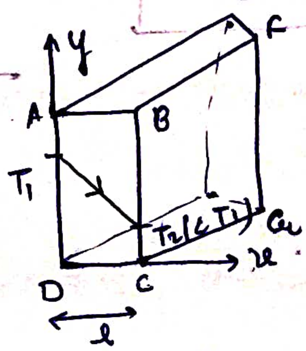
1 Sept

Heat and Mass Transfer. (BME-27)

Modes
- Conduction
  - Special case is convection
- Radiation

Laws
- Fundamental
- Subsidiary



Fourier law of Heat conduction

$$Q \propto dT/dx$$

$Q \propto A$ (area $\perp^r$ to the dir^n of heat transfer)

Assumption: Steady state
- Homogeneous
- Isotropic
- Unidirectional heat transfer
- Boundary surface are isothermal in character.
- There is no heat generation.

$$\boxed{Q = -kA\frac{dT}{dx}}$$

k → proportionality constant.

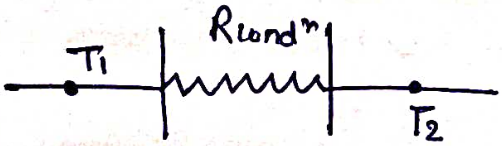
- The temp profile is linear (only for rectangle).
- Negative sign shows the dir^n of decreasing temp.

$$\therefore Q = \frac{KA(T_1 - T_2)}{L}$$

$$Q = \frac{(T_1 - T_2)}{(L/KA)} = \frac{\Delta T}{(L/KA)}$$

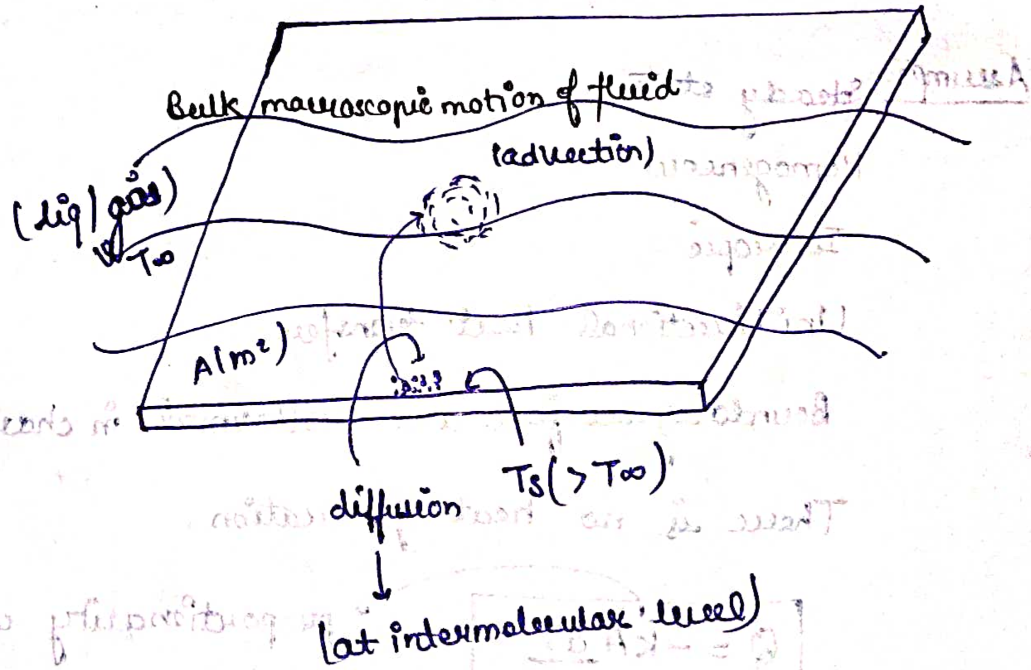
$$I = \frac{\Delta V}{R}$$

$$\boxed{R_{cond^n} = L/KA}$$



2/Sept

Convection:



When a fluid is in communication with a solid with a diff of temp b/w the two, then heat transfer takes place due to bulk macroscopic motion of fluid with reference to the solid surface (advection).

- And intermolecular diffusion occurring randomly at microscopic level (diffusion).

- Cumulative effect of advection and diffusion is the convection.

The law which governs the process of convection is the Newtons Law of Cooling / Heating.

It states that; $Q \propto A$; $Q \propto dT$

$A \downarrow$ projected surface area (area which is in contact to the surface).

$$\boxed{Q = hA(T_s - T_\infty)}$$

h : convection heat transfer coeff.

unit :- $(W/m^2 \cdot K)$ or $(W/m^2 \cdot {}^\circ C)$

complex parameter

- h is neither the property of fluid nor the property of solid unlike thermal conductivity.

- It depends on variety of factors

(1) Temperature under which thermo physical properties of fluid $\rho, \mu, k, \nu, \alpha, \mu$.

(2) The geometry of solid → cylinder, sphere.

(3) Whether flow is internal or external.

(4) whether convection is free, forced or mixed

(5) whether flow is laminar or turbulent.

$$\boxed{h_{gas} < h_{liq}}$$

$\boxed{h_{free} < h_{forced} < h_{mixed}}$

Free convection → Natural convection.

Mixed " → conjugate "

Free Convection	Forced	Mixed
• Cup of coffee.	• Automobile radiator.	• Cooling of electronic equipments.
• Emulsion water heater.	• Window AC	• Hot wire ~~dynamo~~ animo-meter which measures velocity.
• Condenser behind the domestic refrigerator.	• Modern water tube boiler.	
• Argon gas in the light bulb.	• Loeffler boiler in melting, cutting and quenching.	
• Wilcox boiler.		

Mode	Air (h)	Water (h)
Free	5	25
Forced	100	500

• During phase change it ranges upto 5000.

Thermal diffusivity (α)

$$\alpha = \frac{k}{\rho C_p}$$

$\left(\nu = \mu / \rho \right) \; (m^2/sec)$

degree of penetration of momentum.

Momentum diffusivity / kinematic viscosity

- It is the ratio of thermal conductivity to the heat capacity.

thermal conductivity → how well a material can store heat.

heat capacity → how well a material can store heat.

- It can be defined as the degree of penetration of heat.

$$\alpha = \frac{W/m \cdot K}{\frac{Kg}{m^3} \; J/Kg \cdot K} = m^2/s$$

Radiation :-

The thermal radiation induced by manifestion of heat is k/a thermal radiation.

All the bodies above absolute zero temp exhibits tendency to emit as well as receive energy by virtue of their temp on account of changes in electronic configuration of their constituent atoms or molecules, the energy thus emitted or received is defined as thermal radiation.

- Unlike conduction and convection, the propagation of thermal radiation does not need intermediate medium. Infact its best shown where there is ~~vacuum~~

vacuum, say example sun, candle, bulb, heat received from furnace, boiler, IC engine, combustion chamber.

This law which governs the thermal radiation is known as

Stefan Boltzmann Law :

This is theoretical cum empirical law which govern the thermal radiation. It states that the ~~thermal~~ rate of heat transfer is directly proportional to the area of the body when the diff b/w fourth power of abs. temp of the body and environment.

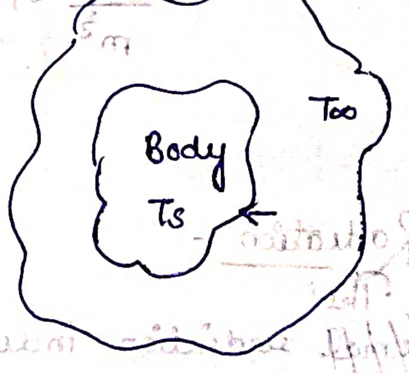
$$Q \propto A(T_s^4 - T_\infty^4)$$

$$\boxed{Q = \sigma \epsilon A(T_s^4 - T_\infty^4)}$$

ϵ → emissivity

$\sigma = 5.6697 \times 10^{-8}\ W/m^2K^4$

Stefan Boltzmann constant



Quasi-linearization :-

$$Q = \sigma \epsilon A(T_s^4 - T_\infty^4)$$

$$Q = \sigma \epsilon A \cdot (T_s^2 - T_\infty^2)(T_s^2 + T_\infty^2)$$

$$Q = \sigma \epsilon A \cdot (T_s - T_\infty)(T_s + T_\infty)(T_s^2 + T_\infty^2)$$

$$h_{rad} = \sigma \epsilon (T_s + T_\infty)(T_s^2 + T_\infty^2)$$

$$Q = h_{rad} A (T_s - T_\infty)$$

$$\boxed{Q = \frac{(T_s - T_\infty)}{1/h_{rad} \cdot A}}$$ → linear form.

$$\boxed{R = \frac{1}{h_{rad} \cdot A}}$$

resistance

250 + 273 = 523

273 + 25 = 298

Ques- A hot plate of dimension 50x75cm² cross-section at 2cm thickness has one of its surface ~~is~~ held fixed at 250°C. Air is blown pass at the above surface at 25°C offering a convection heat transfer coeff 25 W/m²·K. The same surface also losing ~~the heat~~ 300W, heat loss by radiation. The material of plate has thermal conductivity 43 W/m·K. Find (I) net rate of heat transfer to the plate into ambient.

(II) The surface temp of the plate which is greater than the temp of second surface given above.

Area = $50 \times 75 cm^2$, thickness = 2cm

$T_s = 250°C = 523 K$

$T_\infty = 25°C = 298 K$

$h = 25 W/m^2K$

$$\boxed{Q_{cond^n} = Q_{conv} + Q_{rad}}$$

Heat transfer during convection

$$q = hA(T_s - T_\infty)$$

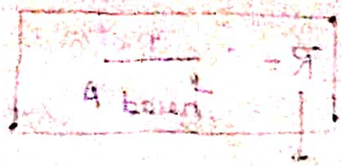
$$Q = 25 \times \left(\frac{50}{100} \times \frac{75}{100}\right)(523 - 298)$$

$$Q = 2109.375 \ W$$

Heat loss during radiation = 300W.

$$K = 43 \ W/mK$$

$$\sigma = 5.6697 \times 10^{-8} \ W/m^2K^4.$$



$$Q_{cond^n} = Q_{conv^n} + Q_{rad}$$

$$\frac{kA(T_1 - T_2)}{L} = hA(T_S - T_\infty) + 300$$

$$= 25 \times \frac{50}{100} \times \frac{75}{100}(250 - 25) + 300$$

$$= 2109.375 + 300$$

$$\frac{kA(T_1 - T_2)}{L} = 2409.375\ W$$

$$\frac{43 \times 0.5 \times 0.75}{0.02}(T_1 - 250) = 2409.375$$

$$(T_1 - 250) = \frac{2409.375 \times 0.02}{43 \times 0.5 \times 0.75}$$

$$T_1 - 250 = 2.988$$

$$\boxed{T_1 = 252.988\ ^\circ C}$$

Taking T_2 as 20

$$\boxed{T_1 = 253.046^\circ C}$$

Effect of temp on thermal conductivity (k)

(A) Solid Surface

(i) Metal

	k (w/m·k)	
Cu	384	Range 10 to hundreds
Brass (alloy of cu)	110	
Steel	54	
S.S.	10	

(ii) Non-Metal

Plastic	0.58	0.1 to 1
Asbestos	0.23	
Wood	0.17	

(3) Liquid

Water	0.60	0.1 to 1
Oil	0.14	

(4) Gas

Air	0.026	0.001 to 0.01
Steam	0.025	

- In case of metals thermal conductivity ranges from 10 to several hundreds.

Therefore solid have better conductivity than liq and gases.

- Thermal conductivity of alloys of any metal will be less than the thermal conductivity of that particular metal.

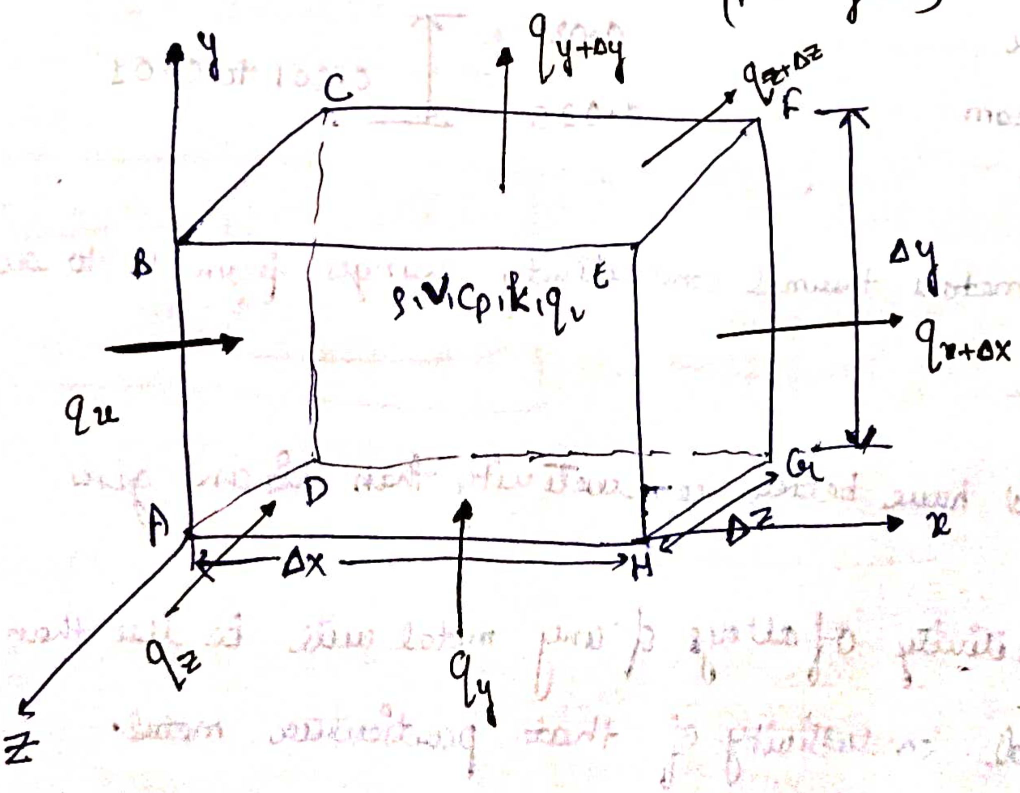
Cu alloys

- Brass: Cu + Zn, 60% + 40%
- Bronze: Cu + Sn, 90% + 10%

- The thermal conductivity of solids decreases with increase in temp.
- Thermal conductivity of liquids may increase or may decrease depending upon liq w.r.t temp.
- Thermal conductivity of gases increases with increase in temp because of particle movement.

8/9/22

General Heat Conduction Eqⁿ in Cartesian Coordinate System
(Rectangular)



$q_v \rightarrow$ volumetric heat genⁿ

1st Law of thermodynamics:

$$Q_{net} = W_{net} + \Delta U$$

Assumptions :- ① The work transition on account of temp change in solid materials is assumed to be negligible.

$$(W_{net} = 0)$$

$$Q_{net} = \cancel{W}^{0} + \cancel{dPE}^{0} + \cancel{dKE}^{0} + \underbrace{dSE}_{\text{stored energy}}$$

② It is assumed that the change in KE and PE is are assumed to be negligible.

$$Q_{net} = dSE$$

$$q_{x\,net} + q_{y\,net} + q_{z\,net} + \underbrace{q_v}_{\text{heat generation (volumetric)}} = dSE$$

$$q_x - q_{x+\Delta x} + q_y - q_{y+\Delta y} + q_z - q_{z+\Delta z} + q_v = dSE$$

Taylor series $f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \cdots\cdots$

$$\cancel{q_x} - \left[\cancel{q_x} + \frac{\partial}{\partial x}(q_x)\frac{\Delta x}{1!} + \frac{\partial^2}{\partial x^2}(q_x)\frac{\Delta x^2}{2!} + \cdots\right] + \cancel{q_y} - \left[\cancel{q_y} + \frac{\partial}{\partial y}(q_y)\frac{\Delta y}{1!} + \cdots\right]$$

$$+ \cancel{q_z} - \left[\cancel{q_z} + \frac{\partial}{\partial z}(q_z)\frac{\Delta z}{1!} + \cdots\right] + q_v = dSE$$

Neglecting higher orders.

$$-\frac{\partial}{\partial x}(q_x)\frac{\Delta x}{1!} - \frac{\partial}{\partial y}(q_y)\frac{\Delta y}{1!} - \frac{\partial}{\partial z}(q_z)\frac{\Delta z}{1!} + q_v = dSE$$

$$-\frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)\Delta x - \frac{\partial}{\partial y}\left(-kA\frac{\partial T}{\partial y}\right)\Delta y - \frac{\partial}{\partial z}\left(-kA\frac{\partial T}{\partial z}\right)\Delta z + q_v = dSE$$

$$\frac{\partial}{\partial x}\left(k_x A\frac{\partial T}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(k_y A\frac{\partial T}{\partial y}\right)\Delta y + \frac{\partial}{\partial z}\left(k_z A\frac{\partial T}{\partial z}\right)\Delta z + q_v = dSE$$

$$\frac{\partial}{\partial x}\left(k_x \Delta z \Delta y \frac{\partial T}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(k_y \Delta x \Delta z \frac{\partial T}{\partial y}\right)\Delta y + \frac{\partial}{\partial z}\left(k_z \Delta x \Delta y \frac{\partial T}{\partial z}\right)\Delta z + q_v = dSE$$

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)\Delta x \Delta y \Delta z + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)\Delta x \Delta y \Delta z + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)\Delta x \Delta y \Delta z$$

$$\cancel{+ q_v = dSE} + q_{gen} V = m C_p \frac{\partial T}{\partial t}$$

Volume, $V = \Delta x \Delta y \Delta z$

$$q_v = q_{gen} \cdot V$$

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)\cancel{V} + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)\cancel{V} + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)\cancel{V} + q_{gen} \cdot \cancel{V} = \rho \cancel{V} C_p \frac{\partial T}{\partial t}$$

$$\boxed{\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + q_{gen} = \rho C_p \frac{\partial T}{\partial t}}$$

1. Let us consider, material is isotropic and homogeneous

$$k_x = k_y = k_z = k$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{q_{gen}}{k} = \frac{\rho C_p}{k}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + q_{gen}/k = \frac{\rho C_p}{k}\frac{\partial T}{\partial t}$$

$$\alpha = k/\rho C_p$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

② Heat transfer is steady state without heat generation and constant thermal conductivity

$$\frac{\partial T}{\partial t} = 0 \text{ and } q_{gen} = 0$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0}$$ → Laplace eqn.

$$\boxed{\nabla^2 T = 0}$$

③ Steady state with heat generation and constant thermal conductivity.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{k} = 0$$

$$\boxed{\nabla^2 T + \frac{q_{gen}}{k} = 0}$$ → Poisson's eqn.

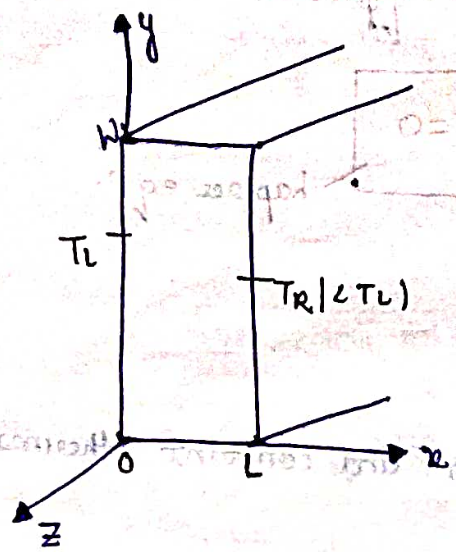
④ Unsteady (Transient) condition without heat generation and constant thermal conductivity

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\boxed{\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$ → Fourier's diffusion eqn.

Boundary Condition :-

$q' = W/m$
$q'' = W/m^2$ (per unit area)

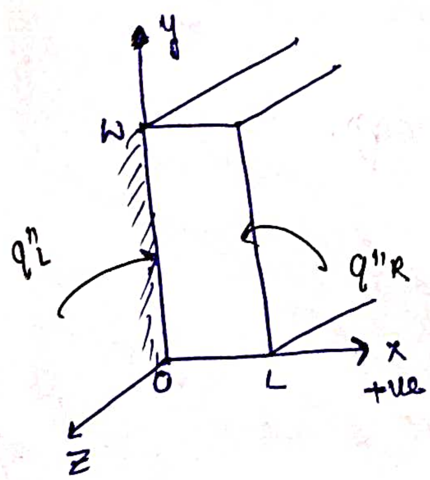


① Dirichlet Boundary Condⁿ (1st kind BC / Prescribed BC) :

$x = 0$; $T = T_L$
$0 \le y < W$
$z = 0$

$x = L$, $T = T_R$
$0 \le y \le W$
$z = 0$

② Neumann Boundary Condⁿ (2nd kind BC / Prescribed Heat Flux BC) :



$x = 0$, $q''_L = -k \dfrac{\partial T(x,y,z,t)}{\partial x}$ (per unit area है तो area से divide हो गया)

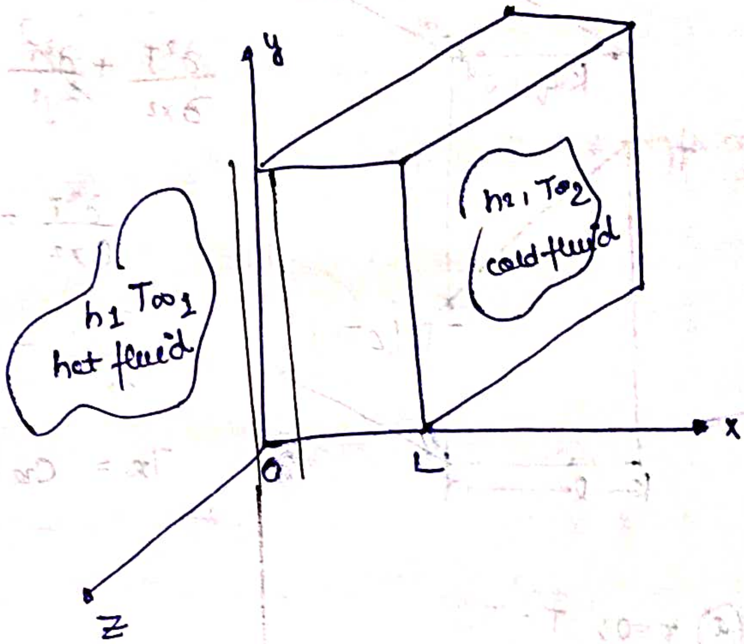
$x = L$, $-q''_R = -k \dfrac{\partial T(x,y,z,t)}{\partial x}$

* Say either boundary is adiabatic or insulated , $q''_L = 0$

$$\boxed{x=0, \quad \frac{\partial T_{x,y,z,t}}{\partial x} = 0}$$

Hence it is also k/a Adiabatic or insulated BC.

③ Newtonian Boundary Condⁿ (3rd kind BC / Combined Conduction & Convection BC / Conjugate BC)



Assuming no radiation

① $x=0;\ h_1 A(T_{\infty 1} - T_{x,y,z,t})$ $= -kA\frac{\partial T_{x,y,z,t}}{\partial x}$

$0 \le y \le W$

$z=0$

Conv in = Condⁿ out

area is same

$$\boxed{x=0;\ h_1(T_{\infty 1} - T_{x,y,z,t}) = -k\frac{\partial T_{x,y,z,t}}{\partial x}}$$

Condⁿ in = Conv out

② $x=L;\ -kA\left(\frac{\partial T_{x,y,z,t}}{\partial x}\right) = h_2 (T_{x,y,z,t} - T_{\infty 2})$

$0 \le y \le W$

$z=0$

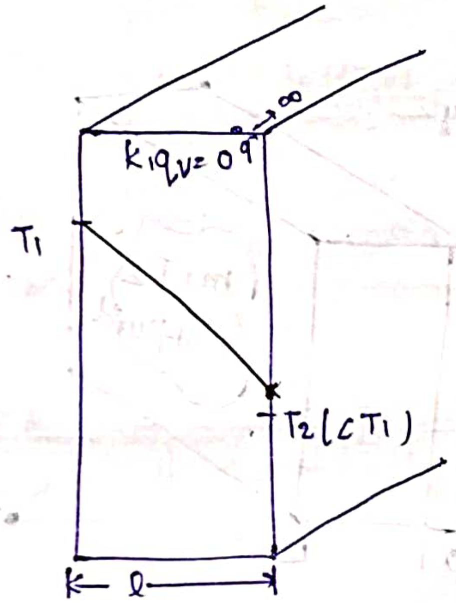
area is same

$$\boxed{-k\left(\frac{\partial T_{x,y,z,t}}{\partial x}\right) = h_2(T_{x,y,z,t} - T_{\infty 2})}$$

15 Sep, 2022

~~Composite Slab~~

1D steady state heat conduction in a slab without heat generation with constant thermal conductivity subjected to Dirichlet Boundary condition:



$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{0} + \cancel{\frac{q_{gen}}{k}}^{0} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}^{0}$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$\frac{\partial T_x}{\partial x} = C \longrightarrow (1)$$

$$T_x = Cx + D \longrightarrow (2)$$

(a) $x = 0;\ T = T_1$

$x = L;\ T = T_2$

When $x = 0,\ T = T_1$

$$T_1 = C(0) + D$$

$$\boxed{D = T_1}$$

When $x = l,\ T = T_2$

$$T_2 = C(L) + D$$

$$C(L) = D - T_2$$

$$C(L) = T_1 - T_2$$

$$C = \left(\frac{T_2 - T_1}{L}\right)$$

$$\cancel{\frac{\partial T_x}{\partial x} = C = \frac{T_1 - T_2}{L}}$$

$$\frac{T_x - T_1}{T_2 - T_1} = \frac{x}{L}$$

$$T_x = \frac{(T_2 - T_1)}{L}x + T_1$$

$y = mx + C$ — Linear profile of temp in rectangle.

Now, $q = -KAC$

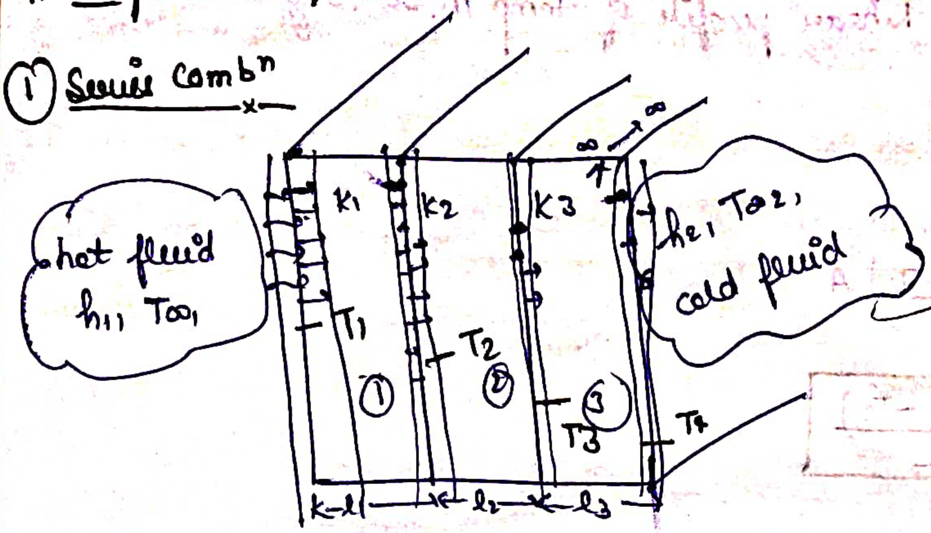
$$q = -k\left(\frac{T_2 - T_1}{L}\right)A$$

$$\boxed{q = \frac{KA(T_1 - T_2)}{L}}$$

Ques → Do the same problem subjected to Newtonian boundary condition.

Composite Slab/ Plates:-

① Series combⁿ



$$q = q_{conv^n 1} = q_{cond^n 1} = q_{cond^n 2} = q_{cond^n 3} = q_{conv^n 2}$$

$$= \frac{(T_{\infty 1} - T_1)}{1/h_1 A} = \frac{T_1 - T_2}{l_1/k_1 A} = \frac{T_2 - T_3}{l_2/k_2 A} = \frac{T_3 - T_4}{l_3/k_3 A} = \frac{T_4 - T_{\infty 2}}{1/h_2 A}$$

Componendo & Dividendo Rule

$$= \frac{(T_{\infty 1} - \cancel{T_1}) + \cancel{T_1} - \cancel{T_2} + \cancel{T_2} - \cancel{T_3} + \cancel{T_3} - \cancel{T_4} + \cancel{T_4} - T_{\infty 2}}{(1/h_1 A) + (l_1/k_1 A) + (l_2/k_2 A) + (l_3/k_3 A) + (1/h_2 A)}$$

$$\boxed{q = \frac{T_{\infty 1} - T_{\infty 2}}{1/h_1 A + l_1/k_1 A + l_2/k_2 A + l_3/k_3 A + 1/h_2 A}} \quad \text{—①}$$

Let us defining U = overall convection heat transfer coeff.

If multiply by appropriate area and overall temp diff must give the same rate of heat transfer as under actual conditions.

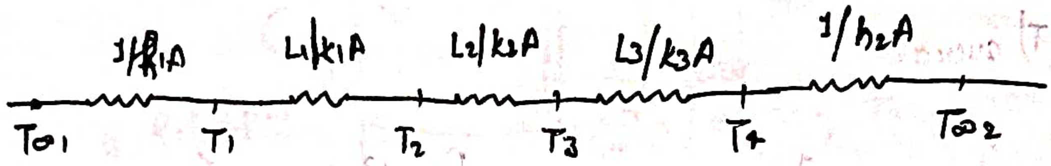
$$q = UA \, (\Delta T)_{overall} \quad \text{—②}$$

Equating ① and ②

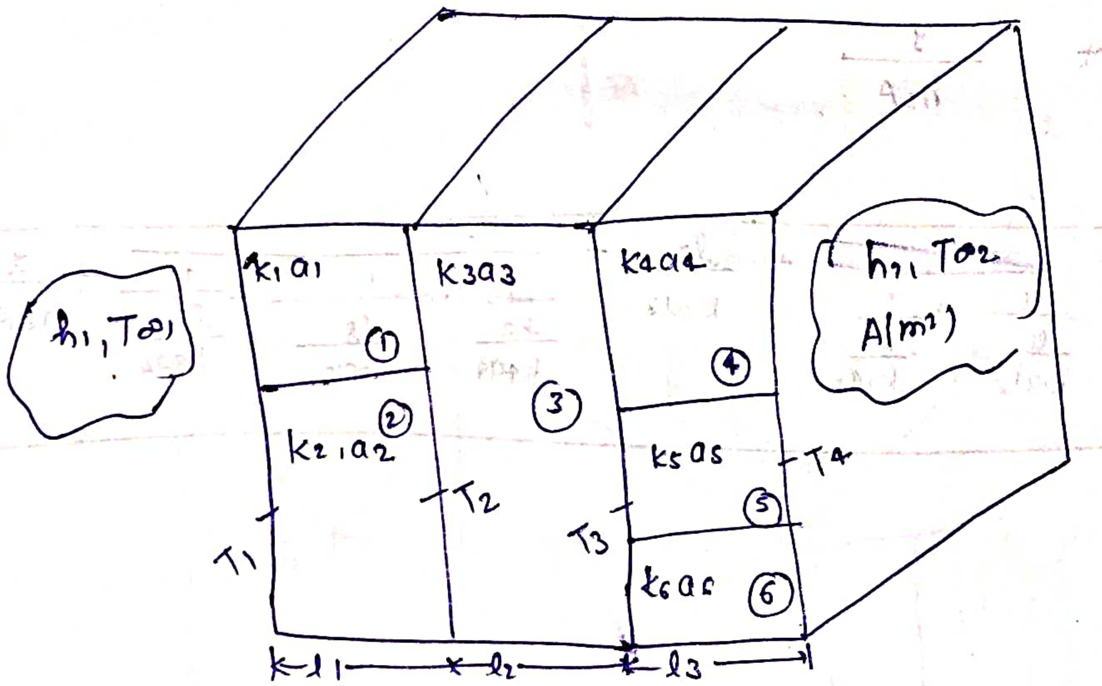
$$UA(\Delta T)_{overall} = \frac{T_{\infty 1} - T_{\infty 2}}{1/h_1A + l_1/k_1A + l_2/k_2A + l_3/k_3A + 1/h_2A}$$

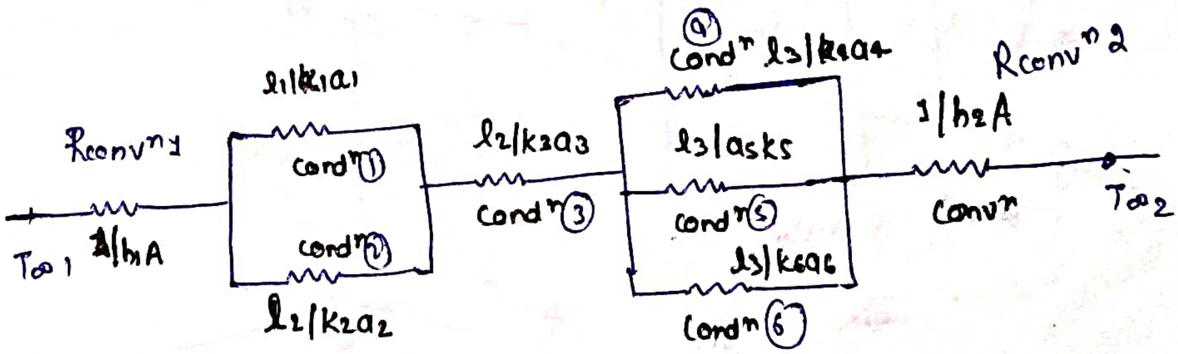
$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{l_1}{k_1} + \frac{l_2}{k_2} + \frac{l_3}{k_3} + \frac{1}{h_2}}$$

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \sum_{i=1}^{n} l_i/k_i + \frac{1}{h_2}}$$



② Series Parallel Combination





$$q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 A} + \frac{1}{\frac{1}{R_{cond^n 1}} + \frac{1}{R_{cond^n 2}}} + R_{cond^n 3} + \frac{1}{\frac{1}{R_{cond^4}} + \frac{1}{R_{cond^5}} + \frac{1}{R_{cond^6}}} + \frac{1}{h_2 A}}$$

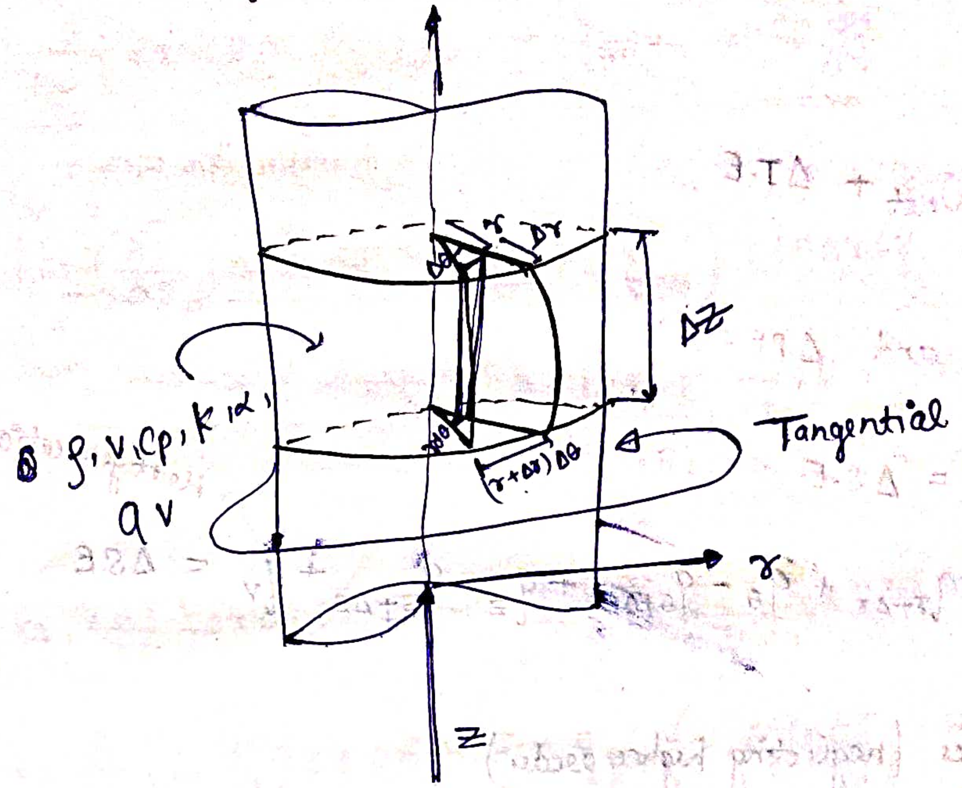
$$q = \frac{T_{\infty 1} - T_{\infty 2}}{1/h_1 A + \frac{1}{1\Big/\frac{l_1}{k_1 a_1} + 1\Big/\frac{l_2}{k_2 a_2}} + \frac{L_2}{k_3 a_3} + \frac{1}{\frac{1}{l_3/k_4 a_4} + \frac{1}{l_3/k_5 a_5} + \frac{1}{l_3/k_6 a_6}} + \frac{1}{h_2 A}}$$

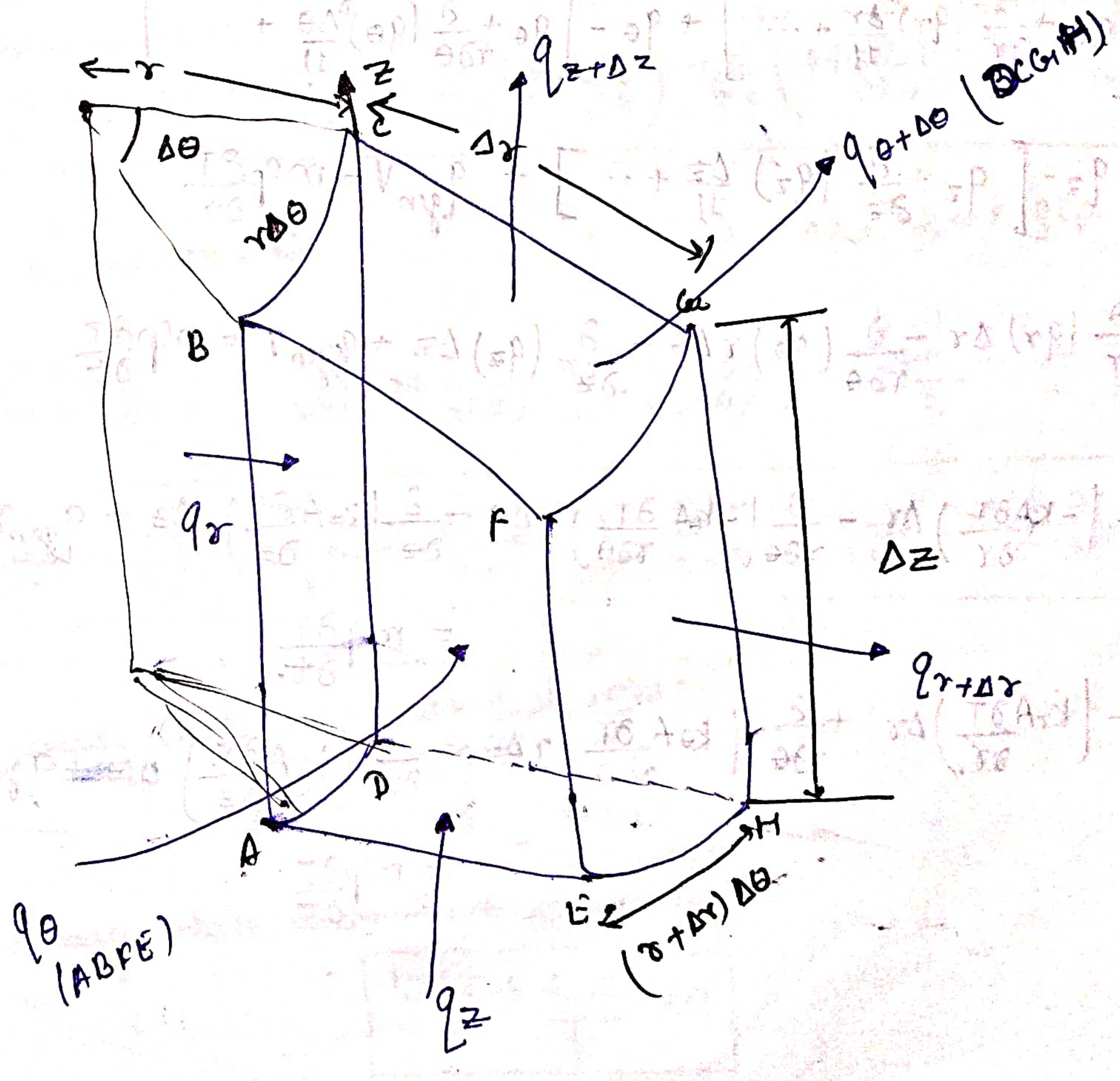
$$q = (UA(\Delta T))_{\text{overall}}$$

$$\frac{1}{UA} = \frac{1}{h_1 A} + \frac{1}{\frac{1}{R_{cond^n 1}} + \frac{1}{R_{cond^n 2}}} + R_{cond^3} + \frac{1}{\frac{1}{R_{cond^n 4}} + \frac{1}{R_{cond^5}} + \frac{1}{R_{cond^6}}} + \frac{1}{h_2 A}$$

$$\boxed{\frac{1}{UA} = \frac{1}{h_1 A} + \frac{1}{\frac{1}{\frac{l_1}{k_1 a_1}} + \frac{1}{\frac{l_2}{k_2 a_2}}} + \frac{l_3}{k_3 a_3} + \frac{1}{\frac{1}{\frac{l_3}{k_4 a_4}} + \frac{1}{\frac{l_3}{k_5 a_5}} + \frac{1}{\frac{l_3}{k_6 a_6}}} + \frac{1}{h_2 A}}$$

General Heat Conduction Eqⁿ in Cylindrical (Polar) Coordinate System :- $T = f(r, \theta, z, t)$





Assumption

The increment in the (θ) direction $(r+\Delta r)\Delta\theta$ is assumed to be negligible.

$$q_{net} = W_{net} + \Delta T.E.$$

① $W_{net} = 0$ and $\Delta PE = 0$

$$\therefore \quad q_{net} = \Delta S.E.$$

Heat generation.

$$q_r - q_{r+\Delta r} + q_\theta - q_{\theta+r\Delta\theta} + q_z - q_{z+\Delta z} + \dot{q}_v = \Delta SE$$

Taylor series (neglecting higher order)

$$q_r - \left[q_r + \frac{\partial}{\partial r}(q_r)\frac{\Delta r}{1!} + \cdots\right] + q_\theta - \left[q_\theta + \frac{\partial}{r\partial\theta}(q_\theta)\frac{r\Delta\theta}{1!} + \cdots\right]$$

$$+ q_z - \left[q_z + \frac{\partial}{\partial z}(q_z)\frac{\Delta z}{1!} + \cdots\right] + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow -\frac{\partial}{\partial r}(q_r)\Delta r - \frac{\partial}{r\partial\theta}(q_\theta)r\Delta\theta - \frac{\partial}{\partial z}(q_z)\Delta z + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow -\frac{\partial}{\partial r}\left(-k_r A\frac{\partial T}{\partial r}\right)\Delta r - \frac{\partial}{r\partial\theta}\left(-k_\theta A\frac{\partial T}{r\partial\theta}\right)r\Delta\theta - \frac{\partial}{\partial z}\left(k_z A\frac{\partial T}{\partial z}\right)\Delta z + q_{gen}V$$
$$= mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial r}\left(k_r A\frac{\partial T}{\partial r}\right)\Delta r + \frac{\partial}{r\partial\theta}\left(k_\theta A\frac{\partial T}{r\partial\theta}\right)r\Delta\theta + \frac{\partial}{\partial z}\left(k_z A\frac{\partial T}{\partial z}\right)\Delta z + q_{gen}V$$
$$= mC_p\frac{\partial T}{\partial t}$$

$$V = r\Delta\theta\Delta r\Delta z$$

$$\Rightarrow \frac{\partial}{\partial r}\left(k_r r\Delta\theta\Delta z\frac{\partial T}{\partial r}\right)\Delta r + \frac{\partial}{r\partial\theta}\left(k_\theta \Delta r\Delta z\frac{\partial T}{r\partial\theta}\right)r\Delta\theta$$

$$+\frac{\partial}{\partial z}\left(k_z r\Delta\theta\Delta r\frac{\partial T}{\partial z}\right)\Delta z + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

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$$\Rightarrow \frac{\partial}{\partial r}\left(k_r r\frac{\partial T}{\partial r}\right)\Delta r\Delta\theta\Delta z + \frac{\partial}{r\partial\theta}\left(k_\theta\frac{\partial T}{r\partial\theta}\right)r\Delta\theta\Delta r\Delta z$$

$$+\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)\Delta z r\Delta\theta\Delta r + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_r r\frac{\partial T}{\partial r}\right)\Delta r\Delta\theta\cancel{\Delta z}\cdot r + \frac{\partial}{r\partial\theta}\left(k_\theta\frac{\partial T}{r\partial\theta}\right)r\Delta\theta\cancel{\Delta r}\Delta z$$

$$+\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)\Delta z\Delta\theta\cancel{\Delta r}\cdot r + q_{gen}\cancel{V} = mC_p\frac{\partial T}{\partial t}$$

$$[m = \rho V$$

$$\Rightarrow \boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(k_r\cdot r\frac{\partial T}{\partial r}\right) + \frac{\partial}{r\partial\theta}\left(k_\theta\frac{\partial T}{r\partial\theta}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + q_{gen} = \rho C_p\frac{\partial T}{\partial t}}$$

① $k_r = k_\theta = k_z = k$ and $\alpha = k/\rho C_p$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right) + \frac{\partial}{r\partial\theta}\left(\frac{\partial T}{r\partial\theta}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{q_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 T}{\partial\theta^2}\right) + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

② Steady state, no heat generation :-

$$\boxed{\nabla^2 T = 0}$$

③ Steady state with heat generation

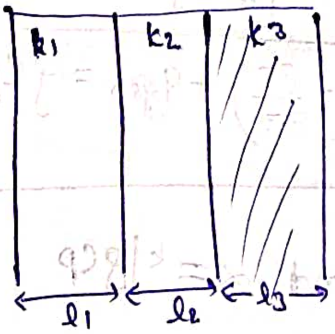
$$\boxed{\nabla^2 T + \frac{q_{gen}}{k} = 0}$$

④ Unsteady state without heat generation:

$$\boxed{\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

16/9/22

Ques- An exterior wall of a house may be approx by a 0.1m layer common brick ($k = 0.7\ W/m°C$) followed by a 0.04m layer of gypsum plaster ($k = 0.48\ W/m°C$). What thickness of loosely packed rock wool insulation ($k = 0.065\ W/m°C$) should be added to reduce the heat loss or (gain) through wall by 80%.



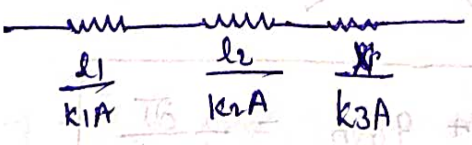
$k_1 = 0.7\ W/m°C$, $l_1 = 0.1m$

$k_2 = 0.48\ W/m°C$, $l_2 = 0.04m$

$k_3 = 0.065\ W/m°C$, $l_3 = ?$

Case ①

$$R_{th} = \frac{l_1}{k_1 A} + \frac{l_2}{k_2 A}$$



$$\Delta T = q R_{th}$$

$$\Delta T = q_1\left[\frac{0.1}{0.7A} + \frac{0.04}{0.48A}\right]$$

$$q_1 = \frac{\Delta T}{\frac{0.1}{0.7A} + \frac{0.04}{0.48A}}$$

$$q_1 = \left[\frac{\Delta T A}{\frac{0.1}{0.7} + \frac{0.04}{0.48}}\right]$$

Case ② Let $l_3 = x$

$$R_{th} = \frac{l_1}{k_1 A} + \frac{l_2}{k_2 A} + \frac{l_3}{k_3 A}$$

$$\Delta T = Q R_{th}$$

$$\Delta T = Q_2 \left[\frac{0.1}{0.7A} + \frac{0.04}{0.48A} + \frac{x}{0.065A} \right]$$

$$Q_2 = \frac{\Delta T}{\frac{0.1}{0.7A} + \frac{0.04}{0.48A} + \frac{x}{0.065A}}$$

$$Q_2 = \left[\frac{\Delta T A}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} \right]$$

Now, according to question,

$$Q_2 = Q_1 - 0.8Q_1 = 0.2Q_1$$

$$\frac{\cancel{\Delta T A}}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} = (0.2) \left[\frac{\cancel{\Delta T A}}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \cancel{\ldots}} \right]$$

$$\frac{1}{0.226190 + \frac{x}{0.065}} = (0.2) \left(\frac{1}{0.226190} \right)$$

$$\frac{1}{0.226190 + \frac{x}{0.065}} = 0.884212$$

$$0.226190 + \frac{x}{0.065} = \frac{1}{0.884212}$$

$$\boxed{x = 0.0588\, m}$$

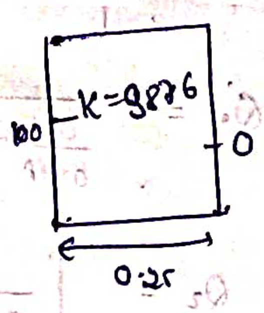
Ques - Consider a slab of thickness $L = 0.25m$. One surface is kept at $100°C$ and the other surface at $0°C$. Determine the net flux across the slab if the slab is made from pure copper $(k = 387.6\ W/mK)$.

$l = 0.25m$

$T_1 = 100°C$

$T_2 = 0°C,$

$k = 387.6\ W/m°K$



$$Q = -kA\frac{\Delta T}{l}$$

$$q = \frac{Q}{A} = -k\frac{\Delta T}{l} = \frac{-(387.6)(100)}{0.25}$$

$$\boxed{\text{flux} = 155040\ W/m^2}$$

Method (2)

$$R = \frac{l}{kA} = \frac{0.25}{(387.6)A}$$

$$\Delta T = Q R_{th}$$

$$100 - 0 = Q\left[\frac{0.25}{(387.6)A}\right]$$

$$100 = \frac{Q}{A}\left[\frac{0.25}{387.6}\right]$$

$$100 = q\left[\frac{0.25}{387.6}\right]$$

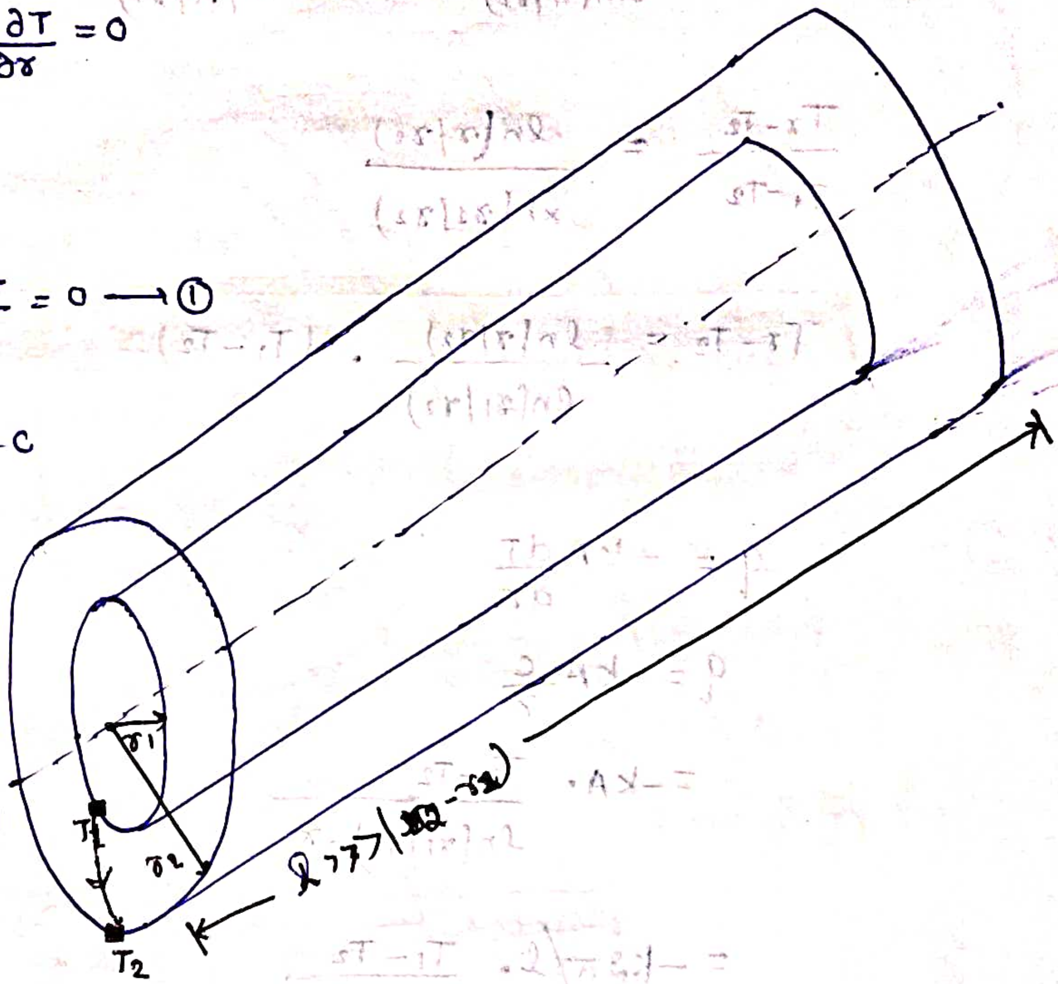
$$\boxed{q = 1.55 \times 10^5\ W/m^2}$$

17/9 :-

1D steady state Heat Conduction in a cylinder without heat generation with constant thermal conductivity subjected to Dirichlet Boundary conditions.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \theta^2}\right)}^{0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{0} + \cancel{\frac{q_{gen}}{k}}^{0} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}^{0}$$

Infinitely large.



$$\frac{1}{r}\frac{\partial}{\partial r} r.\frac{\partial T}{\partial r} = 0$$

$$\frac{1}{r} \neq 0$$

$$\frac{\partial}{\partial r} r.\frac{\partial T}{\partial r} = 0 \longrightarrow (1)$$

$$r.\frac{\partial T}{\partial r} = c$$

$$\frac{\partial T}{\partial r} = c/r \longrightarrow (2)$$

$$T_r = c\ln r + D \longrightarrow (3)$$

@ $r = r_1$; $T = T_1$

& $r = r_2$, $T = T_2$

$$T_1 = c\ln r_1 + D$$

$$T_2 = c\ln r_2 + D$$

$$c\ln\left(\frac{r_1}{r_2}\right) = T_1 - T_2$$

$$\boxed{c = \frac{T_1 - T_2}{\ln(r_1/r_2)}}$$

and,

$$D = T_2 - c\ln r_2$$

$$\boxed{D = T_2 - \frac{(T_1 - T_2)}{\ln(r_1/r_2)}\ln r_2}$$

From eq 3

$$T_r = \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln r + T_2 - \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln r_2$$

$$T_r - T_2 = \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln r - \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln r_2$$

$$\frac{T_r - T_2}{T_1 - T_2} = \frac{\ln(r/r_2)}{\ln(r_1/r_2)}$$

$$T_r - T_2 = \frac{\ln(r/r_2)}{\ln(r_1/r_2)} \cdot (T_1 - T_2)$$

$$q = -kA \frac{dT}{dr}$$

$$q = -kA \frac{c}{r}$$

$$= -kA \cdot \frac{T_1 - T_2}{\ln(r_1/r_2) \cdot r}$$

$$= -k 2\pi \cancel{r} l \cdot \frac{T_1 - T_2}{\ln(r_1/r_2) \cdot \cancel{r}}$$

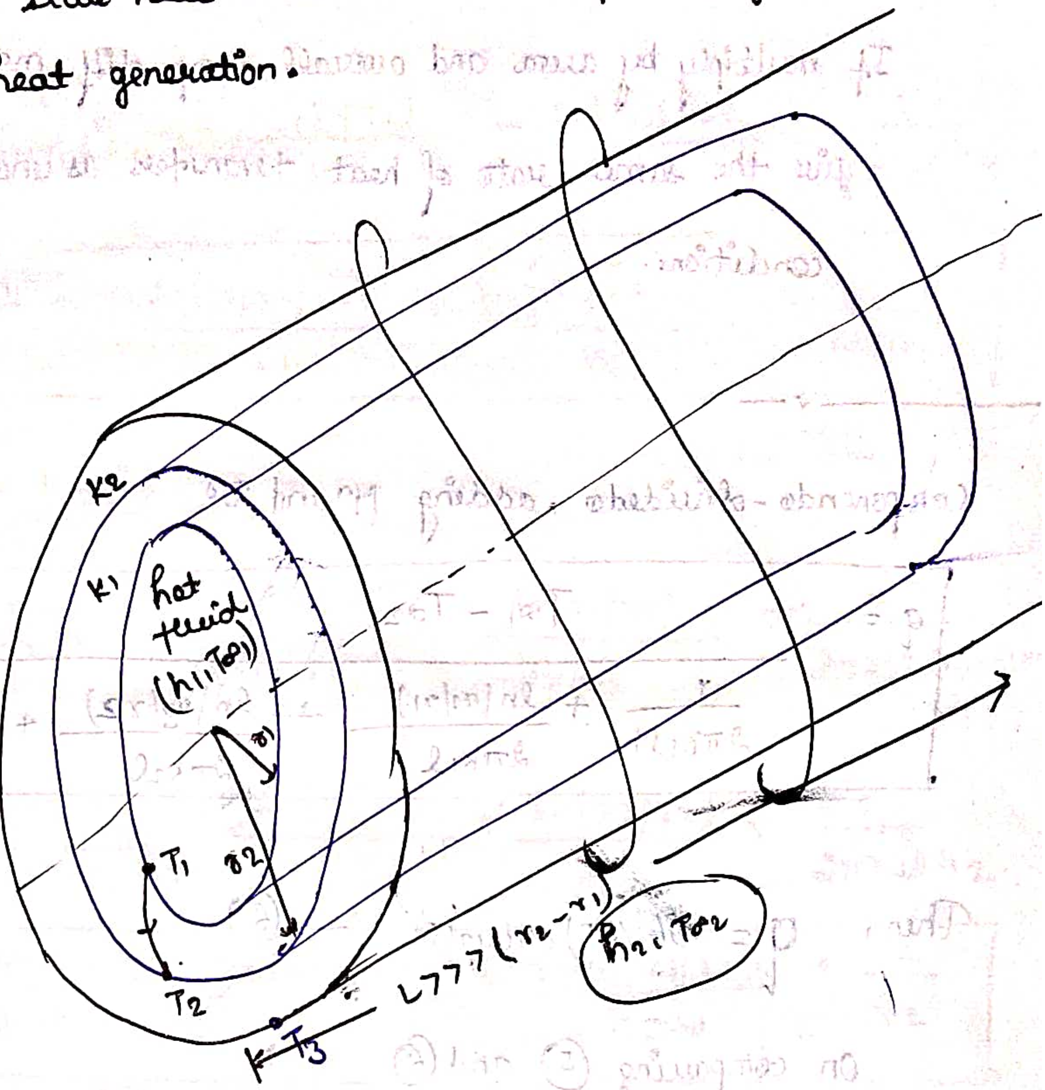
$$\boxed{q = \frac{T_1 - T_2}{\left[\frac{\ln(r_2/r_1)}{2\pi k l}\right]}}$$

in case of cylinder, temp profile is logarithmic.

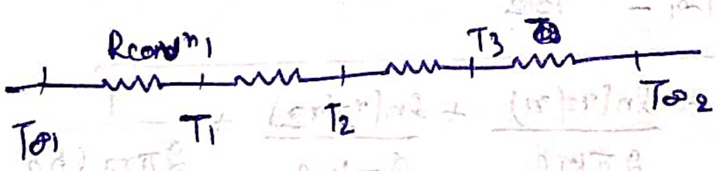
$$\boxed{I = \frac{\Delta V}{R}}$$

Que- Do the same problem with Newtonian Boundary condition.

Que -1D steady state heat conduction in composite cylinder without heat generation.



$q = \frac{\Delta T}{\Sigma R}$



$$q = q_{conv^n 1} = q_{cond^n 1} = q_{cond^n 2} = q_{conv^n 2}.$$

$$q = \frac{T_{\infty 1} - T_1}{\frac{1}{2\pi r_1 l h_1}} = \cancel{\frac{T_1 - T_2}{l / k_1 \cdot 2\pi r_1 l}} = \frac{T_1 - T_2}{\frac{\ln(r_2/r_1)}{2\pi k_1 l}} = \frac{T_2 - T_3}{\frac{\ln(r_3/r_2)}{2\pi k_2 l}}$$

$$= \frac{T_3 - T_{\infty 2}}{\frac{1}{2\pi r_3 l h_2}} \longrightarrow (4)$$

Let us define U = overall heat trans coeff

If multiply by area and overall temp diff must give the same rate of heat transfer as under actual conditions.

---x---

Componendo-dividendo, adding N^r and D^r

$$\boxed{q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{2\pi r_3 l h_2}}} \rightarrow (5)$$

Then, $q = UA(\Delta T)_{overall}$ $\rightarrow (6)$

On comparing (5) and (6)

$$UA(\Delta T)_{overall} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{2\pi r_3 l h_2}}$$

$$\boxed{\frac{1}{UA} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{2\pi r_3 l h_2}} \longrightarrow (7)$$

Let us defining U_i & U_o are the overall heat transfer coeff based on inner and outer radius of the cylinder.

$$\frac{1}{U_i A_i} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{2\pi r_3 l h_2}$$

$$\frac{1}{U_i(\cancel{2\pi r_1 l})} = \frac{1}{\cancel{2\pi} r_1 \cancel{l} h_1} + \frac{\ln(r_2/r_1)}{\cancel{2\pi} k_1 \cancel{l}} + \frac{\ln(r_3/r_2)}{\cancel{2\pi} k_2 \cancel{l}} + \frac{1}{\cancel{2\pi} r_3 \cancel{l} h_2}$$

$$\boxed{\frac{1}{U_i} = \frac{1}{h_1} + \frac{\ln(r_2/r_1)\cdot r_1}{k_1} + \frac{\ln(r_3/r_2)\cdot r_1}{k_2} + \frac{r_1}{r_3\cdot h_2}}$$

$\longrightarrow$ (8)

Based on outer radius

$$\frac{1}{U_o A_o} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{2\pi r_3 l h_2}$$

$$\frac{1}{U_o(2\pi r_3 l)} = \frac{1}{2\pi r_1 l h_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{2\pi r_3 l h_2}$$

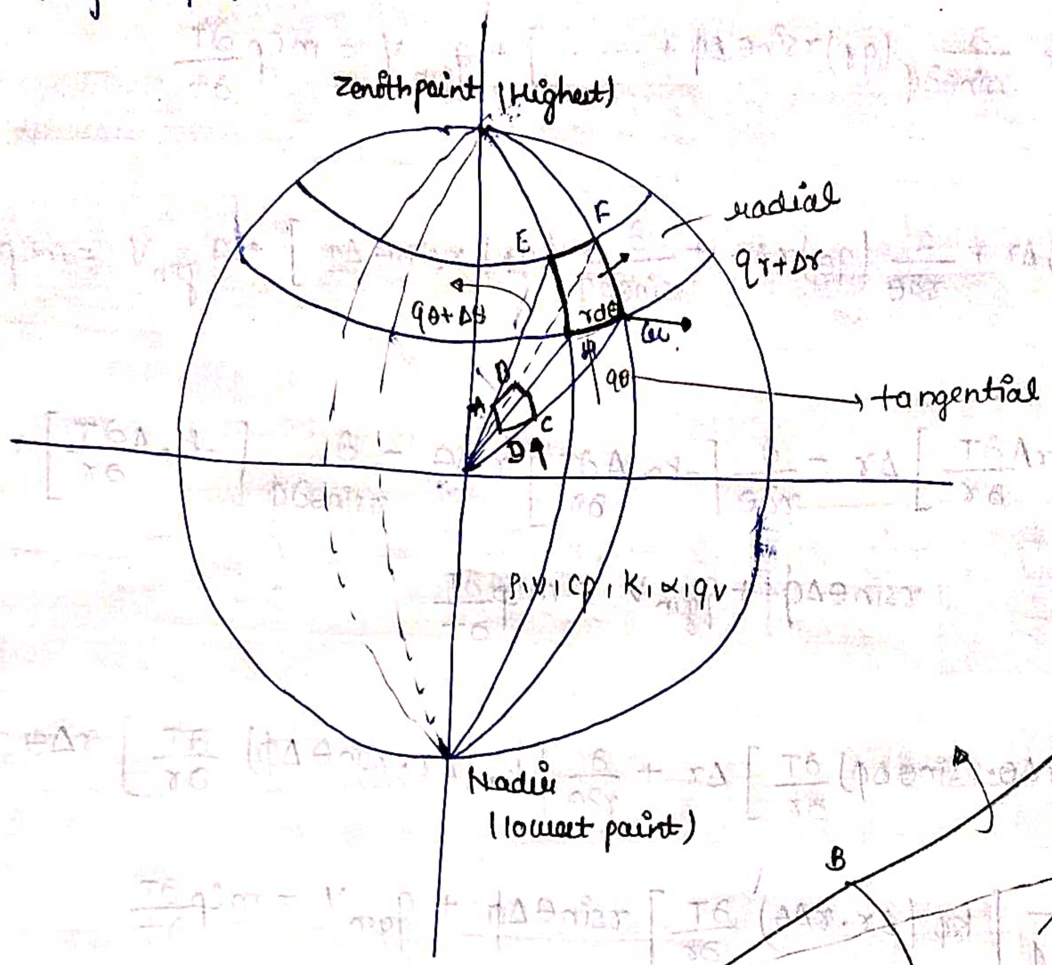
$$\boxed{\frac{1}{U_o} = \frac{r_3}{r_1\cdot h_1} + \frac{r_3\ln(r_2/r_1)}{k_1} + \frac{r_3\ln(r_3/r_2)}{k_2} + \frac{1}{h_2}}$$

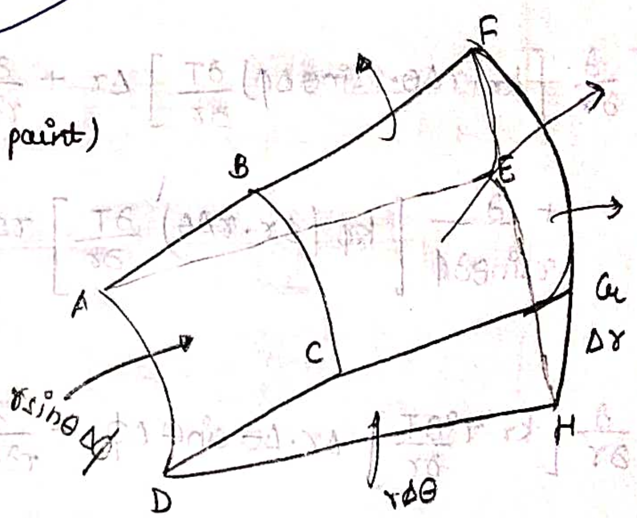
$\longrightarrow$ (9)

22/9

General Heat Conduction Equation in Spherical Coordinate System:

$T = f(r, \theta, \phi, t)$





$V = \Delta r \; r\Delta\theta \; r\sin\theta\Delta\phi$

<u>Assumption</u>, Increment in θ and ϕ direction is negligible.

ΔKE & $\Delta PE \longrightarrow$ negligible.

$$q_{net} = \cancel{W_{net}}^{0} + \Delta SE$$

$$q_r - q_{r+\Delta r} + q_\theta - q_{\theta+r\Delta\theta} + q_\phi - q_{\phi+r\sin\theta\Delta\phi} + q_v = \Delta SE$$

$$q_r - \left[q_r + \frac{\partial}{\partial r} q_r \frac{\Delta r}{1!} + \frac{\partial^2}{\partial r^2} q_r \frac{\Delta r^2}{2!} + \cdots\right] + q_\theta - \left[q_\theta + \frac{\partial}{r d\theta} q_\theta \frac{\cancel{r}\Delta\theta}{1!}\right.$$

$$\left. + \frac{\partial^2}{\partial\theta^2} q_\theta \frac{r\Delta\theta^2}{2!} + \cdots\right] + q_\phi - \left[q_\phi + \frac{\partial}{r\sin\theta\partial\phi} q_\phi \frac{r\sin\theta\Delta\phi}{1!}\right.$$

$$\left. + \frac{\partial^2}{r\sin\theta\partial\phi^2} q_\theta \frac{\Delta r^2}{2!} + \cdots\right] + q_{gen} V = mc_p \frac{\partial T}{\partial t}$$

$$\Rightarrow q_r - \left[q_r + \frac{\partial}{\partial r}(q_r)\Delta r + \cdots\right] + q_\theta - \left[q_\theta + \frac{\partial}{r\partial\theta}(q_\theta)\, r\Delta\theta + \cdots\right]$$

$$+ q_\phi - \left[q_\phi + \frac{\partial}{r\sin\theta\,\partial\phi}(q_\phi)\, r\sin\theta\,\Delta\phi + \cdots\right] + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow -\left[\frac{\partial}{\partial r}(q_r)\Delta r + \frac{\partial}{r\partial\theta}(q_\theta)\, r\Delta\theta + \frac{\partial}{r\sin\theta\,\partial\phi}(q_\phi)\, r\sin\theta\,\Delta\phi\right] + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{-\partial}{\partial r}\left[-k_r A\frac{\partial T}{\partial r}\right]\Delta r - \frac{\partial}{r\partial\theta}\left[-k_\theta A\frac{\partial T}{\partial r}\right] r\Delta\theta - \frac{\partial}{r\sin\theta\,\partial\phi}\left[-k_\phi A\frac{\partial T}{\partial r}\right]$$

$$r\sin\theta\,\Delta\phi + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial r}\left[k_r(r\Delta\theta\cdot r\sin\theta\,\Delta\phi)\frac{\partial T}{\partial r}\right]\Delta r + \frac{\partial}{r\partial\theta}\left[k_\theta(\Delta r\cdot r\sin\theta\,\Delta\phi)\frac{\partial T}{\partial r}\right] r\Delta\theta$$

$$+ \frac{\partial}{r\sin\theta\,\partial\phi}\left[k_\phi(\Delta r\cdot r\Delta\theta)\frac{\partial T}{\partial r}\right] r\sin\theta\,\Delta\phi + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial r}\left[k_r\, r^2\frac{\partial T}{\partial r}\right]\Delta r\cdot\Delta\theta\,\sin\theta\,\Delta\phi + \frac{\partial}{r\partial\theta}\left[k_\theta\sin\theta\frac{\partial T}{r\partial\theta}\right] r^2\Delta\theta\,\Delta r\,\Delta\phi$$

$$+ \frac{\partial}{r\sin\theta\,\partial\phi}\left[k_\phi\frac{\partial T}{r\sin\theta\,\partial\phi}\right]\Delta r\, r^2\Delta\theta\sin\theta + q_{gen}V = mC_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left[k_r\, r^2\frac{\partial T}{\partial r}\right]\cancel{(r^2\Delta r\cdot\Delta\theta\sin\theta\,\Delta\phi)} + \frac{1}{\sin\theta}\frac{\partial}{r\partial\theta}\left[k_\theta\sin\theta\frac{\partial T}{r\partial\theta}\right]$$

$$\cancel{(r^2\Delta\theta\Delta r\Delta\phi\sin\theta)} + \frac{\partial}{r\sin\theta\,\partial\phi}\left[k_\phi\frac{\partial T}{r\sin\theta\,\partial\phi}\right]\cancel{(\Delta r\Delta\theta\, r^2\sin\theta)}$$

$$+ q_{gen}\cancel{V} = \rho V C_p\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left[k_r r^2 \frac{\partial T}{\partial r}\right] + \frac{1}{\sin\theta}\frac{\partial}{r\partial\theta}\left[k_\theta \sin\theta \frac{\partial T}{r\partial\theta}\right] + \frac{\partial}{r\sin\theta\,\partial\phi}\left[k_\phi \frac{\partial T}{r\sin\theta\,\partial\phi}\right] +$$

$$q_{gen} = \rho C_p \frac{\partial T}{\partial t}$$

Assuming material is homogeneous & isotropic

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial T}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial T}{r\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\frac{\partial T}{\partial\phi} + \frac{q_{gen}}{k}$$

$$= \frac{\rho C_p}{k}\frac{\partial T}{\partial t}$$

$$\Rightarrow \boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial T}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} + \frac{q_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

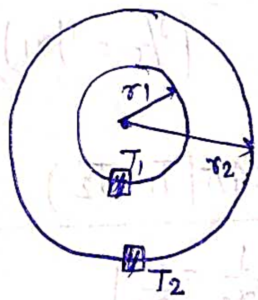
(2) steady state, no ~~q~~ heat generation : $\boxed{\nabla^2 T = 0}$

(3) steady state, with heat generation : $\boxed{\nabla^2 T + \frac{q_{gen}}{k} = 0}$

(4) unsteady state without heat generation : $\boxed{\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$

10 Steady State Sphere without heat generation with constant thermal conductivity subjected to Dirichlet Boundary condition:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right] = 0 \longrightarrow (I)$$



$$\frac{1}{r^2} \neq 0 \Rightarrow \frac{\partial}{\partial r} r^2 \frac{\partial T}{\partial r} = 0 \longrightarrow (II)$$

$$r^2\frac{\partial T}{\partial r} = C \longrightarrow (III)$$

$$\boxed{T_r = -\frac{C}{r} + D} \longrightarrow (IV)$$

(a) $r = r_1 \; ; \; T = T_1$

$r = r_2 ; \; T = T_2$

$$T_1 = -\frac{C}{r_1} + D$$

$$T_2 = -\frac{C}{r_2} + D$$

$- \quad + \quad -$

$$T_1 - T_2 = \frac{C}{r_2} - \frac{C}{r_1}$$

$$C\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = T_1 - T_2$$

$$\boxed{C = \frac{T_1 - T_2}{\frac{1}{r_2} - \frac{1}{r_1}}}$$

$$T_1 = \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_2}\right) r_1} + D$$

$$\boxed{D = T_1 - \frac{(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) r_1}}$$

$$T_r = \frac{T_1 - T_2}{r\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + T_1 - \frac{T_1 - T_2}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) r_1}$$

$$\Rightarrow (T_r - T_1) = \frac{(T_1 - T_2)}{r\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} - \frac{(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) r_1}$$

$$\Rightarrow \frac{T_r - T_1}{T_1 - T_2} = \frac{1}{r}\left[\frac{1}{\frac{1}{r_1} - \frac{1}{r_2}}\right] - \frac{1}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) r_1}$$

$$\boxed{\frac{T_r - T_1}{T_1 - T_2} = \frac{1/r - 1/r_1}{1/r_1 - 1/r_2}}$$

Temp profile in sphere $\rightarrow$ hyperbolic

$$q = -kA\frac{\partial T}{\partial r}$$

$$q = -k(4\pi \cancel{r^2})\frac{(T_1 - T_2)}{(1/r_2 - 1/r_1)\cancel{r^2}}$$

$$q = \frac{-k4\pi(T_1 - T_2)}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

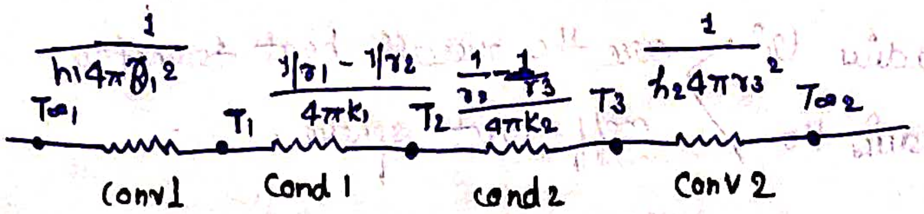
$$q = \frac{T_1 - T_2}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) / 4\pi k}$$

$$\boxed{R_{th\,\text{sphere}} = \frac{1/r_1 - 1/r_2}{4\pi k}}$$

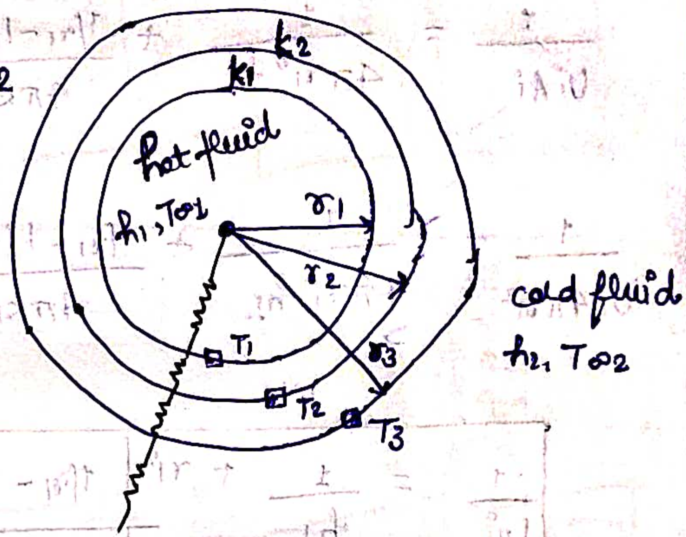
$$\boxed{R_{rec} = l/kA \qquad R_{cy} = \frac{\ln(r_2/r_1)}{2\pi k l}}$$

24/9.

Composite Spheres:-



$$q = q_{conv1} = q_{cond1} = q_{cond2} = q_{conv2}$$



$$= \frac{T_{\infty 1} - T_1}{\frac{1}{h_1 4\pi r_1^2}} = \frac{T_1 - T_2}{\frac{1}{r_1} - \frac{1}{r_2} / 4\pi k_1}$$

$$\boxed{= \frac{T_2 - T_3}{1/r_2 - 1/r_3 / 4\pi k_2} = \frac{T_3 - T_{\infty 2}}{\frac{1}{h_2 4\pi r_3^2}}}$$

Componendo and dividendo

$$q = \frac{T_{\infty 1} - \cancel{T_1} + \cancel{T_1} - \cancel{T_2} + \cancel{T_2} - \cancel{T_3} + \cancel{T_3} - T_{\infty 2}}{\frac{1}{h_1 4\pi r_1^2} + \frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k_1} + \frac{\frac{1}{r_2} - \frac{1}{r_3}}{4\pi k_2} + \frac{1}{h_2 4\pi r_3^2}}$$

$$\boxed{q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{4\pi r_1^2 h_1} + \frac{1/r_1 - 1/r_2}{4\pi k_1} + \frac{1/r_2 - 1/r_3}{4\pi k_2} + \frac{1}{4\pi r_3^2 h_2}}}$$

Now let us consider

$$q = UA(\Delta T)_{over}$$

$$\frac{1}{UA} = \frac{1}{4\pi r_1^2 h_1} + \frac{1/r_1 - 1/r_2}{4\pi k_1} + \frac{1/r_2 - 1/r_3}{4\pi k_2} + \frac{1}{4\pi r_3^2 h_2}$$

For inner radius U_i } are the overall heat transfer
outer radius U_o } coeff wrt sphere.

$$\frac{1}{U_i A_i} = \frac{1}{4\pi r_1^2 h_1} + \frac{1/r_1 - 1/r_2}{4\pi k_1} + \frac{1/r_2 - 1/r_3}{4\pi k_2} + \frac{1}{4\pi r_3^2 h_2}$$

$$\frac{1}{U_i 4\pi r_1^2} = \frac{1}{4\pi r_1^2 h_1} + \frac{1/r_1 - 1/r_2}{4\pi k_1} + \frac{1/r_2 - 1/r_3}{4\pi k_2} + \frac{1}{4\pi r_3^2 h_2}$$

$$\boxed{\frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1^2\left[1/r_1 - 1/r_2\right]}{k_1} + \frac{r_1^2\left[1/r_2 - 1/r_3\right]}{k_2} + \frac{r_1^2}{r_3^2 . h_2}}$$

Similarly

$$\frac{1}{U_o A_o} = \frac{1}{4\pi r_1^2 h_1} + \frac{1/r_1 - 1/r_2}{4\pi k_1} + \frac{1/r_2 - 1/r_3}{4\pi k_2} + \frac{1}{4\pi r_3^2 h_2}$$

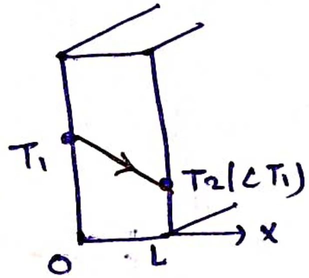
$$\frac{1}{U_o 4\pi r_3^2} = \frac{1}{4\pi r_1^2 h_1} + \frac{1/r_1 - 1/r_2}{4\pi k_1} + \frac{1/r_2 - 1/r_3}{4\pi k_2} + \frac{1}{4\pi r_3^2 h_2}$$

$$\boxed{\frac{1}{U_o} = \frac{r_3^2}{r_1^2 h_1} + \frac{r_3^2\left[1/r_1 - 1/r_2\right]}{k_1} + \frac{r_3^2\left[1/r_2 - 1/r_3\right]}{k_2} + \frac{1}{h_2}}$$

Plate

$$q = -kA\frac{dT}{dx}$$

$$\int_0^L q\,dx = -kA\int_{T_1}^{T_2} dT$$



$$q\,[x]_0^L = -kA\,[T]_{T_1}^{T_2}$$

$$q\,[L-0] = -kA\,[T_2 - T_1]$$

$$\boxed{q = \frac{kA\Delta T}{L}}$$

$$\boxed{\frac{T_x - T_1}{\Delta T} = x/L}$$

$$\boxed{q = hA\Delta T}$$

$$\boxed{q = \frac{\Delta T}{L/kA}}$$ Resistance

$$\boxed{q = \frac{\Delta T}{1/hA}}$$

Linear profile

$$\boxed{\text{flux } q'' = \frac{\Delta T}{L/K}}$$

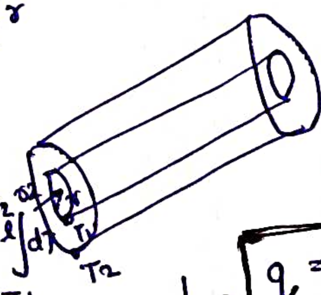
$$\boxed{q'' = \frac{\Delta T}{r\left[\frac{\ln r_2/r_1}{k}\right]}}$$

flux

Cylinder

$$q = -kA\frac{dT}{dr}$$

$$\int_{r_1}^{r_2} q\,dr = -kA\int_{T_1}^{T_2} dT$$



$$\int_{r_1}^{r_2} q\,dr = -k2\pi rl\int_{T_1}^{T_2} dT$$

$$\int_{r_1}^{r_2} q\frac{dr}{r} = -k2\pi l\int_{T_1}^{T_2} dT$$

$$\boxed{q = \frac{\Delta T}{1/h2\pi rl}}$$

$$\boxed{q = hA\Delta T}$$

$$q\,[\ln r]_{r_1}^{r_2} = -2\pi kl\,[T]_{T_1}^{T_2}$$

$$q\,[\ln r_2 - \ln r_1] = 2\pi kl\,(T_1 - T_2)$$

$$\boxed{q = \frac{T_1 - T_2}{\left[\frac{\ln(r_2/r_1)}{2\pi kl}\right]}}$$ resistance

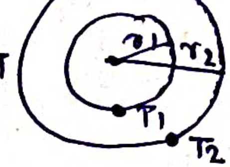
logarithmic temp profile

$$\boxed{\frac{T_r - T_1}{T_2 - T_1} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}}$$

Sphere

$$q = -kA\frac{dT}{dr}$$

$$\int_{r_1}^{r_2} q\frac{dr}{r} = -k4\pi\int_{T_1}^{T_2} dT$$



$$q\left[-\frac{1}{r}\right]_{r_1}^{r_2} = -4\pi k\,[T]_{T_1}^{T_2}$$

$$q\left(-\frac{1}{r_2} + \frac{1}{r_1}\right) = 4\pi k\,(T_1 - T_2)$$

$$q = hA\Delta T$$

$$\boxed{q = \frac{\Delta T_a}{\left[\frac{\frac{1}{r_1} - 1/r_2}{4\pi K}\right]}}$$ hyperbolic temp profile

$$\frac{T_r - T_1}{T_2 - T_1} = \frac{1/r - 1/r_1}{\frac{1}{r_2} - 1/r_1}$$

$$\boxed{q = \frac{\Delta T}{\frac{1}{h4\pi r_2}}}$$ conv resistance

$$\boxed{q'' = \frac{\Delta T}{r^2\left[\frac{\frac{1}{r_1} - \frac{1}{r_2}}{k}\right]}}$$