Shri Mata Vaishno Devi University

School Of Mathematics

B.Tech.- I Semester (Civil, Computer Science, Electrical, Electronics, Mechanical)

Major Exam: Odd Sem 2023-2024

Entry No.:—— Total No. Pages: [2]

Date:——— Total No. Questions: [3]

Course Title: Engineering Mathematics I (MTL-1025)

Time alloted : 3 Hours Total marks: [50]

Attempt all questions.

1. Solve the followings: [Marks 10]

   (i) Find the asymptotes parallel to coordinate axes for the curve

   $$a^2x^2 + b^2x^2 - x^2y^2 = 0.$$

   (ii) If $u = x^2 - y^2, v = 2xy$, then find the value of $\dfrac{\partial(u,v)}{\partial(x,y)}$.

   (iii) Prove that $\displaystyle\int_0^{\frac{\pi}{6}} \sin^7 3x \, dx = \frac{16}{105}$.

   (iv) Evaluate the integral $\displaystyle\int_0^1 (1-x^2)^{-\frac{1}{2}} dx$.

   (v) Explain the rank of a matrix.

2. Do the followings: [Marks 15]

   (i) If $u = u(y-z, z-x, x-y)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.

   (ii) Find the interval of concave upward and concave downword for curve $y = 3x^5 - 40x^3 + 3x - 20$.

   (iii) Prove the following by using the reduction formula

   $$\int_0^\infty \frac{dx}{(1+x^2)^5} = \frac{7\cdot5\cdot3\cdot1}{8\cdot6\cdot4\cdot2}\frac{\pi}{2}.$$

   (iv) Evaluate the integral $\int_0^1 (x \log x)^4 dx$.

   (v) Prove that $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$.

3. Do any five of the following: [Marks 25]

(i) If $u = \sec^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2\cot u$ and find the value of $x^2\dfrac{\partial^2 u}{\partial x^2} + y^2\dfrac{\partial^2 u}{\partial y^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y}$.

(ii) Find the position and nature of the double points of the curve

$$(x+y)^3 - \sqrt{2}(y-x+2)^2 = 0.$$

(iii) In a plane triangle ABC, find the maximum value of $\cos A \cos B \cos C$.

(iv) Prove that

$$\Gamma(m)\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m).$$

Further, deduce that

$$\beta(m,m) = 2^{1-2m}\beta(m,\frac{1}{2}).$$

(v) Find the rank of the matrix $A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$ by reducing it to normal form.

(vi) Verify Cayley-Hamilton Theorem for the matrix $A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$, and hence find the inverse of A.