{(2\pi r_2 l)} (T_r - T_{\infty 2})_{r=r_2}$$

$$-k \left( \frac{dT}{dr} \right)_{r=r_2} = h_2 (T_r - T_{\infty 2})_{r=r_2} \quad \text{—— (III)}$$

Put $T_r$ from equation (I) to (II) & (III) we get

$$h_1 (T_{\infty 1} - (C \ln r_1 + D)) = -k \frac{C}{r_1}$$

$$h_2 (C \ln r_2 + D - T_{\infty 2}) = -k \frac{C}{r_2}$$

$$h_1 T_{\infty_1} - h_1 C \ln r_1 - h_1 D = -k \frac{C}{r_1} \quad \times h_2 \text{ —— (IV)}$$

$$-h_2 T_{\infty_2} + h_2 C \ln r_2 + h_2 D = -k \frac{C}{r_2} \quad \times h_1 \text{ —— (V)}$$

$$h_1 h_2 T_{\infty_1} - h_1 h_2 C \ln r_1 - h_1 h_2 \cancel{D} = -k \frac{C}{r_1} h_2 \text{ —— (VI)}$$

$$-h_1 h_2 T_{\infty_2} + h_1 h_2 C \ln r_2 + h_1 \cancel{h_2 D} = -k \frac{C}{r_2} \cdot h_1 \text{ —— (VII)}$$

$$h_1 h_2 (T_{\infty_1} - T_{\infty_2}) - h_1 h_2 C (\ln r_1 - \ln r_2) = -kC\left(\frac{h_2}{r_1} + \frac{h_1}{r_2}\right)$$

$$(T_{\infty_1} - T_{\infty_2}) - C \ln(r_1/r_2) = -kC\left(\frac{1}{h_1 r_1} + \frac{1}{h_2 r_2}\right)$$

$$(T_{\infty_1} - T_{\infty_2}) = -kC\left(\frac{1}{h_1 r_1} + \frac{1}{h_2 r_2}\right) + C \ln\left(\frac{r_1}{r_2}\right)$$

$$(T_{\infty_2} - T_{\infty_1}) = C\left[\frac{k}{h_1 r_1} + \frac{k}{h_2 r_2} + \ln\left(\frac{r_2}{r_1}\right)\right]$$

$$C = \frac{T_{\infty_2} - T_{\infty_1}}{\left[\frac{k}{h_1 r_1} + \frac{k}{h_2 r_2} + \ln(r_2/r_1)\right]}$$

Put value of C in equ$^n$ (IV) we get

$$h_1 T_{\infty_1} - h_1 C \ln r_1 - h_1 D = -k \frac{C}{r_1}$$

$$h_1 T_{\infty_1} - h_1 D = -C\left(\frac{k}{r_1} - h_1 \ln r_1\right)$$

$$T_{\infty_1} - D = -C\left(\frac{k}{h_1 r_1} - \ln r_1\right)$$

$$D = T_{\infty_1} + \frac{(T_{\infty_2} - T_{\infty_1})\left(-\ln r_1 + \frac{k}{r_1 h_1}\right)}{\left(\frac{k}{r_1 h_1} + \frac{k}{h_2 r_2}\right) + \ln(r_2/r_1)}$$

Now Put C & D in eq$^n$

$$T_r = C \ln r + D$$

$$T_r = \frac{(T_{\infty 2} - T_{\infty 1}) \ln r}{\left(\frac{k}{r_1 h_1} + \frac{k}{h_2 r_2}\right) + \ln(r_2/r_1)} + T_{\infty 1} + \frac{(T_{\infty 2} - T_{\infty 1})\left(-\ln r_1 + \frac{k}{r_1 h_1}\right)}{\left(\frac{k}{h_1 r_1} + \frac{k}{h_2 r_2}\right) + \ln(r_2/r_1)}$$

$$\boxed{\frac{T_r - T_{\infty 1}}{T_{\infty 2} - T_{\infty 1}} = \frac{(\ln r - \ln r_1) + \frac{k}{r_1 h_1}}{\left[\frac{k}{r_1 h_1} + \frac{k}{h_2 r_2} + \ln\left(\frac{r_2}{r_1}\right)\right]}}$$

$$q = -k\, 2\pi r l \frac{dT_r}{dr}$$

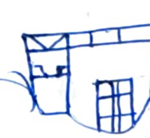
$$= -k \cdot 2\pi \cancel{r} l \frac{C}{\cancel{r}}$$

$$= \frac{2\pi k l (T_{\infty 1} - T_{\infty 2})}{\ln(r_2/r_1) + \cancel{k\left(\frac{1}{h_1 r_1} + \frac{1}{h_2 r_2}\right)} \to 0}$$

let $h$ is very large.

$$q = \frac{2\pi k l (T_{\infty 1} - T_{\infty 2})}{\ln(r_2/r_1)}$$

$$\boxed{q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{\ln(r_2/r_1)}{2\pi k l}}}$$



$$R_{th} = \frac{\ln(r_2/r_1)}{2\pi k l}$$