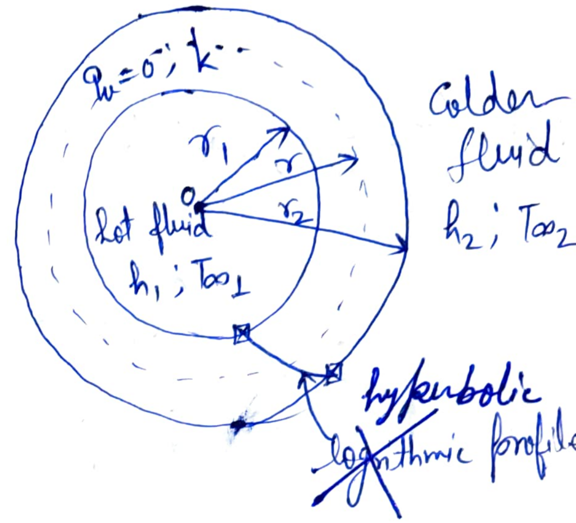
Problem 2:

Solving Problem 1 but with Newtonian BC's! →

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \quad \text{—— (1)}$$



$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

on integrating

$$r^2\frac{dT}{dr} = C$$

$$\frac{dT}{dr} = \frac{C}{r^2}$$

again integrating

$$T_r = -\frac{C}{r} + D \quad \text{—— (2)}$$

BC's @ $r = r_1$; $h_1 \cancel{4\pi} r_1^{\cancel{2}}\left[T_{\infty 1} - T_r\right]_{r=r_1} = -k\,\cancel{4\pi r_1^2}\left[\frac{dT(r)}{dr}\right]_{r=r_1}$ —— (3)

@ $r = r_2$; $-k\,4\pi r_2^2\left[\frac{dT}{dr}\right]_{r=r_2} = h_2\,4\pi r_2^2\left[T_r - T_{\infty 2}\right]_{r=r_2}$ —— (4)

Put the value of 'Tr' from equation (2) to equ^n (3)

$$\frac{h_1}{k}\left[T_{\infty 1} - \left(-\frac{C}{r_1} + D\right)\right] = -\frac{d}{dr}\left[-\frac{C}{r_1} + D\right]$$

$$h_1 T_{\infty 1} + h_1\frac{C}{r_1} - h_1 D = +\frac{kC}{r_1^2} + 0 \quad \text{—— (5)}$$

$$h_2\left(-\frac{C}{r_2} + D - T_{\infty 2}\right)_{r=r_2} = -k\frac{d}{dr}\left(-\frac{C}{r_2} + D\right)_{r=r_2}$$

$$h_2\left(-\frac{C}{r_2} + D - T_{\infty 2}\right) = +k\frac{C}{r_2^2}$$

$$-h_2 T_{\infty 2} + h_2 D - \frac{h_2 C}{r_2} = +\frac{kC}{r_2^2} \quad \text{—— (6)}$$

$$h_1 h_2 T_{\infty 1} + h_1 h_2 \frac{C}{r_1} - \cancel{h_1 h_2 D} = \frac{h_2 k C}{r_1^2}$$

$$-h_1 h_2 \frac{C}{r_2} + \cancel{h_1 h_2 D} - h_1 h_2 T_{\infty 2} = +\frac{k C h_1}{r_2^2}$$

$$h_1 h_2 (T_{\infty 1} - T_{\infty 2}) + h_1 h_2 C \left(\frac{1}{r_1} - \frac{1}{r_2}\right) = kC\left(\frac{h_2}{r_1^2} + \frac{h_1}{r_2^2}\right)$$

$$\boxed{C = \frac{(T_{\infty 1} - T_{\infty 2})}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\left(\frac{k}{h_1 r_1^2} + \frac{k}{h_2 r_2^2}\right)}}$$

$$T_r = -\frac{C}{r} + D$$

$$T_r = -\frac{1}{r}\left[\frac{(T_{\infty 1} - T_{\infty 2})}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\left(\frac{1}{h_1 r_1^2} + \frac{1}{h_2 r_2^2}\right)}\right] + T_{\infty 1} + \frac{(T_{\infty 2} - T_{\infty 1})\left(\frac{1}{r_1} + \frac{k}{h_1 r_1^2}\right)}{\left(\frac{k}{h_1 r_1^2} + \frac{k}{h_2 r_2^2}\right) + \frac{1}{r_1} - \frac{1}{r_2}}$$

$$\frac{T_r - T_{\infty 1}}{T_{\infty 1} - T_{\infty 2}} = \frac{\frac{1}{r_1} - \frac{1}{r} + \cancelto{0}{\frac{k}{h_1 r_1^2}}}{\frac{1}{r_1} - \frac{1}{r_2} + \cancelto{0}{\frac{k}{h_1 r_1^2}} + \cancelto{0}{\frac{k}{h_2 r_2^2}}}$$

$$\boxed{\frac{T_r - T_{\infty 1}}{T_{\infty 1} - T_{\infty 2}} = \left(\frac{1/r_1 - 1/r}{1/r_1 - 1/r_2}\right)}$$

$$q_r = -k 4\pi r^2 \frac{dT}{dr} \Rightarrow -k 4\pi \cancel{r^2} \frac{C}{\cancel{r^2}}$$

$$q_r = -k 4\pi \frac{T_{\infty 1} - T_{\infty 2}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\left(\frac{k}{h_1 r_1^2} + \frac{k}{h_2 r_2^2}\right)}$$

$$q_r = \frac{4\pi (T_{\infty 2} - T_{\infty 1})}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\left(\frac{1}{h_1 r_1^2} + \frac{1}{h_2 r_2^2}\right)}$$