docker run — will start up a new container and run a process within that new container.

Run a process in a new container. docker run starts a process with its own file system, its own networking, and its own isolated process tree. The IMAGE which starts the process may define defaults related to the process that will be run in the container, the networking to expose, and more, but docker run gives final control to the operator or administrator who starts the container from the image. For that reason docker run has more options than any other Docker command.

docker exec — will execute a command in a container is already running.

- -i, --interactive[=false] Keep STDIN open even if not attached
- -t, --tty[=false] Allocate a pseudo-TTY

'run' starts a new container, 'exec' starts a new process inside an existing, running, container