Continuous Topology Simplification of Planar Vector Fields

Name: Keshav Chandak (IMT2021003)

Name: Akash Perla (IMT2021530)

<u>Introduction:</u>

This research paper provides an overview of the use of topological methods for the visualization of planar vector fields. It explains that these methods extract critical points and integrate streamlines to create a synthetic graph representation of vector data, which is a reliable structural depiction. Vector field topology is defined as the graph built up of all first-order critical points, closed orbits and some integral curves connecting them, called separatrices.

<u>Critical Points:</u>

Critical points are special points in a vector field where the velocity is zero or undefined. These points are important because they can be used to extract features such as separatrices and periodic orbits, which provide insight into the behavior of the vector field.

There are **four types of critical points**: sources, sinks, saddles, and centers. A **source** is a critical point where all nearby trajectories move away from the point. A **sink** is a critical point where all nearby trajectories move towards the point. A **saddle** is a critical point where some nearby trajectories move towards the point while others move away from it. Finally, a **center** is a critical point where nearby trajectories move in closed orbits around the point.

The classification of critical points can be used to understand the topology of vector fields and to identify important features such as separatrices and periodic orbits. These features can be used to simplify and visualize complex vector fields, making them easier to interpret and analyze.

We can define **Poincare index** of a simple closed curve which measures the number of rotations of the vector field while traveling along the curve in positive direction. A saddle point has index -1 whereas every other critical point has index +1. Note that the index of a closed orbit is always +1.

$$\text{index}_\gamma = \frac{1}{2\pi}\oint_\gamma \mathrm{d}\phi, \quad \text{where} \quad \phi = \arctan\frac{v_y}{v_x}.$$

The index of a triangle cell is given by the sum of the angle changes. Let φ_0 ,φ_1 and ϕ_2 be the angle coordinates [0, 2π] of the vectors v_0 ,v_1 and v_2 defined at the vertices of a linearly interpolated triangle. The index of this triangle T is given by:

$$\text{index}(T) \quad = \quad \frac{1}{2\pi}\left(\Delta(\phi_0,\phi_1)+\Delta(\phi_1,\phi_2)+\Delta(\phi_2,\phi_0)\right)$$

$$\text{where} \quad \Delta(\phi_i,\phi_j) = \begin{cases} \phi_j-\phi_i+2\pi & \text{if } \phi_j-\phi_i < -\pi, \\ \phi_j-\phi_i & \text{if } |\phi_j-\phi_i| < \pi, \\ \phi_j-\phi_i-2\pi & \text{if } \phi_j-\phi_i > +\pi, \end{cases}$$

Our topology **simplification** approach involves parameter dependent topology, where small parameter changes can cause changes in stable states. We use local bifurcations, such as pairwise annihilation of critical points and Hopf bifurcations, to reduce critical points while maintaining consistency.

The **proposed method** for preprocessing in this research involves computing the topological graph, which provides information for pairing critical points. The process starts with **identifying all critical points** in the grid and integrating the separatrices from each saddle point. The separatrices are checked to see if they leave the grid, reach a critical point, or form a closed orbit. Closed orbits are detected accurately using a separate scheme. If a critical point is reached, it is identified among the set of all critical points and marked as connected. If a closed orbit is reached, it is processed after the complete topology computation. Once all separatrices have been integrated, the method looks for sinks or sources that are not connected to any critical points and associates them with the separatrix surrounding the cycle that contains them, if any. The **pairing strategy** involves simplifying the topology of the field by removing pairs of first-order critical points of opposite indices, such as a saddle point and a source or sink, while maintaining consistency with the original structure. Criteria such as connectivity, Euclidean distance between critical points, length of the separatrix, degree of a critical point, vorticity, and magnitude of the vector field are considered to determine the relevance of critical points and to decide which pairs can be removed. The method uses a combination of threshold values for **Euclidean distance** and **vector field magnitude** to determine which critical points can be simplified concurrently. This preprocessing step reduces the number of singularities in the field and **simplifies** the resulting topology while **preserving** the important features of the original structure.

The **local deformation** step is crucial for simplifying vector fields by removing critical points. It involves deforming the field around singular points while maintaining the interpolation scheme and grid structure. Only vector values at grid vertices are modified. The process computes intersections of the line connecting critical points with triangulation edges. The closest grid vertex to the second critical point is included in a temporary list, forming a well-shaped deformation domain. Marked vertices are internal vertices, and a cell group includes all incident cells. New vector values for internal vertices are determined using angular constraints, considering angle coordinates at boundary vertices and field magnitude on exterior edges. The mean value of field magnitude on exterior edges, computed using linear interpolant, is used as the new vector value. Simplification may not be possible in some cases but can be mitigated by simplifying farther apart critical point pairs. Overall, this step ensures controlled vector field modification while preserving important features of the original structure.

<u>**Our method for finding new vector values for internal vertices with different types of constraints is as follows:**</u>

<u>**- Initialization:**</u>

For each internal vertex, we compute the interval of fixed constraints. If the interval is empty, we interrupt the process. If there are no fixed constraints, we set the interval to [0,2PI].

<u>**- Iterations:**</u>

We repeat the following steps until either we succeed or reach the maximum number of iterations (MAX_NB_ITERATIONS):

- For each internal vertex, we compute the mean vector of the incident vertices.

- If the interval of constraints is not empty and the mean vector is within the interval, we set the current value to the mean vector.

- If the interval of constraints is not empty and the mean vector is outside the interval, we set the current value to the best approximation of the mean vector within the interval.

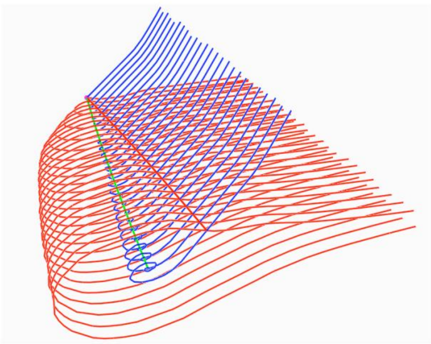
- If the interval of constraints is empty, we set succeeded to false.

- If the mean vector is within the fixed constraints, we set the current value to the mean vector.

- If the mean vector is outside the fixed constraints, we set the current value to the best approximation of the mean vector within the interval.

If any of the internal vertices have incompatible fixed constraints, the process is interrupted during initialization, and we move to the next pair. If the iterative process fails to determine compatible angular constraints for all internal vertices, we maintain the current pair and move to the next.

The continuous deformation of the vector field during the removal of critical points can be visualized by linearly interpolating the values of modified vertices between the original vector values and the modified values. This depicts the intermediate topology in the vicinity of the critical points as they merge and eventually disappear, as shown in figure below: -



Conclusion:

The method presented in the study simplifies the topology of turbulent planar vector fields while maintaining structural consistency with the original data. It involves successive local modifications of the vector field to remove pairs of critical points based on graphical and numerical criteria. The method can be seen as a continuous process that forces local bifurcations, resulting in the merging and annihilation of critical points with opposite indices. The algorithm was tested on a numerical dataset from a computational fluid dynamics (CFD) simulation, demonstrating its ability to filter numerical noise and remove small-scale structural features. This simplification clarifies the depiction of the topology and makes it easier to interpret.