## Shri Mata Vaishno Devi University

## **School Of Mathematics**

B.Tech.- I Semester (Civil, Computer Science, Electrical, Electronics, Mechanical)

Major Exam: Odd Sem 2023-2024

Entry No.:—
Date:——

Total No. Pages: [2]
Total No. Questions: [3]

Course Title: Engineering Mathematics I (MTL-1025)

Time alloted: 3 Hours

Total marks: [50]

## Attempt all questions.

1. Solve the followings:

[Marks 10]

(i) Find the asymptotes parallel to coordinate axes for the curve

$$a^2x^2 + b^2x^2 - x^2y^2 = 0$$

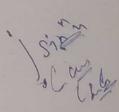
- (ii) If  $u = x^2 y^2$ , v = 2xy, then find the value of  $\frac{\partial(u,v)}{\partial(x,y)}$
- (iii) Prove that  $\int_0^{\frac{\pi}{6}} \sin^7 3x \ dx = \frac{16}{105}$
- (iv) Evaluate the integral  $\int_0^1 (1-x^2)^{-\frac{1}{2}} dx$ .
- (x) Explain the rank of a matrix.
- 2. Do the followings:

[Marks 15]

- (i) If u = u(y z, z x, x y), then prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial y} = 0$ .
- (ii) Find the interval of concave upward and concave downword for curve  $y = 3x^5 40x^3 + 3x 20$ .
- (iii) Prove the following by using the reduction formula

$$\int_0^\infty \frac{dx}{(1+x^2)^5} = \frac{7 \cdot 5 \cdot 3 \cdot 1}{8 \cdot 6 \cdot 4 \cdot 2} \frac{\pi}{2}$$

- (iv) Evaluate the integral  $\int_0^1 (x \log x)^4 dx$
- (x) Prove that  $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$ .



3. Do any five of the following:

[Marks 25]. :

- (i) If  $u = \sec^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ , then show that  $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$  and find the value of  $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x\partial y}$ .
- (ii) Find the position and nature of the double points of the curve

$$(x+y)^3 - \sqrt{2}(y-x+2)^2 = 0.$$

- (iii) In a plane triangle ABC, find the maximum value of  $\cos A \cos B \cos C$ .
- (iv) Prove that

$$\Gamma(m)\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m).$$

Further, deduce that

$$\beta(m,m) = 2^{1-2m}\beta(m,\frac{1}{2}).$$

- (**Y**) Find the rank of the matrix  $A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$  by reducing it to normal form.
- (vi) Verify Cayley-Hamilton Theorem for the matrix  $A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ , and hence find the inverse of A.