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Assignment -1

Q Test for consistency and solve.

i) $2x-3y+7z=5, \quad 3x+y-3z=13, \quad 2x+19y-47z=32$

Sol^n $A \cdot X = B$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

\# $[A:B]$

$$\left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$R_1 = R_1 - R_2$

$$\left[\begin{array}{ccc:c} -1 & -4 & 10 & -8 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$R_2 \to R_2 + 3R_1$

$R_3 \to R_3 + 2R_1$

$$\left[\begin{array}{ccc:c} -1 & -4 & 10 & 8 \\ 0 & -11 & 27 & -11 \\ 0 & 11 & -27 & 32 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc:c} -1 & -4 & 10 & 8 \\ 0 & -11 & -27 & -11 \\ 0 & 0 & [illegible] & 5 \end{array}\right]$$

$\to \rho(A) = 2$

$\rho(A:B) = 3$

$\boxed{\rho(A) \neq \rho(A:B)\ [illegible]\ 3}$

It is inconsistent.

Hence, ~~it is consistent & have unique solution.~~

ii) $2x - y + 3z = 8 \quad -x + 2y + z = 4 \quad 3x + y - 4z = 0$

Sol^n

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$\rightarrow [A:B] \left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$R_1 \rightarrow R_1 + R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 4 & 12 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 + R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 4 & 12 \\ 0 & 3 & 5 & 16 \\ 0 & -2 & -16 & -36 \end{array}\right]$$

$R_2 \rightarrow R_2 + R_3$

$$\left[\begin{array}{ccc:c} 1 & 1 & 4 & 12 \\ 0 & 1 & -11 & -20 \\ 0 & -2 & -16 & -36 \end{array}\right]$$

$R_3 \rightarrow R_3 + 2R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 4 & 12 \\ 0 & 1 & -11 & -20 \\ 0 & 0 & -38 & -76 \end{array}\right]$$

$\begin{array}{r} 16 \\ 22 \\ \hline 38 \end{array}$

$\rho(A) = 3$

$\rho(A:B) = 3$

$\begin{array}{r} 40 \\ 36 \end{array}$

Since, $\rho(A) = \rho(A:B) = n = 3 \rightarrow$ It is consistent and have unique sol^n.

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{vmatrix}$$

$2(-8-1) + 1(4-3) + 3(-1-6)$

$-18 + 1 - 21 = -38$

$$\Delta x = \begin{vmatrix} 8 & -1 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -4 \end{vmatrix} \Rightarrow 8(-9) - 4(+1) \Rightarrow -76$$

$x = \frac{\cancel{-76}\ 2}{\cancel{-38}} \quad x = 2$

$$\Delta y = \begin{vmatrix} 2 & 8 & 3 \\ -1 & 4 & 1 \\ 3 & 0 & 4 \end{vmatrix} = 2(16) - 8(-7) + 3(-12)$$

$32 + 56 - 36$

$y = \frac{\cancel{52}\ 26}{\cancel{-38}\ 19} \quad \frac{-26}{19} \quad 52$

$$\Delta z = \begin{vmatrix} 2 & -1 & 8 \\ -1 & 2 & 4 \\ 3 & 1 & 0 \end{vmatrix} = 2(-4) + 1(-12) + 8(-7)$$

$-8 - 12 - 56$

-76

$\Delta z = \frac{-76}{-38} = +2$

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$\frac{\Delta x}{\Delta} \to x$

iv) $4x - y = 12$, $-x + 5y - 2z = 0$, $-2x + 4z = -8$

Sol $[A:B] = \left|\begin{array}{ccc:c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right|$

$\Delta = 4(20) + 1(-4-4)$
$80 - 8 \Rightarrow 72$

~~$R_1 \to R_1 + 3R_2$~~

$\left|\begin{array}{ccc:c} \cancel{1} & \cancel{14} & \cancel{-6} & \cancel{12} \\ \cancel{-1} & \cancel{5} & \cancel{-2} & 0 \\ \cancel{-2} & \cancel{0} & \cancel{4} & -8 \end{array}\right|$

$R_1 \leftrightarrow R_2$

$\left|\begin{array}{ccc:c} -1 & 5 & -2 & 0 \\ 4 & -1 & 0 & 12 \\ -2 & 0 & 4 & -8 \end{array}\right|$

~~$R_2 \to R_2 + R_1$~~
~~$R_3 \to R_3 + 2R_1$~~

$\left|\begin{array}{ccc:c} 1 & 14 & -6 & 12 \\ 0 & 19 & \cancel{-8} & \cancel{12} \\ 0 & \cancel{28} & -8 & 16 \end{array}\right|$

$R_2 \to R_2 + 2R_3$

$\left|\begin{array}{ccc:c} -1 & 5 & -2 & 0 \\ 0 & -1 & 8 & -4 \\ -2 & 0 & 4 & -8 \end{array}\right|$

$R_3 \to R_3 - 2R_1$

$\left|\begin{array}{ccc:c} -1 & 5 & -2 & 0 \\ 0 & -1 & 8 & -4 \\ 0 & -10 & 8 & -8 \end{array}\right|$

$R_3 \to R_3 - 10R_2$

$\left|\begin{array}{ccc:c} -1 & 5 & -2 & 0 \\ 0 & -1 & 8 & -4 \\ 0 & 0 & -62 & 32 \end{array}\right|$

$\rho(A) = \rho(A:B) = 3$

∴ It is consistent & having unique solⁿ.

$\Delta x \Rightarrow \begin{vmatrix} 12 & -1 & 0 \\ 0 & 5 & -2 \\ -8 & 0 & 4 \end{vmatrix} \Rightarrow 12(20) + 1(-16) \Rightarrow 240 - 16 \Rightarrow 224$, $x = \frac{224}{72}$

$\Delta y = \begin{vmatrix} 4 & 12 & 0 \\ -1 & 0 & -2 \\ 2 & -8 & 4 \end{vmatrix} \Rightarrow 4(-16) - 12(-4+4) \Rightarrow -48$ $\quad y = \frac{-48}{72}$

$\Delta z = \begin{vmatrix} 4 & -1 & 12 \\ -1 & 5 & 0 \\ 2 & 0 & -8 \end{vmatrix} \Rightarrow 4(-40) + 1(8) + 12(-10) \Rightarrow -272$ $\quad z = \frac{-272}{72}$

$\Rightarrow -160 + 8 - 120$

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Q for what values of λ & μ the given system of equations

$x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has

i) no solution ii) unique sol? iii) infinite number of solutions

Sol $[A:B] = \left|\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right|$

$R_3 \to R_3 - R_2$

$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right|$

$R_2 \to R_2 - R_1$

$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right|$

ii) for unique soln:

$\lambda \neq 3$ & $\mu \neq 10$

a) i) for no soln:

$\lambda = 3$ & $\mu \neq 10$ ~~or $\lambda \neq 3$~~

iii) Infinite number of soln:

$\lambda = 3$, $\mu = 10$.

Q for what values of λ the given eqn $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a soln & solve them completely in each case

Sol $[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$ $\begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{array}\right]$

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$R_3 \to R_3 - 3R_2$

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{array}\right|$$

→ for system having solⁿ:

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2$

$\lambda(\lambda-2) - 1(\lambda-2)$

$(\lambda-1)(\lambda-2) = 0$

$\lambda = 1,\ \lambda = 2$

Q Find the solⁿ of the system of eqⁿ $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array} \quad \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A:B) = \rho(A) = 3$,

$\rho(A) = 3$ = no. of variables → has unique solⁿ

↳ Trivial solⁿ

$x = y = z = 0$.

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* find for what values of λ the given eq^n $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$, may possess non-trivial soln & solve them completely in each case.

Sol. $A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$

$\rightarrow |A| = \begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix}$

$3(-2\lambda+12) - 1(4\lambda+6\lambda) - \lambda(16-4\lambda)$

$\Rightarrow -6\lambda+36 - 10\lambda - 16\lambda - 4\lambda^2 = 0$

$\Rightarrow -32\lambda+36-4\lambda^2 = 0$

$4\lambda^2+32\lambda-36=0$

$\lambda^2+8\lambda-9=0$

$\lambda^2+9\lambda-\lambda-9=0$

$\lambda(\lambda+9)-1(\lambda+9)=0$

$(\lambda+9)(\lambda-1)=0$

$\lambda=-9, \lambda=1.$

~~for all values of~~ λ except ~~-9, 1, the eq^n~~ has ~~non-trivial soln.~~

* for $\lambda = -9$ & 1 the system of eqn has non-trivial soln or infinite soln.

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Assignment-02

Are the following sets of vectors linearly independent or dependent?

i) [1 0 0], [1 1 0], [1 1 1]

Sol^n $V = c_1[1\,0\,0] + c_2[1\,1\,0] + c_3[1\,1\,1]$

$c_1 + c_2 + c_3 = 0$

$c_2 + c_3 = 0$

$c_3 = 0$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow$ Rank $= 3 = n \Rightarrow$ Trivial soln.

$\Rightarrow$ Linear Independent.

ii) [7 -3 11 -6], [-56 24 -88 48]

$V = c_1[7\ -3\ 11\ -6] + c_2[-56\ 24\ -88\ 48]$

$7c_1 - 56c_2 = 0$

$-3c_1 + 24c_2 = 0$

$11c_1 - 88c_2 = 0$

$-6c_1 + 48c_2 = 0$

$Y = \begin{vmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ -6 & 48 \end{vmatrix}$

$R_1 + R_4$

$\begin{vmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ 1 & -8 \end{vmatrix}$

$R_2 \to R_2 + 3R_4$ & $R_3 \to R_3 - 11R_4$

$\Rightarrow \begin{vmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 1 & -8 \end{vmatrix}$

$\Rightarrow R_1 = R_1 - 7R_4$

$\Rightarrow \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -8 \end{vmatrix}$ $R_4 \leftrightarrow R_1$ $\Rightarrow \begin{vmatrix} 1 & -8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{vmatrix}$

$\rho(Y) < 2 \Rightarrow$ Infinite soln

$\therefore$ The vector is linearly dependentent

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$$166 - 16 = 144$$

$$192 - 144 = 48$$

$[-1, 5, 0]$, $[16\ 8\ -3]$, $[-64\ 56\ 9]$

$V = c_1[-1\ 5\ 0] + c_2[16\ 8\ -3] + c_3[-64\ 56\ 9]$

$$-c_1 + 16c_2 - 64c_3 = 0$$
$$5c_1 + 8c_2 + 56c_3 = 0$$
$$-3c_2 + 9c_3 = 0$$

$$V = \begin{vmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{vmatrix}$$

$|V| = -1(72 + 168) - 5(144 - 192)$

$\Rightarrow -240 + 240 = 0$

$|V| \neq 0$, Rank $(V) < 3$ $\therefore$

$\therefore$ system has Infinite soln.

$\rightarrow$ Linear dependent.

$[1\ -1\ 1]$, $[1\ 1\ -1]$, $[-1\ 1\ 1]$, $[0\ 1\ 0]$

$V = c_1[1\ -1\ 1] + c_2[1\ 1\ -1] + c_3[-1\ 1\ 1] + c_4[0\ 1\ 0]$

$c_1 + c_2 - c_3 = 0$ $\rightarrow$ No of unknown $= 4$

$-c_1 + c_2 + c_3 + c_4 = 0$.

$c_1 - c_2 + c_3 = 0$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Rank $= 3 \neq n$.

$\rightarrow$ Infinite soln $\rightarrow$ Linearly dependent.

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[2 -4] [1 9] [3 5]

$V = c_1[2, -4] + c_2[1\ 9] + c_3[3\ 5]$

$2c_1 + c_2 + 3c_3 = 0$

$-4c_1 + 9c_2 + 5c_3 = 0$

$$\begin{vmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{vmatrix} \quad R_2 \to R_2 + 2R_1 \Rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 11 & 11 \end{bmatrix}$$

No. of unknown = 3, Rank ⇒ 2

∴ Infinite solⁿ

∴ Linearly dependent.

[3 -2 0 4], [5 0 0 1], [-6 1 0 1], [2 0 0 3]

Ansⁿ $c_1[3\ -2\ 0\ 4] + c_2[5\ 0\ 0\ 1] + c_3[-6\ 1\ 0\ 1] + c_4[2\ 0\ 0\ 3]$

$3c_1 + 5c_2 - 6c_3 + 2c_4 = 0$

$-2c_1 + c_3 = 0$

$4c_1 + c_2 + c_3 + c_4 = 0$

$$\begin{vmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 \end{vmatrix} = V$$

$|V| = 0$

⇒ Rank < 4 ≠ no. of unknown.

∴ Infinite solⁿ or non-trivial solⁿ

⇒ Linearly dependent.

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Assignment-3

Find the eign values & eign vectors of following matrices.

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \Rightarrow (A-\lambda I) = \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix}$$

$|A-\lambda I| = 0$

$$\Rightarrow (-2-\lambda)\left[(-\lambda+\lambda^2)-12\right] - 2\left[-2\lambda-6\right] - 3\left[-4+1-\lambda\right] = 0$$

$$\Rightarrow 2\lambda - 2\lambda^2 + \lambda^2 - \lambda^3 + 24 + 12\lambda + 4\lambda + 12 + 9 + 3\lambda = 0$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\Rightarrow (\lambda-5)(\lambda^2+6\lambda+9) = 0$$

$$(\lambda-5)\left[\lambda^2+3\lambda+3\lambda+9\right] = 0$$

$$(\lambda-5)\left[\lambda(\lambda+3)+3(\lambda+3)\right] = 0$$

$$\Rightarrow (\lambda-5)(\lambda+3)(\lambda+3) = 0$$

$$\Rightarrow \lambda = 5 \text{ or } \lambda = -3.$$

At $\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} -1 & -2 & -5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$ & $R_3 \to R_3 - 7R_1$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}$$

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$R_1 \to R_1 \times -1$

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = K$

$-8x_2 - 16x_3 = 0 \Rightarrow -8x_2 = 16x_3$

$x_2 = -2x_3 = -2K.$

$x_1 + 2x_2 + 5x_3 = 0$

$\Rightarrow x_1 + 2(-2K) + 5(K) = 0$

$x_1 = -K.$

$$\Rightarrow \begin{bmatrix} -K \\ -2K \\ K \end{bmatrix} = K \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

for $\lambda = -3$

$$\begin{bmatrix} -5 & 2 & -3 \\ 2 & -2 & -6 \\ -1 & -2 & -3 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & -2 & -3 \\ 2 & -2 & -6 \\ -5 & 2 & -3 \end{bmatrix}$$

$R_3 \to R_3 - 5R_1$ & $R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} -1 & -2 & -3 \\ 0 & -6 & -12 \\ 0 & 12 & 12 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} -1 & -2 & -3 \\ 0 & -6 & -12 \\ 0 & 0 & -12 \end{bmatrix} = 0$$

$-3+15$

$12-24$

$R_1 \to R_1 \times (-1) \Rightarrow$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & -12 \\ 0 & 0 & -12 \end{bmatrix}$$

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$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & -12 \\ 0 & 0 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x_1 + 2x_2 + 3x_3 = 0$ | $-6x_2 - 12x_3 = 0$, $x_2 = -2x_3$ | $-12x_3 = 0$

$x_3 = k$

$\Rightarrow x_2 = -2k$, $x_1 - 4k + 3k = 0$

$x_1 = k$

$\Rightarrow \begin{bmatrix} k \\ -2k \\ k \end{bmatrix} \Rightarrow k \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$\therefore$ Eign values are $5, -3$ & corresponding Eign vectors are $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

2. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = A$

Sol $\cancel{(A - \lambda I) = \begin{bmatrix} 3-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}}$

$\Rightarrow |A - \lambda I| = 0$

$\Rightarrow (3-\lambda)[(1-\lambda)(1-\lambda)] + 1[+2(1-\lambda)]$

$\Rightarrow (3-\lambda)[1+\lambda^2-2\lambda] + 2 - 2\lambda$

$\Rightarrow \cancel{3+3\lambda^2-6\lambda-\lambda-\lambda^3+2\lambda^2+2-2\lambda}$

$\Rightarrow \cancel{\lambda^3-5\lambda^2+9\lambda-5=0}$

$\cancel{(\lambda-1)(\lambda^2-4\lambda+5)=0}$

$\cancel{(\lambda-1)(\lambda^2}$

$\Rightarrow \cancel{3+3\lambda^2-6\lambda+2-2\lambda=0}$

$\Rightarrow \cancel{3\lambda^2-8\lambda+5=0}$

$\Rightarrow \cancel{3\lambda^2-3\lambda-5\lambda+5=0}$

$\Rightarrow \cancel{3\lambda(\lambda-1)-5(\lambda-1)=0}$

$\Rightarrow \cancel{(3\lambda-5)(\lambda-1)=0}$

$\Rightarrow \cancel{\lambda = \frac{5}{3} \text{ or } \lambda = 1}$

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~~At $\lambda = 3$~~

$$\begin{vmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow (4-\lambda)\left((1-\lambda)^2\right) + 1\left(2(1-\lambda)\right)$

$\Rightarrow (4-\lambda)\ (1+\lambda^2-2\lambda) + (2-2\lambda)$

$\Rightarrow 4+4\lambda^2-8\lambda-\lambda-\lambda^3+2\lambda^2+2-2\lambda$

$\Rightarrow -\lambda^3+6\lambda^2-11\lambda+6=0$

$-\lambda^2(\lambda-1)+5\lambda(\lambda-1)-6(\lambda-1)$

$\Rightarrow (\lambda-1)(\lambda^2-5\lambda+6)=0$

$\Rightarrow (\lambda-1)(\lambda-2)(\lambda-3)=0$

for $\lambda=1$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 + x_3 = 0 \Rightarrow x_2 = 0$

$\Rightarrow x_3 = k \Rightarrow x_1 = -\frac{1}{3}k$

$$\Rightarrow \begin{bmatrix} -1/3 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda=2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \to R_2 + R_1 \Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = k \Rightarrow \quad -x_2 + x_3 = 0 \qquad 2x_1 + x_3 = 0$

$x_2 = x_3 = k \qquad x_1 = -\frac{1}{2}x_3 = -\frac{1}{2}k$

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$$k\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix},$$

for $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 + 2R_1$ $\quad$ $R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + x_3 = 0$, $-2x_2 + 2x_3 = 0$ $\quad x_3 = k$

$x_2 = x_3$, $\quad x_1 = -x_3$

$$k\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

3. $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$\Rightarrow (A-\lambda I) = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & 0-\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$

$|A-\lambda I| = 0$

$\Rightarrow (5-\lambda)[(0-\lambda)(3-\lambda)] = 0$

$(5-\lambda)[-3\lambda+\lambda^2] = 0$

$-15\lambda+5\lambda^2+3\lambda^2-\lambda^3 = 0$

$\lambda^3-8\lambda^2+15\lambda = 0$

$(\lambda-0)(\lambda^2-8\lambda+15) = 0$

$(\lambda-0)(\lambda^2-5\lambda-3\lambda+15) = 0$

$(\lambda-0)(\lambda(\lambda-5)-3(\lambda-5)) = 0$

$(\lambda-0)(\lambda-3)(\lambda-5) = 0$ $\quad\Rightarrow \lambda = 0, 3, 5$

At $\lambda = 0$

$\Rightarrow \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$ $\quad\Rightarrow R_1 \leftrightarrow R_3$ $\quad\Rightarrow \begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 0 \\ 5 & 0 & 0 \end{bmatrix}$

$R_3 \to R_3 + 5R_1$

$\begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ $\quad\Rightarrow R_2 \leftrightarrow R_3$

$\begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 15 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x_2 = 0$

$-x_1+3x_3 = 0 \quad x_1 = 3x_3$

$\Rightarrow \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, grid

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At $\lambda = 3$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \qquad \bullet\ R_1 \leftrightarrow R_3 \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_1$ $\qquad$ $R_1 \to R_1 \times -1$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad x_1 = k_1 \quad x_3 = k_2 \quad x_3 = 0$$

$$\Rightarrow k_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

At $\lambda = 5$ $\qquad$ $R_3 \leftrightarrow R_1$ $\qquad$ $\bullet\ R_1 \to R_1 \times (-1)$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 0 & -2 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + x_3 = 0 \quad x_1 = -x_3, \quad x_2 = 0$

$$\Rightarrow \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$

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4. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$

$\Rightarrow (A - \lambda I) = 0 \Rightarrow \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$

$\Rightarrow |A - \lambda I| = 0 \Rightarrow (0-\lambda)\left[(3-\lambda)(-2-\lambda)\right] = 0$

$(0-\lambda)\left[-6-3\lambda+2\lambda+\lambda^2\right] = 0$

$\Rightarrow +6\lambda+3\lambda^2 \cancel{\ } -2\lambda^2-\lambda^3 = 0$

$\lambda^3 - \lambda^2 - 6\lambda = 0$

$\lambda(\lambda^2 - \lambda - 6) = 0$

$\lambda(\lambda^2 - 3\lambda + 2\lambda - 6) = 0$

$\lambda(\lambda(\lambda-3) + 2(\lambda-3)) = 0$

$\Rightarrow \lambda(\lambda-3)(\lambda+2) = 0 \qquad \Rightarrow \lambda = 0, 3, -2.$

At $\lambda = 0$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \Rightarrow \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad R_2 \leftrightarrow R_3$

$\Rightarrow \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$3x_2 + 4x_3 = 0 \qquad x_2 = \frac{-4}{3}x_3, \quad x_1 = 0$

$\begin{bmatrix} 0 \\ -4/3 \\ 1 \end{bmatrix}$

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At $\lambda = 3$,

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \xrightarrow{R_1 \to -\frac{1}{3}R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \xrightarrow{R_3 \to R_3 + \frac{5}{4}R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x_1 = K_1 \text{ \& } x_3 = K_2 \text{ \& } x_2 = 0$$

$$\Rightarrow \begin{bmatrix} K_1 \\ 0 \\ K_2 \end{bmatrix}, \quad K_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + K_2 \begin{bmatrix} K_1 \\ 0 \\ 1 \end{bmatrix}$$

At $\lambda = -2$

$$\Rightarrow \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1 \times -\frac{1}{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 0$, $x_2 + 4x_3 = 0$
$x_2 = -4x_3$
$x_3 = K$, $x_2 = -4K$.

$$\Rightarrow \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix}$$

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Q. For the following matrix find one eign value without calculating & justify your answer.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

A. i) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$ $\Rightarrow$ $|A| = 0$

as $R_1 = R_2 = R_3$.

As we know that product of eign values = determinant of vector

Since determinat is 0, so we can conclude that one of its eign value will be 0.

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Assignment - 4

find the rank of the matrix A by reducing in row reduced echloon form:

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}.$$

$R_4 \to R_4 - (R_3 + R_2)$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\left.\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\}$$ There is three non-zero row present
∴ rank of Matrix A = 3.

Let , $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, find the eign values & eign vectors of A^{-1} & $A + 4I$.

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Sol $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$R_1 \to R_1 + R_2$

$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 1 & 1 \\ 0 & -3 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

$R_2 \to \frac{1}{3} R_3$

$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ -1/3 & -2/3 \end{bmatrix}$

$R_1 \to R_1 + R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\begin{bmatrix} 2/3 & 1/3 \\ -1/3 & -2/3 \end{bmatrix}$

$R_2 \to (-1) \times R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \to A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

character eqn of A^{-1} is $|A - \lambda I| = 0$

$\frac{1}{3} \left(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} \right) = \frac{1}{3} \left[(2-\lambda)^2 - 1 \right] = 0$

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$\Rightarrow \lambda^2 + 4 - 4\lambda - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda-3) - 1(\lambda-3) = 0$

$(\lambda-1)(\lambda-3) = 0$

$\Rightarrow \lambda = 1, \lambda = 3$

for $\lambda = 1$.

$$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ let $x_2 = K$

$x_1 + x_2 = 0 \Rightarrow x_1 = -x_2 = -K$

$$\Rightarrow \begin{bmatrix} K \\ -K \end{bmatrix} \Rightarrow K\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$ put $K=1 \Rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ //

for $\lambda = 3$.

$$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x_1 + x_2 = 0 \quad x_2 = K \Rightarrow x_1 = x_2 = K$

$$\Rightarrow \begin{bmatrix} K \\ K \end{bmatrix} \Rightarrow K\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ put $K=1 \Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ //

$A + 4I =$ $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} = B$

$B - \lambda I$

$= \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$

$|B - \lambda I| = 0$

$[(6-\lambda)^2 - 1] = 0$

$\lambda^2 + 36 - 12\lambda - 1 = 0$

$\lambda^2 - 12\lambda - 35 = 0$

$\lambda^2 - 7\lambda - 5\lambda - 35 = 0$

$\lambda(\lambda-7) - 5(\lambda-7) = 0$

$(\lambda-5)(\lambda-7) = 0$

$\lambda = 5, \quad \lambda = 7$

for $\lambda = 5$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 + R_1$

$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_1 - x_2 = 0 \quad x_1 = x_2 = K$

$K\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Put $K = 1$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda = 7$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - R_1$

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$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x_1 - x_2 = 0 \quad x_1 = -x_2 \quad x_2 = k \quad x_1 = -k$

$\Rightarrow k\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ put $k=1 \Rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ //

Q. 4 Solve by Gauss-Seidel method (Trace three iteration)

$3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$
with initial values $x(0)=0,\ y(0)=0,\ z(0)=0.$

Sol^n By Gauss-seidel method,

$$x_1 = \frac{7.85 + 0.1y + 0.2z}{3} \xrightarrow{y=0,\, z=0} x_1 = \frac{7.85}{3} = 2.616$$

$$y_1 = \frac{-19.3 - 0.1x + 0.3z}{7} \xrightarrow[z=0]{x=2.616} \frac{-19.3 - 0.1(2.616)}{7} = -2.7945$$

$$z_1 = \frac{71.4 - 0.3x + 0.2y}{10} \xrightarrow[y=-2.7945]{x=2.616} \frac{71.4 - 0.3(2.616) + 0.2(-2.7945)}{10}$$

$= 7.00563$

$$x_2 = \frac{7.85 + 0.1 \times y_1 + 0.2 \times z_1}{3} = \frac{7.85 + 0.1(-2.7945) + 0.2(7.00563)}{3}$$

$= 2.9905$

$$y_2 = \frac{-19.3 + 0.3(z_1) + 0.3z}{7} = \frac{-19.3 + 0.3(7.0056) + 0.1(2.9905)}{7}$$

$= -2.41418$

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$$z_2 = \frac{71.4 - 0.3x_2 + 0.2y_2}{10} = \frac{71.4 - 0.3(2.9905) + 0.2(-2.41418)}{10}$$

$$= 7.0020014$$

$$x_3 = \frac{7.85 + 0.1y_2 + 0.2z_2}{3} = \frac{7.85 + 0.1(-2.041418) + 0.2(7.00020)}{3}$$

$$= 3.0029.$$

$$y_3 = \frac{-19.3 + 0.3(z_2) + 0.1(x_3)}{7} = \frac{-19.3 + 0.3 \times 7.0020 + 0.1 \times 3.0029}{7}$$

$$= -2.41415$$

$$z_3 = \frac{71.4 - 0.3(x_3) + 0.2(y_3)}{10} = \frac{71.4 - 0.3 \times 3.0029 + 0.2 \times -2.41415}{10}$$

$$= 7.00183.$$

x_1, y_1, z_1 = 2.616, -2.794, 7.005
x_2, y_2, z_2 = 2.9905, -2.41418, 7.002
x_3, y_3, z_3 = 3.0024, -2.41415, 7.001.

Define consistency and in consistency of system of equation, then solve the eq^n in consistence.

$$x + 3y + 2z = 0$$
$$2x - y + 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

$AX = 0$ → since it is Homogeneous eq^n it is always consistent.

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$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$, $R_4 \to R_4 + 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$.
No. of variable $= 3$
$\rho(A) < n \to$ Infinite soln.

For soln:

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \to x + 3y + 2z = 0$$

$-7y - z = 0$

$z = -7y$

$x + 3y - 7y = 0$

$x = 4y$ let $y = k$

$x = 4k$, $z = 7k$, $y = k$

$$\begin{bmatrix} 4k \\ k \\ 7k \end{bmatrix} \text{ Ans}$$

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Q. Determine whether the function $T: P_2 \to P_2$ is linear Transformation or not, where $T(a+bx+cx^2) = (a+1) + (b+1)x + (c+1)x^2$

Ans. As, $T(a+bx+cx^2) = (a+1) + (b+1)x + (c+1)x^2$.

for Linear Transformation we have to check.

$\to T(u+v) = T(u) + T(v)$

$\to cT(u) = T(cu)$

$T(\cancel{u(x)+v(x)})$ $u(x) = px^2 + qx + r$

$v(x) = lx^2 + mx + n$

$T(u(x)+v(x)) = T((px^2+qx+r) + (lx^2+mx+n))$

$= T((p+l)x^2 + (q+m)x + (r+n))$

$= (p+l+1)x^2 + (q+m+1)x + (r+n+1)$ ✓

$T(u(x)) = (p+1)x^2 + (q+1)x + (r+1)$

$T(v(x)) = (l+1)x^2 + (m+1)x + (n+1)$

$T(u(x)) + T(v(x)) = (p+1+l+1)x^2 + (q+1+m+1)x + (r+1+n+1)$

$= (p+l+2)x^2 + (q+m+2)x + (r+n+2)$

As $T(u(x)+v(x)) \neq T(u(x)) + T(v(x))$

T is not a linear Transformation.

Q. Determine whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis determine the dimension of ~~the~~ and the basis of ~~sub~~ space spanned by S.

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(ii) Checking Spanning.

$$c_1\begin{pmatrix}1\\2\\3\end{pmatrix} + c_2\begin{pmatrix}3\\1\\0\end{pmatrix} + c_3\begin{pmatrix}-2\\1\\3\end{pmatrix} = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$$

where c_1, c_2 & c_3 are scalers & $(v_1, v_2, v_3) \in V$.

$$\begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix}\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$$

$AX = B$

$$[A:B] = \begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & -9 & 9\end{bmatrix}$$

$R_3 \to R_3 - \frac{9}{5}R_2$.

$$\begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & 0 & 0\end{bmatrix}$$

$\rho(A) = 2 <$ no. of variable

system has ∞ soln.

There exist a linear dependent.

$\therefore$ S span $\mathbb{R}^3$.

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Q8 Using Jacob's method (perform 3 iteration) solve the eq^n $3x-6y+2z=23$, $-4x+y-z=-15$, $x-3y+7z=16$. with initial values $x=1, y=1, z=1$.

Sol^n

$3x-6y+2z=23$

$-4x+y-z=-15$

$x-3y+7z=16$.

$x_1 = \frac{23+6y_0-2z_0}{3} = \frac{23+6\times1-2\times1}{3} = \frac{27}{3} = 9$

$y_1 = -15+z_0+4x_0 = -15+1+4\times1 = -10$

$z_1 = \frac{16-x_0+3y_0}{7} = \frac{16-1+3(1)}{7} = \frac{18}{7} = 2.5714$

$x_2 = \frac{23+6y_1-2z_1}{3} = \frac{23+6\times-10-2\times2.5714}{3} = -13.7142$

$y_2 = -15+z_1+4x_1 = -15+2.5714+4\times9 = 23.5714$

$z_2 = \frac{16-x_1+3y_1}{7} = \frac{16-9+3\times-10}{7} = -3.2857$

$x_3 = \frac{23+6y_2-2z_2}{3} = \frac{23+6(23.5714)-2(-3.2857)}{3}$

$= 51.999$

$y_3 = -15+z_2+4x_2 = -15+(-3.2857)+4(-13.7142)$

$= -69.1425$

$z_3 = \frac{16-x_2+3y_2}{7} = \frac{16-(-13.7142)+3(23.5714)}{7}$

$= 14.2040$

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Explain one application of matrix operation in image programming with example.

Ans Matrix plays fundamental role in image processing due to their ability to represent and manipulate data efficiently.
It helps in filtering, transformation & enhancement.
for ex: · Convolution, a common operation applies a filter matrix (kernel) to an image matrix to produce effects like blurring, sharpening or edge detection.

Q Give a brief description of linear transformation for computer vision for rotating 2-D image.

Ans Linear Transformation plays an important role in computer graphics to reposition the graphic on the screen & change their size or orientation.
Linear transformation rotates an image about a specified point (usually the center) by a given angle. It involves multiplying the coordinates of each pixel in the original image by a rotation matrix to obtain the coordinates of the corresponding pixel in rotated image.
Linear transformation through rotation is crucial in task like image alignment, object detection & feature extraction in computer vision applications.

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