Different Circuit Elements

(1) Resistance (R)

↳ Opposition offered by a substance to the flow of current in a circuit is called as it's resistance

⊙ A wire is said to have a resistance of 1Ω when a potential difference of 1V across it causes a current of 1A in it.

↳ factors on which the resistance depends :-

a) Temperature $(T\uparrow, R\uparrow)$
b) length of a conductor (l)
c) Cross-sectional area $(A\uparrow, R\downarrow)$
d) Specific Resistance / Resistivity (ρ)

⊙ Specific resistance is the resistance of the material with length(l) of 1m & area of cross-section (A) of $1\ m^2$.

$$\Rightarrow \boxed{R = \frac{\rho l}{A}}$$

(2) Conductance (G)

$G = \frac{1}{R}$ ∴ Unit is ohm^{-1} or mho or siemens

↳ Resistance in Series

$V = V_1 + V_2 + V_3$

$\Rightarrow IR_{eq} = IR_1 + IR_2 + IR_3$

$\Rightarrow \boxed{R_{eq} = R_1 + R_2 + R_3}$

Imp
⊙ Voltage Divider Rule

$V_1 = \frac{VR_1}{R_1 + R_2 + R_3}$; $V_2 = \frac{VR_2}{R_1 + R_2 + R_3}$; $V_3 = \frac{VR_3}{R_1 + R_2 + R_3}$

↳ Resistance in Parallel

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Imp
⊙ Current Divider Rule

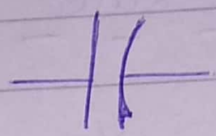
$I_1 = I \frac{R_1}{R_1 + R_2}$ & $I_2 = I \frac{R_2}{R_1 + R_2}$

Capacitor

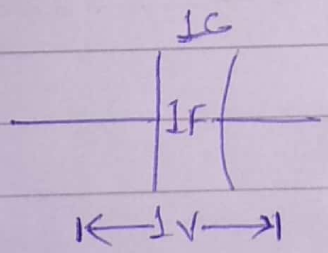
Two conducting plates separated by a dielectric medium to store charge is called a capacitor.

& Capacitance → It is the ability to store charge inside a capacitor.

Unit → farad (F)

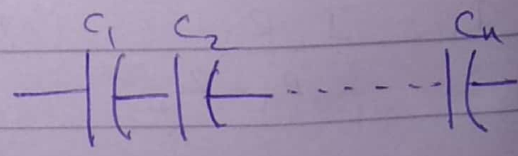
Symbol → 

↳ A capacitor is said to have capacitance of 1F if a charge of 1C accumulate on each plate when a potential difference of 1V is applied across it.



↳ Capacitance in series

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots\cdots \frac{1}{C_n}$$



↳ Capacitance in parallel

$$C_T = C_1 + C_2 + C_3 \cdots C_n$$

Inductor

It consists of a large number of turns having either a ferromagnetic core (or) air core.

& Inductance → It is the property of an inductor that opposes any sudden change in the amount of current flowing.

In series

$L_T = L_1 + L_2 + L_3 - - - L_n$

In parallel

Laws:-

OHM's Law

"The ratio of the potential difference across the ends of a conductor to the current flowing in it is constant provided the physical conditions should not change."

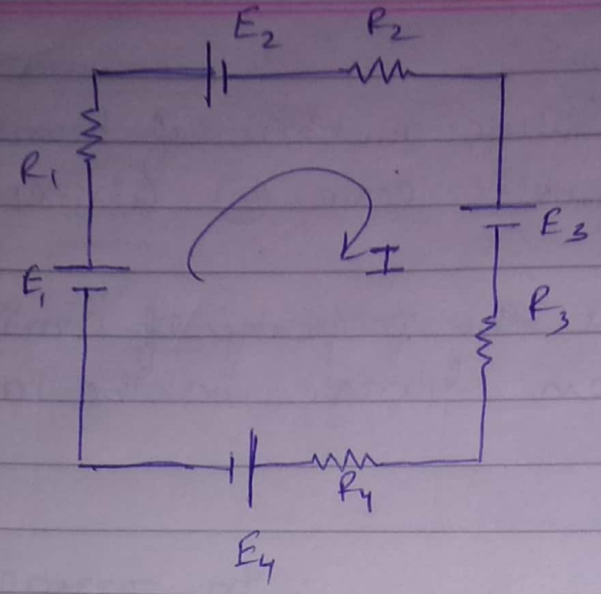
Kirchoff's Law

(i) Kirchoff's Current Law

The Algebraic sum of current flowing towards a junction in an electric circuit is 0 (zero).

(ii) Kirchoff's voltage Law

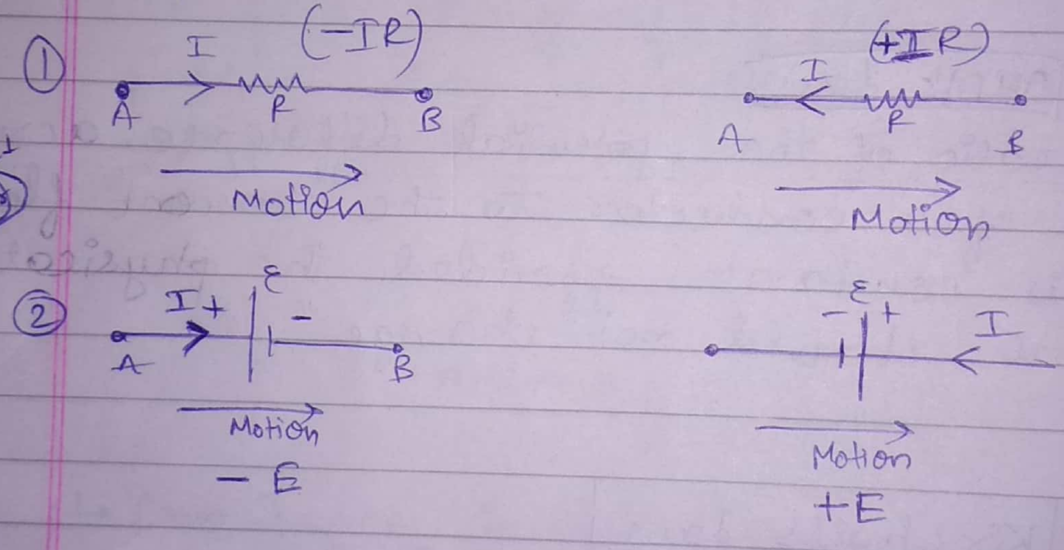
In a closed mesh or loop, the algebraic sum of all the emfs & the voltage drops is equal to zero (0).



V.V.Imp

Sign Convention

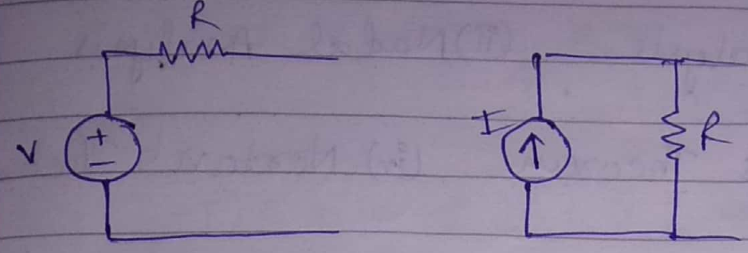
$\mu = \mu_0 \mu_r$



★ **Ideal voltage Source** → It is that ~~output~~ voltage source, it's output remains constant whatever be the load current.

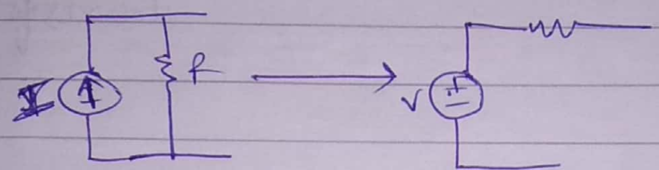
★ **Ideal Current Source** → It is that current source, whose internal resistance is infinite so that it supplies constant current in the loop.

SOURCE CONVERSION



$I = \frac{V}{R}$

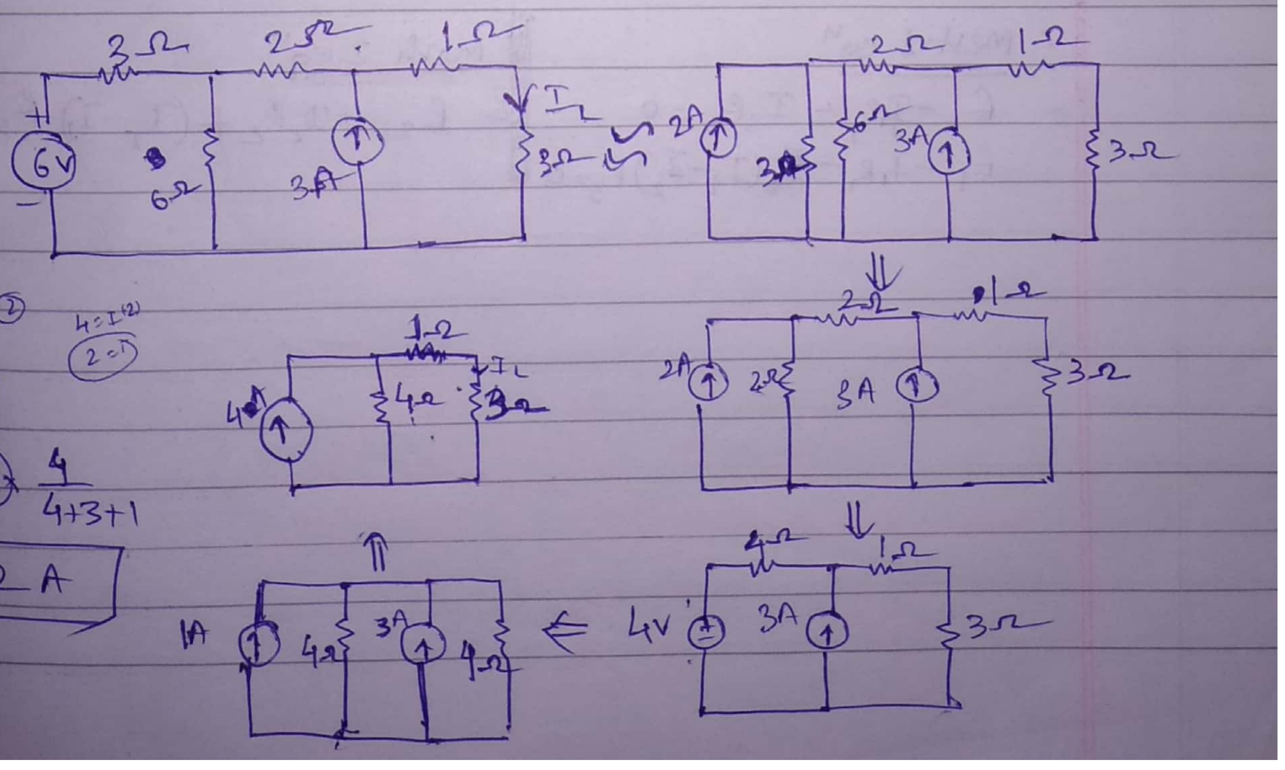
∴ for



for $\boxed{V = IR}$

↳ voltage source with a series resistance can be converted into an equivalent current source with parallel ~~source~~ resistance & vice versa.

Q Using source conversion find the current I_L.



$I_L = 4 \times \frac{4}{4+3+1}$

$\boxed{I_L = 2A}$

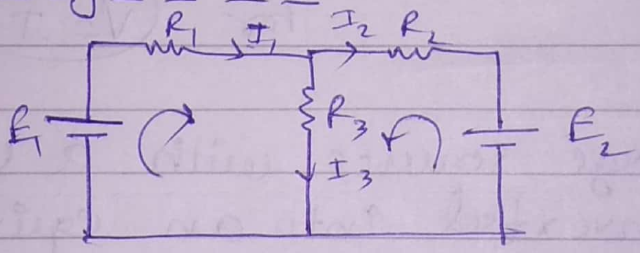
Analysis of Electric circuit

(i) Mesh Analysis (ii) Nodal Analysis

(iii) Thevenin's Theorem (iv) Norton's Theorem

(v) Superposition Theorem (vi) Star-delta & delta-star transformation.

Mesh Analysis :-



$I_3 \rightarrow I_1 - I_2$

Steps |

1) find the no. of mesh
2) write mesh eqⁿs by using KVL

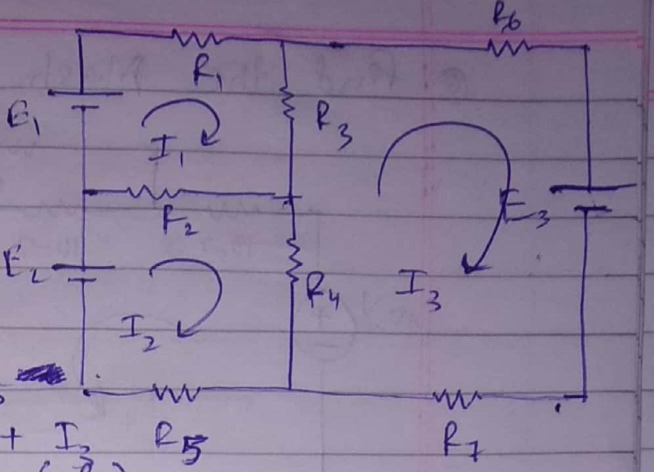
∴ Mesh 1 eqⁿ

$E_1 - I_1R_1 - I_3R_3 = 0$

$E_1 - I_1R_1 - (I_1 - I_2)R_3 = 0$

& Mesh 2 eqⁿ

$E_2 + I_2R_2 + (I_2 - I_1)R_3 = 0$



Mesh-1

$$E_1 = I_1R_1 + (I_1 - I_3)R_3 + \cancel{I_4}R_2 \quad (I_1 - I_2)$$
$$= I_1(R_1 + R_2 + R_3) - I_2R_2 - R_3I_3 \quad —(1)$$

Mesh-2

$$E_2 = (I_2 - I_1)R_2 + (I_2 - I_3)R_4 + I_2R_5$$
$$= I_1(-R_2) + I_2(R_2 + R_4 + R_5) + I_3(-R_4) \quad R_5 \quad —(2)$$

Mesh-3

$$-E_3 = -I_3R_7 - (I_3 - I_2)R_4 - (I_3 - I_1)R_3 - I_3R_6$$

$$-E_3 = I_1(R_3) + I_2(R_4) + I_3(-R_7 - R_4 - R_3 - R_6) \quad —(3)$$

$$\therefore \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

Here

R_1 = Self resistance of Mesh-1

R_2 = Self resistance of Mesh-2

⋮

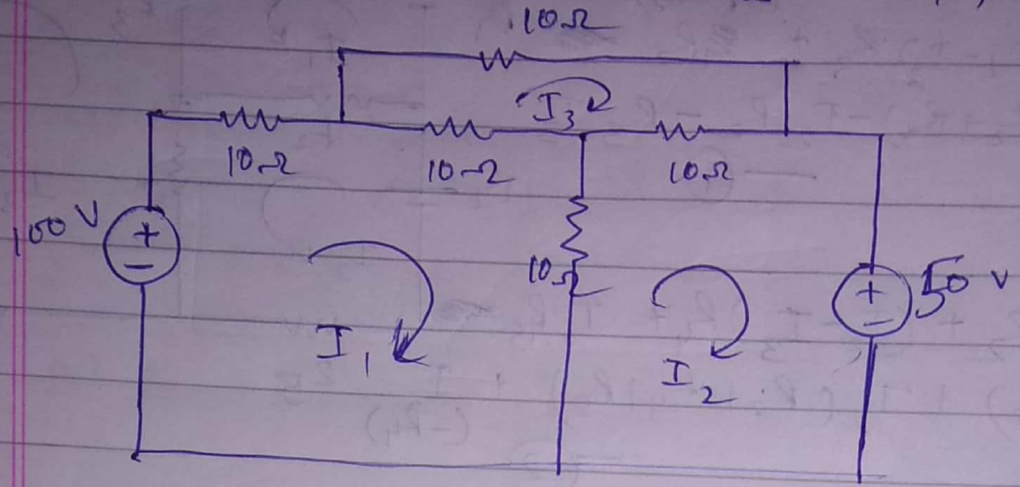
& $R_{12} = R_{21}$

$R_{23} = R_{32}$

⋮

$I_1 = \frac{\Delta_1}{\Delta}$ & $I_2 = \frac{\Delta_2}{\Delta}$ & $I_3 = \frac{\Delta_3}{\Delta}$

© find the Mesh currents I_1, I_2, I_3



Mesh 1

$100 - I_1 10 - (I_1 - I_3) 10 - (I_1 - I_2) 10 = 0$

$\Rightarrow I_1(-30) + I_2(10) + I_3(10) = -100$

$\Rightarrow 30 I_1 - 10 I_2 - 10 I_3 = 100$

Mesh 2

$-50 - (I_2 - I_1) 10 - (I_2 - I_3) 10 = 0$

$+10 I_1$

Mesh-3

$-10 I_3 - (I_3 - I_1) 10 - (I_3 - I_2) 10 = 0$

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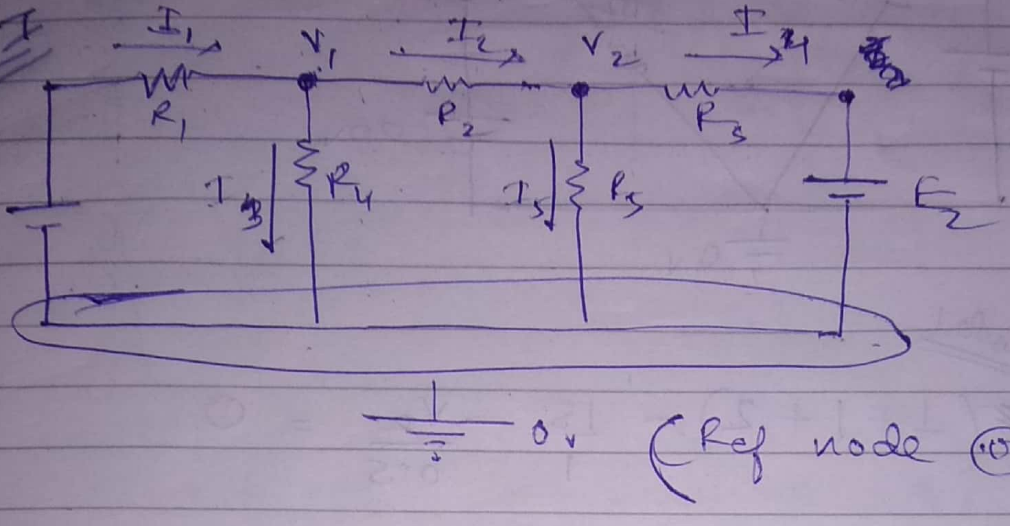
Nodal Analysis :-

Case I



0 V (Ref node or Datum node)

At node 1

$I_1 = I_2 + I_3$ — (1) ; $I_1 = \frac{E_1 - V_1}{R_1}$; $I_3 = \frac{V_1 - 0}{R_4} = \frac{V_1}{R_4}$

$I_2 = \frac{V_1 - V_2}{R_2}$

$\Rightarrow V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_2}{R_2} - \frac{E_1}{R_1}$ — (2)

At node 2

$I_2 = I_5 + I_4$ — (3) ; $I_2 = \frac{V_1 - V_2}{R_2}$; $I_4 = \frac{V_2 - E_2}{R_3}$

$I_5 = \frac{V_2 - 0}{R_5} = \frac{V_2}{R_5}$

$\Rightarrow V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_1}{R_2} - \frac{E_2}{R_3} = 0$ — (4)

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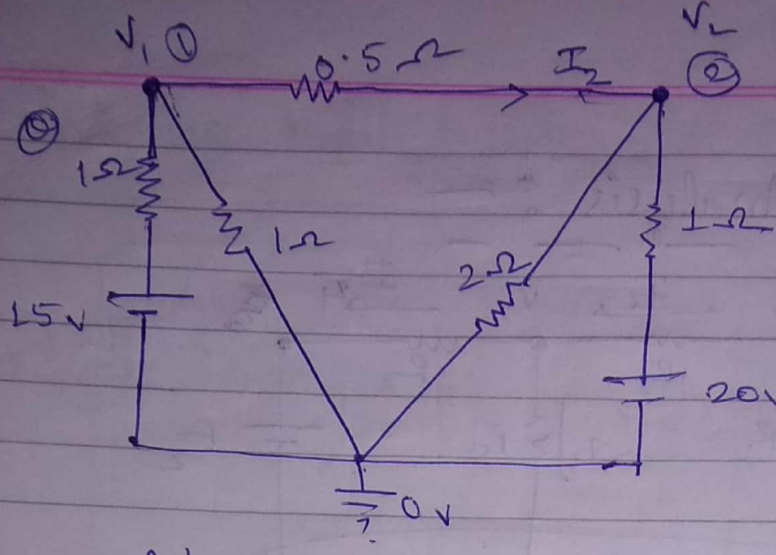


At node 1

$$V_1\left(1+1+2\right) - \frac{15}{1} - \frac{V_2}{0.5} = 0$$

$$4V_1 - 2V_2 = 15$$

At node 2

$$V_2\left(\frac{1}{1}+\frac{1}{0.5}+\frac{1}{2}\right) - \frac{V_1}{0.5} - \frac{20}{1} = 0$$

$$(3.5)V_2 - 2V_1 = 20$$

$$-2V_1 + (3.5)V_2 = 20 \;)\times 2$$

$\cancel{6V_1 - 4V_2 - 6V_1 + 10.5V_2 = 30+60}$

$\cancel{6.5V_2 = 90}$

$\cancel{V_2 = \frac{900}{65}}$

$\cancel{3V_1 - 2\times\frac{900}{65}}$

$$\cancel{4V_1} - 2V_2 - \cancel{4V_1} + 7V_2 = 15+40$$

$$5V_2 = 55$$

$$V_2 = 11\text{ V}$$

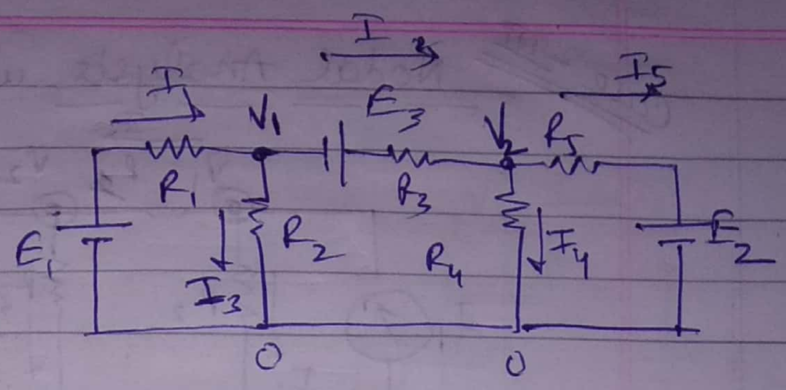
& $4(V_1) - 2(11) = 15$

$$V_1 = \frac{37}{4}\text{ V}$$

Nodal Analysis :-

Case - II

When a battery is present in b/w 2 nodes.



At node - 1

$$I_1 = I_3 + I_2$$

$$I_1 = \frac{E_1 - V_1}{R_1} \; ; \; I_3 = \frac{V_1 - 0}{R_2} \; ; \; I_2 = \frac{V_1 + E_3 - V_2}{R_3}$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{E_1}{R_1} + \frac{E_3}{R_3} - \frac{V_2}{R_3} = 0$$

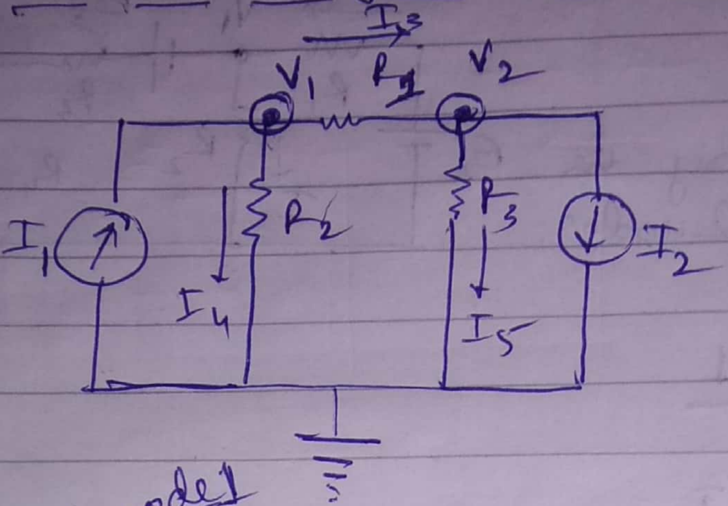
At node - 2

$$I_2 = I_4 + I_5$$

$$I_2 = \frac{V_1 + E_3 - V_2}{R_3} \; ; \; I_4 = \frac{V_2 - 0}{R_4} \; ; \; I_5 = \frac{V_2 - E_2}{R_5}$$

$$V_2\left(\frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_4}\right) - \frac{V_1}{R_3} - \frac{E_2}{R_5} - \frac{E_3}{R_3} = 0$$

Case - III Nodal Analysis with current source:



At node 1

$$I_1 = I_4 + I_3 \quad \text{—— (1)}$$

$$I_4 = \frac{V_1 - 0}{R_2} = \frac{V_1}{R_2}$$

$$\& \; I_3 = \frac{V_1 - V_2}{R_1}$$

$$I_1 = \frac{V_1}{R_2} + \frac{V_1}{R_1} - \frac{V_2}{R_1}$$

$$= V_1\left(\frac{1}{R_2} + \frac{1}{R_1}\right) - \frac{V_2}{R_1} \quad \text{—— (2)}$$

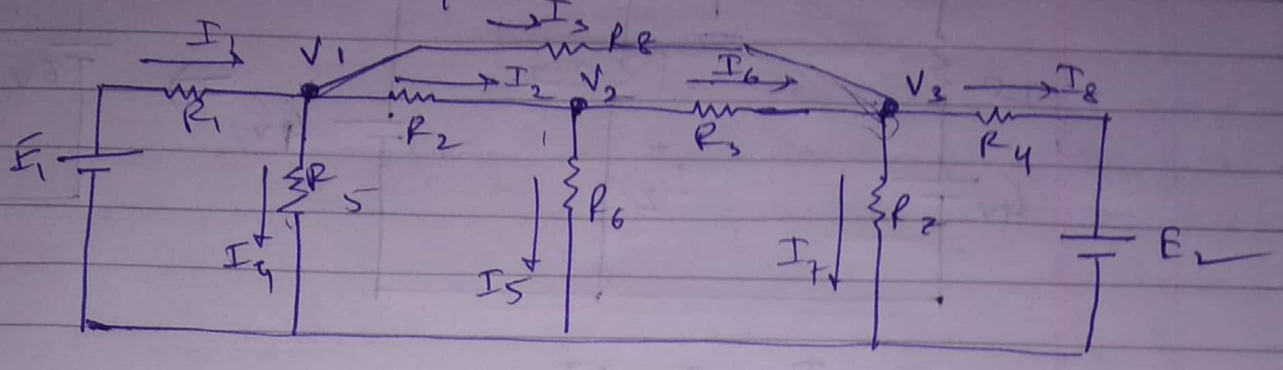
At node 2

$$I_3 = I_2 + I_5 \quad \text{—— (3)}$$

$$I_3 = \frac{V_1 - V_2}{R_1} \; ; \; I_5 = \frac{V_2 - 0}{R_3}$$

$$V_2\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - \frac{V_1}{R_1} = -I_2 \quad \text{—— (4)}$$

⑥ Write the node equations.



At node 1

$$V_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_8}\right)-\frac{E_1}{R_1}-\frac{V_2}{R_2}-\frac{V_3}{R_8}=0$$

At node 2

$$V_2\left(\frac{1}{R_2}+\frac{1}{R_6}+\frac{1}{R_3}\right)-\frac{V_1}{R_R}-\frac{V_3}{R_3}=0$$

At node 3

$$V_3\left(\frac{1}{R_5}+\frac{1}{R_8}+\frac{1}{R_7}+\frac{1}{R_4}\right)-\frac{E_2}{R_4}-\frac{V_2}{R_3}-\frac{V_1}{R_8}=0$$

$$I_3+I_6=I_7+I_8$$

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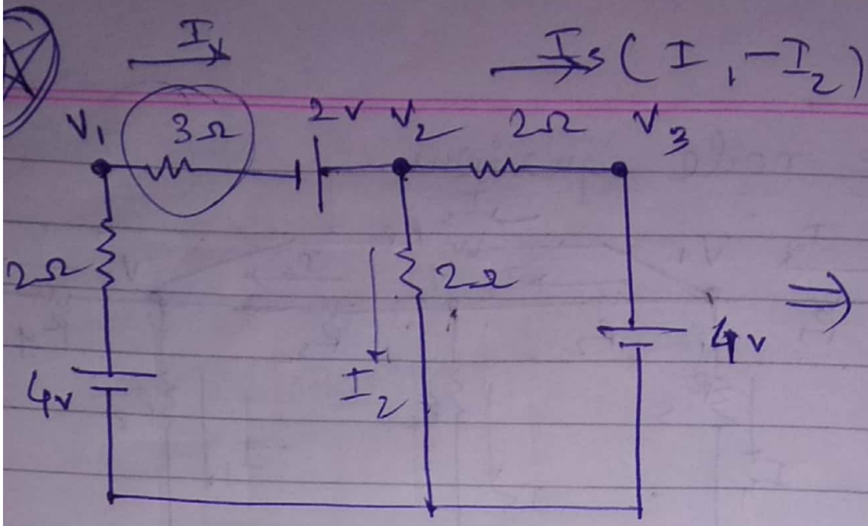

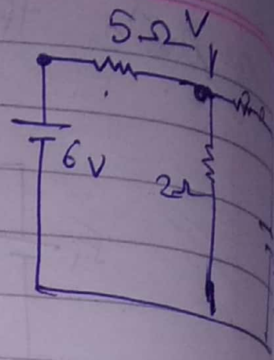

$I_1 = I_2 + I_3$

$\cancel{\frac{V_1+2-V_2}{3} = \frac{V_2-0}{2}} +$

$\cancel{I_1 = I_2 + I_1 - I_2}$

$I_1 = I_2 + I_3$

$\cancel{V_1\left(\frac{1}{3}+\frac{1}{2}\right) + \frac{2}{3} - \frac{4}{2} - \frac{V_2}{3} = 0}$

$V_1\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right) - \left(\frac{4+2}{5}\right) - \frac{4}{2}$

$V_1 = \frac{8}{3}$

$I_{3\Omega} = 0.67A$

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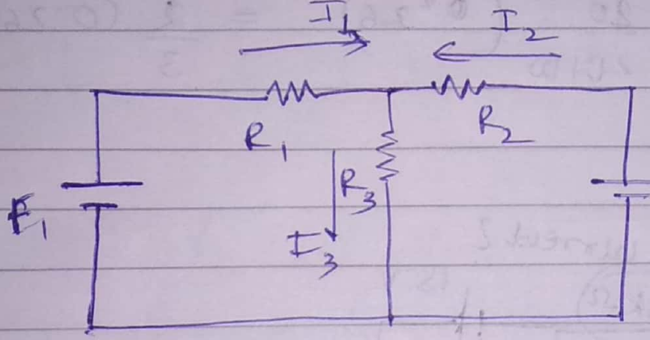
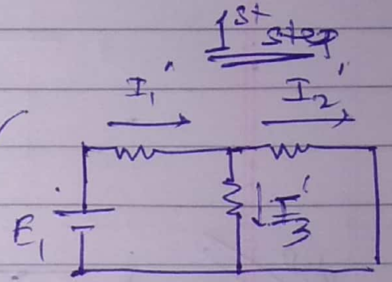
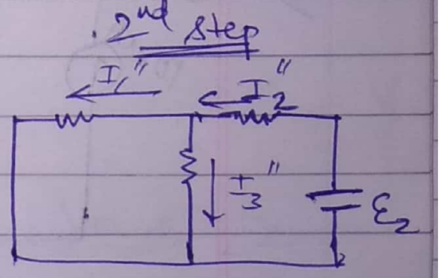
V.IMP

⊛ Superposition Theorem

In a network containing more than one source of emf, the current which flows at any time is the sum of all the currents which would flow at that point if each emf sources were considered separately & all other emf sources replaced for the time being by resistances equal to their internal resistance.







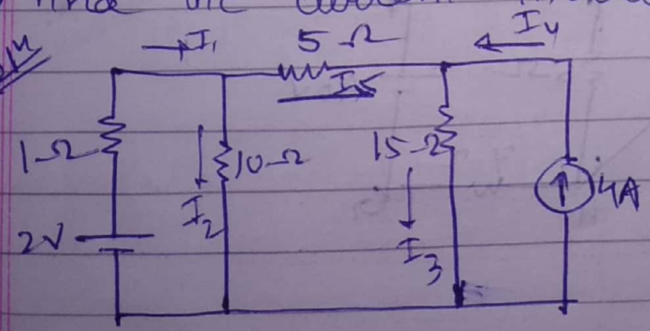
$\Rightarrow I_1 = I_1' - I_1''$

$I_2 = I_2'' - I_2'$

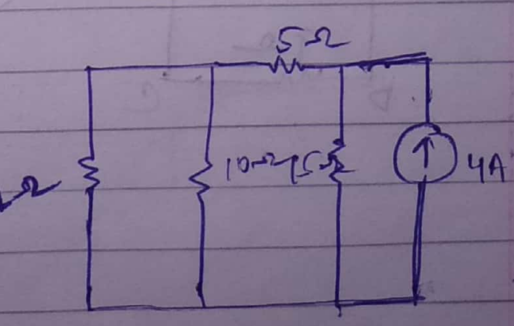
$I_3 = I_3' + I_3''$.

⊙ Find the current through 10 Ω rheostat.

Sol^n



Step 1 :-



$\frac{1}{1} + \frac{1}{10}$

$\frac{11}{10}$ $\frac{10}{11} + 5$

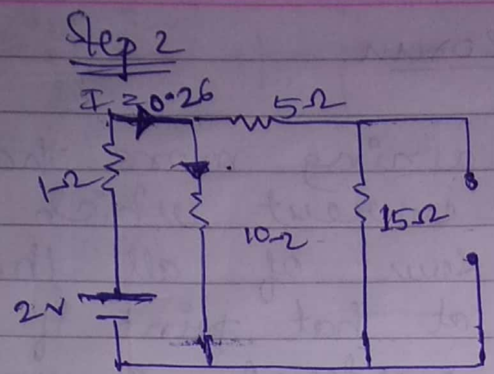
$\frac{10+55}{11}$

$\frac{65}{11}$

$\frac{11}{65} + \frac{1}{15}$

65

Step 2



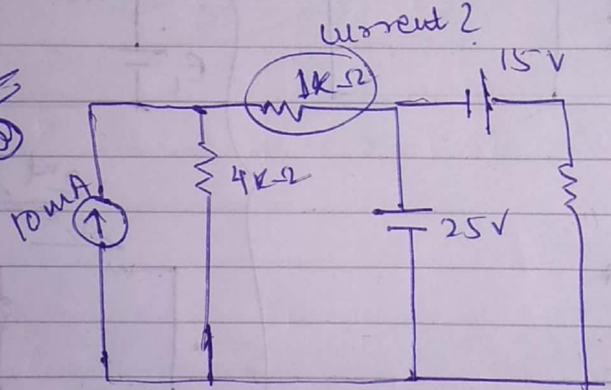
$R_{eq} = \frac{23}{3}$ & $I = \frac{2}{23} \times 3 = 0.26\text{ A}$

$\therefore I_{10\Omega} = \frac{10}{20+10}(0.26) = \frac{2}{3}(0.26) = 0.0866$

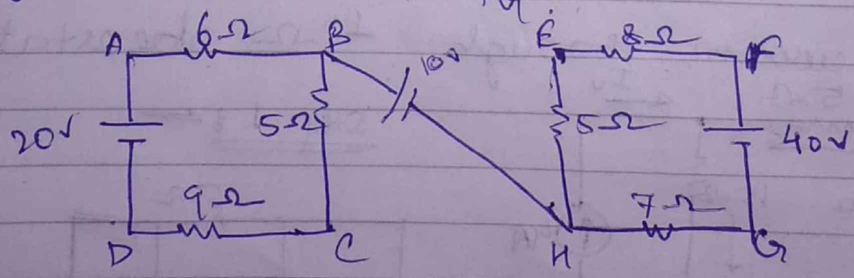
$= 0.1732$

HW

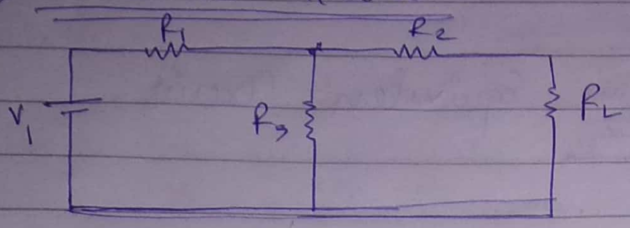
(a)



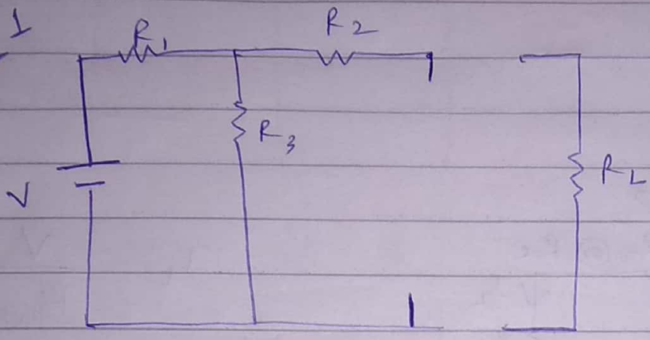
(b) find V_{CE} & V_{AG}



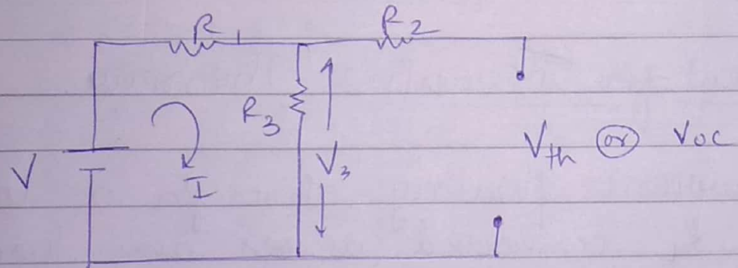
Thevenin's Theorem



Step 1



Step 2 Find V_{th} or V_{oc}



$\therefore V_{th}$ or $V_{oc} = V_3$

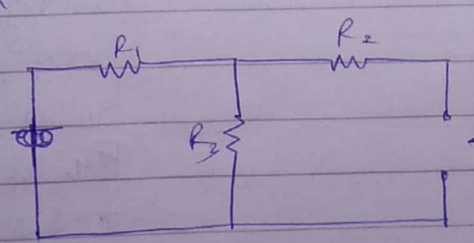
$\Rightarrow I = \left(\frac{V}{R_1+R_3}\right)$

$\Rightarrow V_{th} = I R_3$

& get V_{th}.

Step 3 find R_{th}

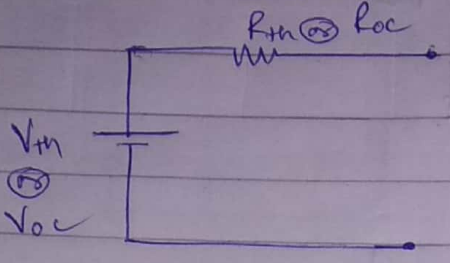
Here;



$R_{eq} = R_{th} = \frac{R_1 R_3}{R_1+R_3} + R_2$

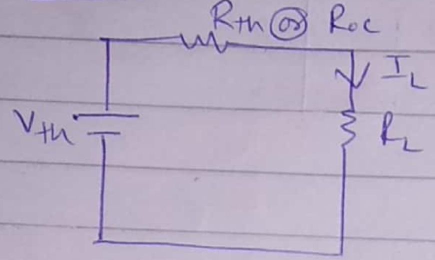
Step IV

Draw Thevenin's Equivalent Circuit.



Step V

Insert R_L



$$I_L = \frac{V}{R_{th}+R_L}$$

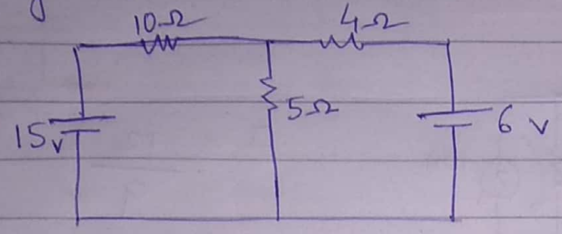
↳ Statement for Thevenin's Theorem

"The current flowing through a load resistance R_L connected across any two terminals 'A' & 'B' of a linear active network is given by

$$I_L = \frac{V_{th}}{R_{th}+R_L} \text{ ; where}$$

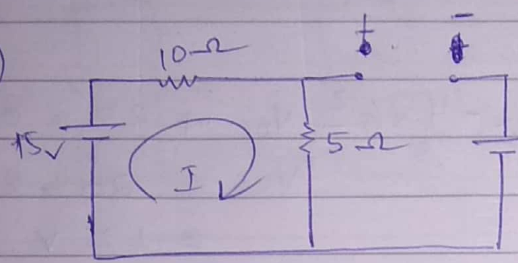
↳ V_{th} or V_{oc} ⇒ open circuit voltage across the 2 terminals from where R_L is removed and R_{th} is the Thevenin's resistance, which is the resistance of the network as viewed back into the open circuited network from terminals 'A' & 'B' with all voltage sources replaced by their internal resistance.

© Find the current in 4 ohm resistance by using Thevenin's Theorem.



Sol^n

1)



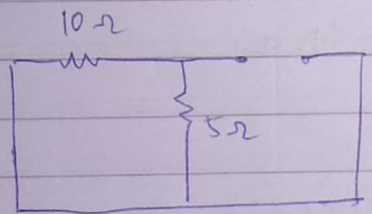
$V_{th} + 6 = 5$

or

$-V_{oc} - 6 = -5$

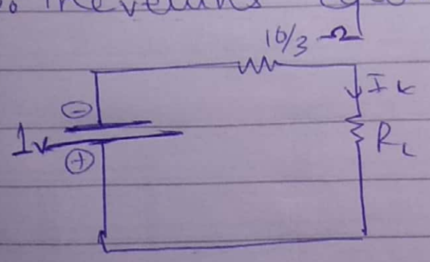
$\Rightarrow V_{oc} = -1\ V$

$\frac{15}{15} = 1\ A$



$R_{th} = \frac{50}{15} = \frac{10}{3}\ \Omega$

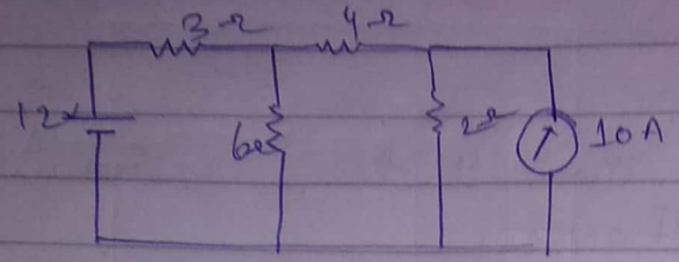
∴ Thevenin's Equivalent circuit

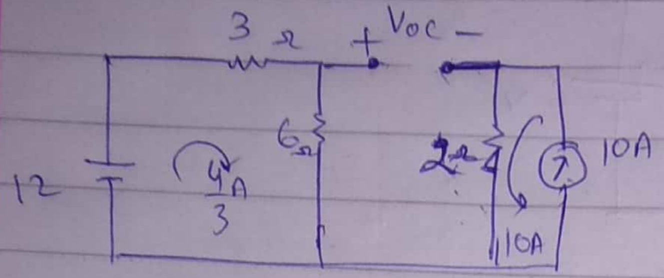


$I_L = \frac{-1}{\frac{10}{3} + 4}$

$= -\frac{3}{22} = -0.136\ A$

(8) Find current in 4 Ω resistor.



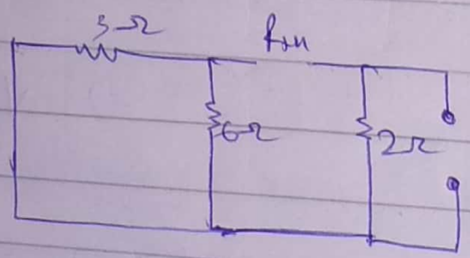


$$I = \frac{12}{9} = \frac{4}{3}\ A$$

$$-\frac{4}{3} \times 6 - V_{oc} + 20 = 0$$

$$V_{oc} = 20 - 8$$

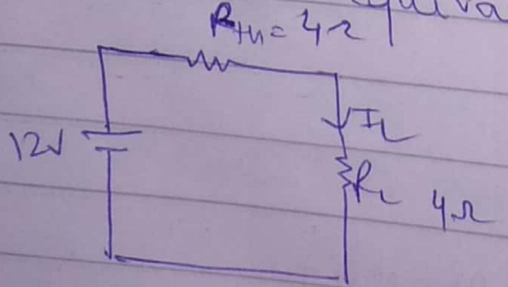
$$= 12\ V$$



$$R_{th} = (3 \| 6) + 2$$

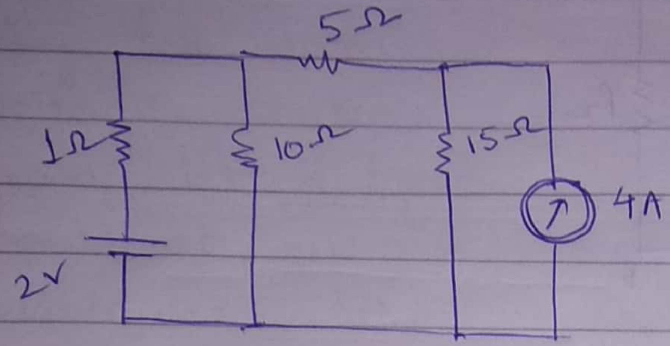
$$= 4\ \Omega$$

∴ Thevenins Equivalent circuit



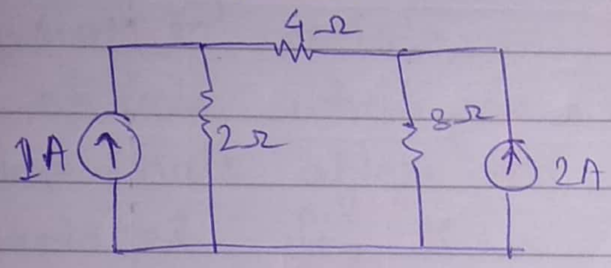
$$I_L = \frac{12}{8} = \frac{3}{2} = 1.5A$$

Q) find the current in 10 Ω resistor
By superposition thm.



Q)

current



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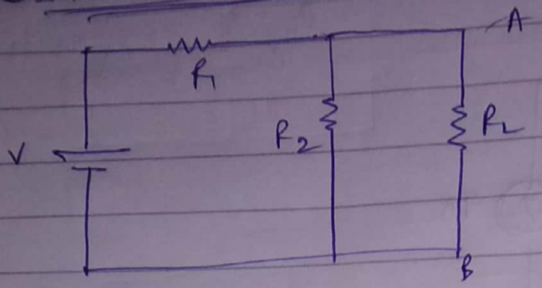
V. Imp

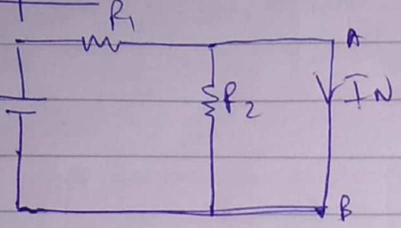
Norton's Theorem



Step (1)

Remove R_L

Step (2)

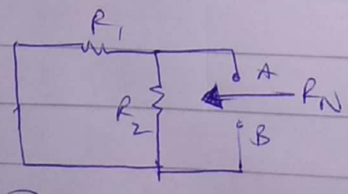


Step (3)

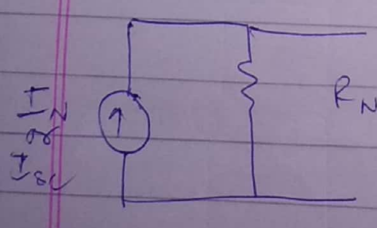
find I_N or I_{sc} $\quad I_N = \frac{V}{R_1}$

Step (4)

Find $R_N = R_{th}$

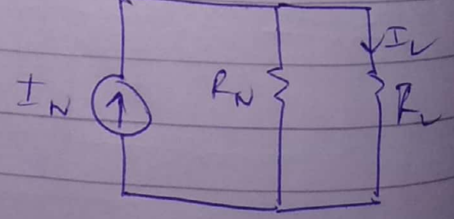


Step (5)



$R_N = R_1 \parallel R_2$

Step (6)

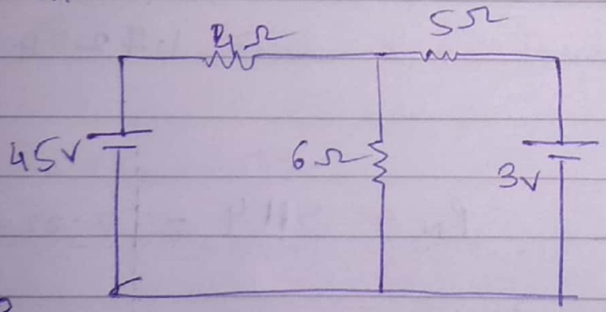


$$I_L = I_N \times \frac{R_N}{R_L + R_N}$$

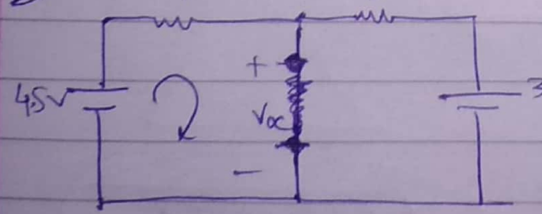
Statement

"Any two terminal active network containing voltage sources & resistances when viewed from it's output terminals is equivalent to a constant current source with a parallel resistance. The constant current is equal to the current which would flow in a short circuit path placed across the terminals & parallel resistance is the resistance of the network when viewed from these open circuited terminals after all voltage & current sources replaced by their internal resistance".

Q) Find current in 6Ω resistor using Norton's Thm.



Theve
1st



$4.5 - 4I - 5I - 3 = 0$

$9I = 1.5$

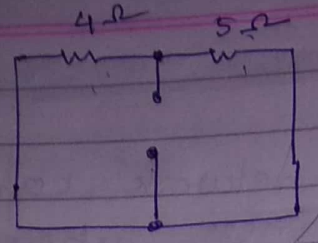
$I = 0.167$

$\therefore \; 4.5 - 4 \times I - V_{oc} = 0$

$V_{oc} = 4.5 - 4 \times 0.167$

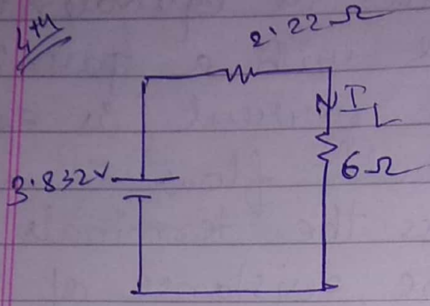
$= 4.5 - 0.668$

$= 3.832$ ✓



$R_{th} = 4 \| 5 = 2.22\ \Omega$

4th



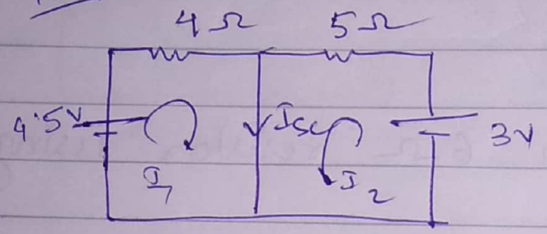
$3.83 - 2.22I - 6I = 0$

$I = \frac{3.83}{8.22}$

$= 0.466\ A$

Norton

1st

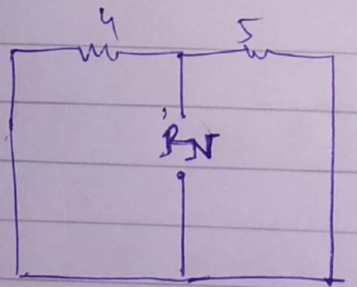


$I_{sc} = I_1 + I_2$

$= \frac{4.5}{4} + \frac{3}{5}$

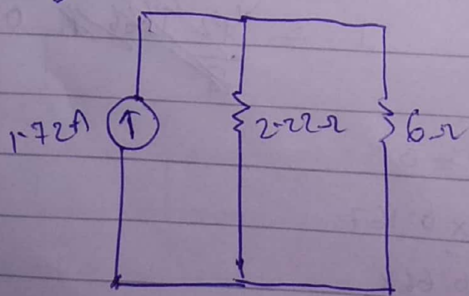
$= 1.72\ A$

2nd



$R_N = 5 \| 4 = 2.22\ \Omega$

3rd



$\frac{2.22}{6 + 2.22} \times 1.72$

$\frac{2.22}{8.22} \times 1.72$

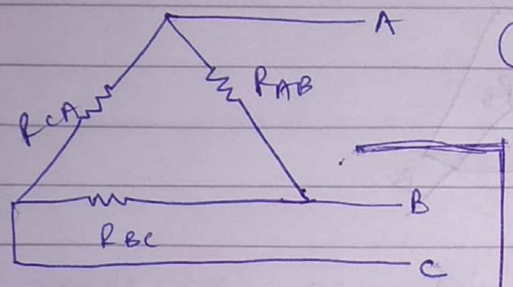
$= 0.464$

Star/Delta & Delta/Star transformations

In some cases resistances are neither connected in series nor connected in parallel. In those cases the resistances may be connected in star (or) Delta.

∴ for simplification we either convert star – delta (or) from delta – star.

$\underline{\Delta}$

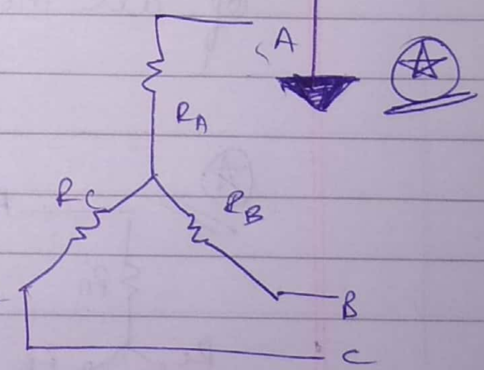


(1) Resistance b/w terminal A&B in Δ

$$= R_{AB} \parallel (R_{CA} + R_{BC})$$

$$= \frac{R_{AB} \times (R_{CA} + R_{BC})}{R_{AB} + R_{CA} + R_{BC}} \quad \text{— (1)}$$

(2) Resistance b/w terminal A & B in (★)



$= R_A + R_B$ — (2)

So;

$$R_A + R_B = \frac{R_{AB} \times (R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \quad \text{— (3)}$$

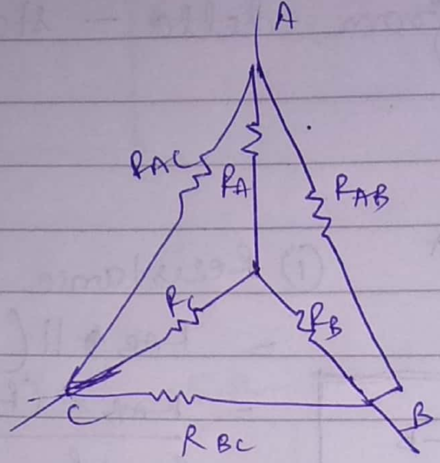
$$R_B + R_C = \frac{R_{BC} \times (R_{CA} + R_{AB})}{R_{BC} + R_{CA} + R_{AB}} \quad \text{— (4)}$$

$$R_C + R_A = \frac{R_{CA} \times (R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad \text{— (5)}$$

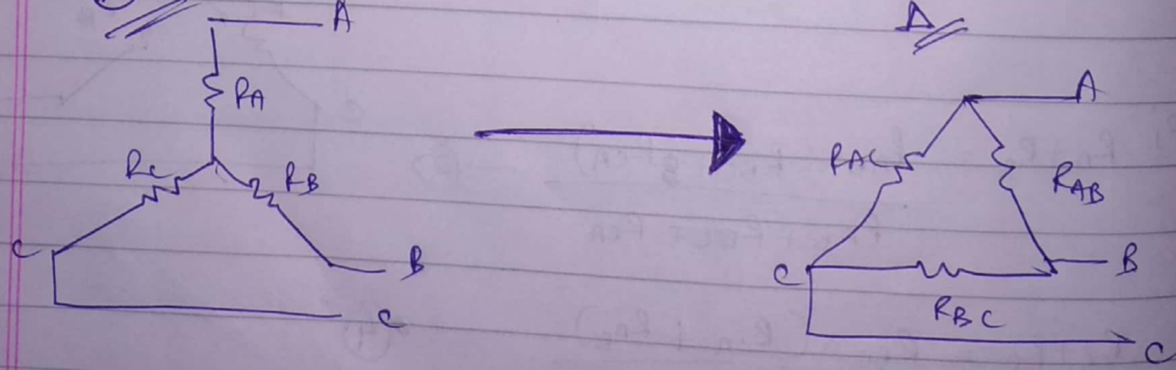
Sub eqn (4) from (3) & add it to (5)

$$R_A = \frac{R_{AB} \cdot R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \quad (6) \; ; \quad R_B = \frac{R_{AB} \cdot R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \; ; \quad (7)$$

$$R_C = \frac{R_{AC} \cdot R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \quad (8)$$



∴ (★)→ An arm's resistance in star pattern is equal to the product of the adjacent arm's resistance, divided by the sum of all the arm's resistances in the Δ pattern.



Dividing eqn (6) by (7)

$$\frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}} \Rightarrow R_{CA} = \frac{R_A \times R_{BC}}{R_B}$$

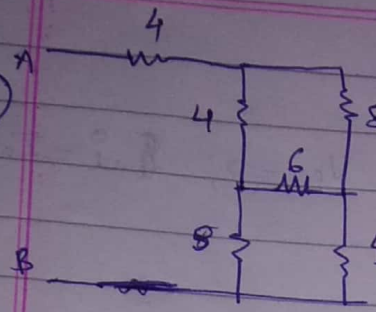
$\Rightarrow R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$

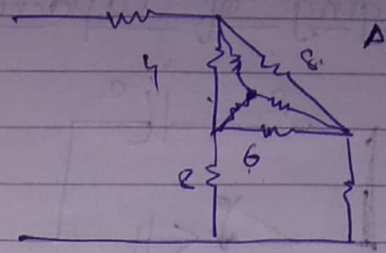
$R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B}$

$R_{AB} = R_A + R_B + \frac{R_A \cdot R_B}{R_C}$

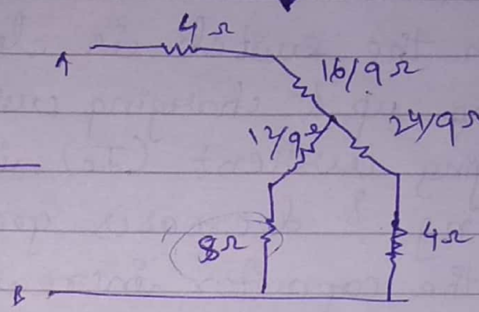
∴ ☆ → Resistance b/w 2 terminals of Δ is equal to (sum of star resistances connected to those terminals) + (product of the same two resistances) divided by the 3rd star resistance.

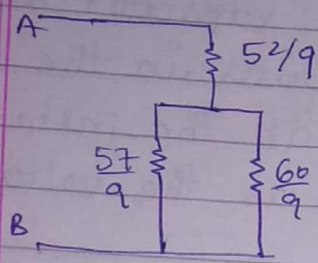
(1)





$\frac{4\times 8}{9\;18}$





$\frac{16+36}{9}$

$\frac{52}{9}$

$\frac{12+8}{9}$ $\quad \frac{72+12}{9}$ $\quad \frac{9}{57}+\frac{9}{60}$

45

$\frac{12}{57}$ $\quad \frac{84}{9}$ $\quad 9\left(\frac{84+60}{3420}\right)$

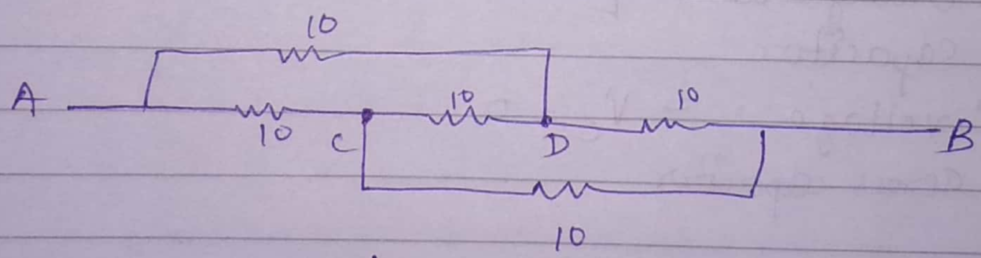
$9(0.017+0.166)$

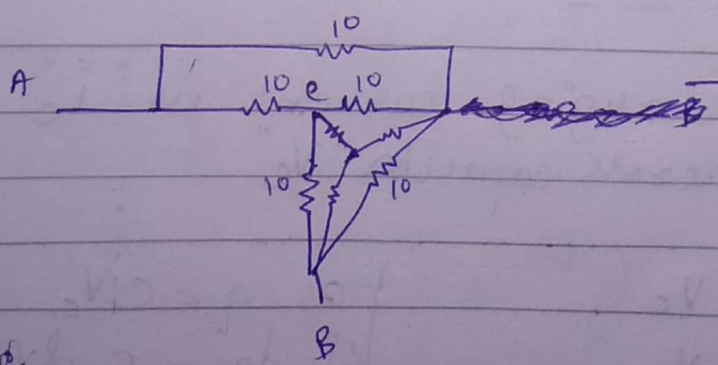
$\frac{87}{9}$

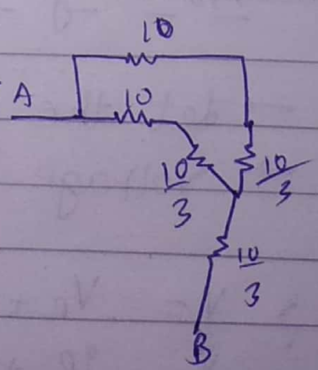
$\frac{24}{9}+4$

$\frac{24+36}{9}$

(2)

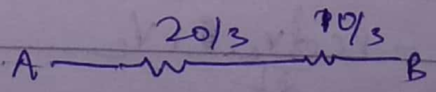


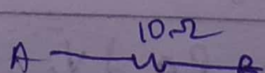




$\frac{40}{3}$

$\frac{100}{300}$





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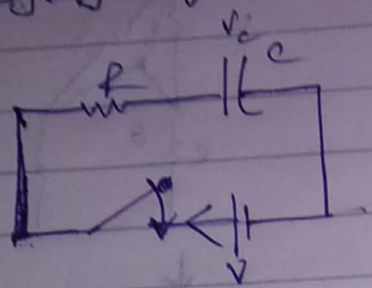
V.V. Imp

Charging of Capacitor



$q = 0;\ V_c = 0;\ i = I_{max}$

↳ When the switch is closed, the capacitor starts charging up & charging current flows in the ckt.
↳ Charging current (I_c) is max at the instant of switching & decreases gradually as the voltage across the capacitor increases.
↳ When capacitor voltage $= V$, $I_c = 0$

At Switching instant

→ current = max^m ; $I_{max} = \frac{V}{R}$

→ Charge in capacitor $= q = 0$
→ voltage across capacitor $= V_c = 0$

At any instant

→ Let the charging current is "i_c"
→ voltage across capacitor "V_c"

$V = V_R + V_c$
$V = iR + V_c$
$V = RC\frac{dV_c}{dt} + V_c$

as $q = CV_c$
$i = \frac{dq}{dt} = C\frac{dV_c}{dt}$

$V = V_c + RC\frac{dV_c}{dt}$

$\Rightarrow \dfrac{dV_c}{dt} = \dfrac{V - V_c}{RC}$

$\dfrac{dV_c}{V - V_c} = \dfrac{dt}{RC}$

$\int_0 \dfrac{-dV_c}{V_0 - V_c} = \int -\dfrac{dt}{RC}$ (Multiplying (−1) on both sides & integrating)

$\ln|V - V_c| = -\dfrac{t}{RC} + K$

& at $t = 0$; $V_c = 0$

$\ln V = K$

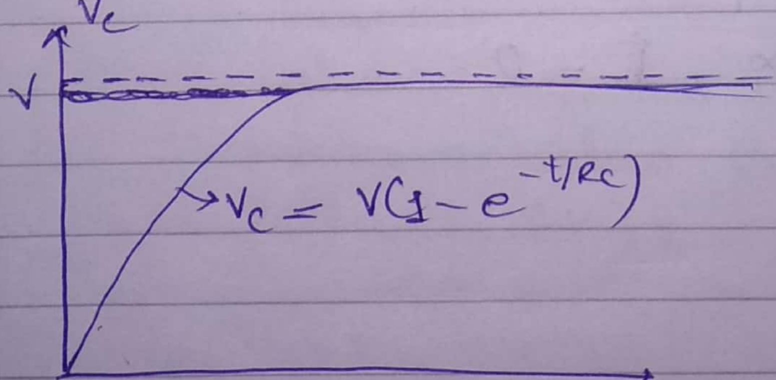
$\Rightarrow \ln|V - V_c| = -\dfrac{t}{RC} + \ln V$

$\ln\left|\dfrac{V - V_c}{V}\right| = -\dfrac{t}{RC}$

$\dfrac{V - V_c}{V} = e^{-t/RC}$

$V - V_c = Ve^{-t/RC}$

$\boxed{V_c = V(1 - e^{-t/RC})}$



& $q = Q(1 - e^{-t/RC})$

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Time Constant (λ)

Time constant is the time required for the capacitor voltage to reach to 0.632 times or 63.2% of it's steady value (or supplied voltage) "V".

$$V_c = V(1-e^{-t/RC})$$

$$t = \lambda = RC$$

$$\Rightarrow V_c = V(1-e^{-1})$$

$$\boxed{V_c = 0.632\,V}$$

Current in the capacitor (I_c)

$$V = V_c + IR$$

$$V - V_c = Ve^{-t/RC} = IR$$

$$\Rightarrow I = \frac{Ve^{-t/RC}}{R}$$

$$\boxed{I = I_{max}\, e^{-t/RC}}$$

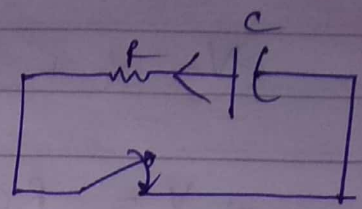
$t=0$; $I = I_{max}$

$t=\infty$, $I = 0$.

Discharging of a Capacitor :—

$t = 0$; $I = I_{max}$; $V_c = V$

$t = \infty$; $I = 0$; $V_c = 0$



At any instant

$$IR + V_c = 0$$

$$RC\frac{dv_c}{dt} = -V_c$$

$$\frac{dv_c}{V_c} = -\frac{dt}{RC}$$

$$\ln v_c = -\frac{t}{RC} + k$$

at $t = 0$; $V_c = V$

$$k = \ln V$$

$$\Rightarrow \ln V_c = -\frac{t}{RC} + \ln V$$

$$\ln \frac{V_c}{V} = -\frac{t}{RC}$$

$$\boxed{V_c = Ve^{-t/RC}}$$

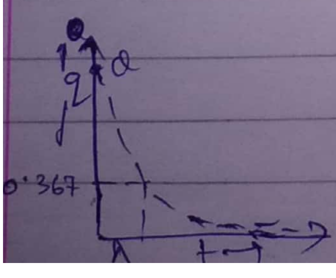
Time constant

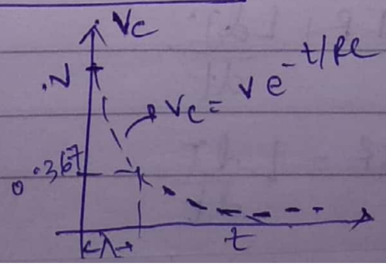
It is the time taken by the capacitor to discharge from maximum value to 36.7% or 0.367 of the max voltage (V).

$$t = \lambda = RC$$

$$V_c = Ve^{-1}$$

$$\boxed{V_c = 0.367\ V}$$





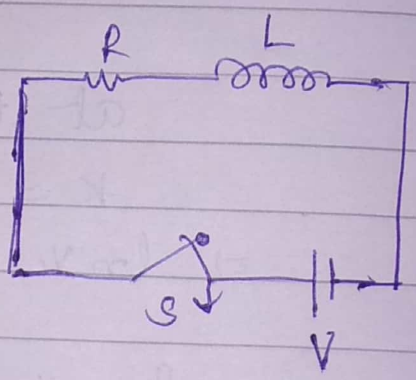
Rate of rise of voltage in the capacitor

$V = V_c + RC\frac{dV_c}{dt}$; when $i=0$, $V_c=0$.

$\Rightarrow V = RC\frac{dV_c}{dt}$ $\Rightarrow \boxed{\frac{dV_c}{dt} = \frac{V}{RC}}$

Closing of an inductive circuit

When the switch is closed at $t=0$, due to the presence of inductor, the current in the circuit will be zero



$V = V_R + V_L$

$V = IR + V_L$

If $I=0$; $V_R = 0$

$\Rightarrow \boxed{V = V_L}$

$V_L = L\frac{di}{dt} = V$

$\therefore$ as $i\uparrow$; $V_R\uparrow$; $V_L\downarrow$; $\frac{di}{dt}\downarrow$ (as L = const.)

- Consider that the current rises from zero (0) to $I = I_{max} = \frac{V}{R}$ in a small time "t".
- At any instant the current is increasing at the rate $\frac{di}{dt}$

$V = V_R + V_L$

$V = IR + L\frac{dI}{dt}$

$V - IR = L\frac{dI}{dt}$

$$\Rightarrow \int \frac{dI}{V-IR} = \int \frac{dt}{L}$$

$$\Rightarrow -R\int \frac{dI}{V-IR} = -\frac{R}{L}\int dt$$

$$\Rightarrow \ln|V-IR| = -\frac{R}{L}t + k$$

$\therefore$ as at $t=0$; $I=0$.

$$\Rightarrow \ln V = k$$

$$\Rightarrow \ln|V-IR| = -\frac{R}{L}t + \ln V$$

$$\Rightarrow \ln\left|\frac{V-IR}{V}\right| = -\frac{R}{L}t$$

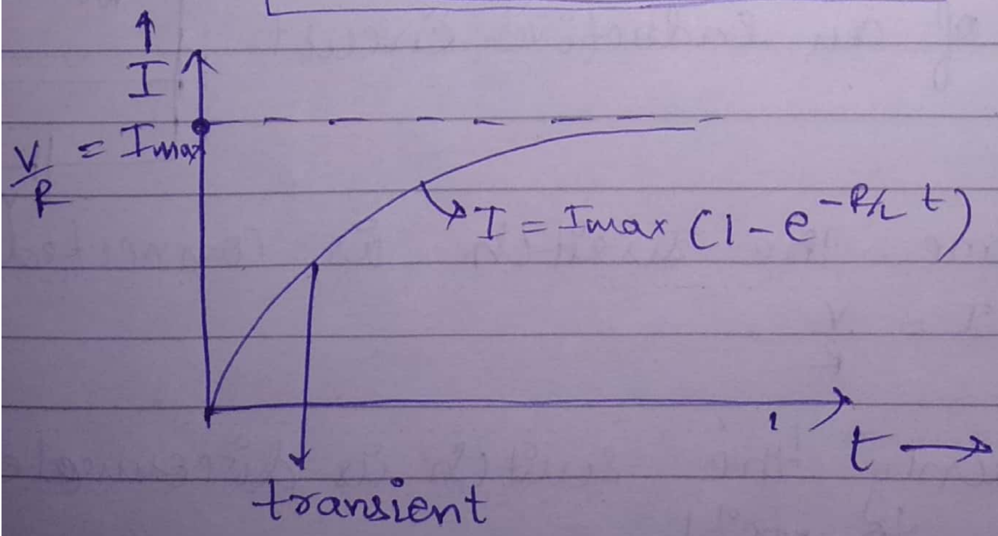
$$\Rightarrow V-IR = Ve^{-\frac{R}{L}t}$$

$$V - Ve^{-\frac{R}{L}t} = IR$$

$$V(1-e^{-\frac{R}{L}t}) = IR$$

$$\Rightarrow I = \frac{V}{R}(1-e^{-\frac{R}{L}t})$$

$$\Rightarrow \boxed{I = I_{max}(1-e^{-\frac{R}{L}t})}$$



Time Constant

$\tau = t = L/R$

$I = I_{max}(1 - e^{-1})$

$\Rightarrow I = 0.632\, I_{max}$

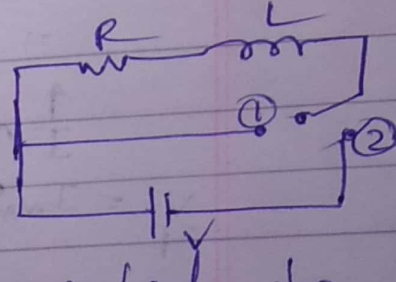
∴ It is the time taken by ~~an~~ ~~inductor~~ current to reach 0.632 times of it's maximum or final stage value.

Initial ~~Rise~~ rate of rise of current

$$V = IR + L\frac{dI}{dt}$$

$I = 0;\ V_R = 0$

$$V = L\frac{dI}{dt} \Rightarrow \boxed{\frac{dI}{dt} = \frac{V}{L}}$$



Breaking of an inductive circuit

(1) A long time the switch is connected to pt 2 ∴ $I = \frac{V}{R}$.

(2) Now suddenly the switch is disconnected from pos^n 2 to pos^n 1.

$$IR + L\frac{dI}{dt} = 0$$

$\Rightarrow \frac{dI}{I} = -\frac{R}{L} dt$

$\Rightarrow \ln I = -\frac{R}{L} t + k$

$\therefore$ as at $t=0$; $I = \frac{V}{R} = I_{max}$

$\Rightarrow k = \ln I_{max}$

& $\boxed{I = I_{max} e^{-\frac{R}{L}t}}$

Time constant

$t = \lambda = \frac{L}{R}$

$I = I_{max} e^{-1}$

$\boxed{I = I_{max} (0.37)}$

↳ It is the time reqd. for the current to fall to (0.37) times of its initial steady value 'I_{max}'.

PTO

Q A $2\mu F$ capacitor is connected by closing a switch to a supply of 100 v through a $1M\Omega$ series resistor.

(i) Calculate λ = ~~...~~ $RC = 2$

(ii) Initial charging current = $\frac{100}{...} = 10^{-4}$

$\frac{10^{-6}}{10^{6}}$

(iii) Initial rate of rise of capacitor = ~~...~~ $\frac{100}{RC}$ = ~~...~~ $\frac{100}{10^6 \times 2 \times 10^{-6}} = 50$ V/s

2×10^{-12}

10×10^{-3}

(iv) Voltage across capacitor after 6sec of closing switch.

$$V = 100\,(1 - e^{-\frac{6}{2}})$$
$$= 100\,(1 - e^{-3})$$
$$= 100\,(1 - 0.05)$$
$$= 100\,(0.95)$$
$$= 95.1\,V$$

Q A coil having resistance of $15\,\Omega$ & $L = 10H$ is connected to a constant DC 75 V supply.

(i) find rate of change of current at the instant of closing the switch.

$L\frac{dI}{dt} + 15I = 75$

at $I = 0$, $V_R = 0$

$\Rightarrow \frac{dI}{dt} = \frac{75}{L}$

(ii) final steady value $= \frac{V}{R} = 5A$

(iii) $\lambda = \frac{R}{L} = 0.67\,s$

(iv) Time taken for the current to reach a value of 4A.

$$4 = I\,(1 - e^{-R/L\,t})$$

$I = 1.078s$

AC - voltage

It is that voltage whose magnitude & polarity changes wrt time.

AC - current

It is that current whose magnitude & polarity changes wrt time

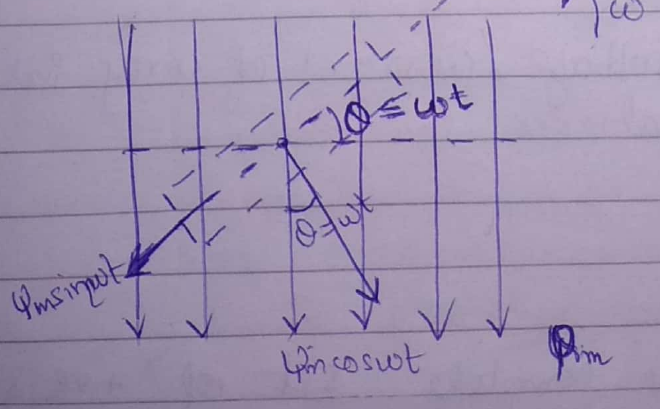
↳ A sinusoidal alternating voltage is generated by rotating a coil with a constant velocity in a uniform magnetic field.

$$e = E_m \sin \omega t = E_m \sin \theta \quad \text{(why sine ?)}$$

↳ (i) sine wave produce lesser copper & iron loss for a given output. ∴ Efficiency is more.

(ii) sine waves produces least disturbance in phone lines

Generation of Alternating voltage



(i) Consider a rectangular coil which is rotating in an uniform magnetic field of angular velocity "ω".

(ii) Alternating voltage is generated according to faraday's laws of electromagnetic induction

$\therefore$ Total flux linkage = No. of turns x flux linkage
$= n \times \phi_m \cos\omega t$

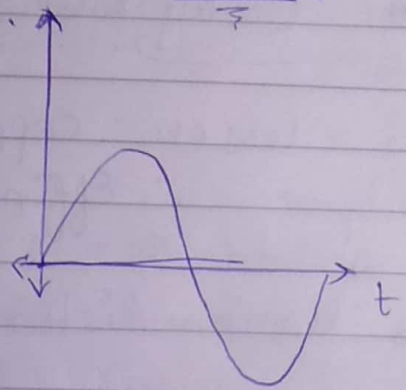
EMF induced = Rate of change of flux linkage
$= \frac{d}{dt}(n\phi_m \cos\omega t)$

$\Rightarrow \quad e = -n\phi_m \omega \sin\omega t$

$\Rightarrow \quad E_m = n\phi_m \omega$

$\Rightarrow \boxed{e = E_m \sin\omega t = E_m \sin\theta}$

Terminology



(i) Wave shape or Wave form
~~Fw~~ It is the curve obtained by taking the instantaneous values of voltage or current on ordinate against time as abscissa is called as ~~a~~ waveform.

(ii) Instantaneous value
The value of voltage / current of any instant is called inst. value

(iii) Cycle → It is the complete set of +ve & -ve values of a particular quantity.
1 cycle = 2π radians or $360°$

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Inst + Avg / Max or Peak

(iv) Alternation

One half-cycle of an alternating quantity is called alternation

(v) Time period

Time required to complete one cycle.

(vi) Amplitude

It is the max or min value of the alternating value.

Different forms of an alternating quantity.

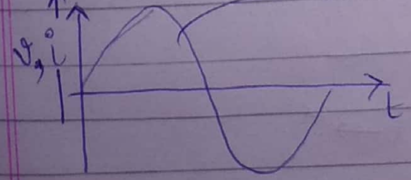
$e = E_m \sin\omega t = E_m \sin 2\pi f t = E_m \sin \frac{2\pi t}{T}$
$= E_m \sin\theta$

Values of an alternating quantity

(i) Instantaneous value.
(ii) Maximum / Peak value.
(iii) Average value → It is the arithmetical average of all the values of an alternating quantity over 1 cycle.

→ For symmetrical wave, since the average value is zero (over 1 cycle), average value of 1 alternation is taken into account

Average value of a sinusoidal



∴ Avg value of + half of a cycle :—

$$= \frac{\text{Area of one alternation}}{\text{Base of one alternation}}$$

∴ Area of 1 alternation $= \int_0^{\pi} I \, d\theta$

$$= \int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= I_m \left[-\cos\theta \right]_0^{\pi}$$

$$= I_m \left[-\cos\pi + \cos 0 \right]$$

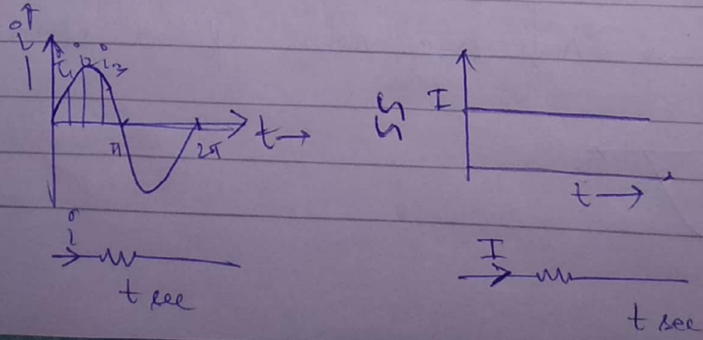
$$= 2I_m$$

∴ Avg value of alt. current $= \frac{2I_m}{\pi} = 0.632\, I_m$

& Avg value of alt. voltage $= 0.632\, E_M$

⊛ RMS value or Effective value

↳ It is that steady state DC value which when flowing through a given resistance for a given time produces the same amount of Heat as produced by the alternating current when flowing through the same resistance for the same time.



$i^2 R t$ $\qquad\qquad$ $I^2 R t$

$$i_1^2 R\, t/n + i_2^2 R\, t/n + i_3^2 R\, t/n \cdots i_n^2 R\, t/n = I^2 R t$$

$$\frac{Rt}{n}\left(i_1^2 + i_2^2 \cdots i_n^2\right) = I^2 \cancel{Rt}$$

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots i_n^2}{n}}$$

↳ for symmetrical wave, RMS value can be calculated for half cycle or full cycle. But for unsymmetrical wave

RMS value of sinusoidal alternating quantity

$$\text{RMS value} = \sqrt{\frac{(\text{Ar. of } \tfrac{1}{2}\text{ cycle of wave})^2}{\text{Base of } \tfrac{1}{2}\text{ cycle of squared wave}}}$$

∴ Ar. of ½ cycle of wave :

$$= \int_0^{\pi} I_m^2 \sin^2 \omega t \; d\omega t$$

$$= I_m^2 \int_0^{\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta$$

$$= I_m^2 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$= I_m^2 \left[\left(\frac{\pi}{2} - 0\right) - 0\right]$$

$$= \frac{\pi I_m^2}{2}$$

$\therefore$ RMS value: $\sqrt{\dfrac{\frac{\cancel{\pi} I_m^2}{2}}{\cancel{\pi}}} = \dfrac{I_m}{\sqrt{2}}$

$\rightarrow \boxed{I = 0.707\ I_m}$

Nece

(A) <u>form factor & Peak factor</u>

$ff = \dfrac{\text{RMS value}}{\text{Avg. value}}$

for ~~a~~ a sinusoidal wave.

$\text{form factor} = \dfrac{0.707\ I_m}{0.632\ I_m} = 1.11$

$\boxed{f.f. = 1.11}$

& $\text{Peak factor} = \dfrac{\text{Max}^m \text{ value}}{\text{RMS value}}$

$= \dfrac{I_m}{0.707\ I_m} = 1.414$

$\boxed{P.f. = 1.414}$

(Q) $I = 141.4 \sin 314t \simeq A \sin \omega t$.

$A = 141.4$ & $\omega = 314$

* $I_m = 141.4$ A
* $T = \frac{2\pi}{314} = \frac{2 \times \cancel{314}}{\cancel{314}} = \frac{\cancel{100}}{2} = 0.02$ s & $f = \frac{1}{T} = 50\ s^{-1}$ or Hz
*

8. An alternating current of frequency 60 Hz has a max. value of 120 Amp.
   (i) Write down the eqⁿ for the instantaneous value.
   (ii) Taking time from the instant the current is zero(0), and becoming +ve, find the instantaneous value after $\frac{1}{360}$ s.
   (iii) Time taken to reach 96 Amp for the 1st time

$\omega = 2\pi f = 2 \times 3.14 \times 60 = 376.8\ s^{-1}$

$I_m = 120$

(i) ∴ $I = I_m \sin \omega t$

$= 120 \sin (376.8) t$

(ii) $I = 120 \sin \left(376.8 \times \frac{1}{360} \right) = 103.9$ A

(iii) $96 = 120 \sin (376.8\ t)$

$53.13 = 376.8\ t$

$\Rightarrow t = 0.00246$ s

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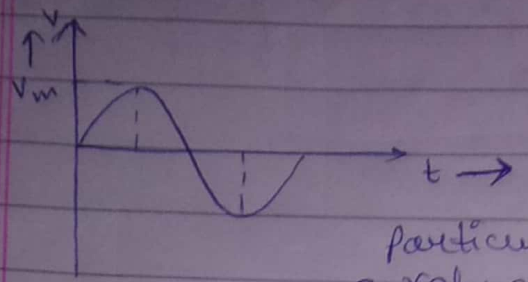
Phase :-



$v = V_m \sin \omega t$

Phase of a particular value of an alternating quantity is defined as the fractional part of time-p or cycle through which the quantity has advanced from selected zero (0) point of reference.

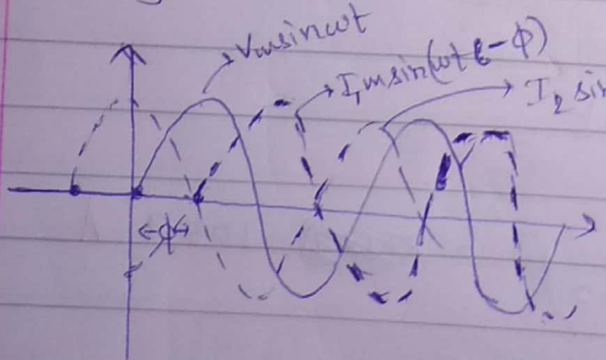
When said, $V_m \sin \omega t = V_m$

$\rightarrow \sin \omega t = 1$

$\omega t = 90°$ or $\frac{\pi}{2}$ rad

$\therefore$ phase $= \frac{\pi}{2}$ rad.

When two alternating quantities having same frequency have different zero points, then they are said to have phase difference.



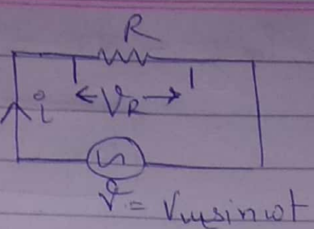
$\therefore$ $v = V_m \sin \omega t$

$I_1 = I_m \sin(\omega t - \phi) \rightarrow$ lagging

$I_2 = I_m \sin(\omega t + \phi) \rightarrow$ leading

Let $v = V_m \sin\omega t$

$v_R = iR = V_m \sin\omega t$

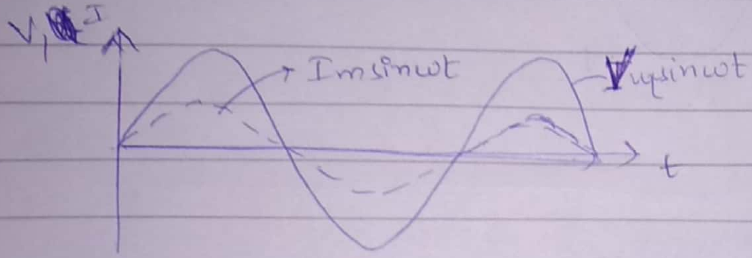


$\Rightarrow i = \frac{V_m}{R} \sin\omega t$

$\Rightarrow \boxed{i = I_m \sin\omega t}$

$I_m = \frac{V_m}{R}$

$\Rightarrow \frac{V_m}{I_m} = \frac{\sqrt{2} V}{\sqrt{2} I} = \frac{V}{I} = R$



↳ In an AC circuit, with resistance only, the applied voltage & current are in phase with each other.

→ <u>Instantaneous Power (P)</u>

$P = v \cdot i$

$= V_m I_m \sin^2\omega t$

$= V_m I_m \left(\frac{1 - \cos 2\omega t}{2}\right)$

& $P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P\,dt$

$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \int_0^{2\pi} \cancel{\cos 2\omega t} \; 0$

$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V \cdot I$

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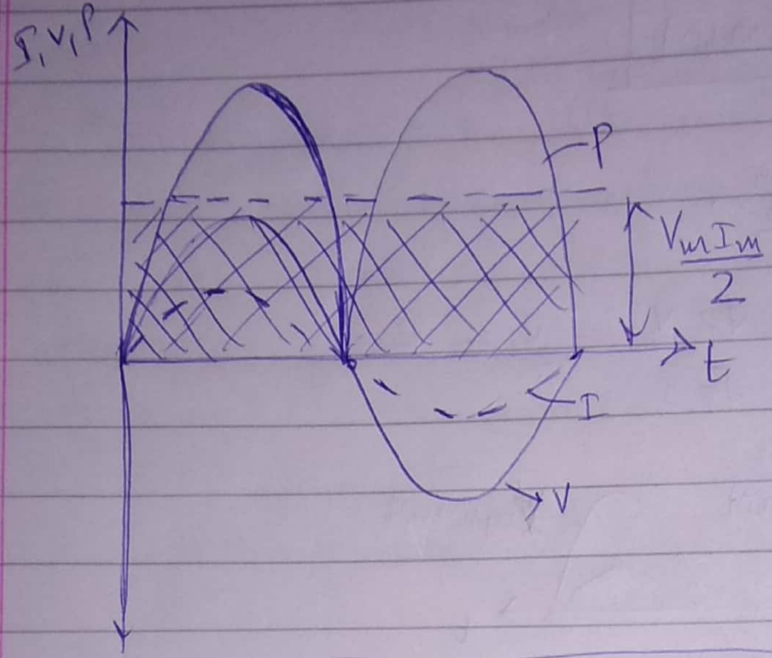
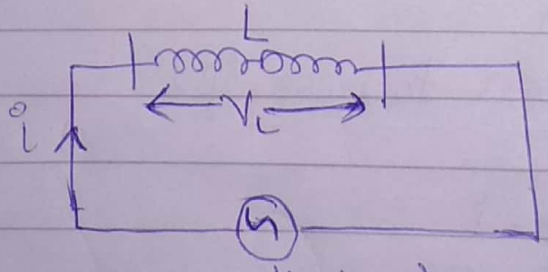
$P_{avg} = VI = IRI$

$$\boxed{P_{avg} = I^2R}$$



✪ AC with Inductance

When an inductor is connected across an AC supply, an EMF is induced called as Back-em



$v = V_m \sin \omega t$ —— (1)

$$\Rightarrow V = L\frac{dI}{dt}$$

$$V_m \sin \omega t = L\frac{dI}{dt}$$

$$\Rightarrow \frac{dI}{dt} = \frac{V_m}{L} \sin \omega t$$

$$\Rightarrow \int dI = \frac{V_m}{L}\int_0 \sin\omega t\,(dt)$$

$$\Rightarrow I = \frac{V_m}{\omega L}\left[-\cos\omega t\right]$$

$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{--- (2)}$$

$\therefore$ Current of an inductor lags behind voltage by $\frac{\pi}{2}$ rad or 90°.

Inductive Reactance (X_L)

$$I_m = \frac{V_m}{\omega L} \Rightarrow \frac{V_m}{I_m} = \omega L = 2\pi f L = X_L$$

↳ Opposition offered to the flow of current is called inductive reactance.

Instantaneous Power

$$P = VI$$

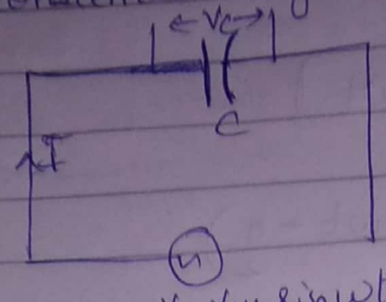
$$= V_m \sin\omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= V_m I_m (\sin\omega t)(-\cos\omega t)$$

$$= -\frac{V_m I_m}{2}\sin 2\omega t$$

$$\therefore P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} P\,dt = -\frac{V_m I_m}{4\pi}\int^{2\pi} \cancel{\sin 2\omega t}^{\,0} = 0$$

NOTE: Power consumed by a pure inductor is '0'



$V = V_m \sin\omega t$

In an AC-circuit with inductance only, the applied voltage leads the current or the current lags the voltage exactly by an angle of $\pi/2$ rad or $90°$

$v = V_m \sin\omega t$

$$I = \frac{dQ}{dt} = \frac{d}{dt}(CV_c) = C\frac{dV_c}{dt} = C\frac{d(V_m \sin\omega t)}{dt}$$

$$= CV_m\omega\cos\omega t$$

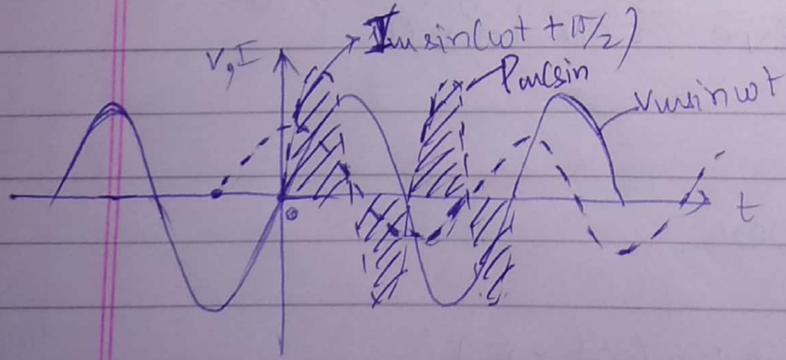
$$= CV_m\,\omega\cos\omega t$$

$$\Rightarrow I = I_m \cos\omega t$$

or

$$= I_m \sin(\omega t + \pi/2)$$

$$\rightarrow I = I_m \sin(\omega t + \pi/2)$$



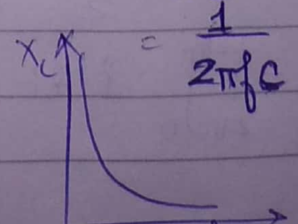
Capacitive Reactance

$I_m = CV_m\omega$

$$\frac{V_m}{I_m} = \frac{1}{\omega C} = X_C$$

$\Rightarrow X_C = \frac{1}{\omega C} \Rightarrow$ Capacitive reactance

$$= \frac{1}{2\pi f C}$$



Instantaneous Power

$$P = VI$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t + \pi/2)$$

$$= V_m \sin\omega t \cdot I_m \cos\omega t$$

$$= \frac{V_m I_m}{2} (\sin 2\omega t)$$

$$\therefore P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} P\,dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \ldots$$

$$= \frac{V_m I_m}{4\pi} \cancel{\int_0^{2\pi} \sin 2\omega t} \; 0$$

$$\boxed{P_{avg} = 0}$$

NOTE

↳ In an AC-circuit with capacitance only the applied voltage lags the current or the current leads the voltage by exactly $\pi/2$ or $90°$.