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Line intigral, surface intrigral and volume intrigral –

- Line intigral → $\int_a^b \vec{A}\cdot dl$

- Surface intigral → $\int_s \vec{A}\cdot ds$ / $\iint \vec{A}\cdot dx\,dy$

- Volume intigral → $\int_v \vec{A}\cdot dv$ / $\iiint \vec{A}\cdot dx\,dy\,dz$

- $\oint_s \vec{A}\,\vec{ds} = \int_v (\vec{\nabla}\cdot\vec{A})\,dv$

$$\nabla A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- Ex $= \vec{A} = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\,\hat{z}$

$$\nabla A = 0 + 2x + 2y = 2(x+y)$$

Now, $\int(\vec{\nabla}\cdot\vec{A})\,dv = \int 2(x+y)\,dx\,dy\,dz$

$$= 2\left[\frac{x^2}{2}yz + \frac{xy^2}{2}z\right] = \left[x^2yz + xy^2z\right]_0^1 = \underline{\underline{2}}$$

→ ① $dy\,dz\,\hat{x}$ $(x=1)$
② $dy\,dz(-\hat{x})$ $(x=0)$
③ $dx\,dy(\hat{z})$ $(z=1)$
④ $dx\,dy(-\hat{z})$ $(z=0)$
⑤ $dx\,dz(\hat{y})$ $(y=1)$
⑥ $dx\,dz(-\hat{y})$ $(y=0)$

→ $\int A\cdot dy\,dz\,\hat{x} = \int y^2x + (2xy+z^2)\hat{y} + (2yz)\hat{z}$

$(\because x=1)$

$$= \int_0^1 y^2x\,dy\,dz = \frac{y^3}{3}z = \underline{\underline{1/3}} \quad ①$$

→ Same as priviously → $\underline{\underline{-1/3}}$ ②

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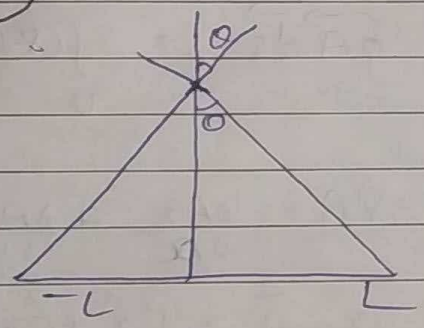
(3) at $z=1$, (1)
(4) at $z=0$, (0)
(5) at $y=1$, (1/2)
(6) at $y=0$, (1/2)

(1)+(2)+(3)+(4)+(5)+(6) = 2 R.M.S

So, L.M.S = R.M.S

- Superposition of charge

- 

$$E = \int dE = \int_{-L}^{L} \frac{\lambda\, dx}{4\pi\epsilon_0 r^2} \cos\theta \quad \text{(important)}$$

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Electrostatic

- Del operator :-

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

→ ∇ operator is vector operator.
→ it can act as –
(i) over a scalar funct^n (f)
∇f = Gradient

(ii) over a vector funct^n ($\vec{A}$) by a dot product

$\nabla \cdot \vec{A}$ ⇒ Divergence.

(iii) over a vector funct^n ($\vec{A}$) by a cross product

$\nabla \times \vec{A}$ ⇒ curl

- Gradient → Del with scalar.
  Ex- temp, pressure, potential.

Ex- $f(x, y, z)$

$$\text{Grad } f = \nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

Gradient of scalar funct^n is vector.

→ Physical significance :-

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$|\nabla T|$:- magnitude of gradient of a scalar funct^n measure the slope/rate of change.

Q. find gradient of scalar funct^n –

$f(x,y,z) = x+y+z$

⇒ Gradient $= \nabla f$

$\cancel{= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(x\hat{i}+y\hat{j}+z\hat{k})}$

$= \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$

$= \hat{i}\frac{\partial(x+y+z)}{\partial x} + \hat{j}\frac{\partial(x+y+z)}{\partial y} + \hat{k}\frac{\partial(x+y+z)}{\partial z}$

$= \hat{i}\cdot 1 + \hat{j}\cdot 1 + \hat{k}\cdot 1$

$= \hat{i}+\hat{j}+\hat{k}$

Imp

Q. find the Gradient of mag. of posit^n vector.

⇒ $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$|\vec{r}| = \sqrt{x^2+y^2+z^2} = f(x,y,z)$

$\nabla|\vec{r}| = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$

$= \hat{i}\frac{\partial(\sqrt{x^2+y^2+z^2})}{\partial x} + \hat{j}\frac{\partial(\sqrt{x^2+y^2+z^2})}{\partial y} + \hat{k}\frac{\partial(\sqrt{x^2+y^2+z^2})}{\partial z}$

$$\hat{i}\cdot\frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2x + \hat{j}\cdot\frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2y + \hat{k}\cdot\frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2z$$

$$\frac{\cancel{2}}{\cancel{2}\sqrt{x^2+y^2+z^2}}\left[\hat{i}\cdot x + \hat{j}\cdot y + \hat{k}\cdot z\right]$$

So, $\nabla|\vec{r}| = \dfrac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \dfrac{\vec{r}}{|\vec{r}|} = \hat{r}$ Ans

- Divergence :-

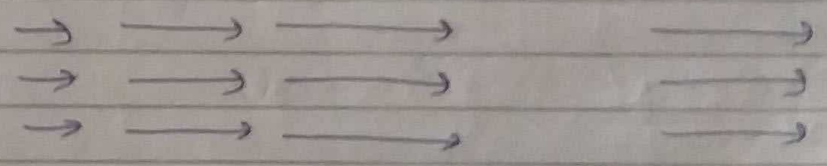
$$V(x,y,z) = \hat{i}v_x + \hat{j}v_y + \hat{k}v_z$$

$$\text{Div}(V) = \nabla\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

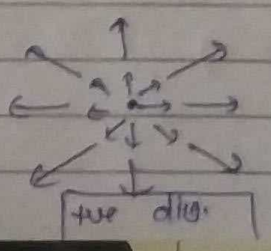
(it is a scalar.)

→ • $\nabla\cdot\vec{v} = 0$, The vector f^n is called Solenoidal vector.

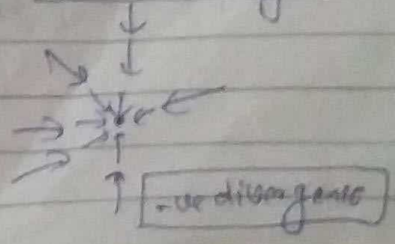
→ Physical significance :-



+ve divergence



Zero divergence



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→ $\nabla \cdot \vec{v}$ measures how much a vector f^n spread.

- Curl :-

$$\vec{A} = A_x \hat{j} + A_y \hat{k} + A_z \hat{k}$$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

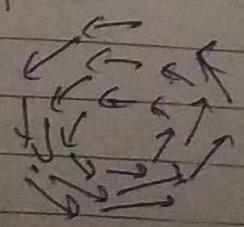
$$\vec{\nabla} \times \vec{A} = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

→ Curl of vector is a vector.

→ $\vec{\nabla} \times \vec{A} = 0$, then the vector field is irrotational.

$\vec{\nabla} \times \vec{E} = 0$ so $\vec{E} = -\Delta V$

<u>Physical Significance</u> :-



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Curl of a vector field measure how much the vector curl around.

Q. find a curl of $\vec{A} = xy\hat{i} + yz\hat{j} + zx\hat{k}$ at $(1,1,1)$

$$\Rightarrow \vec{\nabla} \times \vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(xy\hat{i} + yz\hat{j} + zx\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & yz & zx \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial(zx)}{\partial y} - \frac{\partial(yz)}{\partial z}\right) - \hat{j}\left(\frac{\partial(zx)}{\partial x} - \frac{\partial(xy)}{\partial z}\right) + \hat{k}\left(\frac{\partial(yz)}{\partial x} - \frac{\partial(xy)}{\partial y}\right)$$

$$= -y\hat{i} - z\hat{j} - x\hat{k}$$

$$= -\hat{i} - \hat{j} - \hat{k}$$

- Gauss theorum and stokes theorum $\Longrightarrow$

Gauss theorem is also called fundamental theoreom of divergence.

$$* \boxed{\oint_S \vec{A} \cdot \vec{ds} = \int_v (\nabla \cdot \vec{A})\, dv}$$

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- The surface intigral of a vector over a closed surface is equal to vol. intigral of divergence of that vector enclosed by the surface.

<u>Stokes theorem</u> → (fundamental Theorem of curl)

$$\boxed{\oint_L \vec{A}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{A})\cdot ds}$$

The line intigral of a vector over a closed curve is equal to the surface intigral of curl of that vector enclosed by the curve.

- Coulumb's law :-

$$\boxed{f = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}}$$

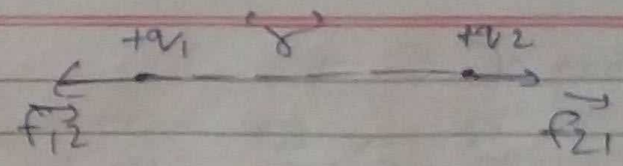
$\frac{1}{4\pi\varepsilon_0} = K = 9\times10^9\ Nm^2/C^2$

↓ Permi Tivity of free space

In vector →

$$* \boxed{f = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r^2}\cdot\hat{r}}$$

- E.S force follow inverse square law
- E.S. force are central force.
- E.S. forces are conservative in nature.



$$\vec{f}_{21} = K \frac{q_1 q_2}{r^2} (+\hat{r})$$

$$\vec{f}_{12} = K \frac{q_1 q_2}{r^2} (-\hat{r})$$

$$* \boxed{\vec{f}_{21} = -\vec{f}_{12}}$$

Electrostatic force obey Newton's third Law.

- Electric field →

$$f = qE$$

$$E = f/q$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$$

In vector →

$$* \boxed{\vec{E} = K \frac{Q}{r^2} \hat{r}}$$

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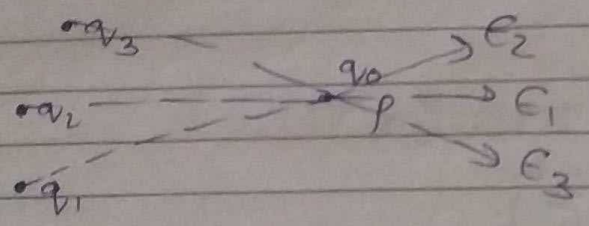
Superposition of $E \rightarrow$

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \quad \text{(vector sum)}$$



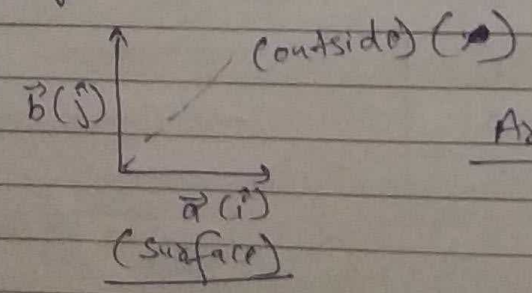
- Electric flux :—

(Uniform electric field)

$\phi = EA\cos\theta$

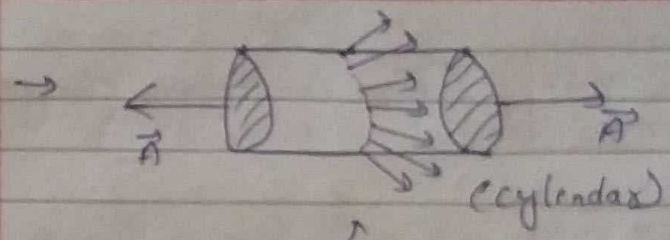
$$\boxed{\oint E \cdot dA = \phi}$$

The no. of Electric field lines passing normally through a surface is called as electric flux.

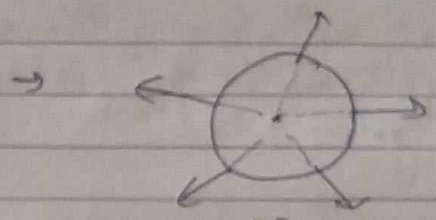


Area vector

→ in Symmetrical object, Area vector always outside normally.

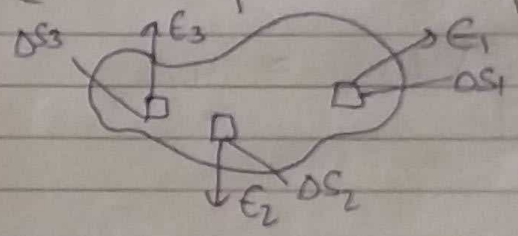


(cylendar)



(sphere)

(Non- uniform electric field)



$$\Delta \phi_1 = \vec{E}_1 \cdot \vec{\Delta S}_1$$
$$\Delta \phi_2 = \vec{E}_2 \cdot \Delta \vec{S}_2$$
$$\Delta \phi_3 = \vec{E}_3 \cdot \vec{\Delta} S_3$$
$$\vdots$$
$$\Delta \phi_n = \vec{E}_n \cdot \vec{\Delta} S_n$$

$$\phi_{net} = \vec{E}_1 \cdot \vec{\Delta S}_1 + \vec{E}_2 \cdot \vec{\Delta S}_2 + \vec{E}_3 \cdot \vec{\Delta S}_3 - - - + \vec{E}_n \cdot \vec{\Delta S}_n$$

$$\phi_{net} = \lim_{\substack{n \to \infty \\ \Delta S \to 0}} \sum_{i=1}^{n} \vec{E}_i \cdot \Delta \vec{S}_i$$

$$\boxed{\phi_{net} = \oint_S \vec{E} \cdot \vec{ds}}$$

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$\rightarrow \boxed{\oint E \cdot ds = \frac{q_{net}}{\varepsilon_0}}$ (for continuous charge distributn)

Gauss state "The net E.F passing through the enclosed surface is equal to the $1/\varepsilon_0$ times the enclosed net charge".

$\boxed{\lambda = \frac{q}{L}, \ \sigma = \frac{q}{A}, \ \rho = \frac{q}{v}}$ charge distributn

- Differential form of gauss law

$$\oint_s \vec{E} \cdot \vec{ds} = \int (\vec{\nabla} \cdot \vec{E}) \, dv = q/\varepsilon_0$$

$$\int (\vec{\nabla} \cdot \vec{E}) \, dv = q/\varepsilon_0$$

($\because$) Volume charge distributn $\Rightarrow$

$$q = \int \rho \, dv$$

$$\int (\vec{\nabla} \cdot \vec{E}) \, dv - \frac{1}{\varepsilon_0} \int \rho \, dv = 0$$

$$\vec{\nabla} \cdot \vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

$$* \ \boxed{\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0}$$

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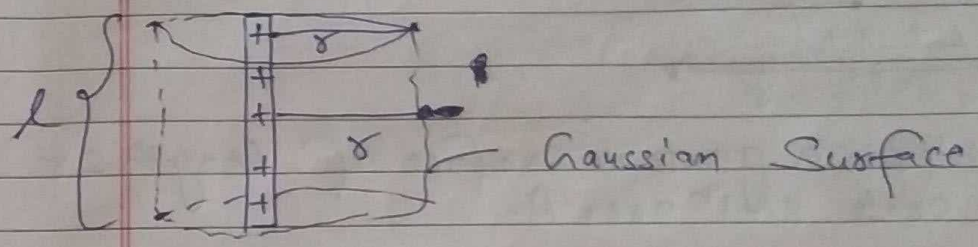
- Electric field due to infinity long charged wire :-



$\because \quad \phi = \oint E \cdot d\vec{s} = q/\varepsilon_0$

$$\boxed{\text{flux} = E \cdot 2\pi r l}$$

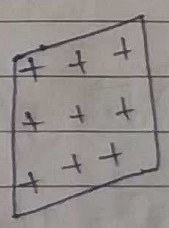
$$E \cdot 2\pi r l = q/\varepsilon_0$$

$$E = \frac{q}{2\pi r l \varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{r}}$$

$E \propto \frac{1}{r}$

- Electric field due to infinity (uniformly) charged plane sheet :-



surface charge density $= \sigma = q/A$

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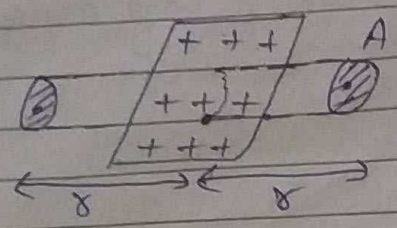


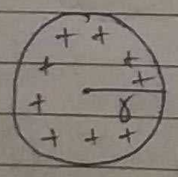
Consider a gaussian surface of length 2r and cross sect^n area A.

$$\oint \vec{E}\cdot\vec{ds} = q/\varepsilon_0 = \phi$$

$$\phi = 2EA = q/\varepsilon_0 = \frac{\sigma A}{\varepsilon_0}$$

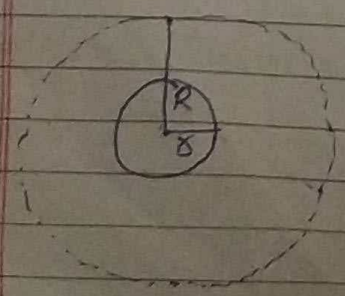
$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}}$$

- E.f. Due to uniformly charged sphere :-



(i) outside (ii) Surface (iii) inside

(i) outside →



$$\text{flux} = \phi = \oint \vec{E}\cdot\vec{ds} = q/\varepsilon_0$$

$$E\int_0^R ds = E\cdot 4\pi r^2 = q/\varepsilon_0$$

$$E\cdot 4\pi R^2 = \frac{\rho v}{\varepsilon_0}$$

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$$E \cdot \cancel{4\pi R^2} = \frac{\rho \times \cancel{4}/3 \cancel{\pi} R^{\cancel{3}}}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho R}{3\varepsilon_0}\hat{R}} \quad \text{or} \quad \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\hat{R}}$$

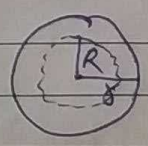
(ii) at surface ($r = R$)

$$\oint \vec{E}\cdot\vec{ds} = \phi = q/\varepsilon_0$$

$$E \cdot 4\pi r^2 = q/\varepsilon_0$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$

(iii) inside the sphere ($R < r$)

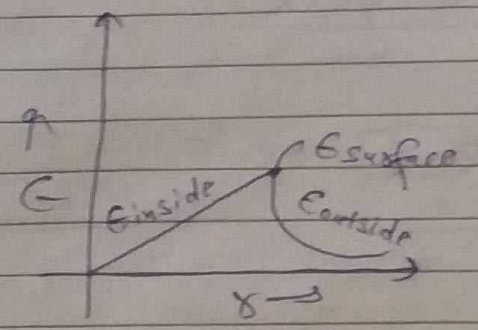


$$q' = \frac{qR^3}{r^3}$$

$$\phi = \oint E \cdot ds = q'/\varepsilon_0$$

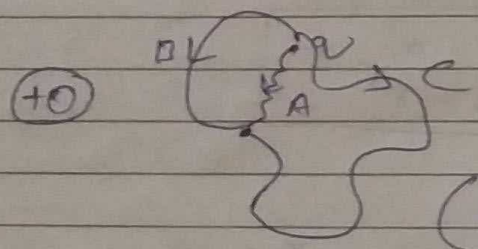
$$E \cdot 4\pi \cancel{R^2} = \frac{qR^{\cancel{3}}}{r^2\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{qR}{r^2}}$$



- Relat^n btw Electric field and potential →

Electrostatic field are conservative in nature



(beat same bal)

$$\vec{\nabla} \times \vec{E} = 0$$

Means

$$\oint \vec{E} \cdot d\vec{l} = 0$$

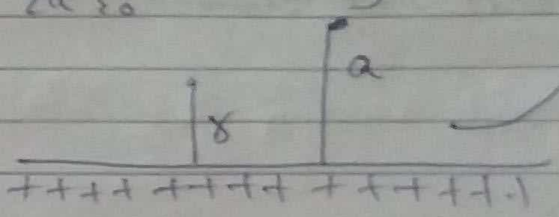
We can define a scalar (Potential) funct^n for $\vec{E}$ →

$$\boxed{\vec{E} = -\nabla V} = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$$

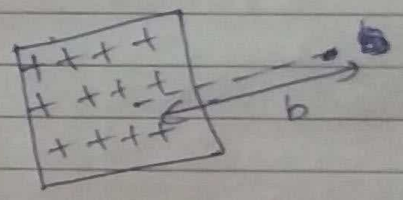
$$V = -\int_{\infty\,(ref.)}^{r} \vec{E} \cdot d\vec{r}$$

- Electric potential due to infinitely long uniform wire

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(a/r\right)$$



- Electric potential due to infinitely long charged plane -



$$E = \frac{\sigma}{2\varepsilon_0}$$

So, $V = \int E \cdot dr$

$$V = -\frac{\sigma}{2\varepsilon_0} \int_{ref}^{b} dr$$

$$V = -\frac{\sigma}{2\varepsilon_0} \left[r \right]_{ref\,(a)}^{b}$$

$$V = -\frac{\sigma}{2\varepsilon_0} (b-a)$$

$$\boxed{V = \frac{\sigma}{2\varepsilon_0} (a-b)}$$

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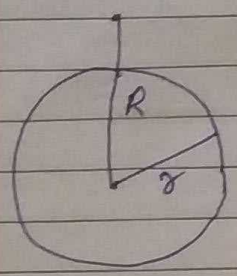
- Electric potential due to solid sphere in outside, inside and surface



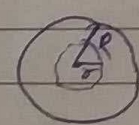
→ $E_{outside} = \dfrac{q}{4\pi\varepsilon_0 R^2}$

$$V = -\int_{r}^{R} E \cdot dr$$

$$V = \frac{-q}{4\pi\varepsilon_0} \int_{R}^{r} \frac{dr}{r^2}$$

$$V = \frac{-q}{4\pi\varepsilon_0} \left[\frac{-1}{r}\right]_{R}^{r}$$

$$V = \frac{+q}{4\pi\varepsilon_0} \left[\frac{1}{r} - \frac{1}{R}\right]$$



→ $E_{inside} = \dfrac{qr}{4\pi\varepsilon_0 R^3}$

$$V = -\oint \vec{E} \cdot d\vec{r}$$

$$V = \frac{-qr}{4\pi\varepsilon_0 R^3} \int_{R}^{r} r\,dr$$

- Dipole moment → $+q \xrightarrow{d} -q$

$$\boxed{p = q \cdot d}$$

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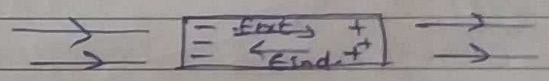
$$V = \frac{-q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}\right]_R^r$$

$$V = \frac{-q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

Dielectrics →

- it is a insulator.
- in the presence of external electric field, the dielectric get polarized. (ex – Glass, Wood, Air, water)

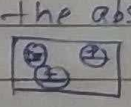
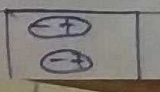
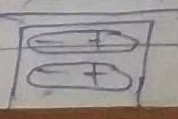


- $E_{induced}$ is opposite direct^n of $E_{external}$

$$E_{ind} < E_{ext}.$$

→ $E_{net} = \vec{E}_{ext} - \vec{E}_{ind}$

- Types of Dielectics →

	(i) Non polar	(ii) polar
(i)	made from non-polar molecule	made from polar molecules.
(ii)	ex – H_2, N_2, O_2	ex – NaCl, HCl, H_2O
(iii)	donot have parmanent electric dipole moment.	have parmanent electric dipole moment.
(iv)	In the absence of E.f.  $p=0$	$E_{ext}=0$ $p=0$
	$E_{ext} \neq 0$ $\xrightarrow{E_{ext}}$  $p \neq 0$	$E_{ext} \neq 0$ $\xrightarrow{E_{ext}}$  $p \neq 0$

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Polarizat^n vector is defined by dipole moment per volume

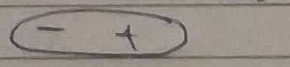
$$\boxed{\vec{P} = \frac{\vec{p}}{v}}$$

- Gauss law in Dielectric & Displacement vector :-

Volume charged density (ρ)

$$\rho = \underbrace{\rho_b}_{\text{bounded charged}} + \underbrace{\rho_f}_{\text{free charged}}$$

Bounded charged



$\vec{P} = \frac{\vec{p}}{v}$

and $\boxed{\rho_b = -\nabla \cdot \vec{P}}$

free charged

ρ_f

(remain unchanged after Introducing E_{ext}).

$$\boxed{\rho_T = -\nabla \cdot \vec{P} + \rho_f}$$

So,

$$\oint E \cdot dA = q/\varepsilon_0$$

$$\nabla \cdot E = \rho/\varepsilon_0$$

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$\rho = \rho_f + \rho_b$

$\rho = \rho_f - \nabla \cdot \vec{P}$

So,

$$\nabla \cdot \vec{E} = \frac{\rho_f + (-\nabla \cdot \vec{P})}{\varepsilon_0}$$

$$\rho_f = \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P})$$

$\boxed{\nabla \cdot \vec{D} = \rho_f}$ here $D = \varepsilon_0 \vec{E} + \vec{P}$

↳ Displacement vector

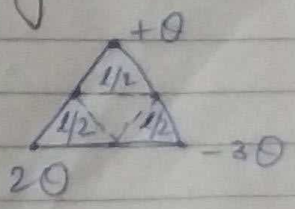
- Work done →

→ $-W_E = -\int_{r_i}^{r_f} f_E \cdot dr = (U_f - U_i) = W_{ext} = q(V_f - V_i)$

Electrostatic

Q. Three charges $+Q, +2Q, -3Q$ are placed at corners of an equitorial triangle of length 'l'. find the work done in shifting the charges to mid-points of their sides.

Pair $= \frac{n(n-1)}{2} = 3$



$$U_i = \frac{K2Q^2}{l} - \frac{K6Q^2}{l} - \frac{K3Q^2}{l}$$

$$\boxed{U_i = -\frac{7KQ^2}{l}}$$

$\cdot\ U = \frac{kQ_1Q_2}{r}$ $\cdot\ V = \frac{kq}{r}$

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$$U_f = \frac{k4Q^2}{l} - \frac{12kQ^2}{l} - \frac{6kQ^2}{l}$$

$$\boxed{U_f = \frac{-14kQ^2}{l}}$$

$$W_{ext} = U_i - U_f$$

$$= \frac{-7kQ^2}{l} + \frac{14kQ^2}{l}$$

$$\boxed{W_{ext} = \frac{7kQ^2}{l}}$$ Ans

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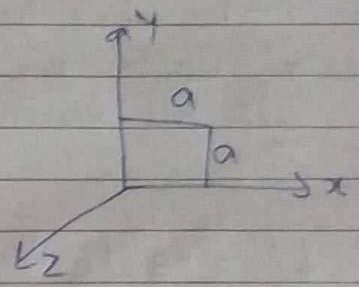
Q. A square loop of wire with side of length 'a' lie in 1st Quadrant of x-y plane w.r.t. one corner as the origin in the region. This is non uniform time dependent. Magnetic field $\vec{B}$ $Ky^3t^2\hat{z}$. find the induced emf.



$\Rightarrow$ $B = \frac{-d\phi}{dt}$

$\phi = \iint B\,dx\,dy$

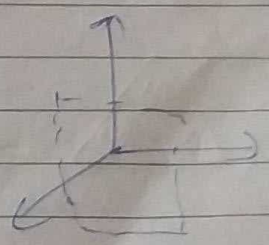
$= Kt^2 \int y^3 dy \int dx$

$= \frac{Kt^2 y^4}{4}$

$\frac{d\phi}{dt} = \frac{Ka^5 t}{2} = \epsilon$

Q. A square loop of wire with side of length a lie in all Quadrant

$\boxed{\phi = 0}$



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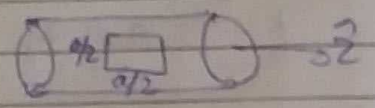
Q. A long solenoid of ~~region~~ radius ~~radius~~ 'a', is driven by an A.C., so that field inside solenoid is $B(t) = B_0 \cos(\omega t)\hat{z}$. A square loop of wire of ~~region~~ length $a/2$, and resistance 'R' is placed inside the solenoid, and coincide with it. Find the induced Emf and current as a factor of time? ~~factor of time~~ If the loop has resistance R'?



$\Rightarrow \phi_m = \int B\, ds$

$= \int B_0 \cos(\omega t)(ds)$

$= B_0 \cos(\omega t) \int ds$

$\phi_m = B_0 \cos(\omega t) \cdot \frac{a^2}{4}$

$\xi = -\frac{d\phi}{dt} = + B_0 \sin(\omega t) \cdot \omega \cdot \frac{a^2}{4}$

$$\boxed{\epsilon = \frac{B_0 a^2 \omega \sin(\omega t)}{4}}$$

Important

if,

$B = B_0 s^2 \cos(\omega t)\, \hat{z}$

Circular loop of radius $a/2$.

$$\phi_m = \int B \cdot ds$$

$$= \int_0^{2\pi} \int_0^{a/2} B_0 s^2 \cos(\omega t)\, s\, d\phi\, ds$$

$$\epsilon = \frac{d\phi}{dt} = B_0$$

Quantum macanics

Quantum Mechanics

Imp

Q. We move from classical to Quantum to study nature of e^- and subatomic particles.

→ $f = ma$

→ $\int_{-\infty}^{+\infty} |\psi|^2 dx = 1$

→ $\int \psi_n \psi_m = \delta_{mn}$

Q. At time $t=0$, a particle is represented by the wave funct^n.

$$\psi(x,0) = \begin{cases} \frac{Ax}{a} & , \quad 0 \le x \le a \\ \frac{A(b-x)}{(b-a)} & , \quad a \le x \le b \\ 0 & , \quad \text{otherwise} \end{cases}$$

Step-1 - Normalization.

$$\int_{-\infty}^{+\infty} |\psi|^2 dx = 1$$

$$\int_0^a \frac{A^2 x^2}{a^2} dx + \int_a^b A^2 \frac{(b-x)^2}{(b-a)^2} dx = 1$$

$$A^2 \left[\frac{x^3}{3} \right]_0^a + \frac{A^2}{(b-a)^2} \left[\frac{(b-x)^3}{3} \right]_a^b$$

$$A^2\left[\frac{a^3}{3} - \frac{1}{(b-a)^2}\cdot\left[\frac{b^3}{3} - \frac{b^3}{3}\right]\right]$$

$$A^2\left[\frac{a^3}{3} + \frac{1}{(b-a)^2}\cdot\frac{(b-a)^3}{3}\right]$$

$$A^2\left[\frac{a^3}{3} + \frac{b-a}{3}\right] = 1$$

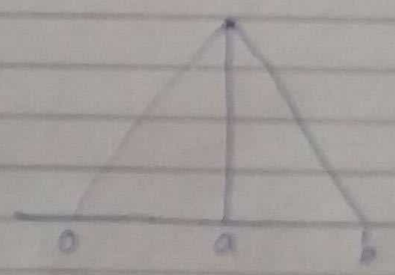
$$A^2\left(\frac{b}{3}\right) = 1$$

$$A^2 b = 3$$

$$\boxed{A = \sqrt{\frac{3}{b}}}$$

Step-2 :-

sketch $\psi(x,0)$ as a funct^n of x.



maximum probability of finding particle at $x = a$