

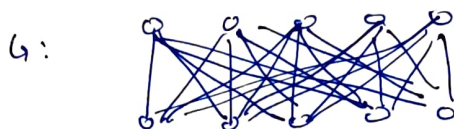
Solution 1: Direct proof :-

Using the condition in the for-loop, we just need to ensure that every edge of G , i.e.

$\forall e \in E(G) : \exists e = (u, v)$ in G . Either $e \in H$, in which case $d_H(u, v) = 1$, else it was not considered.

This is because by description of Algorithm, H already consists a path $u \rightarrow v : |u \rightarrow v| \leq K$.

Solution 2: Increase in ~~vertices~~ $\&$ Increase in edges should be $O(G^2)$.



This is what is called a complete Bipartite Graph.

Solution 3: Proof:



We know that a tree has $n-1$ edges.

m.sp. of $K = n-1$.

since maximum distance in H is $\leq n-1$, and since minimum distance in G is 1. Hence H is a spanning tree.

Solution 4: Proof by contradiction! Assume converse (i.e. $|C| \leq K+1$)

Take any u, v in C : but $\therefore \text{dist}_G(u, v) \leq K+1, \therefore \leq K$

But this is not possible because ~~we are not aware of~~ the condition inside for-loop in the given algorithm. Hence ~~our~~ our assumption is wrong. Therefore $\forall C \in H$: $|H| > K+1$.