

# 5 Card Trick

Akash Suryawanshi

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## Abstract

Suggested a protocol that magician and her assistant should follow to successfully implement the 5 card trick.

## 1 The Protocol

5 cards are picked by the audience, 4 of which are shown to the magician by her assistant. Since a deck of cards has 4 sets of different shapes (clubs, diamonds, hearts, spades), each containing 13 cards, there must be at least two cards which have the same shape (Pigeonhole principle). Thus, the assistant can use this to communicate the shape of the card to the magician. Next, we have 3 cards remaining. We need to use these three cards so as to communicate the number to the magician. Notice that if we arrange the cards in a circular pattern then the maximum distance (maximum number of places apart) between any two cards is 6.



Also, we can arrange the 3 cards we are left with in  $3! = 6$  ways. Let us call these cards A, B, C. In order to use their permutation to find the number, the magician and assistant need to decide which card is A, B, and C respectively. They decide beforehand that the order of preference is clubs - diamonds - hearts - spades and for number it is increasing. Hence, after the assistant places the cards XYZ, the magician calculates the priorities of the cards and assigns A to lowest priority, C to highest priority and B to the remaining card. The numbers associated with the permutation are as follows: ABC = 1, ACB = 2, BAC = 3, BCA = 4, CAB = 5, CBA = 6. For implementing this, the assistant shows the smaller (or smallest) card if the

difference between the cards of same colour is less than 7, else she shows the larger card. The final number is decided after adding the number obtained from the permutation to the number of the larger card (which is shown to the magician first). Hence, the magician has the information on the colour as well as the number, which completes the trick.

## **2 Why is it not possible to do this trick with 4 cards ?**

This is because there is no guarantee that at least two of the cards have the same colour. And if every card is of distinct colour, then we are not able to communicate the colour of the card to the magician.