



Digital Assignment – 2

1. (a) Find the moment generating function(MGF) of the distribution defined by $dF = \frac{1}{2}e^{-|x|}dx, -\infty < x < \infty$ and hence find its variance by using such MGF.

(b) Let X be a random variable with $E[X] = 1$, and $E[X(X-1)] = 4$. Find $\text{Var}(X)$ and $\text{Var}(2-3X)$.

2. An experiment consists of three independent tosses of a fair coin. Let X denotes the number of heads, Y denotes the number of head runs, Z denote the length of head runs, a head run being defined as consecutive occurrence of at least two heads, its length being the number of heads occurring together in three tosses of the coin. Find the probability function of

- (i) X ,
- (ii) Y ,
- (iii) Z ,
- (iv) $X+Y$,
- (v) XY and construct the probability table

3. Two statistically independent components X and Y are connected in a circuit.

The failure times of X and Y are given by $f(x, y) = 0.2e^{-(0.4x+0.5y)}, x, y \geq 0$.

Find the probability that the circuit fails in less than 3 years if

- (a) components are connected in parallel
- (b) components are connected in series.

4. In a certain city district, the need for money to buy drugs is stated as the reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district,

- (i) Exactly 2 resulted from the need for money to buy drugs.
- (ii) At most 3 resulted from the need for money to buy drugs.

5. The number of cars arriving per hour at a car service station is assumed to follow a Poisson distribution with a mean 5. Find the mean arrival rate in a 3 hour period and also the probability that more than 20 cars will arrive in a 3 hour period.

6. In an industrial process, the diameter of a ball bearing is an important measurement. The buyer sets specifications for the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standard deviation $\sigma = 0.005$. On average, how many manufactured ball bearings will be scrapped?

7. The life length X of an electronic component follows an exponential distribution. There are two processes by which the component may be manufactured. The expected life length of the component is 100h if process I is used to manufacture, while it is 150h if process II is used. The cost of manufacturing a single component by process I is Rs.10, while it is Rs.20 for process II. Moreover if the component lasts less than the guaranteed life of 200h, a loss of Rs.50 is to be borne by the manufacturer. Which process is advantageous to the manufacturer?