Tutorial - 1

Piyush 7/10/22

Q-1:- Derive general heat conduction equation in cartesion coordinates?

9.5/10

Ans:- Assumption :-

(i) work transition on account of tempretures change in the material is assumed to be negligible.

(ii) It is assumed that change in kinetic energy and potential energy is assumed to be negligible.

$$Q_{net} = \cancel{\omega}^{0} + \Delta SE$$

$$Q_{net} = \Delta SE$$

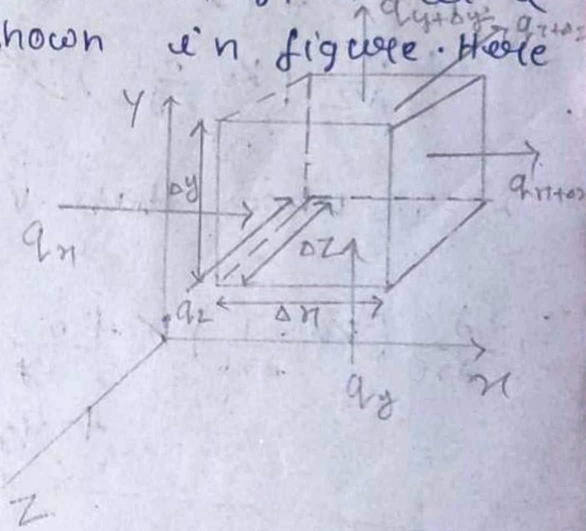
Let us consider a rectangular element of heaving the dimentional as $\Delta x, \Delta y, \Delta z$ at a distance x, y, z, as shown in figure. Here

ρ = density of the material of element

V = volume of the considerd element

C_p = Specific heat of the element

K = Thermal conductivity of the material



q_v = Heat generated rate in the element of volume V

q_{gen} = Rate of heat generation per unit volume.

$\therefore$ From the conservation of energy.

$$q_{x\,net} + q_{y\,net} + q_{z\,net} + q_v = \Delta SE$$

$$q_x - q_{x+\Delta x} + q_y - q_{y+\Delta y} + q_z + q_{z+\Delta z} + q_v = \Delta SE$$

Applying the taylor's series expansion -

$$q_x - \left[q_x + \frac{\partial}{\partial x}(q_x)\frac{\Delta x}{1!} + \frac{\partial^2}{\partial x^2}(q_x)\frac{\Delta x^2}{2!} + \cdots\right] + q_y - \left[q_y + \frac{\partial}{\partial y}(q_y)\frac{\Delta y}{1!} + \frac{\partial^2}{\partial^2 y}(q_y)\frac{\Delta y^2}{2!} + \cdots\right] + q_z - \left[q_z + \frac{\partial}{\partial z}(q_z)\frac{\Delta z}{1!} + \frac{\partial^2}{\partial z^2}(q_z)\frac{\Delta z^2}{2!} + \cdots\right] + q_v$$

$$= \Delta SE \quad \text{— (1)}$$

Neglecting the heigher power of $\Delta x, \Delta y, \Delta z$ from eqⁿ (i) we get —

$$\Rightarrow -\frac{\partial}{\partial x}(q_x)\Delta x - \frac{\partial}{\partial y}(q_y)\Delta y - \frac{\partial}{\partial z}(q_z)\Delta z + q_v = \Delta SE$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(-K_x A \frac{\partial T}{\partial x}\right)\Delta x - \frac{\partial}{\partial y}\left(-K_y A \frac{\partial T}{\partial y}\right)\Delta y - \frac{\partial}{\partial z}\left(-K_z A \frac{\partial T}{\partial z}\right) + q_v = \Delta SE$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(-K_x \Delta y \Delta z \frac{\partial T}{\partial x}\right)\Delta x - \frac{\partial}{\partial y}\left(-K_y \Delta x \Delta z \frac{\partial T}{\partial y}\right)\Delta y - \frac{\partial}{\partial z}\left(-K_z \Delta x . \Delta y \frac{\partial T}{\partial z}\right)\Delta z + q_{gen} * V = m C_p \frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right)\Delta x \Delta y \Delta z + \frac{\partial}{\partial y}\left(K_y \frac{\partial T}{\partial y}\right)\Delta x \Delta y \Delta z + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right)\Delta x \Delta y \Delta z + q_{gen} * V = \rho V C_p \frac{\partial T}{\partial t}$$

$\Rightarrow$ we know that $V = \Delta x . \Delta y . \Delta z$

$$\Rightarrow \frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right)V + \frac{\partial}{\partial y}\left(K_y \frac{\partial T}{\partial y}\right)V + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right)V + q_{gen} * V = \rho V C_p \frac{\partial T}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right) + q_{gen} = \rho C_p \frac{\partial T}{\partial t}}$$

This is called the general heat conduction equation in cartesian coordinates.

Let us consider the material is homogeneous isotropic.

$$K_x = K_y = K_z = K$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

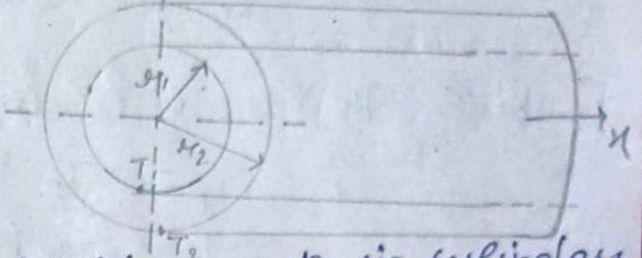
where $\alpha = \frac{K}{\rho C_p}$ and α is known as thermal diffisivity.

Q-2 - Derive expression for temperature distribution under one dimensional steady state heat conduction for cylinder.

Answer :- Steady state unidirectional heat flow through cylinder of uniform conductivity without heat generation —

Let us consider a hollow cylinder whose inside and outer radii are r_1 and r_2 respectively.

Thermal conductivity of the material of the cylinder is K



∴ General heat conduction eq^n in cylinder coordinate is —

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{— (i)}$$

But here —

$$\frac{\partial T}{\partial t} = 0 , \quad \frac{\partial T}{\partial \theta} = 0 \text{ and } \frac{\partial T}{\partial Z} = 0$$

$$q_{gen} = 0$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0$$

$$r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r} = 0 \quad \text{— (ii)}$$

$$\frac{\partial}{\partial r}\left(r \cdot \frac{\partial T}{\partial r}\right) = 0$$

Integrating the above eq^n —

$$r\frac{\partial T}{\partial r} = C_1 \quad \text{— (iii)}$$

$$\frac{\partial T}{\partial r} = \frac{C_1}{r} \quad \text{— (iv)}$$

Integrating once more —

$$T_r = C_1 \log_e r + C_2 \quad \text{— (v)}$$

Now using the boundary condition —

at $r = r_1$, $T = T_1$

at $r = r_2$, $T = T_2$

$$\therefore \quad T_1 = C_1 \log_e r_1 + C_2 \quad \text{---(vi)}$$

$$T_2 = C_1 \log_e r_2 + C_2 \quad \text{---(vii)}$$

From eq^n (vi) and (vii)

$$T_2 - T_1 = C_1 \log_e \left(\frac{r_2}{r_1}\right)$$

$$C_1 = \frac{T_2 - T_1}{\cancel{C_1} \log_e (r_2/r_1)} \Rightarrow \frac{T_1 - T_2}{\log_e \left(\frac{r_1}{r_2}\right)}$$

$\therefore$ putting the value of C_1 in eq^n (vi), we get

$$C_2 = T_1 - \frac{T_2 - T_1}{\log_e \left(\frac{r_2}{r_1}\right)} \times \log_e (r_1)$$

$$= \frac{T_1 \log(r_2) - \cancel{T_1 \log(r_1)} - T_2 \log_e r_1 + \cancel{T_1 \log_e r_1}}{\log_e (r_2/r_1)}$$

$$C_2 = \frac{T_1 \log_e(r_2) - T_2 \log_e(r_1)}{\log_e (r_2/r_1)}$$

Putting the value of C_1 & C_2 in eq^n (5) we get

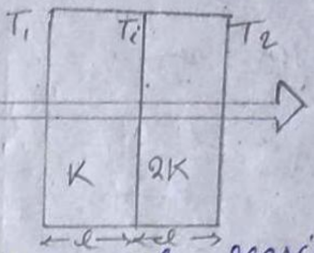
$$T_r = \frac{T_2 - T_1}{\log_e (r_2/r_1)} \times \log_e r + \frac{T_1 \log_e(r_2) - T_2 \log_e r_1}{\log_e (r_2/r_1)}$$

$$T_r = \frac{T_2 \log_e(r) + T_1 \log_e(r_2) - T_1 \log_e(r) - T_2 \log_e r_1}{\log_e (r_2/r_1)}$$

$$\left[T_r = \frac{1}{\log_e (r_2/r_1)} \left[T_1 \log_e \left(\frac{r_2}{r}\right) + T_2 \log_e \left(\frac{r}{r_1}\right) \right] \right]$$

this is the temp^r distribution eq^n of one dimensional stedy state of unidirectional heat flow through cylinder.

Q-3 Heat transfer through a composite wall is shown in figure. Both the section of wall have equal thickness. The conductivity of one is K and that of other is 2K. The left face of the wall is at 600K and the right face is at 300K. The interface temp^r T_i (in K) on the composite wall is?



Soln:- Heat flow rate -

$$Q = \frac{\Delta T}{R_{th}}$$

where R_{th} is equivalent thermal resistance.

here, $T_1 = 600K$, $T_2 = 300K$

$$Q = \frac{T_1 - T_2}{\frac{l}{KA} + \frac{l}{2KA}} = \frac{600-300}{\frac{3}{2}\frac{l}{KA}} \Rightarrow \frac{200}{\frac{l}{KA}}$$

$$Q_1 = \frac{T_1 - T_i}{\frac{l}{KA}} = \frac{600 - T_i}{l/KA}$$

$\because Q = Q_1$ (Because heat flow rate is same)

$$\frac{200}{\cancel{l/KA}} = \frac{600 - T_i}{\cancel{l/KA}}$$

$$\boxed{T_i = 400K}$$

Q-4 ⇒ The temp^r of inner and outer surface of the boiler wall made of 20 mm thick steel and covered with an insulating material of 5 mm are 300°c and 50°c respectively. If the thermal conductivity of steel and insulating material are 58 w/m°c and 0.116 w/m°c respectively, determine the rate of heat flow per unit area of the boiler wall.

Solution:- given that -

$l_1 = 0.02m$

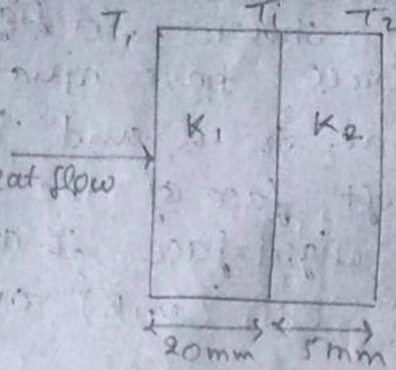
$l_2 = 0.005m$

$T_1 = 300°c$

$T_2 = 50°c$

$K_1 = 58$ W/m°c

$K_2 = 0.116$ W/m°c



heat flow rate,

$$Q = \frac{\Delta T}{R_{th}}$$

$$Q = \frac{T_1 - T_2}{\frac{l_1}{K_1 A} + \frac{l_2}{K_2 A}} \Rightarrow \frac{300-50}{\frac{0.02}{58 \times A} + \frac{0.005}{0.116 \times A}}$$

$$\text{heat flux} \Rightarrow \frac{Q}{A} \Rightarrow \frac{250}{\left[\frac{0.02}{58} + \frac{0.005}{0.116}\right]}$$

$$\left[\frac{Q}{A} \simeq 5754 \ \text{W/m}^2\right]$$

Q-5- Heat flow through a composite slab, as shown in figure. The depth of the slab is 1m. The K values are in W/mK. Find

(I) The overall thermal resistance in K/W

(II) Heat flow rate through composite slab when temp diff. across extreme end of composite slab is 100K.

given :- $l_1 = 0.5m$

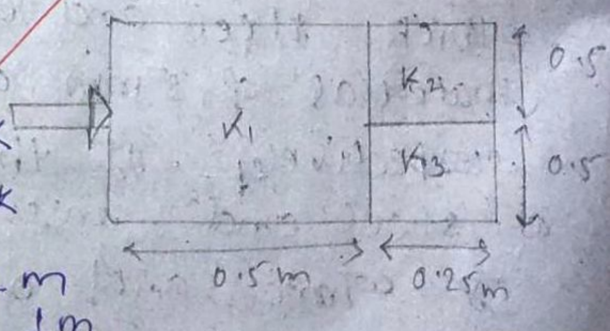
$l_2 = l_3 = 0.25m$

$K_1 = 0.02$ W/mK

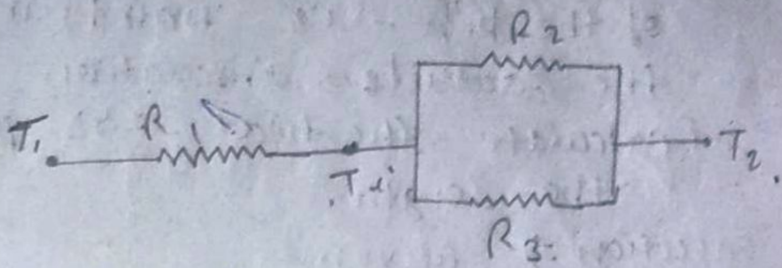
$K_2 = 0.10$ W/mK

depth of the slab $b = 1m$

height of the slab $h = 1m$



(I) Equivalent electrical circuit :-



$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R' = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{\frac{l_2}{K_2 A_2} * \frac{l_3}{K_3 A_3}}{\frac{l_2}{K_2 A_2} + \frac{l_3}{K_3 A_3}}$$

$$R' = \frac{\left[\frac{0.25}{0.10 \times 0.5 \times 1} \times \frac{0.25}{0.04 \times 0.5 \times 1}\right]}{\left[\frac{0.25}{0.10 \times 0.5} + \frac{0.25}{0.04 \times 0.5}\right]}$$

$$\left\{ R' = 3.571 \ \frac{K}{\omega} \right\}$$

$$R_1 = \frac{l_1}{K_1 A_1} = \frac{0.5}{0.02 \times 1 \times 1} = 25 \ \frac{K}{\omega}$$

$$\therefore \ R_{th} = (R' + R_1) = [3.571 + 25]$$

$$\left\{ R_{th} = 28.571 \ K/\omega \right\}$$

(II) $\Delta T = 100 K$

$\therefore$ heat flow rate $Q = \frac{\Delta T}{R_{th}}$

$$Q = \frac{100}{28.571} = 3.50 \ \omega$$

Q-6 :- A thick called type of stainless steel with 20 mm inner diameter and 40 mm outer diameter is converted with a 30 mm layer of asbestas insalation.

($(K) = 0.2\ w/m°c$). If the inside wall tempr of the pipe is maintained at 600°c and the outside insulation at 1000°c. Calculate the heat loss per meter of the length.

Solution ⇒ Given –

$T_1 = 600°c$

$T_2 = 1000°c$

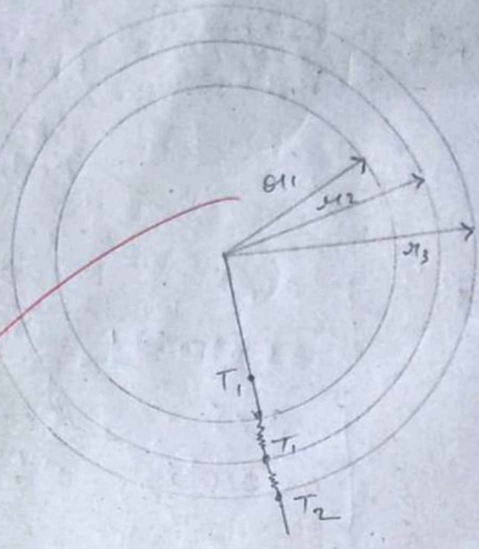
$K_2 = 0.2\ w/m°c$

$r_1 = 0.01\ m$

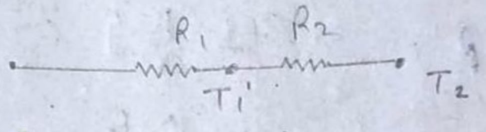
$r_2 = 0.02\ m$

$r_3 = 0.02 + 0.03$

$\Rightarrow 0.05\ m$



Equivalent electrical circuit :–



$$R_{th} = R_1 + R_2$$

$$= \frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln(r_3/r_2)}{2\pi K_2 L}$$

K_1 is not given hence $K_1 = \infty$

$\therefore R_1 = 0, \Rightarrow T_1 = T_i$

$$R_{th} = \frac{\ln(r_3/r_2)}{2\pi K_2 L} = \frac{\ln(0.05/0.02)}{2\pi \times 0.2 \times L}$$

Heat flow rate or heat loss rate –

$$Q = \frac{\Delta T}{R_{th}}$$

$$Q = \frac{1000 - 600}{\frac{\ln(0.05/0.02)}{2\pi \times 0.2 \times L}}$$

$$\frac{Q}{L} = \frac{400}{\frac{\ln(0.05/0.02)}{0.4\pi}}$$

$\left\{ \frac{Q}{L} = 548.57 \, w/m \right\}$

Q-7 :- A steel pipe with 50mm outer dia. is covered with a 6.4 mm asbestus insulation ($K = 0.166$ w/mK) follow by a 25 mm layer of fibre-glass insulation ($K = 0.0485$ w/mK). The pipe wall temp is 393 K and outer insulation temp is 311 K, calculate the interface temp b/w the asbestos and fibre-glass.

Soln :- $d_2 = 50mm$

$r_2 = \frac{50}{2} = 25mm$

$r_2 = 0.025m$

$r_3 = 0.025 + 0.0064$

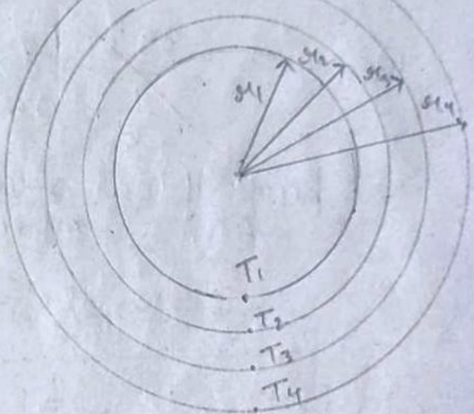
$r_3 = 0.0314 \, m$

$r_4 = 0.0314 + 0.025$

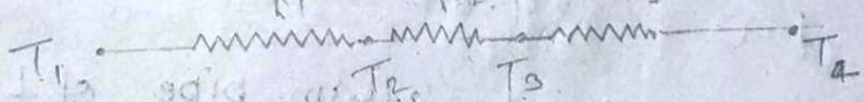
$r_4 = 0.0564 \, mm$

$K_2 = 0.166 \, w/mK$

$K_3 = 0.0485 \, w/mK$



Equivalent electrical circuit –



here K_1 is not given, hence $K_1 = \infty$

$$R_1 = \frac{\log_e (r_2 / r_1)}{2\pi K_1 l}$$

$R_1 = 0 \Rightarrow T_1 = T_2 = 393 K$

$R_{th} = R_2 + R_3$

$$R_{th} = \frac{\log_e (r_3/r_2)}{2\pi K_2 l} + \frac{\log_e (r_4/r_3)}{2\pi K_3 l}$$

$$\Rightarrow \frac{\log_e\left(\frac{0.0314}{0.025}\right)}{2\pi \times 0.166 \times l} + \frac{\log_e\left(\frac{0.0564}{0.0314}\right)}{2\pi \times 0.0485 \times l}$$

$$R_{th} = \frac{1}{l}\left[2.1404\right] K/W$$

$$Q = \frac{\Delta T}{R_{th}} = \frac{393 - 311}{\frac{1}{l} \times 2.1404} = \frac{82}{\frac{1}{l} \times 2.1404} W$$

$$Q_2 = \frac{T_2 - T_3}{\frac{\ln(r_3/r_2)}{2\pi K_2 l}} = \frac{393 - T_3}{\frac{\ln\left(\frac{0.0314}{0.025}\right)}{2\pi \times 0.166 \times l}}$$

$$Q_2 = \frac{393 - T_3}{\frac{1}{l} \times 0.2185} W$$

$\because$ heat flow rate is same through each section -

$$Q = Q_2$$

$$\frac{82}{\left(\frac{1}{l} \times 2.1404\right)} = \frac{393 - T_3}{\frac{1}{l} \times 0.2185}$$

$$\Rightarrow 8.37086 = 393 - T_3$$

$$\{T_3 = 384.63\ K\}$$

Q-8 An insulated steam pipe of 160 mm inner diameter and 180 mm outer diameter is covered with insulation of 40mm thickness and carries steam at 200°C K(pipe) = 29 w/m°c and K (insulation) = 0.23 w/m°c, h_i = 11.6 W/m²°c and h_o = 23.2 w/m²°c of the tempʳ of the air surrounding the pipe is 25°c. calculate the rate of heat loss to the surrounding from the pipe. Calᶜ to rate of heat loss to the surrounding from the pipe of 5m length. Also find the interface temperature.

Solution :- given-

$d_1 = 160\,mm$, $d_2 = 180\,mm$

$h_i = 11.6\ w/m^2{}^\circ c$, $h_o = 23.2\ w/m^2{}^\circ c$

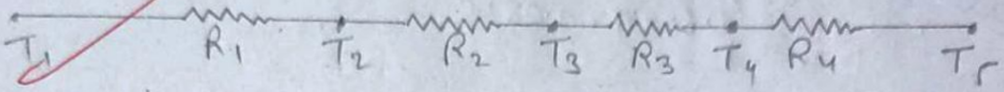
$K_1 = 29\ w/m^\circ c$, $K_2 = 0.23\ w/m^\circ c$

$T_1 = 200^\circ c$, $T_5 = 25^\circ c$

$r_1 = 0.08\ m$, $r_2 = 0.09\ m$, $r_3 = 0.13\ m$

$l = 5m$

Equivalent electrical circuit -



Equivalent thermal resistance -

$$R_{th} = R_1 + R_2 + R_3 + R_4$$

$$R_{th} = \frac{1}{h_i A_i} + \frac{\ln(r_2/r_1)}{2\pi K_1 l} + \frac{\ln(r_3/r_2)}{2\pi K_2 l} + \frac{1}{h_o A_o}$$

$$R_{th} = \frac{1}{11.6 \times 2\pi \times 0.08 \times 5} + \frac{\ln(0.09/0.08)}{2\pi \times 29 \times 5} + \frac{\ln(0.13/0.09)}{2\pi \times 0.23 \times 5} + \frac{1}{23.2 \times 2\pi \times 0.13 \times 5}$$

$$R_{th} = 0.09587544 9\ \frac{K}{\omega}$$

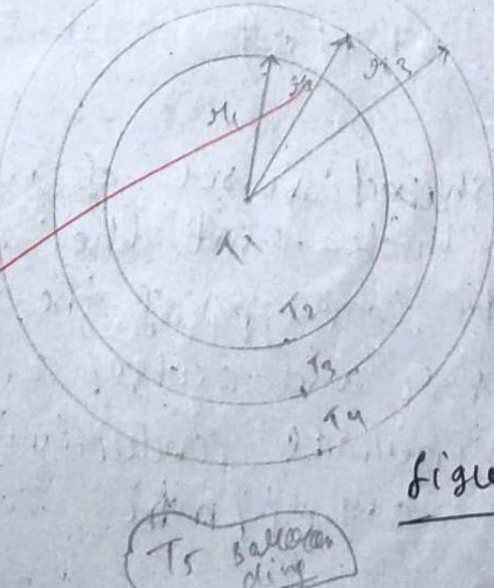


figure -

$$\left\{ Q = \frac{\Delta T}{R_{th}} = \frac{200-25}{0.095875449} = 1825.28 \text{ W} \right\}$$

$$Q = Q_1 = Q_2 = Q_3 = Q_4$$

$$Q_1 = h_i A_i (T_1 - T_2)$$

$$Q_1 = Q = 11.6 \times 2 \times \pi \times 0.08 \times 5 \times (200 - T_2)$$

$$1825.28 = 11.6 \times 2\pi \times 0.08 \times 5 \times (200 - T_2)$$

$$(200 - T_2) = \frac{1825.28}{11.6 \times 2\pi \times 0.08 \times 5}$$

$$\Rightarrow T_2 = 137.39 \,°C$$

$$Q_2 = \frac{(T_2 - T_3)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_1 l}} \Rightarrow \frac{137.39 - T_3}{\frac{\ln\left(\frac{0.09}{0.08}\right)}{2\pi \times 29 \times 5}}$$

$$\because Q = Q_2$$

$$\Rightarrow \frac{137.39 - T_3}{\frac{\ln\left(\frac{0.09}{0.08}\right)}{2\pi \times 29 \times 5}} = 1825.28$$

$$\left[T_3 = 137.15 \,°C \right]$$

Q-9 A spherical shaped vessel of 1.2m diameter is 100mm thick. Find the rate of heat of heat leakage if the temp difference b/w the inner and outer surfaces is 200 °c. Thermal conductivity of material is 0.3 kJ/m°c

Soln:- given - $d_1 = 1.2m$ $r_1 = 0.6m$

thickness $t = 100mm$, $r_2 = 0.7m$

$\Delta T = T_1 - T_2 = 200°C$

$K = 0.3\ kJ/mh°C$

$K = \dfrac{0.3 \times 1000}{3600}\ J/ms°C$

$K = \dfrac{3}{36} = \dfrac{1}{12}\ \dfrac{W}{m°C}$

thermal resistance -

$$R_{th} = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi K}$$

$$= \frac{\left(\frac{1}{0.6} - \frac{1}{0.7}\right)}{4 \times \pi \times \frac{1}{12}}$$

$\therefore R_{th} = 0.22736\ \dfrac{K}{W}$

Rate of heat leakage -

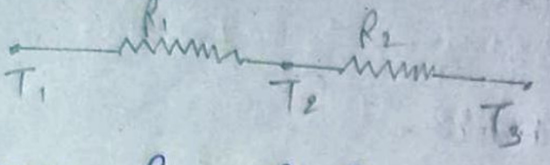
$$Q = \frac{\Delta T}{R_{th}} = \frac{200}{0.22736} \approx 880\ W$$

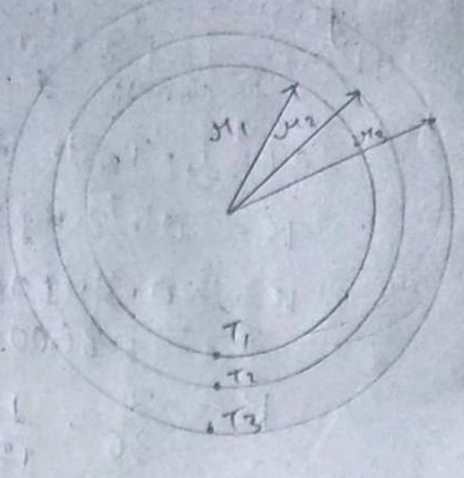
Q:(10) A spherical container having outer diameter 500mm is insulated by 100mm thick layer of material with thermal conductivity $K = 0.03(1 + 0.006T)\ W/m°C$ where T is in °C, if the surface temp of sphere is $-200°C$ and temperature of outer surface is $30°C$. determine the heat flow in!

Soln:- given - $d_1 = 500\ mm$, $t = 100\ mm$

$K = 0.03(1 + 0.006T)\ W/m°C$

Equivalent electrical circuit -





here $R_{th} = R_1 + R_2$

but K of container is not given, it means

$K_1 \to \infty$

$R_1 = 0$

$T_1 = T_2$

$\therefore R_{th} = R_2$

$T_1 = T_2 = -200^\circ C$, $T_3 = 30^\circ C$

$r_2 = 0.25\,m$, $r_3 = 0.35\,m$

$$\therefore Q = -KA\frac{dT}{dr} = -K \times 4\pi r^2 \times \frac{dT}{dr}$$

$$Q\frac{dr}{r^2} = -4\pi K\,dT \quad -(1)$$

integrating eqn (1) we get.

$$Q\int_{r_2}^{r_3}\frac{dr}{r^2} = -4\pi \int_{T_2}^{T_3}(0.03 + 0.03\times 0.006T)\,dT$$

$$Q\left[-\frac{1}{r}\right]_{r_2}^{r_3} = -4\pi\left[0.03T + 9\times10^{-5}T^2\right]_{T_2}^{T_3}$$

$$Q\left[-\frac{1}{0.35} + \frac{1}{0.25}\right] = -4\pi\left[0.03(30-(-200)) + 9\times10^{-5}[(30)^2-(200)^2]\right]$$

$$Q\times\left[\frac{0.1}{0.35\times0.25}\right] = -4\pi[6.9 - 3.519]$$

$$\boxed{Q = -37.176\,W}$$

-ve sign shows that heat is flowing from outside to inside

Where heat flow in -

$$\boxed{Q_{in} = 37.176\ \omega}$$

$T = \frac{T_1 + T_2}{2}$