SFE

Correlation:

X → #
Y → #

X → time spent in Social media
Y → grade

U → Study hrs Per day
V → grade

"direction":

same ↗↗, ↙↙ +ve | positive correlation

opp: ↗↙, ↙↗ −ve correlation

— — no correlation

Variable:

— Simple
— Partial
— Multiple

Z → BMI X → Height, Y → Weight, U →

$r_{xy} \rightarrow$ simple

$r_{x(x)}$

$r_{zy \cdot x} \rightarrow$ partial

$R_{z \cdot xy} \rightarrow$ multiple

Value of r : ρ , τ

$\rho \rightarrow$ spearman

$\tau \rightarrow$ kendall

$r \rightarrow$ Karl pearson

$$\boxed{-1 \le r \le 1}$$

$$\begin{cases} -1 \le r < 0 \longrightarrow -ve \\ 0 < r \le 1 \longrightarrow +ve \end{cases}$$

$\boxed{r = 0} \rightarrow$ no correlation

$r = 1 \rightarrow$ perfect +ve

$r = -1 \rightarrow$ " −ve

$\boxed{r_{xx} = 1}$ $\rightarrow$ $\boxed{r_{xy} = r_{yx}}$

$\boxed{x, y \rightarrow \text{independent}} \Rightarrow r_{xy} = 0$ ✓

Karl-Pearson correlation co-efficient:

X, Y

$$r = r_{XY} = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

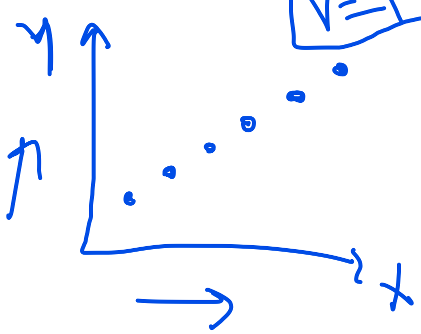
$$\boxed{Cov(X, Y) = E(XY) - E(X)E(Y)}$$

Discrete X, Y , $E(X) = \sum$

Continuous X, Y $E(X) = \int$

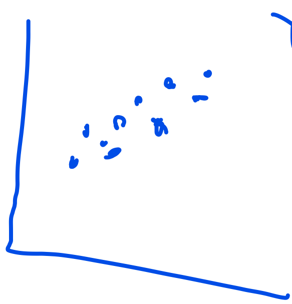
Scatter plot

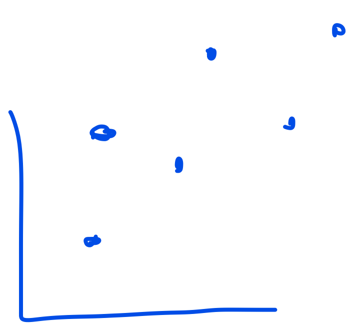
$\boxed{r = 1}$

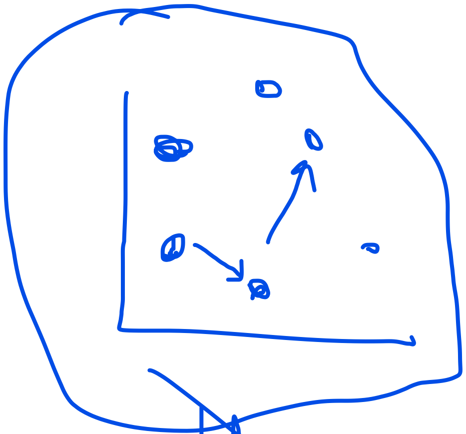


$\boxed{r = -1}$

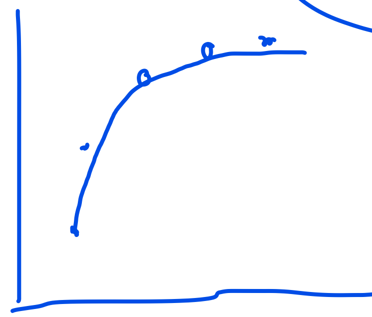
$r = 0.8$







$r = 0$



$r = 0.7$

$r = -0.5$

Karl Pearson's

$$r = \boxed{r_{xy} = \frac{cov(X,Y)}{\sigma_x \sigma_y}}$$

(x_1, y_1)
$\vdots$
(x_N, y_N)

N = # ordered pairs

$$= \frac{N\sum xy - \sum x \sum y}{\sqrt{\left[N\sum x^2 - (\sum x)^2\right]\left[N\sum y^2 - (\sum y)^2\right]}}$$

Eg: $N = 4$

x: 1 2 3 4

y: 5 6 7 8

$\sum x^2 = 1 + 2^2 + 3^2 + 4^2$

$(\sum x)^2 = (10)^2$

$\sum xy = 5 + 12 + 21 + 32$

Spearman's Rank correlation co-efficient:

$-1 \le \rho \le 1$ $\quad \rho_{uv}$

$$\boxed{\rho = \rho_{xy} = 1 - \frac{6\sum d^2}{n(n^2-1)}}$$

$n = \#$ ordered pairs.

(Note: Ranks No "tie")

$n = 5$

	5-1	5-2	5-3	5-4	5-5
X:	70	75	95	85	80
R_x:	5	4	1	2	3
Y:	60	65	100	75	80
R_y:	5	4	1	3	2
	d_1	d_2	d_3	d_4	d_5
$d_i = R_x - R_y$:	0	0	0	-1	1
$d_i^2 = d^2$:	0	0	0	1	1

(x_i, y_i) $\quad$ $n = 5$

$\sum d^2$

$$\sum d^2 = 2$$

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(2)}{5(5^2-1)}$$

$$\boxed{r = 0.4}$$

Modified Rank correlation co-efficient:

(Spearman's Rank correlation)

— This formula used for the Rank case.

$$\rho = 1 - \frac{6\left[\sum d_i^2 + \frac{1}{12}\underbrace{\sum m_j (m_j^2 - 1)}\right]}{n(n^2-1)}$$

(ie) $$\rho = 1 - \frac{6\left[\sum d_i^2 + \frac{1}{12}\sum (m_j^3 - m_j)\right]}{n(n^2-1)}$$

n = # ordered pairs

$d_i = R_x - R_y$

m_j = # jth the Rank

Eg:

1 2 3 4 ~~5 6~~ $\quad$ (4, 5) $\quad$ Avg (2, 3) = 2.5 $\quad$ 75, 75

Avg (4, 5, 6) = 5 $\quad$ 4 5 6

51 $\quad$ 55 $\quad$ 56 $\quad$ ~~70~~

X:	75	90	85	85	75	75
R_x:	5	1	2.5	2.5	5	5
Y:	70	90	80	72	73	71
R_y:	6	1	2	4	3	5
Z:	72	95	95	95	95	70
R_z:	5	2.5	2.5	2.5	2.5	6

$m_1 = 2$

$m_2 = 3$

$m_3 = 4$

Avg (1, 2, 3, 4) = 2.5 $\quad$ 5

$$\rho_{xy} = 1 - \frac{6\left[\sum d_i^2 + \frac{1}{12}\sum(m_j^3 - m_j)\right]}{n(n^2-1)}$$

$d = R_x - R_y$

$n = 6$

$m_1 = 2$, $m_2 = 3$.

~~$m_3 =$~~

$$\rho_{xy} = 1 - \frac{6\left[\sum d^2 + \frac{1}{12}\left\{(2^3-2) + (3^3-3)\right\}\right]}{n(n^2-1)}$$

x, y, z

ρ_{xz}

$\underline{\rho_{xy}}, \underline{\rho_{xz}}, \underline{\cancel{\rho_{yz}}}.$

$\rho_{yx}, \rho_{zx}, \rho_{zy}$

$m_1 = 2$ $\quad m_3 = 4$

$m_2 = 3$

$\cancel{m_3}$

$$\frac{1}{12}\left[(2^3 - 2) + (3^3 - 3) + (4^3 - 4)\right]$$

$\rho_{xz} = ??$

$m_1 = 2, \; m_2 = 3, \; m_3 =$

$$1 - \frac{6\left[\sum d^2 + \frac{1}{12}\sum(m_j^3 - m_j)\right]}{n(n^2 - 1)}$$

$\rho_{xy} = + 0.92$ please check it

0.7142 ??

$\rho_{yz} = 0.657$ please check it

$\rho_{xz} = 0.514$

x → the Rank

z → the Rank

Karl Pearson's correlation coefficients

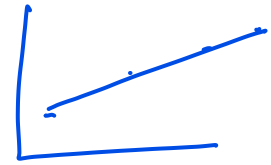
$r_{xy} = ?? = r_{yx}$

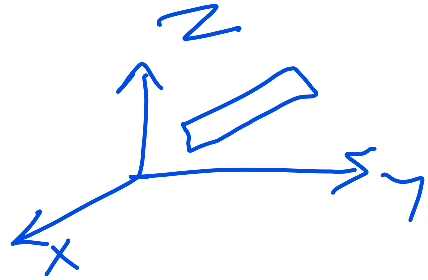
$r_{yz} = ?? = r_{zy}$

$r_{xz} = ?? = r_{zx}$

Multiple linear Regression Equation

$\# \; > 3$





Multiple linear

formula — least square method

$\underbrace{x_1, \; x_2,}$ x_3

x_1 on x_2 & x_3

x_2 on x_1 & x_3 $\quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

x_3 on x_1 & x_2

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $\quad C_{ij} = (-1)^{i+j} M_{ij}$

$r_{xy} = r_{yx}$

$$W = \begin{array}{c|ccc|} & x_1 & x_2 & x_3 \\ x_1 & 1 & r_{12} & r_{13} \\ x_2 & r_{21} & 1 & r_{23} \\ x_3 & r_{31} & r_{32} & 1 \end{array}$$

$$w_{23} = (-1)^{2+3} \begin{vmatrix} 1 & r_{12} \\ r_{31} & r_{32} \end{vmatrix}$$

$$w_{11} = (-1)^{1+1} \begin{vmatrix} 1 & r_{23} \\ r_{32} & 1 \end{vmatrix}$$

X_1 on x_2 and x_3 $\boxed{ax+by+cz=0}$

$$(x_1-\bar{x}_1)\frac{\omega_{11}}{\sigma_1} + (x_2-\bar{x}_2)\frac{\omega_{12}}{\sigma_2} + (x_3-\bar{x}_3)\frac{\omega_{13}}{\sigma_3} = 0$$

x_2 on x_3 and x_1

$$(x_1-\bar{x}_1)\frac{\omega_{21}}{\sigma_1} + (x_2-\bar{x}_2)\frac{\omega_{22}}{\sigma_2} + (x_3-\bar{x}_3)\frac{\omega_{23}}{\sigma_3} = 0$$

x_3 on x_1 and x_2

$$(x_1-\bar{x}_1)\frac{\omega_{31}}{\sigma_1} + (x_2-\bar{x}_2)\frac{\omega_{32}}{\sigma_2} + (x_3-\bar{x}_3)\frac{\omega_{33}}{\sigma_3} = 0$$

$\omega_{ij} = (-1)^{i+j}$ | (ω, deleting i row jth column)

$r_{12} = 0.6$ $\quad \sigma_1 \quad \sigma_2 \quad \sigma_3$

X_1 :

X_2 :

X_3 :

least square Method: (x_1, x_2, x_3)

x_1 on x_2 and x_3

$ax+by+cz=d$

$x = \frac{d}{a} - \frac{b}{a}y - \frac{c}{a}z$

$x = A + By + Cz$

$$X_1 = a + b x_2 + c x_3$$

Normal Eqns

$$\sum x_1 = na + b\sum x_2 + c\sum x_3$$

$$\sum x_1 x_2 = a\sum x_2 + b\sum x_2^2 + c\sum x_2 x_3$$

$$\sum x_1 x_3 = a\sum x_3 + b\sum x_2 x_3 + c\sum x_3^2$$

X_2 on x_3 & x_1

~~Regn~~ $X_2 = D + E x_3 + F X_1$

Normal eqns.

$$\sum x_2 = nD + E\sum x_3 + F\sum x_1$$

$$\sum x_2 x_1 = D\sum x_1 + E\sum x_3 x_1 + F\sum x_1^2$$

$$\sum x_2 x_3 = D\sum x_3 + E\sum x_3^2 + F\sum x_1 x_3$$

x_3 on x_1 & x_2

Regn: $X_3 = G + H X_1 + I X_2$

Normal Eqns:

$$\Sigma x_3 = nG + H \Sigma x_1 + I \Sigma x_2$$
$$\Sigma x_3 x_2 = G \Sigma x_2 + H \Sigma x_1 x_2 + I \Sigma x_2^2$$
$$\Sigma x_3 x_1 = G \Sigma x_1 + H \Sigma x_1^2 + I \Sigma x_1 x_2$$

x_1:	2	3	4	5	6
x_2:	6	8	4	3	3
x_3:	10	6	12	16	8

1) Find all regn. eqns.

2) find x_3 when $x_1 = 10$, $x_2 = 3$

x_1 on x_2 & x_3 $n = 5$

x_2 on x_3 & x_1

x_3 on x_1 & x_2

Soln.

$r_{12} = ?$ $r_{23} = ?$ $r_{13} = ?$

σ_1 σ_2 $\sigma_3 =$

$$x_3 = ax_1 + bx_2 + c$$

$$\boxed{x_3 = 10a + 3b + c} \longrightarrow *$$

$$\sum x_3 = 52, \quad \sum x_1 = 20, \quad \sum x_2 = 24$$

Normal

$$\sum x_3 = a\sum x_1 + b\sum x_2 + nc$$

$$52 = 20a + 24b + 5c \longrightarrow 1$$

$$\underbrace{\sum x_3 x_1} = a\sum x_1^2 + b\sum x_1 x_2 + c\sum x_1$$

$$214 = 90a + 85b + 20c \longrightarrow 2$$