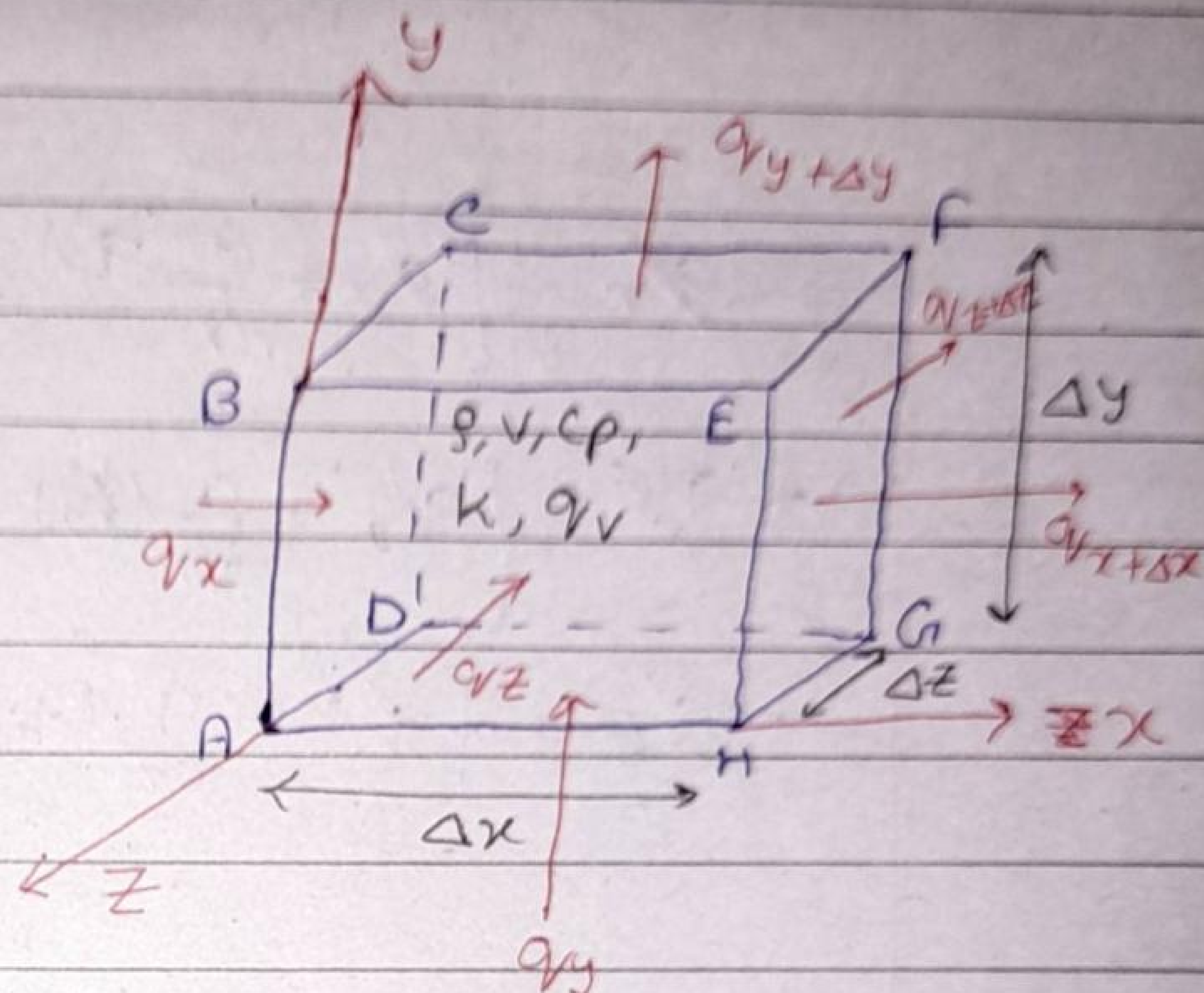
Gen. Heat conduction eqn in cartesian / Rectangular coordinate system -



Applying cons. of energy,

$$Q_{net} = W_{net} + \Delta U$$

Assumption -

① The work transition on account of temp change in a solid material is negligible

② It is assumed that change in KE & PE is assumed to be negligible

$$Q_{net} = \cancel{W}^{0} + d\cancel{PE}^{0} + d\cancel{KE}^{0} + dSE$$

$$Q_{net} = dSE$$

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$$q_{x\,net} + q_{y\,net} + q_{z\,net} + q_v = dSE$$

$$\Rightarrow q_x - q_{x+\Delta x} + q_y - q_{y+\Delta y} + q_z - q_{z+\Delta z} + q_v = dSE$$

$$\Rightarrow \cancel{q_x} - \left[\cancel{q_x} + \frac{\partial}{\partial x}(q_x)\frac{\Delta x}{1!} + \frac{\partial^2}{\partial x^2}(q_x)\frac{\Delta x^2}{2!} + \dots\right]$$
$$+ \cancel{q_y} - \left[\cancel{q_y} + \frac{\partial}{\partial y}(q_y)\frac{\Delta y}{1!} + \frac{\partial^2}{\partial y^2}(q_y)\frac{\Delta y^2}{2!} + \dots\right]$$
$$+ \cancel{q_z} - \left[\cancel{q_z} + \frac{\partial}{\partial z}(q_z)\frac{\Delta z}{1!} + \frac{\partial^2}{\partial z^2}(q_z)\frac{\Delta z^2}{2!} + \dots\right]$$
$$+ q_v = dSE$$

⇒ Neglecting the higher power,

$$-\frac{\partial}{\partial x}(q_x)\frac{\Delta x}{1!} - \frac{\partial}{\partial y}(q_y)\frac{\Delta y}{1!} - \frac{\partial}{\partial z}(q_z)\frac{\Delta z}{1!} + q_v = dSE$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(-k_x A\frac{\partial T}{\partial x}\right)\Delta x - \frac{\partial}{\partial y}\left(-k_y A\frac{\partial T}{\partial y}\right)\Delta y - \frac{\partial}{\partial z}\left(-kA\frac{\partial T}{\partial z}\right)\Delta z + q_v = dSE$$

$$\Rightarrow \frac{\partial}{\partial x}\left(k_x A\frac{\partial T}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(k_y A\frac{\partial T}{\partial y}\right)\Delta y + \frac{\partial}{\partial z}\left(k_z A\frac{\partial T}{\partial z}\right)\Delta z + q_v = dSE$$

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$$\Rightarrow \frac{\partial}{\partial x}\left(k_x \cdot \Delta z\Delta y \cdot \frac{\partial T}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(k_y\,\Delta x\Delta z\frac{\partial T}{\partial y}\right)\Delta y + \frac{\partial}{\partial z}\left(k_z\,\Delta x\Delta y\frac{\partial T}{\partial z}\right)\Delta z + q_v = dSE$$

$$\Rightarrow \frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)\Delta x\Delta y\Delta z + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)\Delta x\Delta y\Delta z + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)\Delta x\Delta y\Delta z + q_v = dSE$$
$$+ q_g V = mc_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)\cancel{\Delta x\Delta y\Delta z} + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)\cancel{\Delta x\Delta y\Delta z} + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)\cancel{\Delta x\Delta y\Delta z} + q_g\cancel{V} = \rho\cancel{V}c_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + q_{gen} = \rho c_p\frac{\partial T}{\partial t}}$$

Let's consider material is homogeneous & is isotropic

$$k_x = k_y = k_z = k$$

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$$\Rightarrow \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q_{gen} = \rho C_p \frac{\partial T}{\partial t}$$

$$\Rightarrow k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial y^2} + k\frac{\partial^2 T}{\partial z^2} + q_{gen} = \rho C_p \frac{\partial T}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

$$\left[\alpha = \frac{k}{\rho C_p}\right]$$

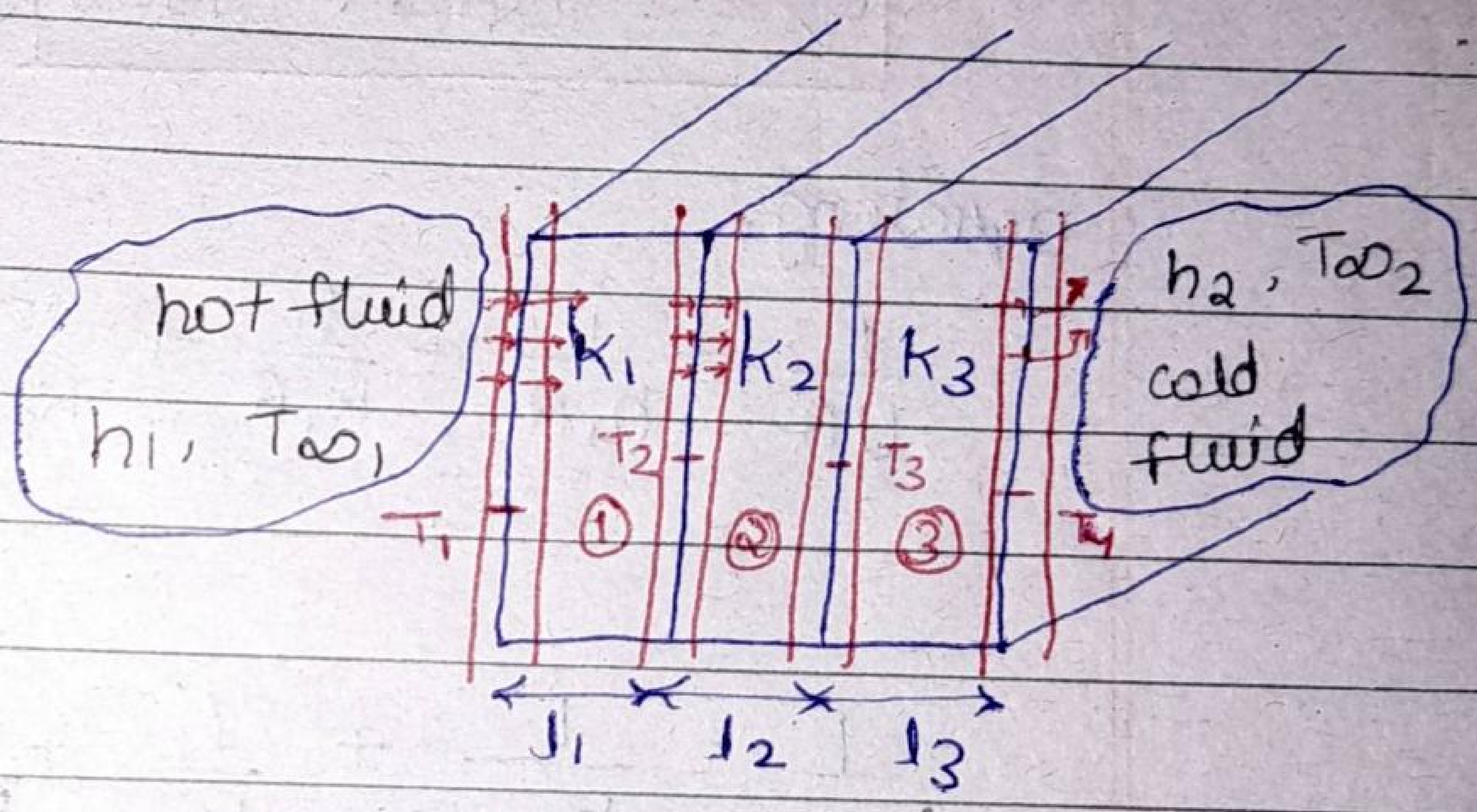
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \cancel{\frac{q_{gen}}{k}}^{\,0} = \frac{1}{\alpha}\cancel{\frac{\partial T}{\partial t}}^{\,0}$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\Rightarrow \boxed{\nabla^2 T = 0}$$ (Laplace eqn)

$$\Rightarrow \boxed{\cancel{\nabla^2 T + \frac{q_{gen}}{k} = 0}}$$

Composite slabs -



$$\cancel{q_{cond^n_1}}$$

$$q = q_{conv^n_1} = q_{cond^n_1} = q_{cond^n_2} = q_{cond^n_3}$$

$$= q_{conv^n_2}$$

$$q = \frac{(T_{\infty_1} - T_1)}{\frac{1}{h_1 A}} = \frac{T_1 - T_2}{l_1/k_1 A} = \frac{T_2 - T_3}{l_2/k_2 A}$$

$$\frac{T_3 - T_4}{l_3/k_3 A} = \frac{T_4 - T_{\infty_2}}{1/h_2 A}$$

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$$q = \frac{T_{\infty 1} - T_1 + T_1 - T_2 + T_2 - T_3 + T_3 - T_4 + T_4 - T_{\infty}}{\frac{1}{h_1 A} + \frac{l_1}{k_1 A} + \frac{l_2}{k_2 A} + \frac{l_3}{k_3 A} + \frac{1}{h_2 A}}$$

$$q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 A} + \frac{l_1}{k_1 A} + \frac{l_2}{k_2 A} + \frac{l_3}{k_3 A} + \frac{1}{h_2 A}}$$

Let's consider U is overall conv^n heat transfer coeff if multiplied by A & overall temp diff.

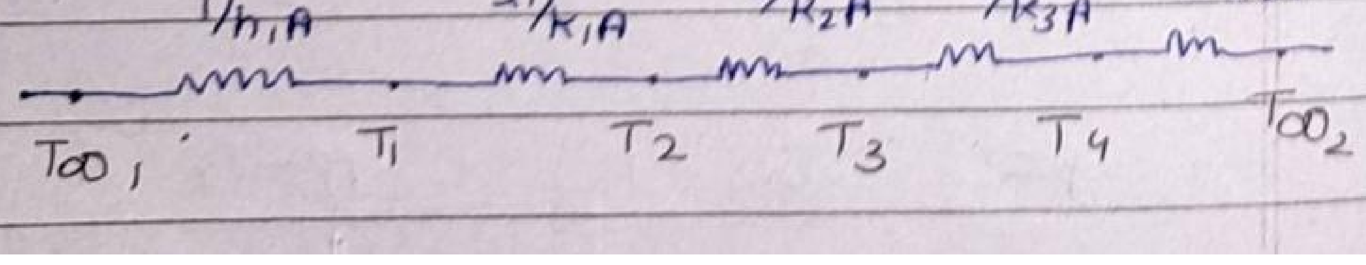
$$\boxed{q = UA(\Delta T)_{overall}}$$

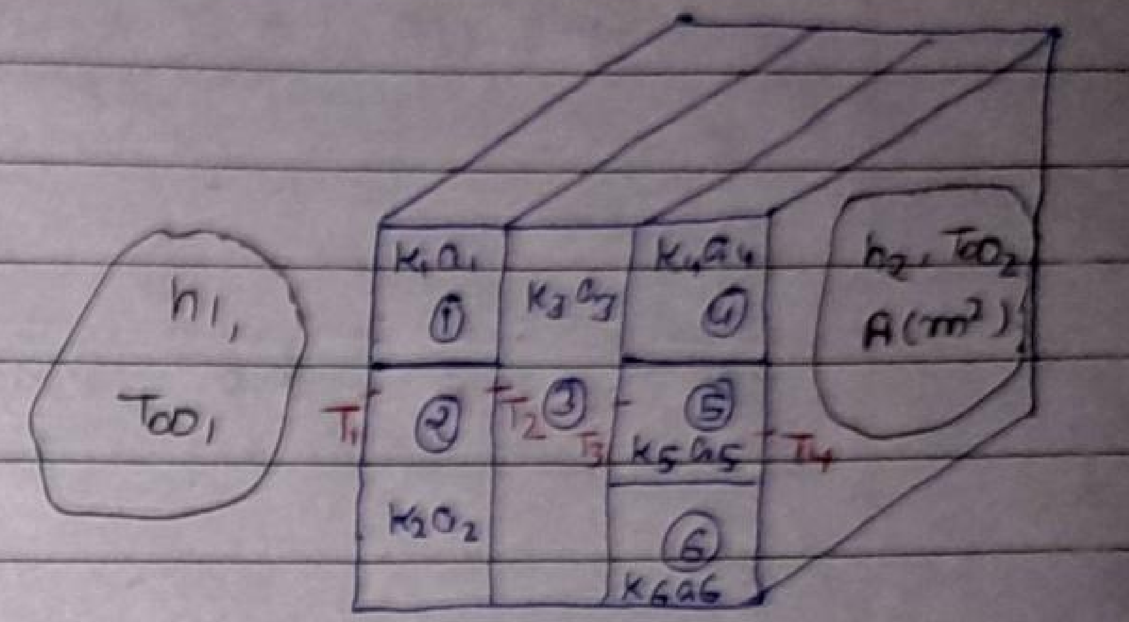
equating,

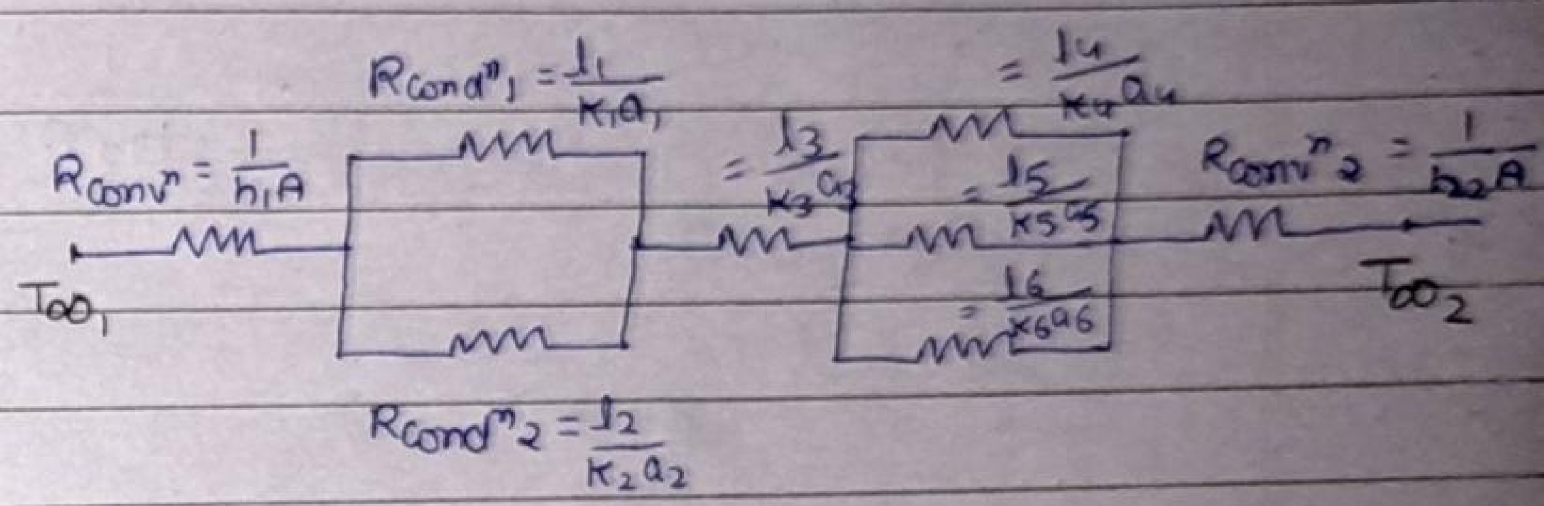
$$\frac{1}{UA} = \frac{1}{h_1 A} + \frac{l_1}{k_1 A} + \frac{l_2}{k_2 A} + \frac{l_3}{k_3 A} + \frac{1}{h_2 A}$$

$$\frac{1}{U} = \frac{1}{h_1} + \frac{l_1}{k_1} + \frac{l_2}{k_2} + \frac{l_3}{k_3} + \frac{1}{h_2}$$

$$\frac{1}{U} = \frac{1}{h_1} + \sum_{i=1}^{n} \frac{l_i}{k_i} + \frac{1}{h_2}$$







$$\frac{1}{UA} = \frac{1}{h_1 A} +$$

$$q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 A} + \frac{1}{\frac{l_1}{k_1 a_1} + \frac{l_2}{k_2 a_2}} + \frac{l_3}{k_3 a_3}}$$

$$q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 A} + \frac{1}{\frac{1}{R_{cond^n 1}} + \frac{1}{R_{cond^n 2}}} + R_{cond^n 3} + \frac{1}{\frac{1}{R_{cond^n 4}} + \frac{1}{R_{cond^n 5}} + \frac{1}{R_{cond^n 6}}} + \frac{1}{h_2 A}}$$

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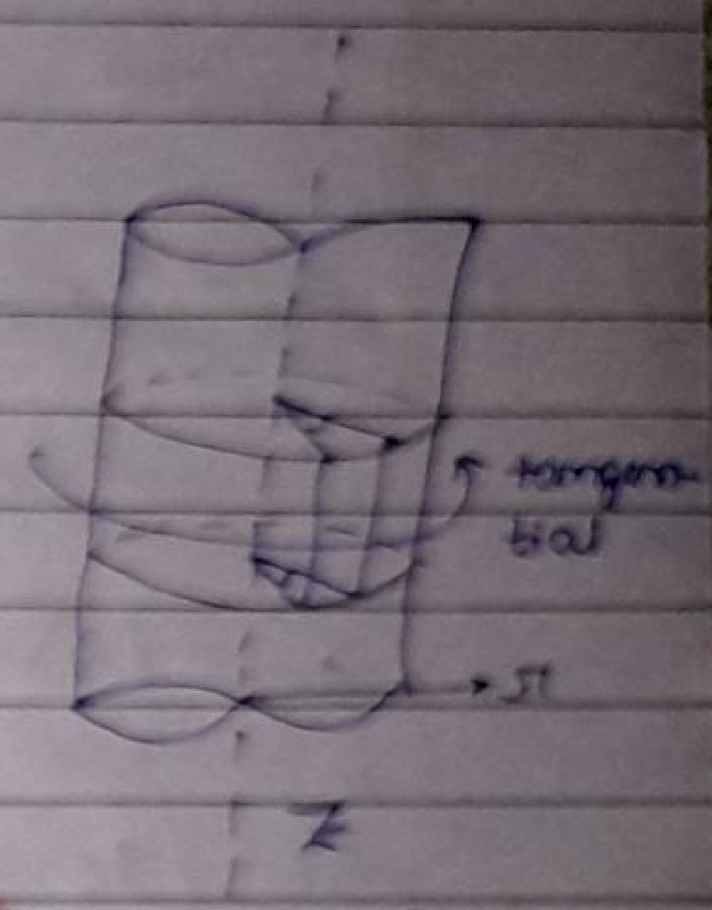
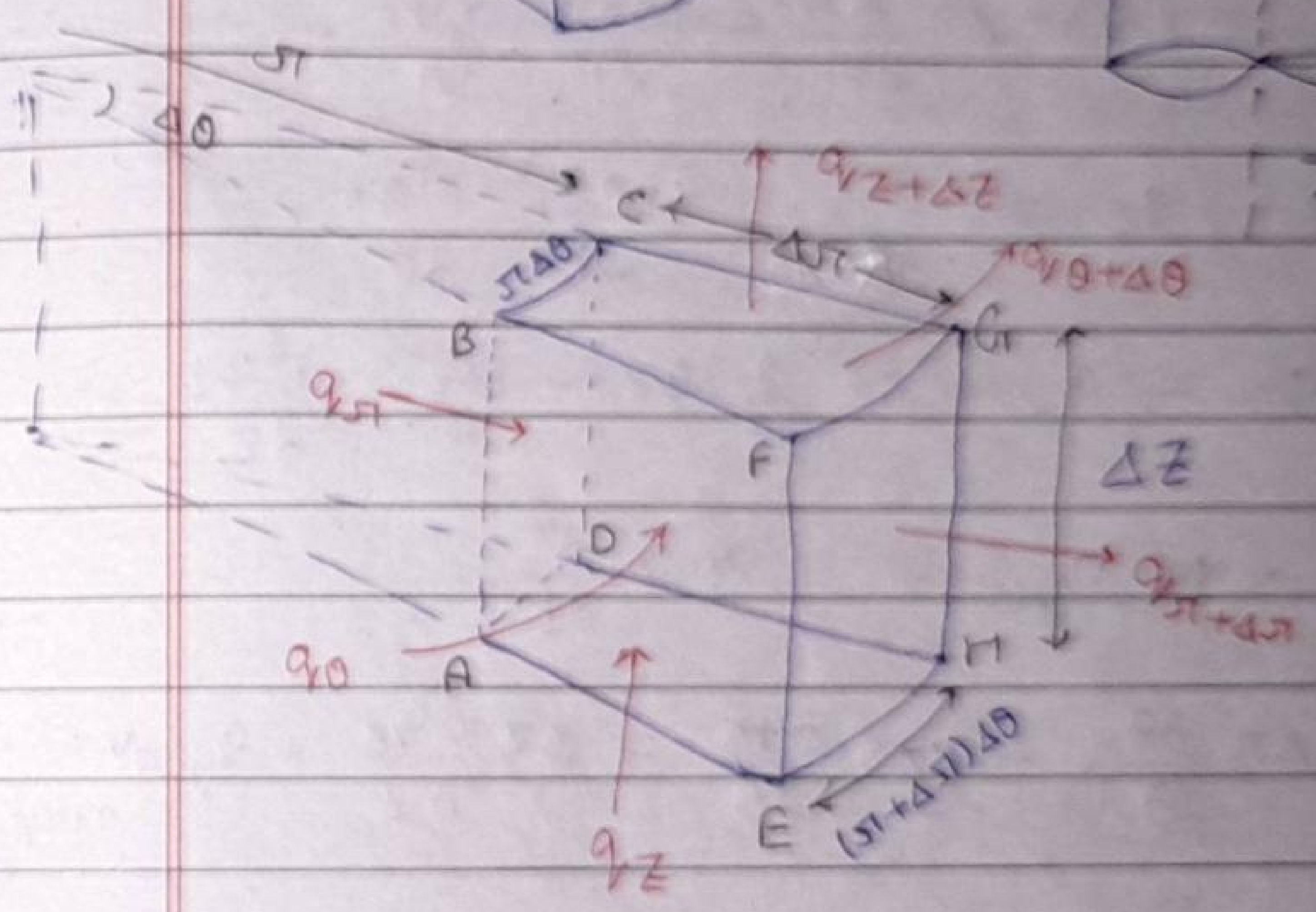
$q = UA\Delta T$

$$\frac{1}{UA} = \frac{1}{h_1 A} + \frac{1}{\frac{k_1 a_1}{l_1} + \frac{k_2 a_2}{l_2}} + \frac{l_3}{k_3 a_3} + \frac{1}{\frac{k_4 a_4}{l_4} + \frac{k_5 a_5}{l_5} + \frac{k_6 a_6}{l_6}} + \frac{1}{h_2 A}$$

General heat conduction





$\rho, v, C_p, k,$
α, q_v

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The increment in θ direction $[(r+\Delta r)\Delta\theta]$ assumed to be negligible.

i.e. $r\Delta\theta \simeq (r+\Delta r)\Delta\theta$

vol. of element $= r\Delta\theta \cdot \Delta r \cdot \Delta Z$

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$\Rightarrow q_{net} = W_{net} + \Delta IE$

$\Rightarrow q_{net} = \Delta SE$ [from assumption]

$\Rightarrow q_{r} - q_{r+\Delta r} + q_{\theta} - q_{\theta+r\Delta\theta}$

$+ q_{z} - q_{z+\Delta z} + q_{v} = \Delta SE$

$\Rightarrow q_r - \left(q_r + \Delta r \frac{\partial q_r}{\partial r} + \frac{(\Delta r)^2}{2!}\frac{\partial^2 q_r}{\partial r} + \cdots\right)$

$+ q_\theta - \left(q_\theta + r\Delta\theta \frac{\partial q_\theta}{\partial\theta} + \frac{(r\Delta\theta)^2}{2!}\frac{\partial^2 q_\theta}{\partial\theta} + \cdots\right)$

$+ q_z - \left(q_z + \Delta z \frac{\partial}{\partial z}(q_z) + \frac{(\Delta z)^2}{2!}\frac{\partial^2 q_z}{\partial z} + \cdots\right)$

$+ q_{gen} V = mc_p \frac{\partial T}{\partial t}$

$\Rightarrow -\Delta r \frac{\partial q_r}{\partial r} - r\Delta\theta \frac{\partial q_\theta}{\partial \theta} - \Delta z \frac{\partial q_z}{\partial z} + q_{gen}V = mc_p \frac{\partial T}{\partial t}$

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$\Rightarrow -\frac{\partial}{\partial r}\left(-k_r A \frac{\partial T}{\partial r}\right)\Delta r - \frac{\partial}{\partial \theta}\left(-k_\theta A \frac{\partial T}{\partial \theta}\right) r\Delta\theta$

$-\frac{\partial}{\partial z}\left(-k_z A \frac{\partial T}{\partial z}\right)\Delta z + q_{gen}V = mc_p \frac{\partial T}{\partial t}$

$\Rightarrow -\frac{\partial}{\partial r}\left(-k_r \times r\Delta\theta \times \Delta z \times \frac{\partial T}{\partial r}\right)\Delta r - \frac{\partial}{\partial\theta}\left(-k_\theta \times \Delta r \Delta z \times \frac{\partial T}{\partial \theta}\right) r\Delta\theta$

$-\frac{\partial}{\partial z}\left(-k_z \times \Delta r \times r\Delta\theta \times \frac{\partial T}{\partial z}\right)\Delta z + q_{gen} \cdot V$

$= mc_p \frac{\partial T}{\partial t}$

$\Rightarrow \Delta r \cdot \Delta\theta \Delta z \cdot \frac{\partial}{\partial r}\left(k_r \cdot r \cdot \frac{\partial T}{\partial r}\right) + \Delta r \cdot \Delta z (r\Delta\theta) \frac{\partial}{\partial\theta}\left(k_\theta \times \frac{\partial T}{\partial \theta}\right)$

$+ \Delta r (r\Delta\theta) \Delta z \frac{\partial}{\partial z}\left(k_z \cdot \frac{\partial T}{\partial z}\right) + q_{gen} V = \rho V c_p \frac{\partial T}{\partial t}$

$\Rightarrow \cancel{\frac{\partial}{\partial r}\left(k_r \times \frac{\partial T}{\partial r}\right)} \cancel{+} \Rightarrow \frac{r}{r}\Delta\theta \cdot \Delta r \cdot \Delta z \frac{\partial}{\partial r}\left(k_r \cdot r \cdot \frac{\partial T}{\partial r}\right)$

$+ \Delta r \cdot \Delta z \cdot (r\Delta\theta) \frac{\partial}{\partial\theta}\left(k_\theta \frac{\partial T}{\partial \theta}\right) + \Delta r \Delta z \cdot (r\Delta\theta) \times$

$\frac{\partial}{\partial z}\left(k_z \cdot \frac{\partial T}{\partial z}\right) + q_{gen} V = \rho V c_p \frac{\partial T}{\partial t}$

$$\Rightarrow \boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(k_r r \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(k_\theta \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + q_{gen} = \rho C_p \frac{\partial T}{\partial t}}$$

Assume material to be homogeneous & isotropic,

$$k_r = k_\theta + k_z = k$$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{q_{gen}}{k} = \frac{\rho C_p}{k}\frac{\partial T}{\partial t}$$

$$\Rightarrow \boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \left(\frac{\partial^2 T}{\partial \theta^2}\right) + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

where, $\alpha = \frac{k}{\rho C_p}$

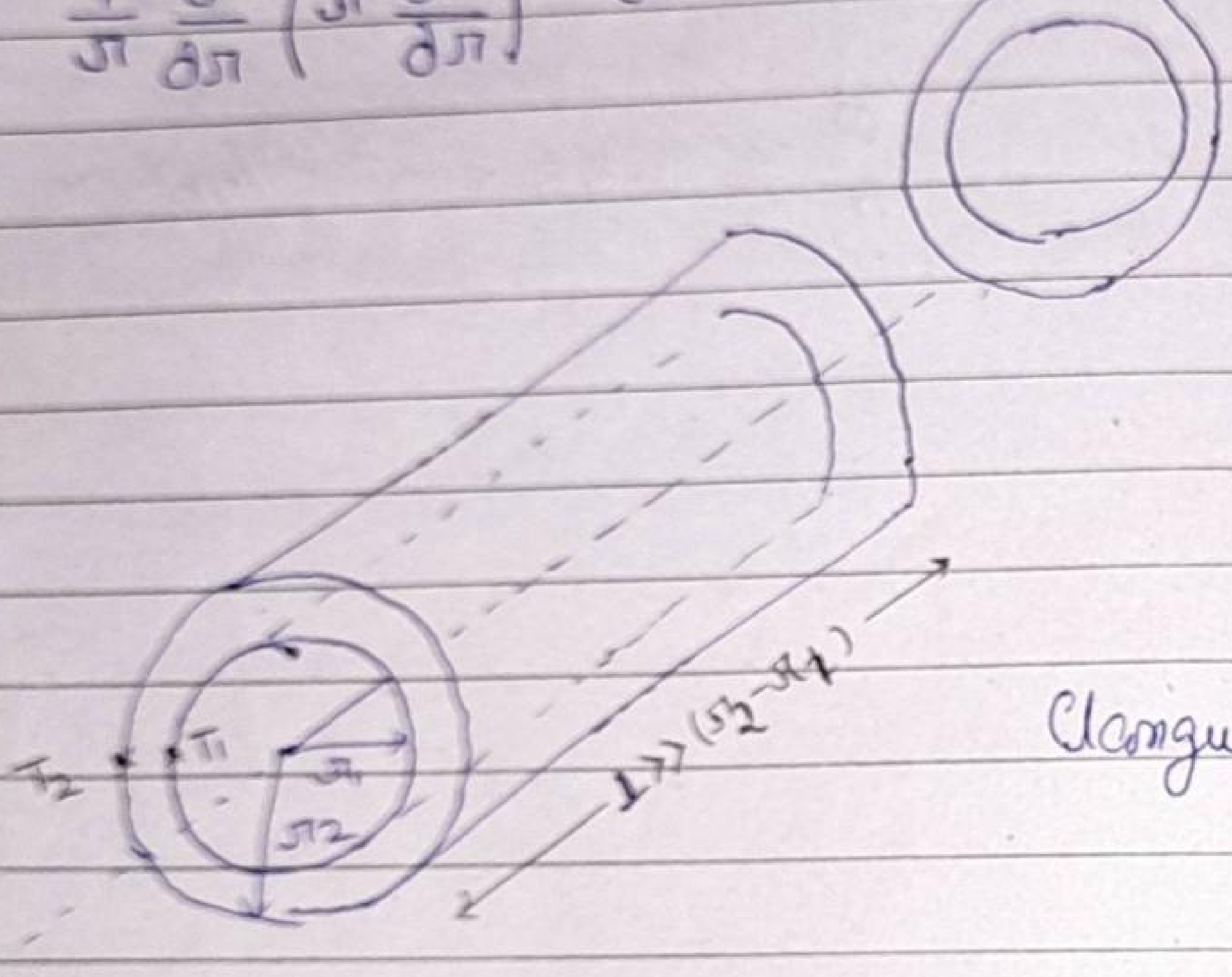
$$\nabla^2 T = 0$$

$$\nabla^2 T + \frac{q_{gen}}{k} = 0$$

$$\nabla^2 T + \frac{1}{\alpha}\frac{\partial T}{\partial t} = 0$$

1-D steady state heat cond^n in cylinder without heat gen. Subjected to dirichlet boundary condition.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$



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$$\frac{1}{r} \neq 0$$

$$\therefore \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \quad \text{--- (i)}$$

$$\Rightarrow \frac{\partial T_1}{\partial r} = \frac{C}{r} \quad \text{--- (ii)}$$

$$\Rightarrow T_r = C \ln r + D \quad \text{--- (iii)}$$

at, $r = r_1$, $T = T_1$

$r = r_2$, $T = T_2$

$$\Rightarrow T_1 = C \ln r_1 + D$$

$$\Rightarrow D = T_1 - C \ln r_1$$

$$\Rightarrow T_2 = C \ln r_2 + D$$

$$\Rightarrow D = T_2 - C \ln r_2$$

$$\therefore T_1 - C \ln r_1 = T_2 - C \ln r_2$$

$$\Rightarrow T_1 - T_2 = C \left[\ln \frac{r_1}{r_2}\right]$$

$$\Rightarrow \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} = C$$

$$\Rightarrow D = T_1 - \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} \ln r_1$$

$$T_r = \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} \ln r + T_1 - \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} \ln r_1$$

$$\Rightarrow (T_r - T_1) = \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} \left[\ln r - \ln r_1\right]$$

$$\Rightarrow \frac{T_r - T_1}{T_1 - T_2} = \frac{\ln (r/r_1)}{\ln (r_1/r_2)}$$

Similarly, $\dfrac{T_r - T_2}{T_1 - T_2} = \dfrac{\ln(r/r_2)}{\ln(r_1/r_2)}$

$q = -KA \dfrac{dT}{dr}$

$q = -KA \dfrac{C}{r}$

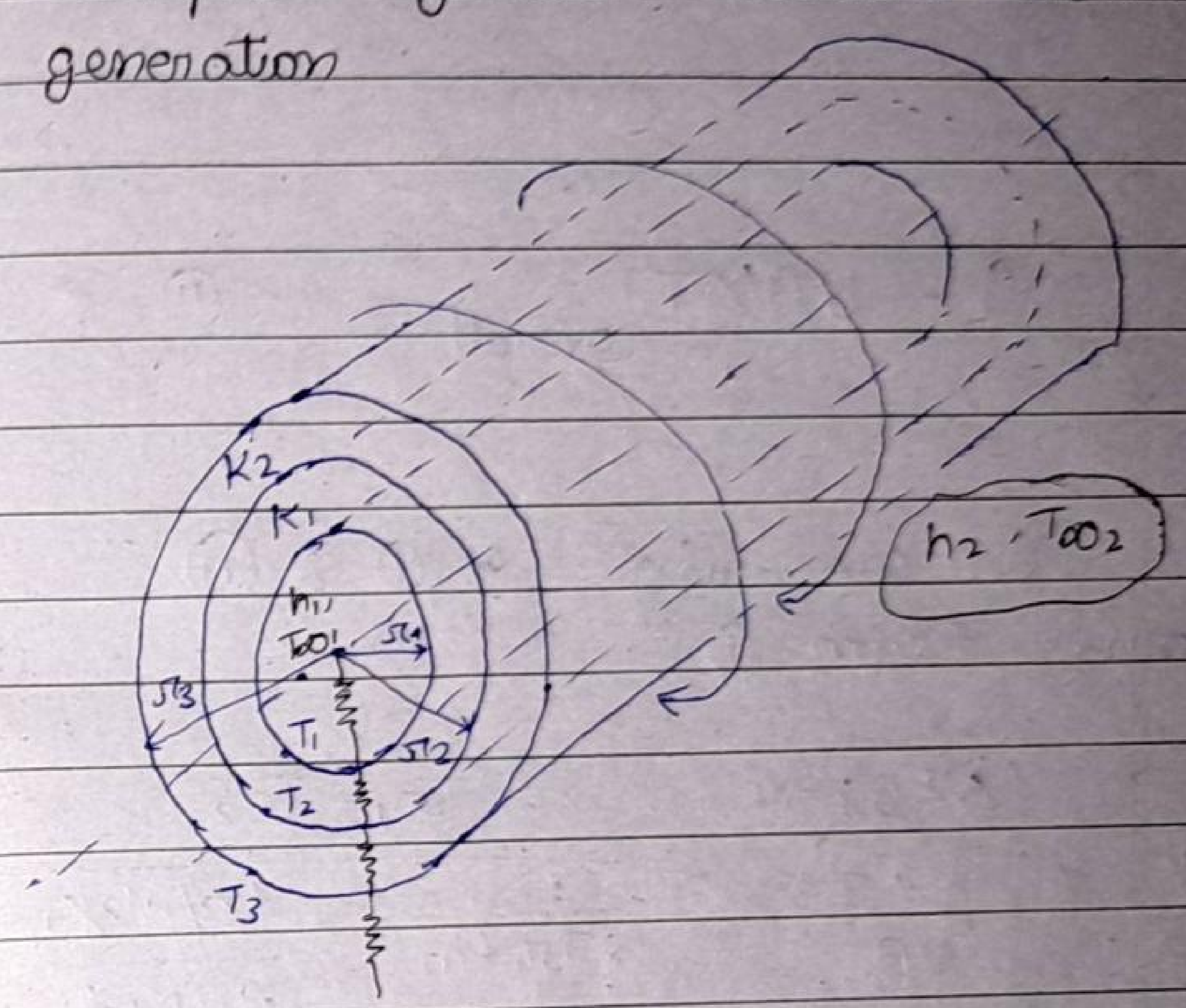
$= -K \times 2\pi r \times l \times \dfrac{(T_1 - T_2)}{r \ln(r_1/r_2)}$

$q = \dfrac{(T_1 - T_2)}{\left[\dfrac{\ln(r_2/r_1)}{2\pi K l}\right]}$

Here, Temperature profile is logarithmic.

Here, $R = \dfrac{\ln(r_2/r_1)}{2\pi K l}$

(Q) 1-D steady state heat cond^n in composite cylinder without heat generation



$q = q_{conv^n_1} = q_{cond^n_1} = q_{cond^n_2} = q_{conv^n_2}$

$q = \dfrac{(T_{\infty 1} - T_1)}{\dfrac{1}{2\pi r_1 l h_1}} = \dfrac{T_1 - T_2}{\dfrac{\ln(r_2/r_1)}{2\pi K_1 l}}$

$= \dfrac{T_2 - T_3}{\left(\dfrac{\ln(r_3/r_2)}{2\pi K_2 l}\right)} = \dfrac{T_3 - T_{\infty 2}}{\dfrac{1}{2\pi r_3 l h_2}}$ ——(1)

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Let's define U is ~~overall~~ overall

$$q = UA(\Delta T)_{overall} \quad \text{— (II)}$$

~~On comparing eq (I) & (II)~~

Now, from (I)

$$q = \frac{(\Delta T)_{overall}}{\frac{1}{UA}} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi K_1 l} + \frac{\ln(r_3/r_2)}{2\pi K_2 l} + \frac{1}{2\pi r_3 l h_2}} \quad \text{— (III)}$$

$$\frac{1}{UA} = \cancel{\ }$$

On comparing (II) & (III)

$$\frac{1}{UA} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi K_1 l} + \frac{\ln(r_3/r_2)}{2\pi K_2 l} + \frac{1}{2\pi r_3 l h_2}$$

Let's define U_i & U_o are the overall heat transfer coeff. based on the inner & outer radius.

$$\frac{1}{U_i A_i} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi K_1 l} + \frac{\ln(r_3/r_2)}{2\pi K_2 l} + \frac{1}{2\pi r_3 l h_2}$$

$$\Rightarrow \frac{1}{U_i \times 2\pi r_1 l} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi K_1 l} + \frac{\ln(r_3/r_2)}{2\pi K_2 l} + \frac{1}{2\pi r_3 l h_2}$$

$$\Rightarrow \frac{1}{U_i \times r_1} = \frac{1}{r_1 h_1} + \frac{\ln(r_2/r_1)}{K_1} + \frac{\ln(r_3/r_2)}{K_2} + \frac{1}{r_3 h_2}$$

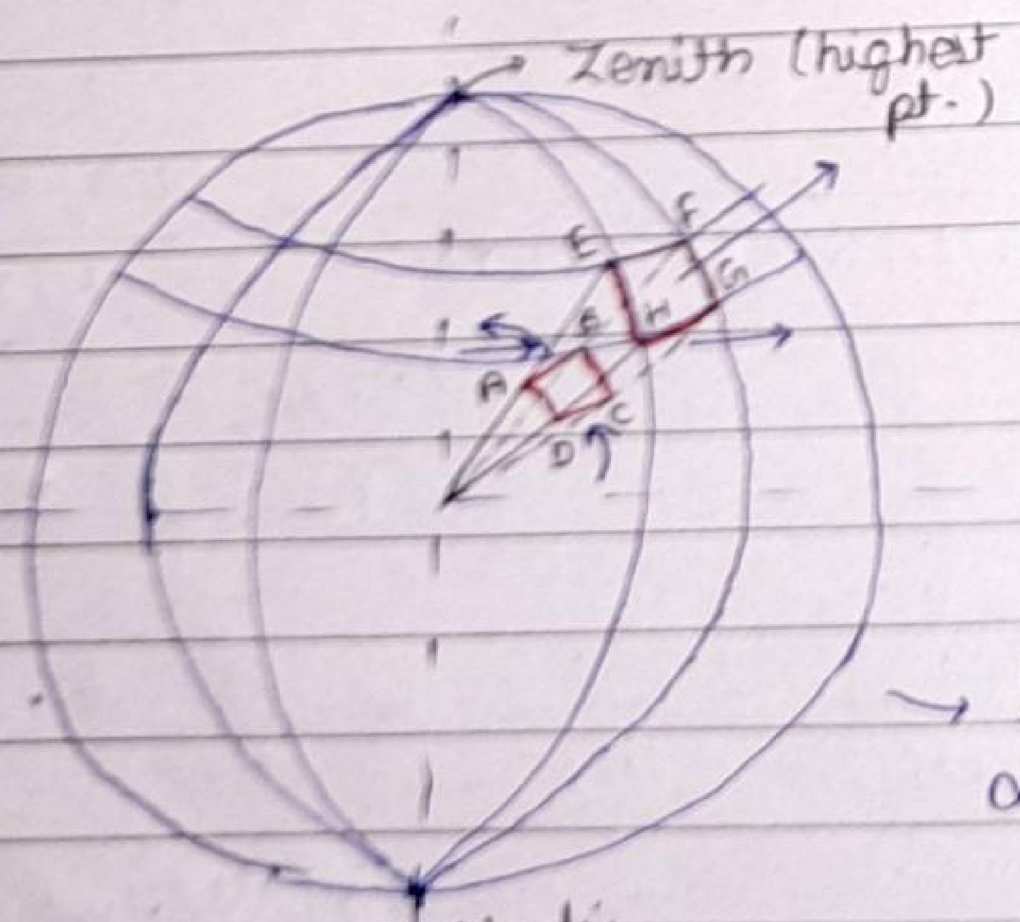
$$\Rightarrow \frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1}{K_1}\ln(r_2/r_1) + \frac{r_1}{K_2}\ln(r_3/r_2) + \frac{r_1}{r_3 h_2}$$

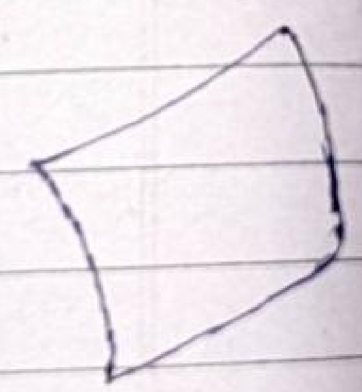
Similarly

$$\Rightarrow \frac{1}{U_0 A_0} \Rightarrow \frac{1}{U_0 \times 2\pi r_3 l} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{2\pi r_3 l h_2}$$

$$\Rightarrow \frac{1}{U_0} = \frac{r_3}{r_1 h_1} + \frac{r_3}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_3}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_2}$$

General heat conduction equation in spherical co-ordinate system –





$\rightarrow \rho, V, C_p, k, \alpha, q_v$

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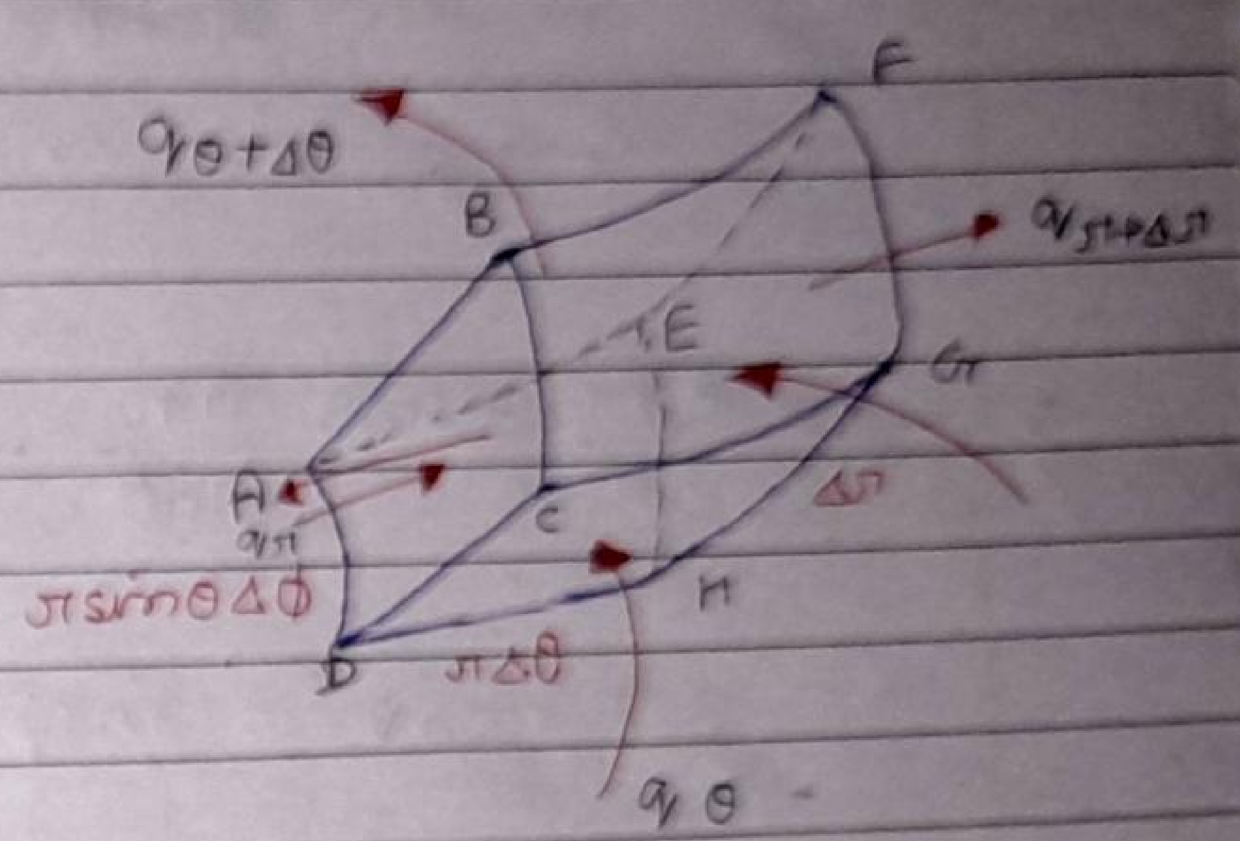
$T = f(r, \theta, \phi, t)$



$$q_{net} = \cancel{W_{net}}^{\,0} + \Delta SE$$

Assumption.

The increment in θ & ϕ direction are neglected.

$$q_r - q_{r+\Delta r} + q_\theta - q_{\theta + r\Delta\theta} + q_\phi - q_{\phi + r\sin\theta\Delta\phi} + q_v = \Delta SE$$

$$\Rightarrow q_r - \left(q_r + \frac{\Delta r}{1!}\frac{\partial}{\partial r}q_r + \dots\right) + q_\theta - \left(q_\theta + \frac{r\Delta\theta}{1!}\frac{\partial}{r\,\partial\theta}q_\theta + \dots\right) +$$

$V = \Delta r \cdot r\Delta\theta \cdot r\sin\theta\,\Delta\phi$

$$q_\phi - \left(q_\phi + \frac{\partial q_\phi}{r\sin\theta\,\partial\phi}\,\frac{r\sin\theta\,\partial\phi}{1!} + \dots\right)$$

$$+ q_{gen}V = mc_p\frac{\partial T}{\partial t}$$

$$\Rightarrow -\Delta r\frac{\partial q_r}{\partial r} - r\Delta\theta\frac{\partial q_\theta}{r\,\partial\theta} - r\sin\theta\,\partial\phi\,\frac{\partial q_\phi}{r\sin\theta\,\partial\phi}$$

$$+ q_{gen}V = mc_p\frac{\partial T}{\partial t}$$

$$\Rightarrow -\Delta r\frac{\partial}{\partial r}\left(-k_r A\frac{\partial T}{\partial r}\right) - r\Delta\theta\frac{\partial}{r\,\partial\theta}\left(-k_\theta A\frac{\partial T}{r\,\partial\theta}\right)$$

$$- r\sin\theta\,\partial\phi\,\frac{\partial}{r\sin\theta\,\partial\phi}\left(-k_\phi A\frac{\partial T}{r\sin\theta\,\partial\phi}\right)$$

$$+ q_{gen}V = mc_p\frac{\partial T}{\partial t}$$

$$\Rightarrow -\Delta r\frac{\partial}{\partial r}\left(-k_r \times r\Delta\theta \times r\sin\theta\,\Delta\phi\,\frac{\partial T}{\partial r}\right)$$

$$- r\Delta\theta\frac{\partial}{r\,\partial\theta}\left(-k_\theta \times \Delta r \times r\sin\theta\,\Delta\phi\,\frac{\partial T}{r\,\partial\theta}\right) -$$

$$r\sin\theta\,\partial\phi\,\frac{\partial}{r\sin\theta\,\partial\phi}\left(-k_\phi\,\Delta r\,r\Delta\theta\,\frac{\partial T}{r\sin\theta\,\partial\phi}\right)$$

$$+ q_{gen}V = mc_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \Delta r \times \Delta\theta\sin\theta\,\Delta\phi\,\frac{\partial}{\partial r}\left(k_r \times r^2 \times \frac{\partial T}{\partial r}\right)$$

$$+ r\Delta\theta \times \Delta r \times r\Delta\phi\,\frac{\partial}{r\,\partial\theta}\left(k_\theta \times \sin\theta \times \frac{\partial T}{r\,\partial\theta}\right)$$

$$+ r\sin\theta\cdot\Delta\phi\cdot\Delta r\cdot r\Delta\theta\,\frac{\partial}{r\sin\theta\,\partial\phi}\left(k_\phi\frac{\partial T}{r\sin\theta\,\partial\phi}\right)$$

$$+ q_{gen} \times V = mc_p\frac{\partial T}{\partial t} = \rho V c_p\frac{\partial T}{\partial t}$$

dividing both sides by $V = \Delta r \cdot r\Delta\theta \cdot r\sin\theta\,\Delta\phi$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(k_r \times r^2 \times \frac{\partial T}{\partial r}\right) +$$

$$\frac{1}{\sin\theta}\frac{\partial}{r\,\partial\theta}\left(k_\theta \times \sin\theta \times \frac{\partial T}{r\,\partial\theta}\right) + \frac{\partial}{r\sin\theta\,\partial\phi}\left(k_\phi\frac{\partial T}{r\sin\theta\,\partial\phi}\right)$$

$$+ q_{gen} = \rho c_p\frac{\partial T}{\partial t}$$

$$\boxed{\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(k_r \times r^2 \times \frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k_\theta \times \sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k_\phi\frac{\partial T}{\partial\phi}\right) + q_{gen} = \rho c_p\frac{\partial T}{\partial t}}$$

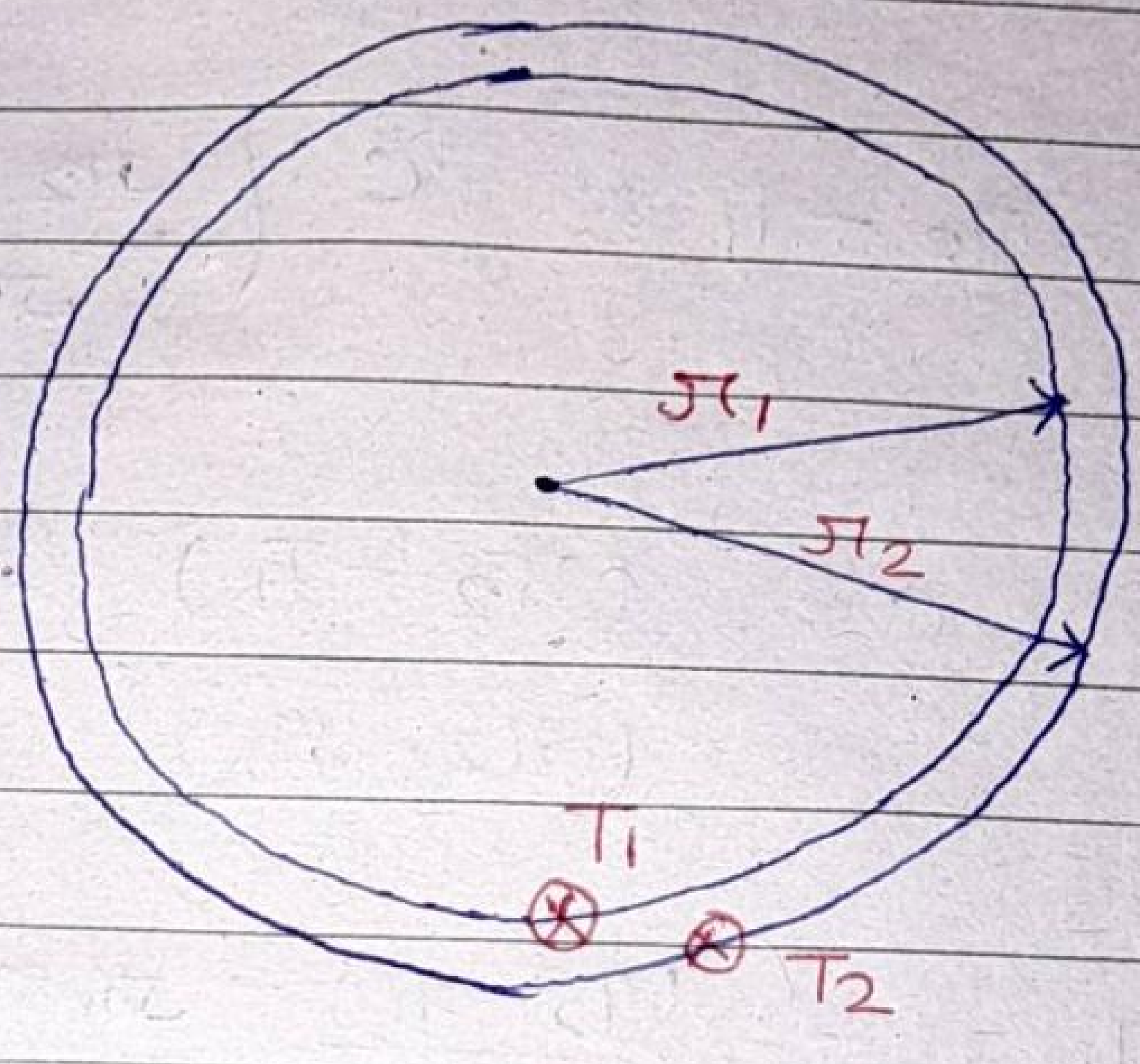
Assume material to be homogeneous & isotropic,

$$K_r = K_\theta = K_\phi = K$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \times \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{q_{gen}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Steady state



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0 \quad \text{— (I)}$$

$\because \frac{1}{r^2}\neq 0 \quad \therefore \frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0$ — (II)

$$r^2\frac{\partial T}{\partial r}=C \quad \text{— (III)}$$

$$\Rightarrow T_r = -\frac{C}{r}+D \quad \text{— (IV)}$$

At, $r=r_1$, $T_r=T_1$

$r=r_2$, $T_r=T_2$

$$\Rightarrow T_1=-\frac{C}{r_1}+D$$

$$\Rightarrow T_2=-\frac{C}{r_2}+D$$

$$\Rightarrow T_2 - T_1 = -\frac{C}{r_2} + \frac{C}{r_1}$$

$$\Rightarrow T_2 - T_1 = C\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow T_2 - T_1 = C\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$\Rightarrow C = \frac{(T_2 - T_1)\, r_1 r_2}{(r_2 - r_1)}$$

$$\therefore T_1 = -\frac{(T_2 - T_1)}{r_2 - r_1}\frac{\cancel{r_1} r_2}{\cancel{r_1}} + D$$

$$\Rightarrow D = T_1 + \frac{(T_2 - T_1)\, r_2}{(r_2 - r_1)}$$

$$\therefore T_r = -\frac{\frac{(T_2 - T_1)\, r_1 r_2}{(r_2 - r_1)}}{r} + T_1 + \frac{(T_2 - T_1)}{(r_2 - r_1)} r_2$$

$$= -\frac{(T_2 - T_1)}{(r_2 - r_1)}\frac{r_1 r_2}{r} + T_1 + \frac{(T_2 - T_1)\, r_2}{(r_2 - r_1)}$$

$$\Rightarrow \frac{(T_r - T_1)}{(T_2 - T_1)} = -\frac{r_1 r_2}{(r_2 - r_1)\, r} + \frac{r_2}{(r_2 - r_1)}$$

$$\Rightarrow \frac{T_r - T_1}{T_2 - T_1} = \frac{r_2}{(r_2 - r_1)}\left[1 - \frac{r_1}{r}\right]$$

from sin,

$$\frac{T_r - T_1}{T_1 - T_2} = \frac{\frac{1}{r} - \frac{1}{r_1}}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$q = -K \times 4\pi r^2 \times \frac{(T_2 - T_1)}{(r_2 - r_1)}\frac{r_1 r_2}{r^2}$$

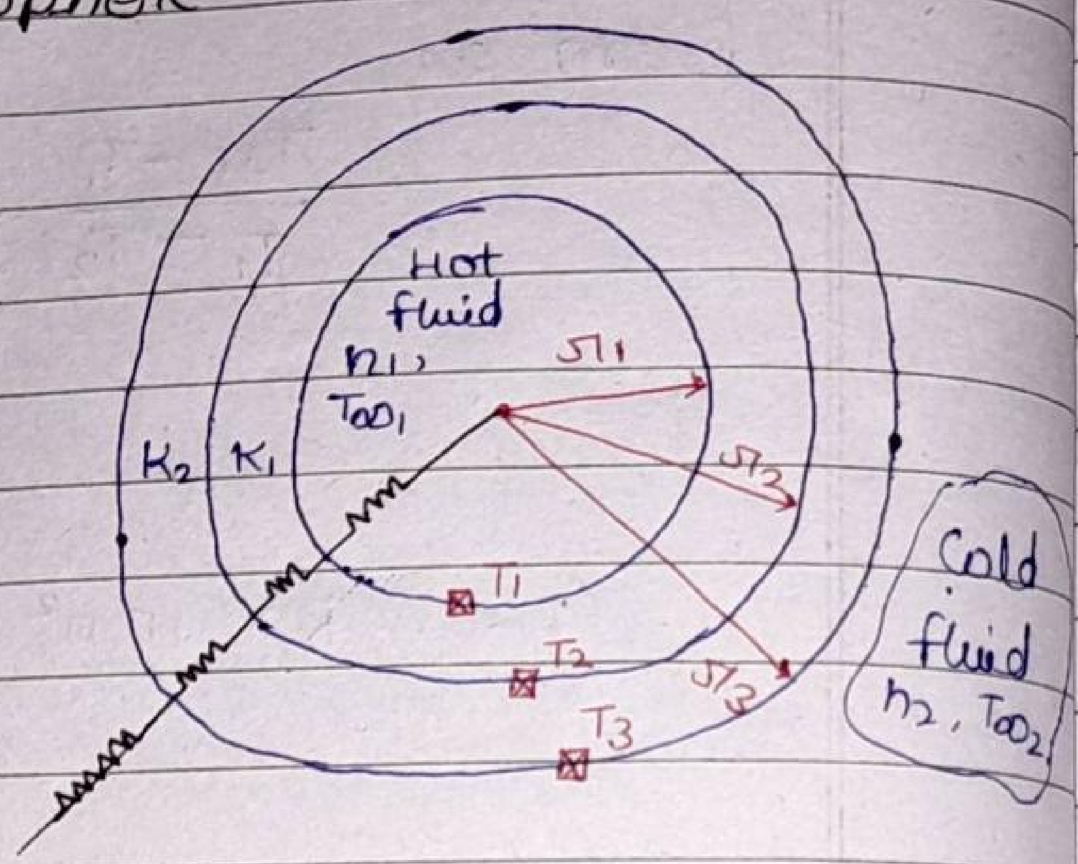
$$q = -4\pi K \times \frac{(T_2 - T_1)}{r_2 - r_1} r_1 r_2$$

$$q = 4\pi K \times \frac{(T_1 - T_2)}{(r_2 - r_1)} r_1 r_2$$

$$\therefore \boxed{q = \frac{T_1 T_2}{\frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi K}}}$$

Here, $R = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi K}$

Composite Sphere -



$T_{\infty 1}$ — T_1 — T_2 — T_3 — $T_{\infty 2}$

$\dfrac{1}{h_1 4\pi r_1^2}$ $\quad \dfrac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K_1}$ $\quad \dfrac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi K_2}$ $\quad \dfrac{1}{h_2 4\pi r_3^2}$

$conv^n 1$ $\quad cond^n 1$ $\quad cond^n 2$ $\quad conv^n 2$

$$q = q_{conv^n 1} = q_{cond^n 1} = q_{cond^n 2} = q_{conv^n 2}$$

$$q = \frac{T_{\infty 1}-T_1}{\frac{1}{h_1 4\pi r_1^2}} = \frac{T_1-T_2}{\frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K_1}} = \frac{T_2-T_3}{\frac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi K_2}}$$

$$= \frac{T_3-T_{\infty 2}}{\frac{1}{h_2 4\pi r_3^2}}$$

$$q = \frac{(T_{\infty 1}-T_1)+(T_1-T_2)+(T_2-T_3)+(T_3-T_{\infty 2})}{\frac{1}{h_1(4\pi r_1^2)} + \frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K_1} + \frac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi K_2}}$$

$$\therefore q = \frac{T_{\infty 1}-T_{\infty 2}}{\frac{1}{h_1(4\pi r_1^2)} + \frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K_1} + \frac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi K_2} + \frac{1}{h_2(4\pi r_3^2)}}$$

Now, $q = UA\Delta T$

Comparing,

$$\frac{1}{UA} = \frac{1}{h_1(4\pi r_1^2)} + \frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K_1} + \frac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi K_2} + \frac{1}{h_2(4\pi r_3^2)}$$

Let's define U_i & U_o are the overall heat transfer coeff. based on inner & outer radius of sphere resp.

$$\frac{1}{U_i A_i} = \frac{1}{h_1(4\pi r_1^2)} + \frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K_1} + \frac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi K_2} + \frac{1}{h_2(4\pi r_3^2)}$$

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$$\Rightarrow \frac{1}{U_i(4\pi r_1^2)} = \frac{1}{h_1(4\pi r_1^2)} + \frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi K_1} + \frac{\frac{1}{r_2} - \frac{1}{r_3}}{4\pi K_2} + \frac{1}{h_2(4\pi r_3^2)}$$

$$\Rightarrow \frac{1}{U_i \times r_1^2} = \frac{1}{h_1 r_1^2} + \frac{\frac{1}{r_1} - \frac{1}{r_2}}{K_1} + \frac{\frac{1}{r_2} - \frac{1}{r_3}}{K_2} + \frac{1}{h_2 r_3^2}$$

$$\Rightarrow \frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1^2}{K_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{r_1^2}{K_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{r_1^2}{h_2 r_3^2}$$