**K-Nearest Neighbour and Hidden Markov Model**

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**Abstract:**

This report presents theoretical analysis of the results obtained on implementing the K-nearest neighbour classifier and Discrete Hidden Markov Model and practically testing them on Image and Speech datasets. This gives an analysis of how both the algorithms perform on the real-world data. Models are tested on real world image and speech datasets. There are some limitations while using these models and that has been discussed in the last section of this report

**1 Introduction**

**K-Nearest Neighbour Classifier:** This is a probably the simplest classifier for classification purpose. It is a non-parametric method so there is no need of estimation of parameters. In this classifier we classify the test example by looking K -nearest neighbouring points to that test example. The class which has maximum number of points similar to that test example is considered as class label for that test example. A commonly used method to test the similarity in varying length sequence is using Dynamic Time Warping. In this particular classifier since the length of the sequence to be matched is different for all the examples of all the classes, DTW distance is used for accounting the similarity between different pairs.

**DTW distance:** This algorithm is used specially in speech recognition It is an algorithm which gives measure of the dissimilarity of the two sequences having varying length. For same length sequences generally, Euclidean distance is preferred.

**Discrete Hidden Markov Model (DHMM classifier):** HMM’s are extensively used in speech recognition, AI and pattern recognition tasks nowadays. They are used everywhere specially to model time series data.

It is a parametric method having parameters λ = (N, M, A, B, π),

where N=No of states,

M=No of unique observation symbols,

A=State transition probability matrix,

B=state observation symbol probability,

Π=Initial state probability

In Discrete Hidden Markov Model, the number of state observation symbols are discrete, whereas in Continuous Hidden Markov Model the state observation symbol probability is a continuous probability density function. It is called Hidden Markov Model because the underlying state sequence of the observation sequence is not known to us. Parameters of the model are estimated using Baum-Welch algorithm which is basically an Expectation Maximisation Algorithm. We build a model for each class having parameters (N, M, A, B, π) and then checks the probability of the test sequence in all the classes and class label is assigned accordingly.

1. Ergodic HMM: In this model there is a finite path from every state to every other state.
2. Non-ergodic HMM: In this model from each state there is a path to only certain states and transition probabilities to other states is zero. For example: Left-Right HMM in which we can only go ahead that any state left behind cannot be visited.

**Assumptions:**

1. It is assumed that the system being modelled is first order Hidden Markov model which means that output at time ‘t’ depends only on the what happened at time ‘t-1’ and not before.
2. It is Discrete Hidden Markov Model.
3. Values of parameters N and M is same for all the different classes.

**2 Baum-Welch Algorithm**

This algorithm is used for estimation for parameters for Hidden Markov Model. It is an Expectation-Maximisation Algorithm.

Procedure:

Step 1: Initialise the parameters λold.

Step 2: Estimate ξt(i,j) and γt(i) using forward (αt(i)) and backward variables (βt(Vk)) corresponding to λold (Expectation Step).

ξt (i,j) = = Probability of being in state ‘i’ at time ‘t’ and state ‘j’ at time ‘t+1’.

γt (i) = = Probability of transition from state ‘i’ at time ‘t’.

Step 3: Re-estimate parameters λnew using ξt(I,j) and γt(i) (Maximisation Step).

=

bi (Vk) =

πi = γ1(i)

Step 4: Repeat Step 2 and Step 3 till convergence. Convergence condition can be

1. Difference of Log likelihood of the data is less than some threshold.
2. We can repeat steps some fixed number of times.

After estimating parameters, we calculate the probability of test sequence in all the classes and the class having maximum probability is assigned as the class label.

P () = =Probability of an observation sequence given λ.

**3 Feature Extraction**

For image dataset: A image was divided into 64\*64 blocks and a 24-dimension colour histogram vector is computed corresponding to each non overlapping block in a image. In this way we got a sequence of vectors or observation symbols corresponding to each image.

For Speech dataset: Speech dataset is given as the 39-dimensional mel frequency cepstral coefficient (MFCC) features extracted frame by frame from utterances for a particular CV segment by multiple people.

**4 Results and Inferences for K-Nearest Neighbour Method**

K represents how much nearest neighbours we have to assign the class label to a particular example.

1. Image Dataset:
2. For K=3

|  |  |  |
| --- | --- | --- |
| 4 | 6 | 6 |
| 44 | 40 | 39 |
| 2 | 4 | 5 |

Table 1: Confusion Matrix

Accuracy: 0.32666666666666666 Precision for class 0 : 0.25 Recall for class 0 : 0.08 F-measure for class 0 ; 0.12121212121212122 Precision for class 1 : 0.3252032520325203 Recall for class 1 : 0.8 F-measure for class 1 ; 0.46242774566473993 Precision for class 2 : 0.45454545454545453 Recall for class 2 : 0.1 F-measure for class 2 ; 0.16393442622950818 Mean Precision: 0.34324956885932495 Mean Recall: 0.32666666666666666 Mean F-measure: 0.24919143103545646

1. For K=5

|  |  |  |
| --- | --- | --- |
| 3 | 7 | 3 |
| 43 | 41 | 39 |
| 4 | 2 | 8 |

Table 2: Confusion Matrix

Accuracy: 0.3466666666666667 Precision for class 0 : 0.23076923076923078 Recall for class 0 : 0.06 F-measure for class 0 ; 0.09523809523809523 Precision for class 1 : 0.3333333333333333 Recall for class 1 : 0.82 F-measure for class 1 ; 0.47398843930635837 Precision for class 2 : 0.5714285714285714 Recall for class 2 : 0.16 F-measure for class 2 ; 0.25 Mean Precision: 0.3785103785103785 Mean Recall: 0.3466666666666666 Mean F-measure: 0.27307551151481785

1. For K=11

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 0 |
| 43 | 48 | 42 |
| 6 | 2 | 8 |

Table 3: Confusion Matrix

Accuracy: 0.38 Precision for class 0 : 1.0 Recall for class 0 : 0.02 F-measure for class 0 ; 0.0392156862745098 Precision for class 1 : 0.3609022556390977 Recall for class 1 : 0.96 F-measure for class 1 ; 0.5245901639344263 Precision for class 2 : 0.5 Recall for class 2 : 0.16 F-measure for class 2 ; 0.24242424242424243 Mean Precision: 0.6203007518796992 Mean Recall: 0.37999999999999995 Mean F-measure: 0.2687433642110595

1. For K=20

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 0 |
| 40 | 48 | 37 |
| 9 | 2 | 13 |

Table 4: Confusion Matrix

Accuracy: 0.41333333333333333 Precision for class 0 : 1.0 Recall for class 0 : 0.02 F-measure for class 0 ; 0.0392156862745098 Precision for class 1 : 0.384 Recall for class 1 : 0.96 F-measure for class 1 ; 0.5485714285714286 Precision for class 2 : 0.5416666666666666 Recall for class 2 : 0.26 F-measure for class 2 ; 0.35135135135135137 Mean Precision: 0.6418888888888888 Mean Recall: 0.41333333333333333 Mean F-measure: 0.3130461553990966

Inference:

The classification accuracy with respect to different values of K is shown below:

Fig 1: Chart showing variation in Accuracy v/s K

As the value of K increases the classification accuracy is not following any particular pattern or distribution. This can be much clearer in KNN on speech Dataset. This is because as we continuously expand the neighbourhood of a point it keeps on jumping in different classes. The value of K for which it maximises accuracy depends on the dataset.

1. Speech Dataset:
2. For K=3

|  |  |  |
| --- | --- | --- |
| 29 | 25 | 8 |
| 6 | 17 | 3 |
| 1 | 4 | 10 |

Table 5: Confusion Matrix

Accuracy: 0.5436893203883495 Precision for class 0 : 0.46774193548387094 Recall for class 0 : 0.8055555555555556 F-measure for class 0 ; 0.5918367346938775 Precision for class 1 : 0.6538461538461539 Recall for class 1 : 0.3695652173913043 F-measure for class 1 ; 0.4722222222222222 Precision for class 2 : 0.6666666666666666 Recall for class 2 : 0.47619047619047616 F-measure for class 2 ; 0.5555555555555556 Mean Precision: 0.5960849186655638 Mean Recall: 0.5504370830457788 Mean F-measure: 0.5398715041572185

1. For K=5

|  |  |  |
| --- | --- | --- |
| 28 | 21 | 7 |
| 8 | 23 | 4 |
| 0 | 2 | 10 |

Table 6: Confusion Matrix

Accuracy: 0.5922330097087378 Precision for class 0 : 0.5 Recall for class 0 : 0.7777777777777778 F-measure for class 0 ; 0.6086956521739131 Precision for class 1 : 0.6571428571428571 Recall for class 1 : 0.5 F-measure for class 1 ; 0.5679012345679013 Precision for class 2 : 0.8333333333333334 Recall for class 2 : 0.47619047619047616 F-measure for class 2 ; 0.6060606060606061 Mean Precision: 0.6634920634920635 Mean Recall: 0.5846560846560847 Mean F-measure: 0.5942191642674736

1. For K=7

|  |  |  |
| --- | --- | --- |
| 26 | 21 | 7 |
| 10 | 21 | 6 |
| 0 | 4 | 8 |

Table 7: Confusion Matrix

Accuracy: 0.5339805825242718 Precision for class 0 : 0.48148148148148145 Recall for class 0 : 0.7222222222222222 F-measure for class 0 ; 0.5777777777777777 Precision for class 1 : 0.5675675675675675 Recall for class 1 : 0.45652173913043476 F-measure for class 1 ; 0.506024096385542 Precision for class 2 : 0.6666666666666666 Recall for class 2 : 0.38095238095238093 F-measure for class 2 ; 0.4848484848484849 Mean Precision: 0.5719052385719051 Mean Recall: 0.519898780768346 Mean F-measure: 0.5228834530039349

1. For K=11

|  |  |  |
| --- | --- | --- |
| 28 | 23 | 8 |
| 8 | 22 | 6 |
| 0 | 1 | 7 |

Table 8: Confusion Matrix

Accuracy: 0.5533980582524272 Precision for class 0 : 0.4745762711864407 Recall for class 0 : 0.7777777777777778 F-measure for class 0 ; 0.5894736842105264 Precision for class 1 : 0.6111111111111112 Recall for class 1 : 0.4782608695652174 F-measure for class 1 ; 0.5365853658536586 Precision for class 2 : 0.875 Recall for class 2 : 0.3333333333333333 F-measure for class 2 ; 0.48275862068965514 Mean Precision: 0.6535624607658507 Mean Recall: 0.5297906602254429 Mean F-measure: 0.5362725569179467

1. For K=17

|  |  |  |
| --- | --- | --- |
| 27 | 19 | 7 |
| 9 | 25 | 6 |
| 0 | 2 | 8 |

Table 9: Confusion Matrix

Accuracy: 0.5825242718446602 Precision for class 0 : 0.5094339622641509 Recall for class 0 : 0.75 F-measure for class 0 ; 0.6067415730337078 Precision for class 1 : 0.625 Recall for class 1 : 0.5434782608695652 F-measure for class 1 ; 0.5813953488372093 Precision for class 2 : 0.8 Recall for class 2 : 0.38095238095238093 F-measure for class 2 ; 0.5161290322580645 Mean Precision: 0.644811320754717 Mean Recall: 0.5581435472739821 Mean F-measure: 0.5680886513763271

1. For K = 19

|  |  |  |
| --- | --- | --- |
| 28 | 17 | 7 |
| 8 | 27 | 7 |
| 0 | 2 | 7 |

Table 10: Confusion Matrix

Accuracy: 0.6019417475728155 Precision for class 0 : 0.5384615384615384 Recall for class 0 : 0.7777777777777778 F-measure for class 0 ; 0.6363636363636364 Precision for class 1 : 0.6428571428571429 Recall for class 1 : 0.5869565217391305 F-measure for class 1 ; 0.6136363636363638 Precision for class 2 : 0.7777777777777778 Recall for class 2 : 0.3333333333333333 F-measure for class 2 ; 0.4666666666666666 Mean Precision: 0.6530321530321531 Mean Recall: 0.5660225442834138 Mean F-measure: 0.5722222222222222

1. For K = 23

|  |  |  |
| --- | --- | --- |
| 24 | 18 | 9 |
| 11 | 27 | 7 |
| 1 | 1 | 5 |

Table 11: Confusion Matrix

Accuracy: 0.5436893203883495 Precision for class 0 : 0.47058823529411764 Recall for class 0 : 0.6666666666666666 F-measure for class 0 ; 0.5517241379310345 Precision for class 1 : 0.6 Recall for class 1 : 0.5869565217391305 F-measure for class 1 ; 0.5934065934065934 Precision for class 2 : 0.7142857142857143 Recall for class 2 : 0.23809523809523808 F-measure for class 2 ; 0.35714285714285715 Mean Precision: 0.5949579831932773 Mean Recall: 0.4972394755003451 Mean F-measure: 0.5007578628268283

1. For K = 31

|  |  |  |
| --- | --- | --- |
| 21 | 16 | 8 |
| 14 | 30 | 7 |
| 1 | 0 | 6 |

Table 12: Confusion Matrix

Accuracy: 0.5533980582524272 Precision for class 0 : 0.4666666666666667 Recall for class 0 : 0.5833333333333334 F-measure for class 0 ; 0.5185185185185186 Precision for class 1 : 0.5882352941176471 Recall for class 1 : 0.6521739130434783 F-measure for class 1 ; 0.6185567010309279 Precision for class 2 : 0.8571428571428571 Recall for class 2 : 0.2857142857142857 F-measure for class 2 ; 0.42857142857142855 Mean Precision: 0.6373482726423902 Mean Recall: 0.5070738440303657 Mean F-measure: 0.5218822160402917

Inference:

Fig 2: Chart showing variation in accuracy w.r.t. K

As the value of K increases the classification accuracy is increasing and decreasing without any particular pattern. This is because as we continuously expand the neighbourhood of a point it keeps on jumping in different classes. The value of K for which it maximises accuracy depends on the dataset.

PS. It is good practice to choose the values of K which are even or multiple of the no of classes because it may lead to the test example having equal probability in multiple classes.

**5 Results and Inferences for Discrete Hidden Markov Model**

Significance of N and M in both models:

In image dataset

N= This has no physical significance but it plays its role in deciding if sky(blue) can come after mountain(brown) in particular class or not.

M= No of different colour entities like sky, trees, walls etc.

For speech dataset

N= different alphabets that are part of the Speech Dataset.

M=Frequency or speed with which different people speaks.

1. Ergodic Discrete Hidden Markov Model for image dataset:
2. For N=5 and M=7

|  |  |  |
| --- | --- | --- |
| 42 | 30 | 33 |
| 1 | 0 | 3 |
| 7 | 20 | 14 |

Table 13: Confusion Matrix

Accuracy: 0.37333333333333335 Precision for class 0 : 0.4 Recall for class 0 : 0.84 F-measure for class 0 ; 0.5419354838709678 Precision for class 1 : 0.0 Recall for class 1 : 0.0 Precision for class 2 : 0.34146341463414637 Recall for class 2 : 0.28 F-measure for class 2 ; 0.3076923076923077 Mean Precision: 0.24715447154471545 Mean Recall: 0.37333333333333335 Mean F-measure: 0.2832092638544252

1. For N=5 and M=11

|  |  |  |
| --- | --- | --- |
| 41 | 29 | 33 |
| 0 | 0 | 0 |
| 9 | 21 | 17 |

Table 14: Confusion Matrix

Accuracy: 0.38666666666666666 Precision for class 0 : 0.39805825242718446 Recall for class 0 : 0.82 F-measure for class 0 ; 0.5359477124183006 Recall for class 1 : 0.0 F-measure for class 1 ; 0.0 Precision for class 2 : 0.3617021276595745 Recall for class 2 : 0.34 F-measure for class 2 ; 0.3505154639175258 Mean Precision: 0.25325346002891963 Mean Recall: 0.38666666666666666 Mean F-measure: 0.29548772544527546

1. For N=5 and M=17

|  |  |  |
| --- | --- | --- |
| 39 | 28 | 31 |
| 0 | 0 | 1 |
| 11 | 22 | 18 |

Table 15: Confusion Matrix

Accuracy: 0.38 Precision for class 0 : 0.3979591836734694 Recall for class 0 : 0.78 F-measure for class 0 ; 0.5270270270270271 Precision for class 1 : 0.0 Recall for class 1 : 0.0 Precision for class 2 : 0.35294117647058826 Recall for class 2 : 0.36 F-measure for class 2 ; 0.3564356435643565 Mean Precision: 0.2503001200480192 Mean Recall: 0.38000000000000006 Mean F-measure: 0.2944875568637945

1. For N=5 and M=23

|  |  |  |
| --- | --- | --- |
| 35 | 24 | 29 |
| 0 | 0 | 0 |
| 15 | 26 | 21 |

Table 16: Confusion Matrix

Accuracy: 0.37333333333333335 Precision for class 0 : 0.3977272727272727 Recall for class 0 : 0.7 F-measure for class 0 ; 0.5072463768115942 Recall for class 1 : 0.0 F-measure for class 1 ; 0.0 Precision for class 2 : 0.3387096774193548 Recall for class 2 : 0.42 F-measure for class 2 ; 0.375 Mean Precision: 0.24547898338220916 Mean Recall: 0.3733333333333333 Mean F-measure: 0.29408212560386476

1. For N = 5 and M = 31

|  |  |  |
| --- | --- | --- |
| 37 | 26 | 22 |
| 0 | 0 | 0 |
| 13 | 24 | 28 |

Table 17: Confusion Matrix

Accuracy: 0.43333333333333335 Precision for class 0 : 0.43529411764705883 Recall for class 0 : 0.74 F-measure for class 0 ; 0.5481481481481482 Recall for class 1 : 0.0 F-measure for class 1 ; 0.0 Precision for class 2 : 0.4307692307692308 Recall for class 2 : 0.56 F-measure for class 2 ; 0.4869565217391305 Mean Precision: 0.2886877828054299 Mean Recall: 0.43333333333333335 Mean F-measure: 0.3450348899624262

1. For N = 3 and M = 7

|  |  |  |
| --- | --- | --- |
| 42 | 30 | 33 |
| 1 | 0 | 3 |
| 7 | 20 | 14 |

Table 18: Confusion Matrix

Accuracy: 0.37333333333333335 Precision for class 0 : 0.4 Recall for class 0 : 0.84 F-measure for class 0 ; 0.5419354838709678 Precision for class 1 : 0.0 Recall for class 1 : 0.0 Precision for class 2 : 0.34146341463414637 Recall for class 2 : 0.28 F-measure for class 2 ; 0.3076923076923077 Mean Precision: 0.24715447154471545 Mean Recall: 0.37333333333333335 Mean F-measure: 0.2832092638544252

Inference:

Fig 3: Ergodic HMM Accuracy varying with M at N=5

As we know that M represents different colour identities present in the dataset like sky(blue), mountains(brown), etc, so as we increase the value of M the accuracy will sometimes increase then decrease and at some particular value it will reach global maximum and that value of M separates the different colour identities in all the classes in a best possible way and will lead to a good classification accuracy.

The first dataset was of Coast images, second was of Kennel outdoor images, and third was of indoor Volleyball Court images. One interesting thing to note is that the number of examples classified in the 2nd class is very less as compared to other classes. This may be because the kennel image set has background resembling to that of coast images and kennel resembling more to volleyball indoor images and as we are considering only colour based features of an image which is resulting in misclassification of images.

1. Non-ergodic Discrete Hidden Markov Model for Speech Dataset:
2. For N=5 and M=11

|  |  |  |
| --- | --- | --- |
| 21 | 20 | 12 |
| 10 | 13 | 4 |
| 5 | 13 | 5 |

Table19: Confusion Matrix

Accuracy: 0.3786407766990291 Precision for class 0 : 0.39622641509433965 Recall for class 0 : 0.5833333333333334 F-measure for class 0 ; 0.4719101123595506 Precision for class 1 : 0.48148148148148145 Recall for class 1 : 0.2826086956521739 F-measure for class 1 ; 0.3561643835616438 Precision for class 2 : 0.21739130434782608 Recall for class 2 : 0.23809523809523808 F-measure for class 2 ; 0.22727272727272724 Mean Precision: 0.3650330669745491 Mean Recall: 0.3680124223602485 Mean F-measure: 0.3517824077313072

1. For N=5 and M=17

|  |  |  |
| --- | --- | --- |
| 16 | 16 | 9 |
| 19 | 23 | 7 |
| 1 | 7 | 5 |

Table 20: Confusion Matrix

Accuracy: 0.42718446601941745 Precision for class 0 : 0.3902439024390244 Recall for class 0 : 0.4444444444444444 F-measure for class 0 ; 0.4155844155844156 Precision for class 1 : 0.46938775510204084 Recall for class 1 : 0.5 F-measure for class 1 ; 0.4842105263157895 Precision for class 2 : 0.38461538461538464 Recall for class 2 : 0.23809523809523808 F-measure for class 2 ; 0.2941176470588235 Mean Precision: 0.4147490140521499 Mean Recall: 0.3941798941798942 Mean F-measure: 0.39797086298634293

1. For N=5 and M=19

|  |  |  |
| --- | --- | --- |
| 18 | 19 | 9 |
| 9 | 6 | 2 |
| 9 | 21 | 10 |

Table 21: Confusion Matrix

Accuracy: 0.3300970873786408 Precision for class 0 : 0.391304347826087 Recall for class 0 : 0.5 F-measure for class 0 ; 0.4390243902439025 Precision for class 1 : 0.35294117647058826 Recall for class 1 : 0.13043478260869565 F-measure for class 1 ; 0.1904761904761905 Precision for class 2 : 0.25 Recall for class 2 : 0.47619047619047616 F-measure for class 2 ; 0.32786885245901637 Mean Precision: 0.33141517476555843 Mean Recall: 0.3688750862663906 Mean F-measure: 0.31912314439303646

1. For N=2 and M=11

|  |  |  |
| --- | --- | --- |
| 10 | 7 | 6 |
| 23 | 34 | 12 |
| 3 | 5 | 3 |

Table 22: Confusion Matrix

Accuracy: 0.4563106796116505 Precision for class 0 : 0.43478260869565216 Recall for class 0 : 0.2777777777777778 F-measure for class 0 ; 0.3389830508474576 Precision for class 1 : 0.4927536231884058 Recall for class 1 : 0.7391304347826086 F-measure for class 1 ; 0.591304347826087 Precision for class 2 : 0.2727272727272727 Recall for class 2 : 0.14285714285714285 F-measure for class 2 ; 0.18749999999999997 Mean Precision: 0.4000878348704435 Mean Recall: 0.3865884518058431 Mean F-measure: 0.37259579955784816

1. For N=3 and M=11

|  |  |  |
| --- | --- | --- |
| 9 | 6 | 4 |
| 21 | 34 | 12 |
| 6 | 6 | 5 |

Table 23: Confusion Matrix

Accuracy: 0.46601941747572817 Precision for class 0 : 0.47368421052631576 Recall for class 0 : 0.25 F-measure for class 0 ; 0.32727272727272727 Precision for class 1 : 0.5074626865671642 Recall for class 1 : 0.7391304347826086 F-measure for class 1 ; 0.6017699115044248 Precision for class 2 : 0.29411764705882354 Recall for class 2 : 0.23809523809523808 F-measure for class 2 ; 0.2631578947368421 Mean Precision: 0.4250881813841012 Mean Recall: 0.4090752242926156 Mean F-measure: 0.3974001778379981

1. For N=4 and M=11

|  |  |  |
| --- | --- | --- |
| 14 | 9 | 9 |
| 13 | 16 | 4 |
| 9 | 21 | 8 |

Table 24: Confusion Matrix

Accuracy: 0.36893203883495146 Precision for class 0 : 0.4375 Recall for class 0 : 0.3888888888888889 F-measure for class 0 ; 0.411764705882353 Precision for class 1 : 0.48484848484848486 Recall for class 1 : 0.34782608695652173 F-measure for class 1 ; 0.4050632911392405 Precision for class 2 : 0.21052631578947367 Recall for class 2 : 0.38095238095238093 F-measure for class 2 ; 0.2711864406779661 Mean Precision: 0.3776249335459862 Mean Recall: 0.37255578559926383 Mean F-measure: 0.3626714792331865

Inferences:

Fig 4: Non-ergodic HMM Accuracy varying with M at N=5

As we know that the significance of M is the different frequency or speed with which the alphabet was spoken. As we have dataset of CV segment spoken by different people it so happens that the value of M for some small group of people is same or very close to each other. So, at those values of M that particular group of people will be classified precisely and will lead to good classification accuracy for that particular value of M.

That’s why we are getting some peaks while varying the value of M for same value of N. So maximum classification accuracy is very likely to come around these maxims as these maxims lead to precisely classify CV segment spoken by a particular group of people. This particular data justifies the claim too as the maximum accuracy with testing so far is around N=3 and M=11(where there is a maxima).

Fig 5: Non-ergodic HMM Accuracy varying with N at M=11

Inference:

As we know that significance of N is the number of alphabets spoken. Speech dataset used was having utterances for a particular CV segment by multiple people. CV segment were of two alphabets. As we can clearly see from the Fig 6 that for a constant value of M=11, we are getting accuracy maximum around N=2 and N=3 which satisfies the theoretical predictions. It also justifies the significance of N in this model.

**6 Limitations while implementing hidden Markov model**

Few limitations encountered while implementation:

1. Forward and backward variables tend to zero: When the time series is too long the value of the forward and backward variables approaches to zero. They become so small that they go below the precision of any machine which makes their value equal to zero. This also makes the probability of the sequence zero which lead to math domain errors while estimating parameters and also lead to misclassify test examples.
2. Initial values of parameters: In a parametric method of classification how fast the parameters converges and how good the solution obtained is also depends on the initial values of the parameters. It might be that you start with some random values of the parameters and end up in some local minima/maxima. So, it is important to initialise parameters appropriately. Well there is no simple solution to this problem. One solution could be to give make all the transitions from one state to another and state observation probabilities equiprobable and let it left to the model to converge to a good estimation.

-------------------------------------------THE END----------------------------------------