UNIT : 03

★ Heat Exchanger -

A heat exchanger is broadly defined as a control volume where in exchange of heat occure b/w 2 or more fluid in a steady flow condition. eg: Refrigerator, Radiator, Power plants (economiser, preheater, pregenerators), milk posturizing plants, evaporator, condenser, etc.

• **Classification of Heat Exchanger -**

Given by TEMA (Tubular Heat Exchanger Manufacturer Association).

1). On the basis of heat transfer process.

a). Direct Contact Type. (Mixing type) (Fluids mix).

b). Indirect contact type (Surface contact type) (Some surface separates the fluids).

Direct contact → eg: cooling tower, spray ponds, jet condenser, open feed water heater

Indirect contact → all types of boiler

2). On the basis of compactness (SAD - Surface area density m^2/m^3). or C

It is defined as the surface area available for heat exchanger per unit volume occupied by the heat exchanger

a). Non-compact, $70 \leq C \leq 500$.

b). Medium compact, $500 \leq C \leq 700$

c). Compact $C > 700$.

for radiator, $C \approx 1100\ m^2/m^3$, gas turbine engine $C = 6600\ m^2/m^3$.

The most compact heat exchanger, $C \approx 20000\ m^2/m^3$. { human LUNGS }

3). On the basis of hot and Cold fluid in the heat exchanger.

a). Recuperator (Transfer Type)

b). Regenerator (Storage type).

In Recuperator, there is no time lag b/w the flow of hot and cold fluid in the exchanger means hot and cold fluid entries into and leaves the exchanger simultaneously

In Regenerator, there is a time lag.

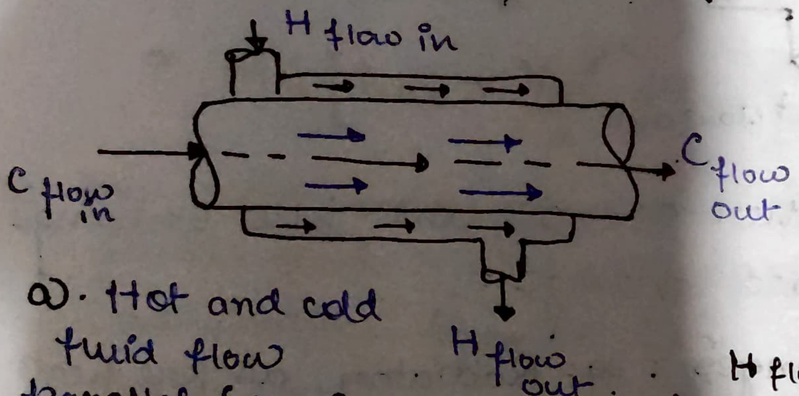
Basically the matrix absorbs or pickup the heat when it comes in contact with hot fluid and leaves to the cold fluid subsequently. The material chosen for matrix must have larger thermal heat capacity (ρC_P).

4). On the basis of relative direction variation of Heat flow -

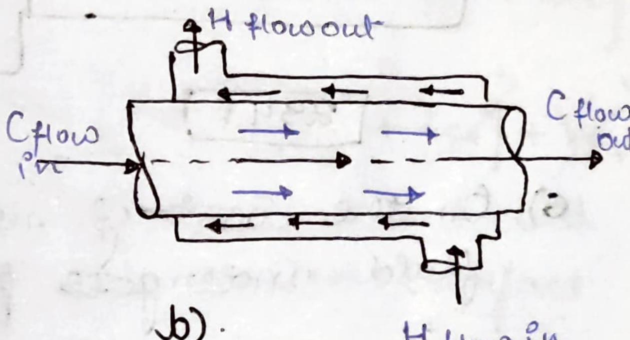
a). Co - current (Parallel flow)

b). Counter current (Counter flow)

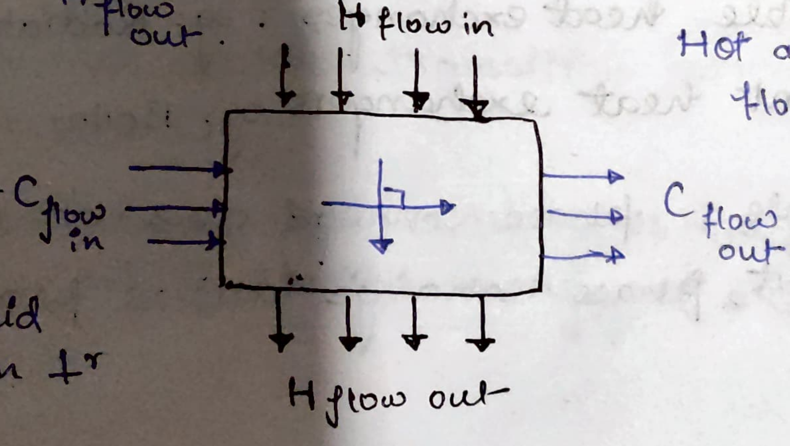
c). Cross flow heat transfer (Transverse flow)



a). Hot and cold fluid flow parallel (same direction)



b). Hot and cold fluid flow in opposite direction

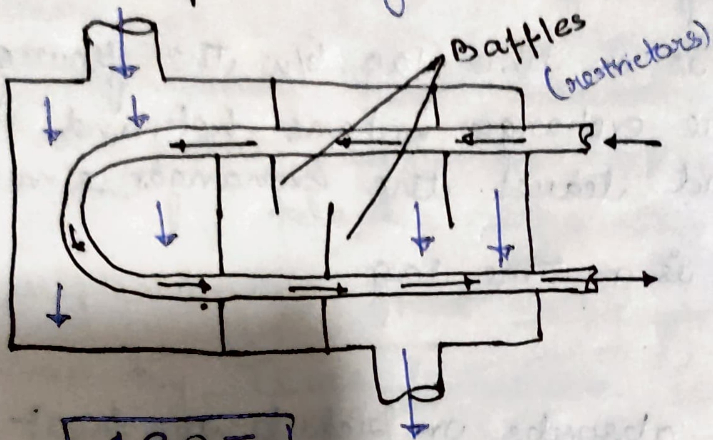


c). Hot and cold fluid fluid flow in direction $\perp^r$ to each other

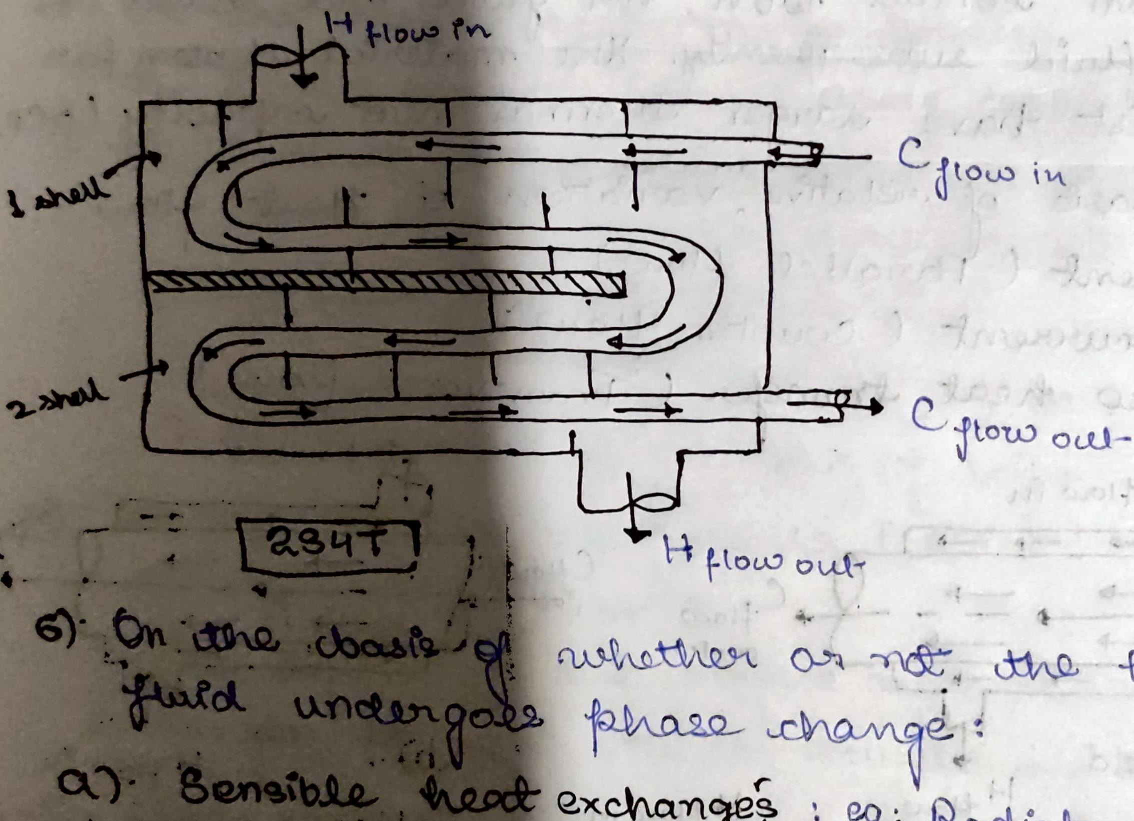
5). On the basis of no. of passes taken by hot and cold fluid -

a). Single pass. eg: parallel, counter, transverse flow.

b). Multipass. eg 1S2T, 2S4T, 3S6T, { S = shell, T = tube }



1S2T



2S4T

6): On the basis of whether or not the heat exchange fluid undergoes phase change:

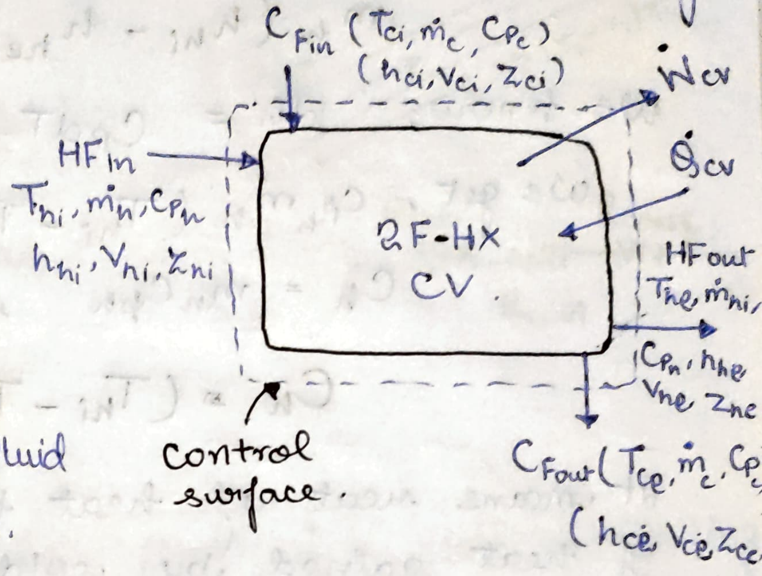
a). Sensible heat exchangers : eg: Radiator, air pre heater

b). Latent heat exchangers eg: Boiler, condenser, evaporat

In sensible, phase change does not take place whereas in latent, phase change takes place,

• Mathematical treatment to Heat Exchangers -

Backbone - equation for two fluid heat exchanger-



• Assumptions

1). The heat exchanger is considered to a control volume involved in a steady state heat exchange.

It is assumed that mass flow rate of hot & cold fluid at entry and exit is same.

2). The exhanging fluids are assumed to have constant physical thermo properties.

$$C_{p_{hi}} = C_{p_{he}} = C_{p_h} \quad / \quad C_{p_{ci}} = C_{p_{ce}} = C_{p_c}.$$

Now, apply Steady flow energy equation,

$$\dot{m}_h\left[h_{hi} + \cancel{\frac{V_{hi}^2}{2}}^{0} + \cancel{Z_{hi}}^{0}\right] + \cancel{\dot{Q}_{cv}}^{0} + \dot{m}_c\left[h_{ci} + \cancel{\frac{V_{ci}^2}{2}}^{0} + \cancel{Z_{ci}}\right]$$

$$= \dot{m}_h\left[h_{he} + \cancel{\frac{V_{he}^2}{2}}^{0} + \cancel{Z_{he}}^{0}\right] + \dot{m}_c\left[h_{ce} + \cancel{\frac{V_{ce}^2}{2}}^{0} + \cancel{Z_{ce}}^{0}\right] + \cancel{\dot{W}_{cv}}$$

3)- The heat exchange occurs only between hot and cold fluids and no heat crossing the control surface. (adiabatic, $\dot{Q}_{cv} = 0$).

4). There is no net work transition across the control surface. ($\dot{W}_{cv} = 0$).

5). The hot and cold fluid undergoes negligible change in kinetic and potential energy. ($\Delta KE = 0$, $\Delta PE = 0$)

Therefore, we get the equation as -

$$\dot{m}_h . h_{hi} + \dot{m}_c h_{ci} = \dot{m}_h h_{he} + \dot{m}_c h_{ce} \quad — ②$$

$$\dot{m}_h (h_{hi} - h_{he}) = \dot{m}_c (h_{ce} - h_{ci}) \quad — ③$$

We know, $dh = C_p dT$.

We get, $C_{p_h} \dot{m}_h (T_{hi} - T_{he}) = \dot{m}_c (T_{ce} - T_{ci}) C_{p_c}$

$C_h = m_h C_{ph}$, $C_c = \dot{m}_c C_{pc}$

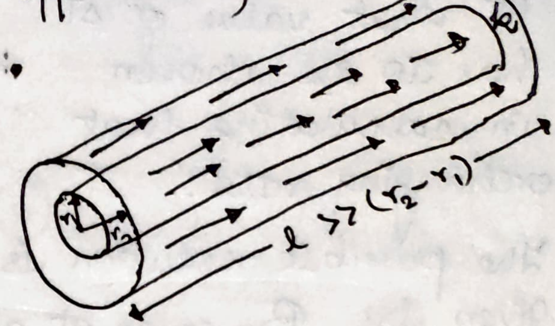
$$C_h (T_{hi} - T_{he}) = C_c (T_{ce} - T_{ci})$$

It means rate of heat lost by hot fluid is rate of heat gained by cold fluid.

$$\boxed{C_h(T_{hi} - T_{he}) = C_c(T_{ce} - T_{ci}) = q}$$

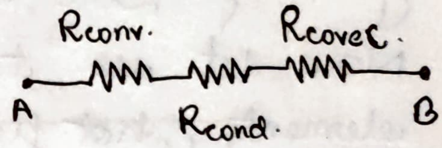
• Concept of 'U' (Overall heat co-effecient)

$$\frac{1}{UA} = \frac{1}{h_1 A} + \frac{\ln(r_2/r_1)}{2\pi k l} + \frac{1}{h_2 A}$$



$$\frac{1}{U_i} = \frac{1}{h_1} + \frac{r_i \ln(r_2/r_1)}{k} + \frac{1}{h_2}\left(\frac{r_1}{r_2}\right)$$

$$\frac{1}{U_0} = \frac{r_2}{h_1 r_1} + \frac{r_2(\ln(r_2/r_1))}{k} + \frac{1}{h_2}.$$



• Fouling/scaling :

Let us consider h_{si} & h_{so} stand for scale and fouling heat transfer coffecient on thickness suface of tube and outer surface of tube.

Then, scaled or fouled heat exchanger will have a reduced heat transfer rate.

Now, let us define U_0' is the overall heat transfer coffecient with scaling.

$$\frac{1}{U_0'} = \frac{r_2}{r_1 h_{si}} + \frac{r_2}{h_1 r_1} + \frac{r_2 \ln(r_2/r_1)}{k} + \frac{1}{h_2} + \frac{1}{h_{so}}.$$

Now defining the fouling factor/ scaling factor / scaling resistance

$$\boxed{R_f = \frac{1}{U_0'} - \frac{1}{U_0}}$$

• Heat Exchanger Analysis – (using LMTD) –

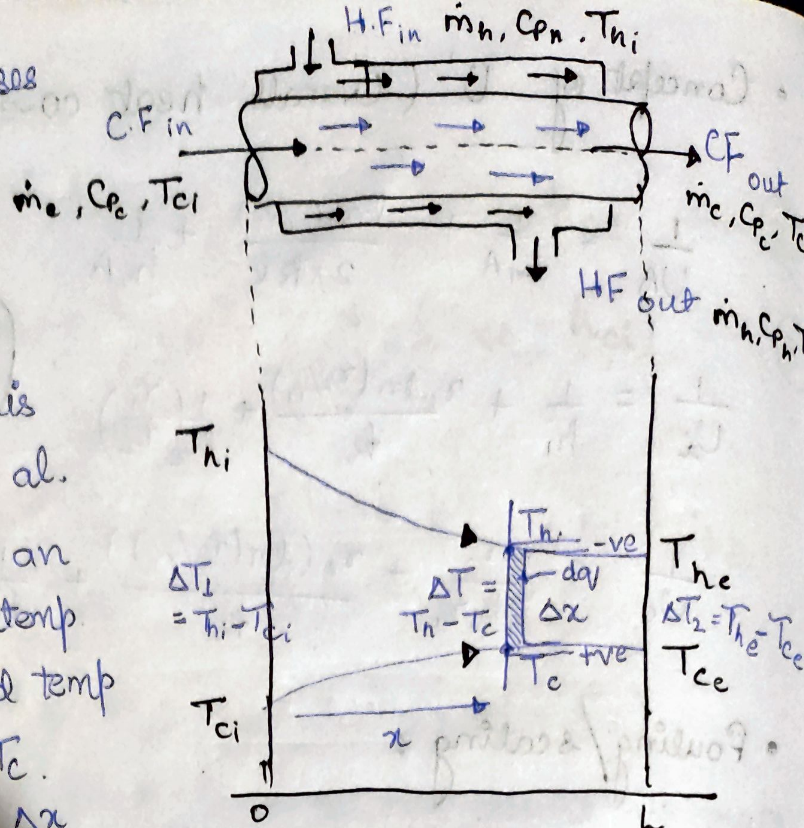
It is noticed that (we will notice), the local temperature of both heat exchanging fluid undergo a constinuous variation in a typic heat exchanger.

Thus, The Temperature difference also turned out of locally varrying locally.

Now, a question arises at what value of ΔT has to be chosen in calculating heat exchanging rate.

The possible solution is given by Bowman et al.

Now let us take an element, hot fluid temp is T_h and cold fluid temp is T_c. $\Delta T = T_h - T_c$.



Element thickness $= \Delta x$

Small heat transfer rate $= dq$.

Assumption -

Overall heat transfer coeffecient is assumed to be uniform.

Both hot and cold fluid have constant thermo physical properties.

Change in KE and PE and assumed to be negligible.

Heat transfer is adiabatic implying negligicale heat transfer across

There is no work transition experienced during heat transfer process.

Temp. is varying only in axial direction, in transverse $=0$

Local temp. difference $\quad \Delta T = T_h - T_c$

$$d\Delta T = dT_h - dT_c \quad \text{—— } 0$$

In the element, there is infinitesimal heat transfer accuring between the two fluids.

$$\therefore \quad dq = -m_h C_{P_h} dT_h \quad \text{—— (I)}$$

$$dq = m_c C_{P_c} dT_c \quad \text{—— (II)}$$

We get $dT_h = -\frac{dq}{C_h}$.

$dT_c = \frac{dq}{C_c}$.

substituting them in (i) and (ii) and eq. 0.

eq. 0 becomes,

$$d\Delta T = -\frac{dq}{C_h} - \frac{dq}{C_c}$$

$$\boxed{d\Delta T = -dq\left[\frac{1}{C_h} + \frac{1}{C_c}\right]} \text{ --- (iii)}$$

The elemental rate of heat exchange,

$$dq = U \underset{\uparrow \, \pi d_o \Delta x}{dA} \Delta T \longrightarrow \text{(iv)}$$

substituting (iv) in (iii)

$$d\Delta T = -U dA \Delta T\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\boxed{\frac{d\Delta T}{\Delta T} = -U dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]} \text{ --- (v)}$$

Now, Integrating for whole length

$$\int_1^2 \frac{d\Delta T}{\Delta T} = \int_0^l -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \text{ --- (vi)}$$

Now applying the Backbone eq: $q = C_h(T_{hi} - T_{he})$

$= C_c(T_{ce} - T_{ci}$

we get $C_h = \frac{q}{T_{hi} - T_{he}}$, $C_c = \frac{q}{T_{ce} - T_{ci}}$.

we get (vi) as, $\ln \frac{\Delta T}{\Delta T_1} = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$

Now putting the values of C_h and C_c.

$$\ln \frac{\Delta T_2}{\Delta T_1} = \frac{-UA}{q}\left[(T_{hi} - T_{he}) + (T_{ce} - T_{ci})\right]$$

(VI) —— $$\ln \frac{\Delta T_2}{\Delta T_1} = \frac{-UA}{q}\left[(T_{hi} - T_{ci}) - (T_{he} - T_{ce})\right]$$

$$q = UA \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$\Delta T_1 = T_{hi} - T_{ci}$
$\Delta T_2 = T_{he} - T_{ce}$

$$\boxed{q = UA\left(\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}\right)}$$

LMTD

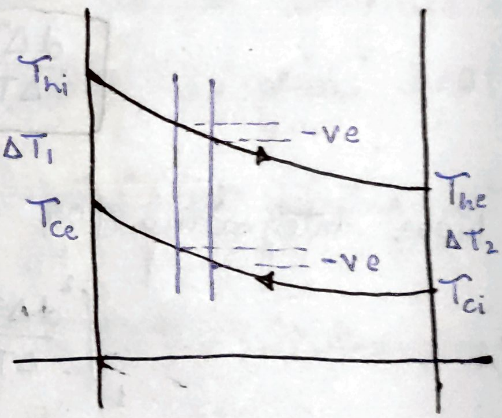
{Logarithmic Mean Temperature Diffience}

$$\therefore \quad \boxed{q = UA\,(LMTD)}$$

so at LMTD, we will calculate the rate of heat transfer in both parallel and counter flow.

Now for counter flow –

(VI) $$\ln \frac{\Delta T_2}{\Delta T_1} = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$



In the element, there is infinitesimal heat transfer occuring b/w two fluids

$$dq = -m_h C_{P_h} dT_h$$
$$dq = -m_c C_{P_c} dT_c$$

we get,

$$m_h C_{P_h} dT_h = dq$$

$$dT_h = \frac{-dq}{C_{PH}}, \quad dT_c = -\frac{dq}{C_c}$$

$$dT_h - dT_c = -dq\left[\frac{1}{C_H} - \frac{1}{C_c}\right]$$

The elementant rate of heat exchange,

$$dq = U\,dA\,\Delta T$$

$$d\Delta T = -U\,dA\,\Delta T\left[\frac{1}{C_H} - \frac{1}{C_c}\right]$$

$$\frac{d\Delta T}{\Delta T} = -U\,dA\left[\frac{1}{C_H} - \frac{1}{C_c}\right]$$

Now integrating for whole length,

$$\int_1^2 \frac{d\Delta T}{\Delta T} = -\int_0^l U\,dA\left[\frac{1}{C_H} - \frac{1}{C_c}\right]$$

we get $\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left[\frac{1}{C_H} - \frac{1}{C_c}\right]$

Now, applying the backbone equation.

$$q = C_h(T_{hi} - T_{he}) = C_c(T_{ce} - T_{ci})$$

$$C_H = \frac{q}{T_{hi} - T_{he}}, \quad C_c = \frac{q}{T_{ce} - T_{ci}}$$

$\therefore$ we get, $\ln\frac{\Delta T_2}{\Delta T_1} = -\frac{UA}{q}\left[T_{hi} - T_{he} - T_{ce} + T_{ci}\right]$

$$\ln\frac{\Delta T_2}{\Delta T_1} = \frac{UA}{q}\left[(T_{hi} - T_{ce}) - (T_{he} - T_{ci})\right]$$

$$\ln\frac{\Delta T_2}{\Delta T_1} = \frac{UA}{q}\left[\Delta T_1 - \Delta T_2\right]$$

$$q = \frac{UA\,(\Delta T_1 - \Delta T_2)}{-\ln(\Delta T_2/\Delta T_1)}$$

$$\boxed{q = \frac{UA(\Delta T_1 - \Delta T_2)}{\ln(\Delta T_1/\Delta T_2)}}$$

where, $\Delta T_1 = T_{hi} - T_{ce}$, $\Delta T_2 = T_{he} - T_{ci}$

$$\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = LMTD$$

$$\boxed{q = UA(LMTD)}$$

For given terminal temperature, LMTD for counter flow will be greater than parallel flow.

$$LMTD_{\rightleftarrows} > LMTD_{\rightrightarrows}, \quad q_{\rightleftarrows} > q_{\rightrightarrows}$$

Ques:- In a single pass, counter current HE, both fluids having indentical heat capacity rates.

$$q = -M_H C_{PH}\, dT_h = -M_C C_{PC}\, dT_C$$

$$q = -C_H\, dT_h = -C_C\, dT_C$$

given, $C_H = C_C$

$$\boxed{q = UA\Delta T_1 = UA\Delta T_2}$$

$$LMTD = AMTD = \frac{1}{2}(\Delta T_1 + \Delta T_2)$$

$$q = UA(AMTD)$$

- Capacity ratio:- It is defined as the ratio of heat capacity of heat tube fluid to that of the shell fluid. It is denoted by R.

$$\boxed{R = \frac{C_t}{C_s}}$$

$$(mC_P)_{tube}(t_2 - t_1) = (mC_P)_{shell}(T_1 - T_2)$$

$$C_T(t_2 - t_1) = C_S(T_1 - T_2)$$

Let us define t_1 and t_2 are of the temp. of tube fluid at inlet and exit and T_1 and T_2 be the temp. of shell fluid at inleet and exit.

$$\boxed{\frac{C_t}{C_s} = \frac{T_1 - T_2}{t_2 - t_1}}$$

- Temperature ratio: It is the ratio of change in temperature of tube fluid to the difference b/w entry temp of the shell and tube.

$$\boxed{P = \frac{t_2 - t_1}{T_1 - t_1}}$$

- Correction factor: It is an indication of degree of departure of the given heat exchange from the ideal single pass counter heat transfer for given terminal temperature. It is denoted by f and it is a function of R and P.

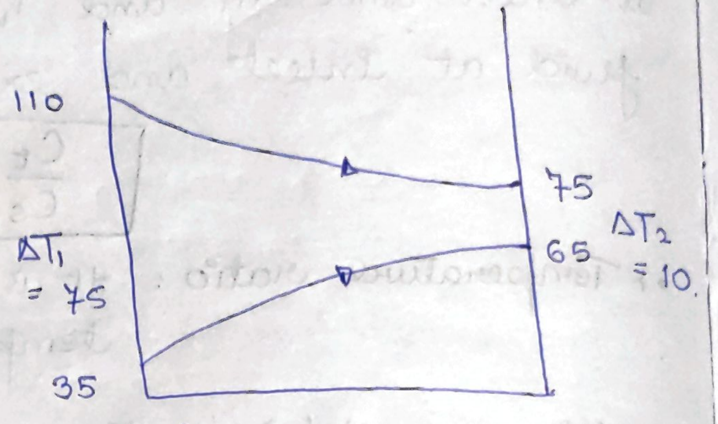
$f(R, P)$, $\boxed{0 \leq f \leq 1}$ ⇒ Ranges from 0 to 1.

eg: Water flowing at a rate of 4080 kg/hr is heated from 35°C to 65°C by making use of hot engine oil having specific heat at constant pressure C_P = 1900 J/kg K entering and leaving the shell of a STHT respectively at 110°C and 75°.

The heat exchanger is a single pass flow current type and has a uniform overall heat transfer coeffecient

$U = 320 \ W/m^2K$. Calculate the design area to be provided for heat transfer for given.

(ii). What would happen to the design area if one uses counter flow heat transfer, keeping all parameters constant.



(parallel flow).

$$LMTD = \frac{(\Delta T_2 - \Delta T_1)}{\ln(\Delta T_2/\Delta T_1)}$$

$$= \frac{75 - 10}{\ln(75/10)}$$

$$LMTD = 32.2596 \ ^\circ C$$

$$q = UA(LMTD).$$

Heat absorbed by cold water, $q = \dot{m}_c C_{p_c} \Delta T$

$$= \frac{4080}{3600} \times 4180 \, (65-35)$$

$$= 1.133 \times 4180 \times 30$$

$$q = 142.137 \ kW$$

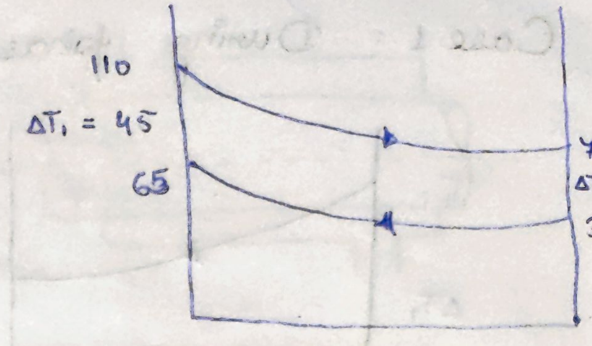
Now equating, $UA(LMTD) = 142.137 \times 10^3$.

$$320 \times A_{parallel} \times 32.2596 = 142.137 \times 10^3.$$

$$A_{parallel} = \frac{142.137 \times 10^3}{320 \times 32.2596}$$

$$A_{parallel} = 13.7689 \ m^2$$

Now, for counter flow,



(Counter flow).

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$= \frac{45 - 40}{\ln(45/40)}$$

$$= 42.45\ ^\circ C.$$

$$q = UA(LMTD)$$

Heat absorbed by cold water $= \dot{m}_c C_{p_c} \Delta T$

$$= 142.137\ kW.$$

$\therefore$ $UA(LMTD) = 142.137 \times 10^3$.

$$A_{counter} = \frac{142.137 \times 10^3}{320 \times 42.45}$$

$$= 10.463\ m^2.$$

$\therefore$ Change in area $= 13.7685 - 10.463$

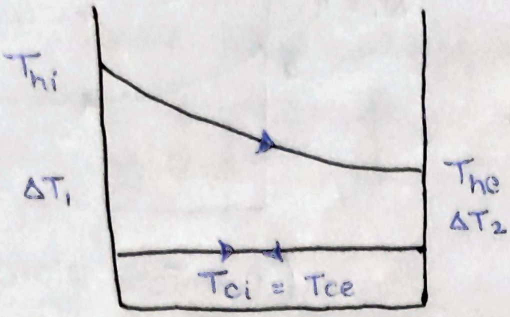
$$= 3.3055\ m^2.$$

% change in area reqd $= \frac{3.3055}{13.7689} \times 100$

$$= 24\ \%.$$

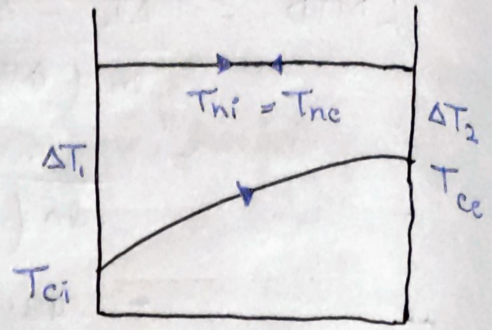
* HX = heat transfer
HE = heat exchanger

Case 1 : During phase change :



Boiler/evaporator

(cold fluid temp. at inlet & outlet remains same)



Condenser.

(hot fluid temp. at inlet & outlet remains same)

(3 temperatures will be given)

If only 2 temperatures are given, LMTD method won't be applied. We will have to use ϵ - NTU method.
(NTU — no. of transfer units)

- Effective of HE : It is the ratio of actual rate of HX possible in a HE to the thermodynamic max limit of rate of HX to be achived.

$$\boxed{\epsilon = \frac{q_{act}}{q_{max}}}$$

$q_{act} \leftarrow C_c(T_{ce} - T_{ci}) = C_H(T_{he} - T_{hi})$

$q_{max} \leftarrow C_{min}(T_{hi} - T_{ci})$

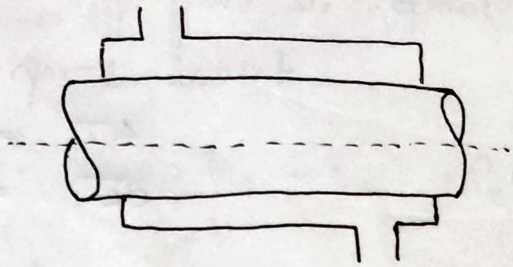
* Heat Capacity Ratio :

$$\boxed{HCR = \frac{C_{min}}{C_{max}}}$$

* NTU : No. of transfer unit is the ratio of to min. heat capacity rate of flowing fluid.

$$\boxed{NTU = \frac{UA}{C_{min}}}$$

It is noticed that



Temp. is varying only in one direction.

Local temp. difference

$$\Delta T = T_h - T_c$$

$$d\Delta T = dT_h - dT_c \quad \text{—— (0)}$$

$$-dq = \dot{m}_c C_{pc} dT_c \quad \text{—— (I)}$$

$$dq = \dot{m}_h C_{ph} dT_h \quad \text{—— (II)}$$

We get $dT_h = \left(\frac{C_H}{dq}\right)^{-1} = \frac{dq}{C_H}$

$$dT_c = \frac{dq}{C_c}$$

substituting the values in eq- (0), we get.

$$d\Delta T = -dq\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \quad \text{—— (III)}$$

The elemental rate of heat exchange

$$dq = U\,dA\,\Delta T.$$

$$\therefore \quad d\Delta T = -U\,dA\,\Delta T\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d\Delta T}{\Delta T} = -U\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Integrating both sides.

$$\int_1^2 \frac{d\Delta T}{\Delta T} = -\int_0^l U\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln\frac{(T_{he} - T_{ce})}{(T_{hi} - T_{ci})} = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{T_{he} - T_{ce}}{T_{hi} - T_{ci}} = e^{-UA\left[\frac{1}{C_c} + \frac{1}{C_H}\right]} \longrightarrow \text{(IV)}$$

using backbone equation, $C_h(T_{hi} - T_{he}) = C_c(T_{ce} - T_{ci})$

$$T_{hi} - T_{he} = \frac{C_c}{C_h}(T_{ce} - T_{ci})$$

$$\boxed{T_{he} = T_{hi} - \frac{C_c}{C_h}(T_{ce} - T_{ci})}$$

putting this in eq - (IV)

$$\boxed{\frac{T_{hi} - \frac{C_c}{C_h}(T_{ce} - T_{ci}) - T_{ce}}{T_{hi} - T_{ci}} = e^{-UA\left[\frac{1}{C_c} + \frac{1}{C_H}\right]}}$$

Now, appling the definition of NTU,

$$UA = NTU \cdot C_{min}$$

effectiveness,
$$\boxed{\epsilon = \frac{1 - e^{-NTU\left(\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right)}}{\left(\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right)}}$$

<u>Case 1:</u> Lets say $C_c \rightarrow C_{min}$ and $C_h \rightarrow C_{max}$.

$$\epsilon = \frac{1 - e^{-NTU\left(1 + \frac{C_c}{C_h}\right)}}{\left(1 + \frac{C_c}{C_h}\right)}$$

$$\boxed{\epsilon = \frac{1 - e^{-NTU\cdot\left(1 + \frac{C_{min}}{C_{max}}\right)}}{\left(1 + \frac{C_{min}}{C_{max}}\right)}}$$

<u>Case 2:</u> Let say $C_c \rightarrow C_{max}$, $C_h \rightarrow C_{min}$.

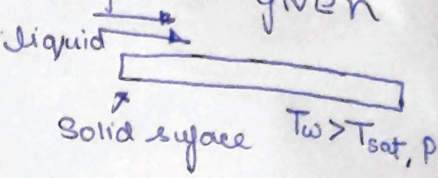
$$\boxed{\epsilon = \frac{1 - e^{-NTU\left(1 + \frac{C_{min}}{C_{max}}\right)}}{\left(1 + \frac{C_{min}}{C_{max}}\right)}}$$

We get the same value of effectiveness for both cases.

The above two observation shows that there is no deviation in the effectiveness of HE for case 1 and case 2.

* Boiling and Condensations :

1). Boiling – It is convection heat transfer process in which the liquid undergoes change in its phase (vapour) when it comes in contact with a solid surface that at a temperature greater than the temperature of saturation of the liquid corresponding to given pressure.



• Classification of Boiling –

a). Subcooled or Local boiling.

b). Saturated or Bulk boiling.

In subcooled, the liquid comes in contact with the surface when its Temp. less than saturation temp. This results in sensible heat addition from the HX surface to the liquid during which vapour bubble forms and collapse in before reacting of the liquid with the ambient. This continues till the liquid reaches its saturation condiⁿ.

While in saturated, liquid comes in contact with the surface at a temp. = saturated temp. Thus, boiling commense without prefixing sensible heat addition.

c). • Pool boiling or Submerged boiling

d). • Flow boiling or Forced convecⁿ boiling.

In pool boiling, the HX surface is completely submerged in liquid.

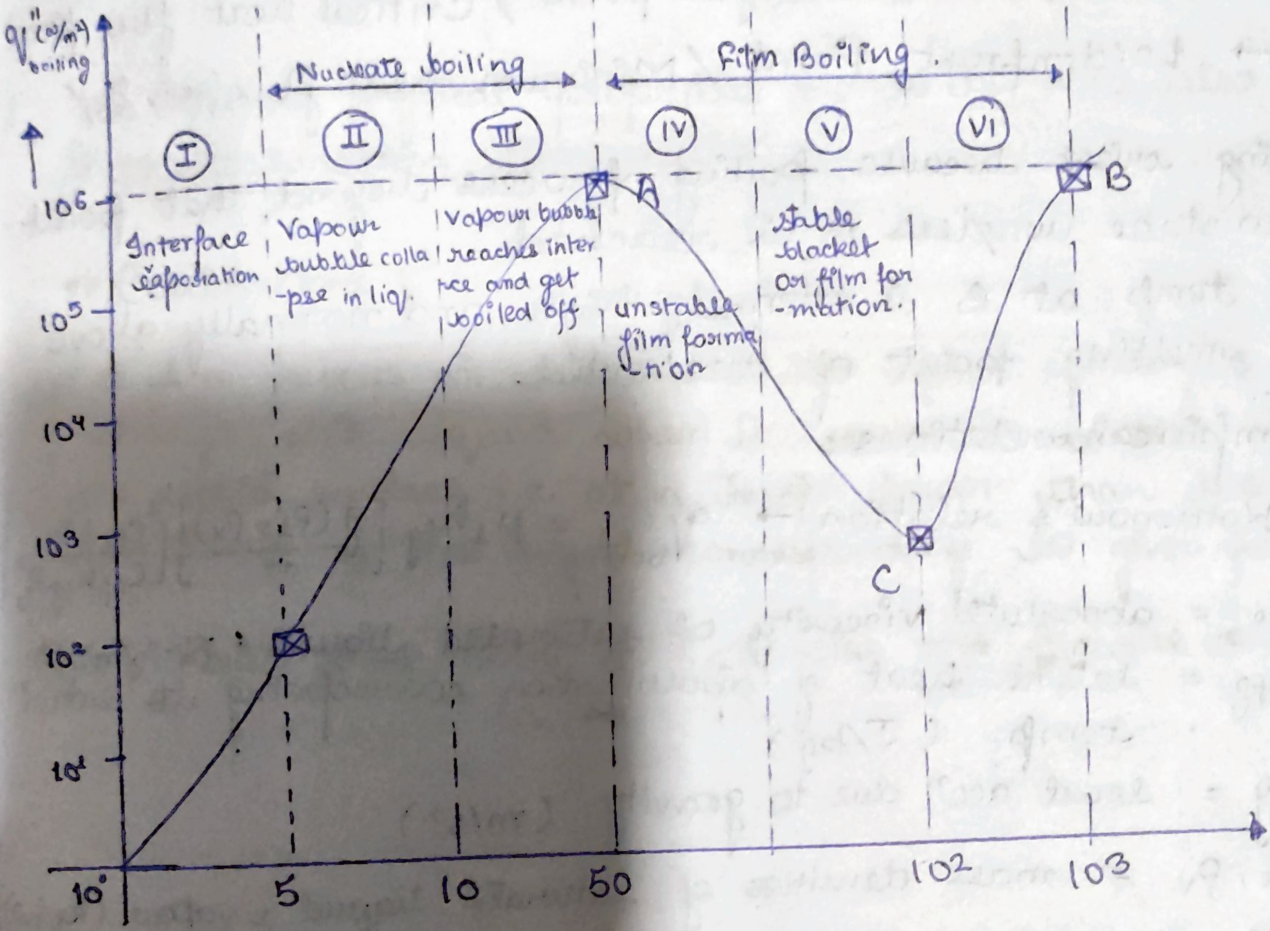
eg :

Whereas, it the forced boiling, the liquid to be boiled is force through a tube or pipe whose inner surface provides the necessary HX to the liquid.

eg : Babcock - Wilcox boiler, all water tube boilers, window AC, etc.

• Boiling Curve : (Pool Boiling Curve / Nukiyama Curve)

Nukiyama conducted exp. on pool boiling by taking an electrical resistance Nichrome heating element immersed in liquid water (at saturating temperature) by varying its surface temp. and measuring the surface heat flux.



$$\Delta T_E = (T_w - T_{sat})\,^\circ C$$

$$q''_{boiling} = \bar{h}\,(T_w - T_{sat})$$

Nukiyama varies the ohmic resistance of the heating element and thus varying ΔT_E and made measurement for boiling heat flux. The plotted pool boiling curve is plotted wrt boiling heat flux (q'') as a function of ΔT_E.

Upto 5°C it is known as interface evaporation.

Upto 10°c, vapour bubble collapse in the liquid.

The film is nothing but vapour bubble formation very much (blanket)

Nucleate boiling is the best regime in whole episode because we get 10^6 W/m^2 at 50°C and then at 1000°C. As 1000°C is too much, hence nucleate boiling is best.

A → boiling crisis point / Max. heat flux point / Burnout point / Peak heat flux point / Critical heat flux point.

C → leidenfrost Point / Minimum heat flux point /

Boiling crises because boiling process beyond that point is unstable unless B is reached.

The temp. at B is extremely high and normally above the melting point of the solid.

* Empirical corelations :

1). Rohsenow's relation → $q''_{boiling} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left[\frac{C_{pl}\,\Delta T_e}{C_{sf}\, h_{fg}\, P_{r_l}^{\,n}}\right]^3$

μ_l = absolute viscosity of saturated liquid (N-s/m^2)

h_{fg} = latent heat of vapourization correspoing to saturn temp. (J/kg)

g = local acc^n due to gravity (m/s^2)

ρ_l, ρ_v = mass densities of saturate liquid & vapour (kg/m

σ = surface tension at liq. - vap. interface.

C_{pl} = specific heat at const. pressure for sat. liquid

ΔT_e = temp. axis (°C)

C_{sf} = empirical const. whose value depends on the nature of surface, surface combination and nature of surface.

P_r = Prandtl' number of saturated liq.

n = 1 for water and 1.7 for other liquid

2). Peak heat flux (Zuber's Relation)

$$CHF = 0.149\, h_{fg}\, \rho_v \left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2} \right]^{1/4} \quad W/m^2.$$

3). Zuber - Berenson relation of leidenfrost point or MHF:

$$MHF = 0.09\, h_{fg}\, \rho_v \left[\frac{\sigma g(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2} \right]^{1/4} \quad W/m^2.$$

The thermo physical properties are to be calculated at the saturation temperature

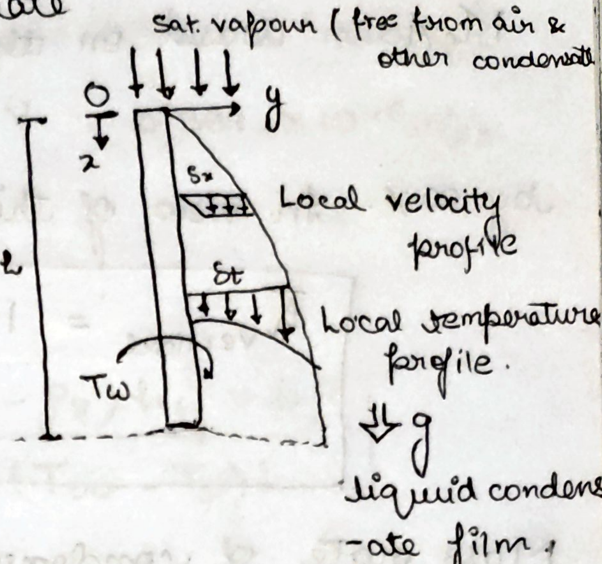
• Condensations :

It is a process in which saturated vapour undergoes a phase change into liquid when it comes in contact with the solid surface i.e. at a Temp. lower than the saturat-ion Temp. of the liquid corresponding to given pressure.

Que: Write short notes of drop wise and filmwise condensation

Wilhem Nuselt Approach:

This approach gives the theory of laminar filmwise condensation over an isothermal vertical flat plate



1. Assumptions -

Nuselt made certain assumptions for filmwise

- The vapour is pure dry saturated and free from air and any other condensate
- Flow of condensate film is laminar.
- local velocity profile is parabolic and local temp. profile is assumed to be linear.
- The themophysical properties of condensing liquid are assumed to be constant.

The analytical solution for thickness of condenstate film at given x from leading edge

$$\boxed{\delta_x = \left[\frac{4\mu_l k_l (T_{sat} - T_w)x}{g\rho_l(\rho_l - \rho_v)h_{fg}}\right]^{1/4}}$$

The local value of condensation heat transfer coeffecient can be given by

$$\boxed{h_x = \frac{k_l}{\delta_x}}$$

$$h_x = \left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}k_l^3}{4\mu_l(T_{sat} - T_w)x}\right]^{1/4}$$

$$\bar{h} = \frac{1}{L}\int_0^L h_x\,dx$$

$$\bar{h}_{vertical} = \frac{4}{3}\left[h_x\right]_{x=l}$$

$$\boxed{\bar{h}_{vertical} = 0.943\left[\frac{g\rho_l(\rho_l-\rho_v)h_{fg}k_l^3}{\mu_l(T_{sat}-T_w)L}\right]^{1/4} \; W/m^2K}$$

McAdam based on the condulded that the understitaate the result by 20%. In view of this McAdam refined the solⁿ for $\bar{h}$

$$\boxed{\bar{h}_{vertical} = 1.1316\left[\frac{g\rho_l(\rho_l-\rho_v)h_{fg}k_l^3}{\mu_l(T_{sat}-T_w)L}\right]^{1/4} \; W/m^2K}$$

Mass rate of condensation,

$$\boxed{m_{condensation} \times h_{fg} = \bar{h}S(T_{sat}-T_w)}$$

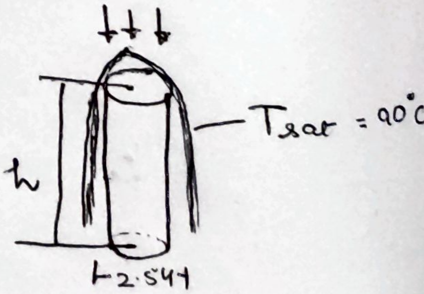
S = surface area, h_{fg} = latent heat

The thermophysical properties are to be evaluated at mean film temperature. $T = (T_{sat}+T_w)/2$

Ques: Saturated steam i.e. pure and free from air and other non-condensate at a temp. 90°C corresponding to a given pressure condenses in a steam condenser comprise a single vertical tube of out. dia 2.54 cm. On its outer surface, the surface temp. of condenser tube is maintained at 70°C. Calculate the design height of the tube of condenser. If it is required to condense 20 kg/hr of steam

$\dot{m}$ = 20 kg/hr = $\frac{20}{3600}$ kg/s.

= 5.556×10^{-3} kg/s.



T_{sat} = 90°C.

$T_{mean} = \frac{90+70}{2} = 80°C.$

* When the temp. mean in problem comes < 100°C. Then we will find the value of thermophysical properties at 100°C.

$k_l = 0.6687\ W/mK$

$\mu = 354 \times 10^{-6}\ N\text{-}s/m^2$.

$\rho_l = 974\ kg/m^3$.

$\nu = 0.364 \times 10^{-6}\ m^2/s$.

$\rho_v = 0.598\ kg/m^3$.

$h_{fg} = 2307.3 \times 10^3\ J/kg$.

$$\bar{h}_{vertical} = 1.1316 \left[\frac{g \rho_l (\rho_l - \rho_v) h_{fg} \times k_l^3}{\mu_l (T_{sat} - T_w) L} \right]$$

* THERMAL RADIATION

→ The energy emitted by any one of the given body is balanced by that received by other body. On the other hand, if bodies are at two different temp. Then there is net propagation of thermal radiation from hotter to colder body.

On account of the fact that the rate of emission from the hotter body dominate the rate of exhaustion from the incident radiation from the neighbouring body. It is also know as Prevost's Theory of Thermal Radiation.

• Prevost's Theory :

1). Theory of propagation of Thermal radiation (also known as Clark Maxwell electromagnetic wave theory)

2). Quantum Mechanics Theory by Max Plank.

Clark Maxwell theory states that all the bodies in the universe are permeated (covered) by some hypothetical ether medium and species of a given body on the accont of change of electronic configuration of them set up vibration in the ether medium which results in continuous electromagnetic waves that travel with speed of light and get incident on colder body.

$$\boxed{\mu = \frac{C_0}{C}}$$

μ = refractive index.

C_0 = speed of light in vaccum.

$\mu = 1$ generally for all engineering gas.

$\mu = 1.33$ for water, 1.5 for glass.

Max plank contradicts the obve theory and stated that the propagation of thermal radiation is not continuous but in instant. It is discreet in the form of carriers or packets of energy called photons.

$$1 \text{ photon} = h\nu = \frac{hC_0}{\lambda} \quad \left\{ \begin{array}{l} h = \text{Plack's constant} \\ = 6.66256 \times 10^{-34} \text{ J-s} \end{array} \right.$$

$$E_{photon} = mC_0^2 \qquad \lambda \to \mathring{A}, \ \nu \to Hz.$$

Now, making use of Einstein's mass energy equivalence

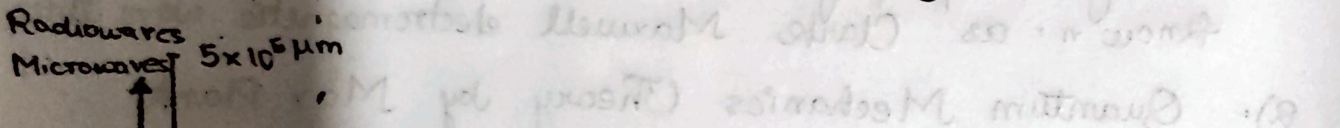
$$\therefore \ h\nu_0 = mC_0^2$$

$$m_{photon} = \frac{h\nu_0}{C_0^2} = \frac{h}{\lambda C_0}$$

$$\boxed{m_{photon} \times C_0 = \frac{h}{\lambda}}$$

$$m_{photon} \times C_0 = \text{momentum of photon}$$

- Wavelength Spectrum Of Themal Radiation



- Irradiation or Incident Radiation:

It is defined as the rate at which thermal radiaⁿ is getting incident

Ques:- Write short note on black body, white body, opaque body, transparent body, grey body and coloured body.

• **Black body**: Black body is a surface that absorbs all radiant energy falling on it. The term arises because incident visible light will be absorbed rather than reflected and therefore the surface will appear black.

The best practical black body is a small body (hole) in a box with a blackened interior because practically none of the radiation entering such a hole could escape inside. A surface covered with lampblack will absorb about 97% of incident light and can be considered as black body.

for a black body. $\tau = 0$, $\alpha = 1$, $\rho = 0$

• **White body**: It is a hypothetical substance whose surface absorbs no electronic radiation of any wavelength, that is, one which exhibits zero absorptivity for all wavelengths.

For a white body, $\tau = 0$ $\alpha = 0$, $\rho = 1$.

where α = absorptivity τ = transmissibility ρ = reflectivity.

• **Opaque body**: An opaque body is one that transmits none of the radiation that reaches it, although some may be reflected. For an opaque object, sum of absorptivity and reflectivity is equal to one.

$$\alpha + \rho = 1 \qquad \tau = 0$$

Eg: All thick metal and non-metallic surfaces, all liquids etc.

• **Transparent body**: When all the irradiation is transmitted through the body, it is called a transparent body.

for transparent body, $\alpha = 0$, $\rho = 0$ and $\tau = 1$
for eg: dry air.

- Grey body: A grey body is a body that emits radiation at each wavelength in a constant ratio less than unity to the emitted by a black body at same temperature. A grey body is a physical body whose absorbtivity does not vary with variation in temperature and wavelength of the incident radiation

Unlike the black body, a grey body is non-ideal emitter or an imperfect radiator. Grey body can absorb some of the energy it receives and also it may reflect some of the energy.

Ques:- Write short note on:

1). Monochromatic emissive power.
2). Wein's law.
3). Concept of black body radiation.
4). Rayleigh
5). Max Planck's law.
6). Total hemispherical emissive power.

Ans:-

- Monochromatic emissive power: The energy emitted by the surface at a given length per unit time per unit area in all direction is known as monochromatic emissive power.

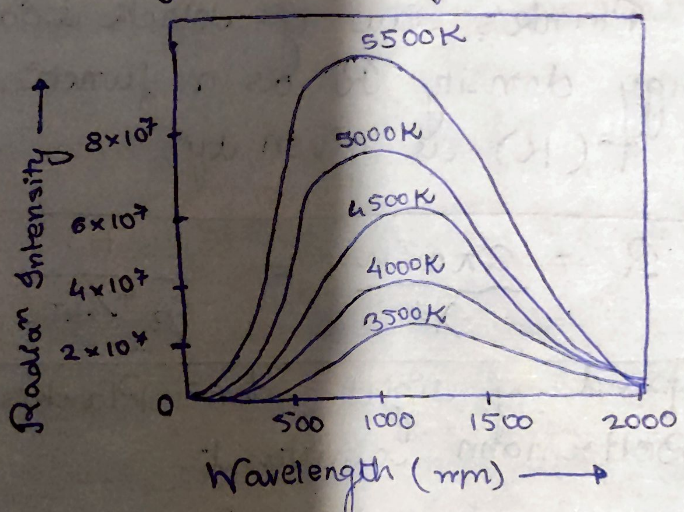
The energy emitted by a black body is a function of it temperature and is not evenly distributed over all wavelength. Its unit is W/m^2.

At a specified temperature, monochromatic emissive power increase with increase in wavelength and attain a maximum value corresponding to particular wavelength. However, further increase in wavelength results in decrease in emissive power.

• **Wein's displacement law:** It states that black body has different peaks of different wavelength at temp. that are inversely proportional to temperature.

$$\lambda_{max} = \frac{b}{T} \quad \begin{cases} b = \text{Wein's disp. constant} \\ \quad = 2.8977 \times 10^{3}\ mK \\ T = \text{temp. in Kelvin} \end{cases}$$

Wilhelm Wein, tells us that objects of different temp. emits spectra that peak at different wavelengths. Hotter object emit radiation of shorter wavelength and hence they appear blue and cooler object emit radiation of longer wavelength so they appear reddish.



• **Black body radiation** – Black body radiation is the thermal electromagnetic radiation within or surrounding, a body in thermodynamic equilibrium with its environment, emitted by a black body. A black body is a hypothetical body that completely absorbs all wavelengths of thermal radiation incident on it (vi). <u>Rayleigh's Law</u>: The law stated that the intensity of the radiation emitted by a black body is directly proportional to the temperature and inversely proportional to the wavelength raised to a power of four. However, this law works for only low frequencies.

Rayleigh - Jean equation can be written as:

$$\boxed{B_\lambda(T) = \frac{2ck_bT}{\lambda^4}}$$

where, B_λ = spectra radiace, k_b = Boltzmann constant

c = speed of light, T = temp. in Kelvin.

(v). <u>Max Planck's law</u>: A mathematical relationship formulated is 1900 by German physicist Max Plack to explain the spectral energy distribution of radiation emitted by a black body.

According to Plank's law of black body radiation, the spectral energy density R as a function as wavelength $\lambda(m)$ and temp. $T(K)$ is given by,

$$\boxed{R = \frac{2\pi c^2 h}{\lambda^5} = \frac{1}{e^{hc/\lambda kT} - 1}}$$

where, c = speed of light. h = Plank's constant

k = Boltzmann constant.

And, The wavelength of the emitted radiation is inversely proporpositonal to its frequency:

$$\boxed{\lambda = \frac{c}{\nu}}$$

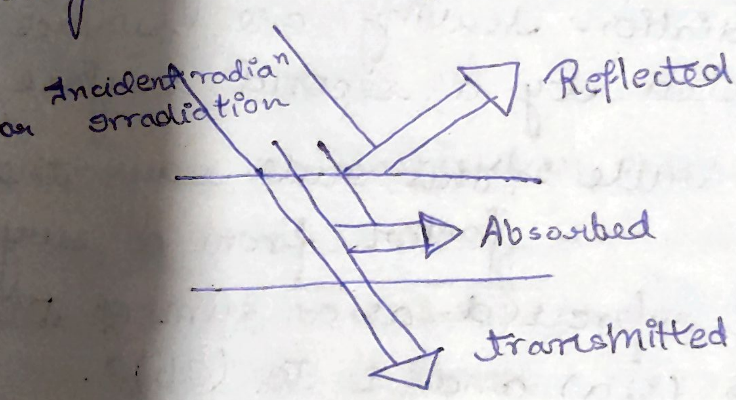
(vi). <u>Total hermispherical Emissive power</u>: The spectral hemispherical emissive power is defined as the rate at which radiation is emitted per unit area at all possible wavelengths and in all possible direction, from a surface per unit wavelength and per unit surface area

Ques: Define :

- Absorptivity : The funcⁿ of irradiation absorbed by the surface is called the absorptivity. It is the ratio of absorbed radiation to incident radiation.
  Its value range : $0 \leq \alpha \leq 1$.

- Reflectivity : The funcⁿ of radiation reflected by the surface is called the reflectivity (ρ).
  It is the ratio of reflected radiation to incident radiation.
  Its value range : $0 \leq \rho \leq 1$.

- Transmissivity : The fraction of radiation transmitted is called the transmissivity.
  It is the ratio of transmitted radiation to incident radiation.
  Its value range : $0 \leq \tau \leq 1$



- Emissivity : The emissivity of an object or surface is a meature for how strongly it interacts with the thermal radiation is terms of emission and absorption. A high emissivity implies a high importance.
  Emissivity is defined as the ratio of the energy radiated from a material's surface to that

radiated from a perfect emitter known as black body at the same temperature and wavelength under the same viewing conditions. It is a dimensionless number b/w 0 to 1.

- **Kirchoff's Law of Radiation**: At a given temp, the ratio of the emissive power of a body to its absorptive power is const. and is equal to the emissive power of a black body at the same temperature.

$$\frac{E}{a} = E_b \quad \text{but} \quad \frac{E}{E_b} = e$$

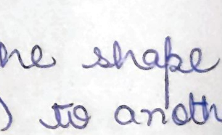
$$\therefore \boxed{a = e}$$

- Ques: Define view factor. also discuss the summation & reciprocating rules of the view factor. Aditionally discuss the shape factor algerbrically and salient features.

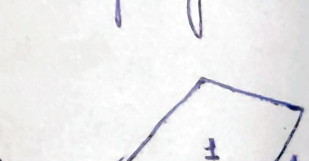
Ans: The view factor is the degree to which heat carried by radiation leaving one surface which is intercepted by a second surface.

- Sumation rule: This rule says that the shape factor from a surface (1) to another (2) can be expressed as a sum of the shape factor from (1) to (2a) and 1 to (2b).

$$\boxed{F_{1\to 2} = F_{1\to 2a} + F_{1\to 2b}}$$



- Reciprocating rule: The reciprocating rule relate to the shape factors from (1) to (2) and that from (2) to (1) as follows.



$$\boxed{A_1 F_{1\to 2} = A_2 F_{2\to 1}}$$

Shape factor for some imp. surface area.

* plane surface = 0 ($F_{11} = 0$)
* Convex surface = 0 ($F_{11} = 0$)
* Concave surface $\neq 0$ ($F_{11} \neq 0$)

• Important points -

* Shape factor only depends on the geometer of a body.
* $0 \leq F_{m-n} \leq 1$
* $A_1 F_{1\to 2} = A_3 F_{1-3} + A_4 F_{4-2}$, if radiation surface A_1 is subdivided into A_3 and A_4.
* $A_1 F_{1-2} = A_1 F_{1-3} + A_1 F_{1-4}$, if receivent surface A_2 is subdivided into A_3 and A_4.
* When one body is completely enclosed inside the other body, then the shape factor of the inner body to outer body is equal to 1.

Que:- Prove that the hemispherical emissive power varies with the fourth power of absolune temp. of the body

$$Q = \sigma T^4$$

Ans:- The total power radiated per unit area over all wavelength of a block body can be obtained by integrating Planck's radiation formula. Thus the radiated power per unit area as a function of wavelength is.

$$\frac{dP}{d\lambda} \times \frac{1}{A} = \frac{2\pi hc^2}{\lambda^5 (e^{hc/kT} - 1)}$$

where P = power radiation, A = surface area of block body.

λ = wavenlength of emitted radia^n, k = Boltzmann's const.

h = Plack's constant, c = speed of light.

T = temp.

$$\frac{d(P/A)}{d\lambda} = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

On integrating both the sides with rs to λ and appling the limit.

$$\int_0^{\infty} \frac{d(P/A)}{d\lambda} = \int_0^{\infty} \left[\frac{2\pi hc^2}{(e^{hc/\lambda kT} - 1)\lambda^5} \right] d\lambda$$

$$\frac{P}{A} = 2\pi h^2 c \int_0^{\infty} \left[\frac{d\lambda}{\lambda^5 (e^{hc/\lambda kT} - 1)} \right] \quad \text{— (I)}$$

Let $x = \frac{hc}{\lambda kt}$, $dx = -\frac{hc}{\lambda^2 kT} d\lambda$ — (II)

$$d\lambda = -\frac{\lambda^2 kT}{hc}, \quad h = \frac{x\lambda kT}{c}, \quad c = \frac{x\lambda kT}{h} \quad \text{— (IV)}$$

Substituting (III) and (IV) in (I), we get

$$\frac{P}{A} = \left(2\pi \frac{x\lambda kT}{c}\right)\left(\frac{x\lambda kT}{h}\right)^2 \int_0^{\infty} \left[-\frac{\lambda^2 kT / hc}{e^x - 1} \right] dx.$$

$$\frac{P}{A} = 2\pi \left(\frac{x^3 \lambda^5 k^4 T^4}{h^3 c^2 \lambda^5} \right) \int_0^{\infty} \left[\frac{x^3}{dx - 1} \right] dx \quad \text{— (V)}$$

Comparing the eq. (V) with standard form of integrals

$$\int_0^{\infty} \left[\frac{x^3}{e^x - 1} \right] dx = \frac{\pi^4}{15}$$

$$\therefore \frac{P}{A} = \frac{2\pi (kT)^4}{h^3 c^2} \frac{\pi^4}{15}$$

$$\frac{P}{A} = \left(\frac{2k^4 \pi^5}{15 h^3 c^2} \right) T^4$$

$$\boxed{\frac{P}{A} = \sigma T^4}$$

$$\sigma = \frac{2k^4 \pi^5}{15 h^3 c^2} = 5.670 (10^8) \, (W/m^2 K^4)$$

• Calculation of interchange factor for common configuration

S.no.	Configuration	Interchange factor$_{12}$	Configuration factor
01.	Infinite parallel planes	$\dfrac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$	1
02.	Cylinders or parallel spheres	$\dfrac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$	1
03.	Small body 1 enclosed by body 2	ϵ_1	1
04.	Long body 1 enclosed by body 2	$\dfrac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$	1
05.	Two rectangle with common sides at right angle to each other	$\epsilon_1\epsilon_2$	1

Ques: Determine the rate of heat loss by red portion from a steel tube of outside dia. 70mm and 3m long at a temp. of 227°C. It the tube is located within a square brick of 0.3m sides at temp. of 27°C. $\epsilon_{steel} = 0.79$

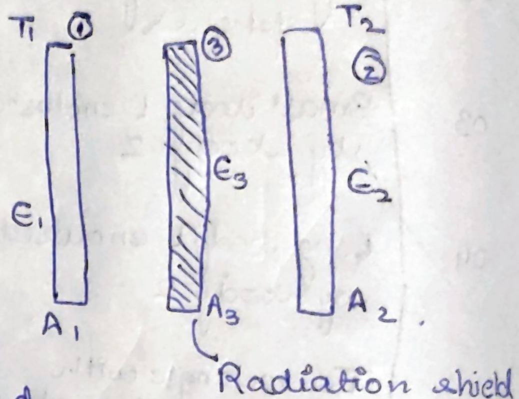
$\epsilon_{brick} = 0.93$.

Radiation Shield :

Radiation shield reduces the radiation heat exchange by effectively increasing the surface resistance without actually removing any heat from the overall system.

eg: Thin sheets of plastics coated with highly reflecting metallic film on both side.

Let us consider, $A_1 = A_2 = A$



With no shiled,

$$Q_{1-2\,net} = \frac{\sigma A(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \quad \text{—} (1)$$

If the radiation shield is inserted they get heat exchange function 1 -3 & 3 - 2

$$Q_{1-3} = \frac{\sigma A(T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} \quad \text{—} (II)$$

$$Q_{3-2} = \frac{\sigma A(T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1} \quad \text{—} (III)$$

Since, the radiation shild does not remove heat from the system,

$$Q_{1-3} = Q_{3-2}$$

$$\frac{\cancel{\sigma A}(T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{\cancel{\sigma A}(T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1}$$

$$(T_1^4 - T_3^4)\left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right) = (T_3^4 - T_2^4)\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right)$$

$$T_1^4(x_1) - T_3^4(x_1) = T_3^4(x_2) - T_2^4(x_2)$$

$$T_3^4(x_1 + x_2) = T_1^4 x_1 + T_2^4 x_2$$

$$T_3^4 = \frac{T_1^4 x_1 + T_2^4 x_2}{x_1 + x_2}$$

$$\boxed{T_3^4 = \frac{T_1^4\left(\frac{1}{\epsilon_3}+\frac{1}{\epsilon_2}-1\right) + T_2^4\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_3}-1\right)}{\left(\frac{1}{\epsilon_3}+\frac{1}{\epsilon_2}-1\right)+\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_3}-1\right)}} \quad \text{— (IV)}$$

Now, net heat exchange with shield :

$$Q_{1-2\,net} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_3}-1\right)+\left(\frac{1}{\epsilon_3}+\frac{1}{\epsilon_2}-1\right)} \quad \text{— (V)}$$

$$\frac{Q_{1-2 \text{ with shield}}}{Q_{1-2 \text{ without shiled}}} = \frac{\dfrac{\cancel{\sigma A\left(T_1^4 - T_2^4\right)}}{\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_3}-1\right)+\left(\frac{1}{\epsilon_3}+\frac{1}{\epsilon_2}-1\right)}}{\dfrac{\cancel{\sigma A\left(T_1^4 - T_2^4\right)}}{\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1\right)}}$$

It is assumed that, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$

$$\frac{Q_{1-2 \text{ with shield}}}{Q_{1-2 \text{ without shiled}}} = \frac{1 + \cancel{1} - \cancel{1}}{1 + \cancel{1} - \cancel{1} + 1 + \cancel{1} - \cancel{1}} = \frac{1}{2}$$

$$\boxed{\frac{Q_{1-2 \text{ with shield}}}{Q_{1-2 \text{ without shield}}} = \frac{1}{2}}$$

Hence, we can conclude that,

$$\boxed{Q_{1-3\,net} = Q_{3-2\,net} = \frac{1}{2} Q_{1-2\,net}}$$

$$\boxed{T_3^4 = \frac{1}{2}\left(T_1^4 + T_2^4\right)}$$

Thus, when one shield is inserted b/w two parallel surfaces the direct radiation heat transfer is halved.

The corresponding value of T_3 will be

$$T_3^4 = \frac{1}{2}(T_1^4 + T_2^4)$$

Say there are 'n' no. of sheets, all the surface resistance will be same, since emissitivity is same there will be two surface resistance for each heat transfer surfaces.

There will be $(n+1)$ space resistances hence total resistance for n shield,

$$R_{n\,shield} = \left[(2n+2)\left(\frac{1-\epsilon}{\epsilon}\right) + (n+1)\right] \Big/ A$$

$$R_{n\,shield} = (n+1)\left(\frac{2}{\epsilon} - 1\right) \Big/ A$$

The radiant heat transfer rate b/w two perfectly parallel plate separated by n shield,

$$Q = \frac{\sigma A (T_1^4 - T_2^4)}{(n+1)\left(\frac{2}{\epsilon} - 1\right)}$$

* Suppose there is no shield i.e. $n = 0$

$$Q = \frac{\sigma A (T_1^4 - T_2^4)}{\left(\frac{2}{\epsilon} - 1\right)}$$

Either it may be asked,

(i). Ratio of rate of heat exchange with n number shield to that of no shield (OR)

(ii). What could be the ratio of resistances without shield to that of with n-number of shield.

$$\boxed{\frac{Q_{n\,shield}}{Q_{without\,shield}} = \frac{R_{without\,shield}}{R_{n\,shield}} = \frac{1}{n+1}}$$

Now eq. 5 $$Q_{1-2\,net} = \frac{\sigma A (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right) + \left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right)}$$

can be written as, $$Q_{1-2\,net} = \frac{\sigma A (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + 2\left(\frac{1}{\epsilon_3}\right) - 2}$$

The above equation can be generalised for a system of two parallel plane separated by 'n' shields of emissivity, $\epsilon_{s1}, \epsilon_{s2} \ldots \ldots \epsilon_{sn}$.

$$\boxed{Q_{1-2\,net} = \frac{\sigma A (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + 2\sum_{i=1}^{n} \frac{1}{\epsilon_{si}} - (n+1)}}$$