UNIT-III "Matter and Radiations"

3.1 ELECTROMAGNETIC WAVES

The transverse time varying electric and magnetic fields propagating in space in a direction perpendicular to the directions of both the electric and magnetic fields are said to constitute electromagnetic waves. The electric and magnetic field vectors are functions of space and time. They oscillate at right angles to each other and also to the direction of wave propagation.

3.1.1 ORIGIN OF ELECTROMAGNETIC WAVES

We know that a steady electric current produces a steady magnetic field. An accelerated charge produces a magnetic field, which varies with time and space. Thus, it follows that *an accelerated charge is a source of electromagnetic waves*. As simple harmonic motion is the most common form of an acceleratory motion, therefore, a charge oscillating simple harmonically with a frequency 'ν' produces sinusoidally varying electric and magnetic fields and then produce electromagnetic waves of same frequency.

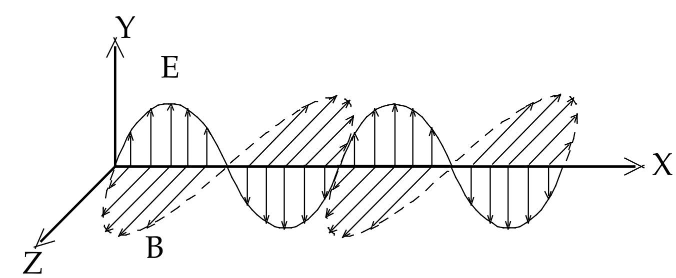

Fig.3.1

3.1.2 HISTORY OF ELECTROMAGNETIC WAVES

In 1865, **Maxwell** predicted the presence of electromagnetic waves. He formulated the theory in terms of four ***Maxwell equations*** which predict that the electromagnetic waves of all frequencies should propagate with the speed of light.

Hertz, in 1888, succeeded in *producing* and *observing* electromagnetic waves of wavelength of the order of 6 m in laboratory.

J.C. Bose, in 1895, succeeded in *producing* and *observing* electromagnetic waves of much *shorter* wavelength of the order of 5 mm − 25 mm.

In the same year, **G. Marconi** succeeded in *transmitting* electromagnetic waves over distances of many kilometers. He was first to establish a *wireless communication* across the *English Channel*.

3.1.3 PROPERTIES OF ELECTROMAGNETIC WAVES

Following are the few properties of electromagnetic waves:

1. They propagate in the form of varying electric and magnetic fields such that the *two fields are perpendicular to each other and also to the direction of propagation* of the wave. This shows their ***transverse nature***.
2. These waves are produced by ***accelerated charges***.
3. Electromagnetic waves do not require any material medium for their propagation.
4. They travel with a velocity

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 3.0 \times 10^8\, ms^{-1}$$

   in free space, which is equal to the ***velocity of light*** in free space. In material medium, their velocity is given by

$$v = \frac{1}{\sqrt{\mu \varepsilon}},$$

   where μ is the ***absolute permeability*** and ε is the ***absolute permittivity*** of the medium.
5. They obey the principle of ***superposition***.
6. Both electric and magnetic fields vary sinusoidally at the same place and at the same time. Thus, *the ratio of the amplitudes of electric and magnetic fields is always constant and is equal to velocity of the electromagnetic waves*. Mathematically,

$$\frac{E_o}{B_o} = c$$

7. The energy in electromagnetic waves is divided ***equally*** between electric field and magnetic field vectors.
8. The electric vector is responsible for optical effects of an electromagnetic wave and is, therefore, called the "***Light vector***".
9. The direction of energy flow per unit area per second along the wave is represented by a vector called the "***Poynting vector***" given by

$$\vec{S} = \frac{1}{\mu_o}(\vec{E} \times \vec{B})$$. Its unit is watt-m^{-2}.

10. The electromagnetic waves ***exerts a force*** on the surface on which it is incident. This is followed from the fact that electromagnetic waves possesses ***momentum*** given by

$$p = \frac{U}{c}$$

where U = energy of the electromagnetic wave propagating with speed c.

11. A propagating electromagnetic wave is offered hindrance by the medium. Such a hindrance is called "***wave impedence***" and its value is given by

$$Z = \mu \frac{E}{B} = \mu c,$$

where μ is ***permeability*** of the medium and 'c' the ***velocity of light***.

For vacuum or free space,

$$Z = \frac{\sqrt{\mu_o}}{\sqrt{\varepsilon_o}} = 376.6\Omega$$

12. The ***field particles*** for carrying out electromagnetic interactions are "***photons***".

<u>SAMPLE PROBLEM</u>:

In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0 × 10^{10} Hz and amplitude 48 Vm^{-1}. (i) What is the wavelength of
the wave? (ii) What is the amplitude of the oscillating magnetic field?

Sol. Here, E_o = 48 Vm^{-1}; f = 2.0 × 10^{10} Hz; c = 3 × 10^8 ms^{-1}

(i) wavelength of wave, $\lambda = \dfrac{c}{f} = \dfrac{3\times10^{8}}{2.0\times10^{10}} = 1.5\times10^{-2}$ m

(ii) Amplitude of magnetic field, $B_o = \dfrac{E_o}{c} = \dfrac{48}{3\times10^{8}} = 1.6\times10^{-7}$ T

3.2 ELECTROMAGNETIC SPECTRUM

The orderly distribution of electromagnetic waves according to their wavelength or frequency is called an electromagnetic spectrum. Electromagnetic spectrum extends form very short γ rays (wavelength ~ 10^{-14} m) to long radiowaves (wavelength ~ 10^{6} m). Fig. 3.2 shows the electromagnetic spectrum we are familiar with. The boundaries separating different regions of spectrum are not sharply defined.

The gamma-ray region and X-ray region ***overlap considerably***.

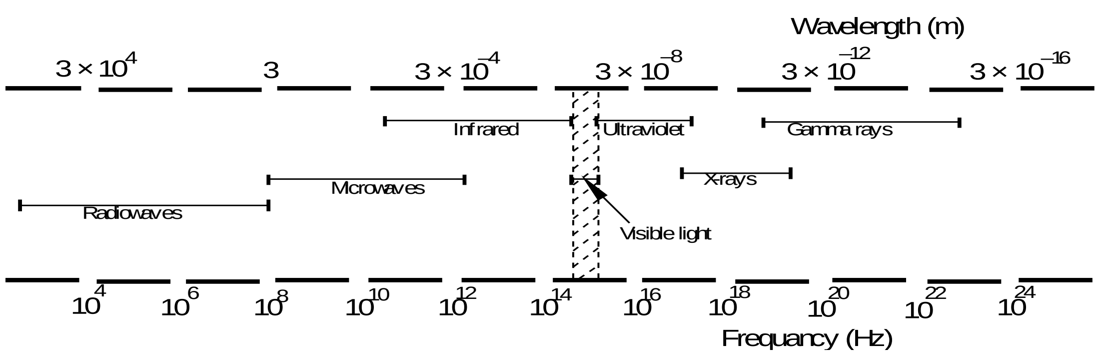

Fig. 3.2

The electromagnetic spectrum may be divided into following main parts:

1. <u>GAMMA RAYS</u>: These waves are of ***nuclear*** origin and have the ***shortest*** wavelengths among the electromagnetic waves (10^{-3} A° to 1 A°). These are highly energetic radiations and are mainly emitted by radioactive substance.

<u>Properties</u>: (i) γ-rays travel with the velocity of light.

(ii) These are highly penetrating and can penetrate through *several centimetres* thick iron block.

(iii) They have got small ionizing power.

(iv) They can produce ***fluorescence*** in a substance like *willemite*.

(v) They can affect a photographic plate.

(vi) They are not ***deflected*** by electric and magnetic fields.

(vii) They ***knock out*** electrons from the surface on which they fall.

(viii) γ-rays produce ***heating*** effect in the surface exposed to them.

<u>Uses</u>: (i) γ-rays are used in ***radiotherapy***. In hospitals, γ-rays are used to treat cancer and tumors.

(ii) Soft γ-rays are used to kill micro-organisms for ***preserving*** foodstuffs for a prolonged time.

(iii) These are used to carry out ***nuclear*** reactions.

2. <u>X-RAYS</u>: X-rays were discovered by the German physicist **W. Rontgen**. These are produced most commonly when fast moving electrons decelerate inside a metal target.

Properties. (i) X-rays have frequency range 8×10^{15} to 3×10^{19} Hz.

(ii) These possess a ***high*** penetrating power.

(iii) They affect the photographic plate very ***intensely***.

(iv) They ***ionize*** the gas through which they pass.

(v) They cause ***fluorescence*** in substances like *zinc sulphide*, *barium platinocyanide*, *calcium tungstate* etc.

(vi) They cast the ***shadows*** of the objects falling in their path.

(vii) They have ***injurious effect*** on human bodies. Long exposures of X-rays to human body causes surface ***sores***.

(viii) X-rays can also cause ***photoelectric effect***.

<u>Uses</u>:

(i) These rays are widely used in ***medical diagnosis***. These are used in surgery for the detection of fractures, diseased organs etc.

(ii) X-rays are used in engineering for ***detecting*** faults, cracks, flaws and gas voids in finished metal products.

(iii) They are used for ***distinguishing*** real diamonds, gems etc. from artificial ones.

(iv) They have been used in ***investigating*** the structure of crystals, constitution and properties of atoms and arrangements and molecules in the complex substances.

(v) They are used in detective departments for *detection of explosives, opium* etc.

3. <u>ULTRAVIOLET RAYS:</u> These rays were discovered by **Ritter** in 1801. They can be produced by ***arcs*** of mercury and iron. The sun emits large amount of ultraviolet radiations. These radiations are harmful to us if absorbed in large amount.

Properties. (i) These rays possess the property of ***synthesizing*** vitamin-D, when the skin is exposed to the sunlight.

(ii) They can affect a photographic plate and cause ***fluorescence*** in certain materials.

(iii) They can undergo *reflection, refraction, interference and polarization.*

(iv) UV rays cannot pass through glass but quartz, fluorite and rock salt are ***transparent*** to them.

<u>Uses</u>:

(i) They are used for ***checking*** the mineral samples due to their property of causing fluorescence.

(ii) They can ***destroy*** bacteria and hence it is used for ***sterilizing*** surgical instruments.

(iii) Their ***absorption spectra*** is used for the study of molecular structure.

(iv) They are used in ***burglar alarms*** by using their property of causing emission of ***photoelectrons***.

4. <u>VISIBLE LIGHT:</u> It is most familiar to us and forms a very narrow part of electromagnetic spectrum. Its frequency ranges from 4×10^{14} Hz. To 8×10^{14} Hz. The visible light is emitted due to ***atomic excitation***. Human eye is sensitive to only visible part of electromagnetic spectrum.

5. <u>INFRA-RED RAYS:</u> These were discovered by **Herschell**. These are emitted by the atoms and molecules of hot bodies. About 60% of the solar radiations are infra-red in nature. Their frequency ranges from 10^{13} to 4×10^{14} Hz.

Properties. (i) These obey laws of ***reflection*** and ***refraction***.

(ii) When ***absorbed*** by molecules, the energy of IR rays gets converted into molecular vibrations.

(iii) They are ***scattered less*** as compared to visible light by the atmosphere.

(iv) When allowed to fall on matter, infra-red rays amounts to an *increase in temperature*.

Uses:

(i) They are used in ***physical therapy***.

(ii) Infra-red rays are used in solar water heaters and cookers.

(iii) Infra-red rays photographs are used for ***weather forecasting***.

(iv) They are used to provide ***electrical energy*** to ***satellites*** by using solar cells.

6. MICROWAVES: The microwaves are produced by oscillating electronic circuits. The frequency range of microwaves is 10^{19} to 3×10^{11} Hz. These are used in ***radar*** and other communication systems. ***Microwave ovens*** are used in cooking.

7. RADIOWAVES: Radiowaves are produced by charges accelerating in ac circuits having an *inductor* and a *capacitor*. The frequency of radiowaves varies from a few hertz to 10^{9} Hz. These are used in ***radio*** and ***T.V. communication***.

The following table shows the summary of the various parts of electromagnetic spectrum with ***frequency range***, ***wavelength range***, ***means of production*** etc. of electromagnetic radiations:

S. No.	NAME	Frequency Range	Wavelength Range	Means of Production	Uses and Applications
1.	Gamma Rays	5×10^{22} Hz to 3×10^{18} Hz	6×10^{-14} m to 1×10^{-10}m	Emitted by radioactive substances	Study of atomic Structure

2.	X-Rays	3×10^{21} Hz to 1×10^{16} Hz	1×10^{-13} m to 3×10^{-8} m	Bombardment of metals by cathode rays	Medical sciences and Crystallography
3.	Ultraviolet Rays	3×10^{17} Hz to 8×10^{14} Hz	6×10^{-10} m to 4×10^{-7} m	Excitation of atoms and Vacuum spark	Studies related to structure of matter and as disinfectant in Hospitals.
4.	Visible Light	8×10^{14} Hz to 4×10^{14} Hz	4×10^{-7} m to 8×10^{-7} m	Arc sources and atomic Excitation	Studies of structure of matter and in human sight.
5.	Infrared Rays	4×10^{14} Hz to 1×10^{13} Hz	8×10^{-7} m to 3×10^{-5}m	Atomic and molecular Excitations	Bad weather photography and medical sciences
6.	Heat Radiations	3×10^{13} Hz to 3×10^{11} Hz	10^{-5} m to 10^{-3} m	Heating of materials	Heating
7.	Microwaves	3×10^{11} Hz to 1×10^{9} Hz	10^{-3} m to 0.3m	Vacuum tunes	Radar and television etc.
8.	Radiowaves	3×10^{7} Hz to 3×10^{4} Hz	10 m to 10^{4} m	Oscillating circuits	Radio communication and television
9.	Electric Waves	60 Hz to 50 Hz	5×10^{6} m to 6×10^{6} m	Weak radiations from sources of ac supply	Lighting

3.3 PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons from a metallic surface when light radiations of suitable frequency are made to incident on it is called photoelectric effect.

The electrons ejected from the metal are called as ***photoelectrons***. Hallwach discovered the phenomenon in 1988. For photoelectric emission the metal used must have low ***work function***. For example, ***Cesium*** is best metal for photoelectric effect.

Note: *Work function of metal is defined as the minimum energy required by an electron to just come out of the metal surface.*

3.3.1 CAUSE OF PHOTOELECTRIC EFFECT

When a photon (of energy greater than work function of metal) strikes the metal surface, it is absorbed by the surface electron and lose its existence. This excites the electron out of the metal surface and the remaining energy is imparted to the electron as its kinetic energy.

3.3.2 EXPERIMENTAL ARRANGEMENT TO STUDY PHOTOELECTRIC EFFECT

The experimental arrangement to study the photoelectric effect is shown in Fig. 3.3.

The metal plates C and A are sealed in an evacuated quartz tube. When light of reasonably short wavelength passes through a transparent window in the wall of the tube and falls on the plate C, called the ***cathode*** or the ***emitter***, the photoelectrons are emitted and are collected by the plate A called the ***anode*** or the ***collector***. The potential difference between the cathode and the anode can be changed with the help of the ***batteries, rheostat,*** and the ***commutator***. The anode potential can be made positive or negative w.r.t. the cathode. These electrons flow in the outer circuit and hence a photocurrent is established and is indicated by the deflection of the microammeter.

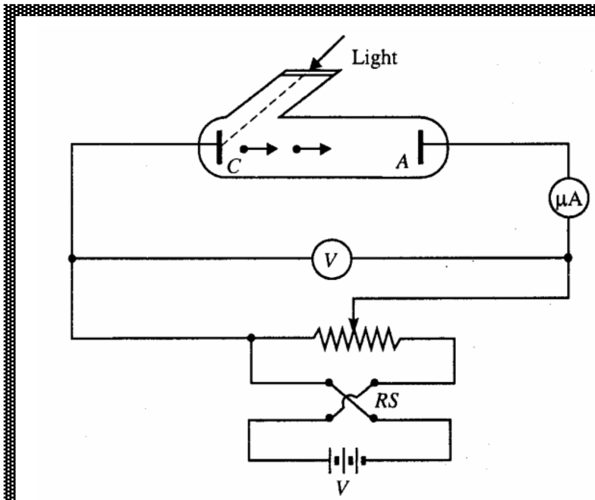

Fig. 3.3

3.3.3 LAWS OF PHOTOELECTRIC EMISSION

Ist Law: The number of photo-electrons emitted per second i.e. photoelectric current is proportional to the intensity of incident light.

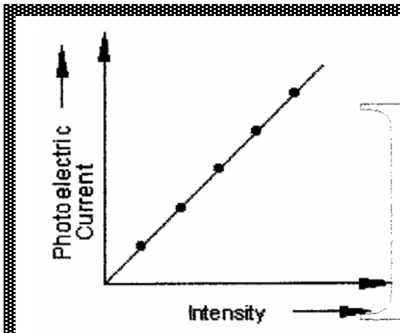

Fig. 3.4

Fig. 3.4 shows the linear variation of photoelectric current with intensity.

IInd Law: For a given material, there exists a certain minimum frequency of incident light below which no photoelectric emission takes place. The frequency is called threshold frequency. The corresponding wavelength is called ***threshold wavelength***. The threshold frequency caries from metal to metal.

IIIrd Law: The maximum kinetic energy of the photoelectrons is independent of the intensity of the light and depends only upon its frequency.

IVth Law: The emission of photoelectrons starts as soon as light falls on metal surface.

3.3.4 FACTORS AFFECTING PHOTOELECTRIC CURRENT

1. *EFFECT OF POTENTIAL:* If the frequency and intensity of the incident radiation is kept constant and potential 'V' is varied, it is found that the photocurrent increases gradually with the increase in positive potential on the plate A until a stage comes when photocurrent becomes maximum and attains a saturation value.

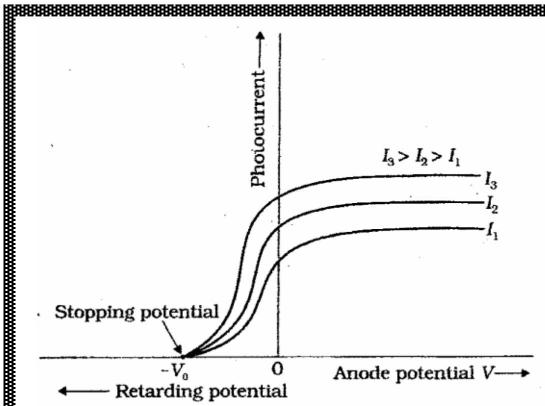

Fig. 3.5

If plate A is applied the negative potential w.r.t. plate C and is increased gradually, we find that the photocurrent decreases rapidly until it becomes zero at a certain negative potential on plate A.

Stopping potential: *The minimum negative potential given to the plate A for which the photoelectric current becomes zero is called the **stopping potential** or **cut-off potential (V_o)**.*

It is observed that the stopping potential remains the same for another radiation of same frequency but of higher intensity (Fig. 3.6).

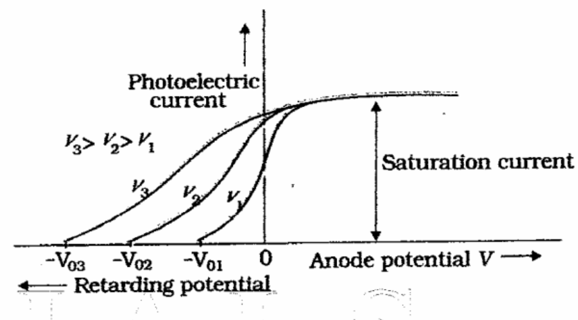

Fig. 3.6

In such a case, the work done by the stopping potential is equal to the maximum kinetic energy of the electrons, i.e.

$$eV_o = \frac{1}{2}mv^2_{max}$$

2. *EFFECT OF FREQUENCY*: When the frequency of incident light is kept fixed and frequency is increased, the photoelectric current saturates at the same value, but the stopping potential increases. If the frequency is decreased and at a particular frequency of incident light, the stopping potential becomes zero. This value of frequency is called ***threshold frequency*** ν_o. *If the frequency of incident light (ν) is less than the threshold frequency (ν_o), no photoelectric emission takes place.*

3.3.5 EINSTEIN'S PHOTOELECTRIC EQUATION

Einstein suggested that photon's energy is used in the following two ways:

(i) A part of its energy is used to free the electron from the atom and away from the metal surface. This energy is equal to the *work function* W_o of the metal.

(ii) The other part is used in giving ***kinetic energy*** $\left(\frac{1}{2}mv^2\right)$ to the electron.

Thus, if the energy of a photon is given by

$$E = h\nu \qquad \text{...... (i)}$$

then $h\nu = W_o + \frac{1}{2}mv^2$ or $\frac{1}{2}mv^2 = h\nu - W_o$ ..... (ii) where 'v' is the velocity of emitted electrons. Equation (ii) is known as ***Einstein's Photoelectric Equation***.

<u>SPECIAL CASE</u>: When the energy of photons is such that it can only liberate the electron from metal, then the kinetic energy of the electron will be zero. Thus, Eq. (ii) reduces to

$$h\nu_o = W_o \qquad \text{..... (iii)}$$

where ν_o = threshold frequency

From (ii), $\frac{1}{2}mv^2 = h\nu - h\nu_o = h(\nu - \nu_o)$

$$\therefore \quad K.E. = h(\nu - \nu_o) \qquad \text{..... (iv)}$$

For a particular metal, work function W_o is constant and hence

$$K.E. = \frac{1}{2}mv^2 \propto \nu$$

or ***$v^2 \propto$ frequency ν of incident light***

Thus the increase in frequency ν of incident light causes ***increase in velocity*** of photoelectrons provided ***intensity of light is constant***.

3.3.6 VARIOUS FORMS OF EINSTEIN'S PHOTELECTRIC EQUATION

(i) $h\nu = W_o + \frac{1}{2}mv^2_{\max}$

(ii) $h\nu = h\nu_o + \frac{1}{2}mv^2_{\max}$ $\quad \therefore \quad (K.E.)_{\max} = h(\nu - \nu_o)$

(iii) Max. K.E. of emitted electrons = $h(\nu - \nu_o) = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_o}\right)$

(iv) $\therefore$ Max. K.E. of emitted electrons = $ch\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right)$

(v) If ν_o is the value of stopping potential, then

$$(K.E.)_{\max} = eV_o = h(\nu - \nu_o) = ch\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right)$$

or $$V_o = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right) \qquad \text{..... (v)}$$

As h, e and c are constants, $V_o \propto \frac{1}{\lambda}$

Fig. 3.7 shows the graph between V_o and ν, which is a straight line, intercepting x-axis at $\nu = \nu_o$. The slope of the graph is given by $\frac{h}{e}$.

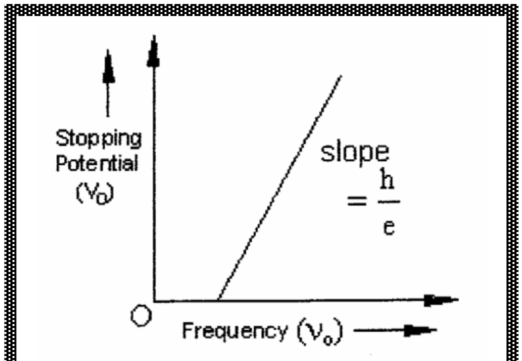

Fig. 3.7

3.3.7 CONDITION FOR PHOTOELECTRIC EMISSION

The condition for photo-electric effect to take place is $h\nu\ (= E) \geq W_o$

or $h\nu \geq h\nu_o$ or $\nu \geq \nu_o$ or $\lambda \leq \lambda_o$ $\left(\because \nu \propto \frac{1}{\lambda}\right)$

where ν_o = threshold frequency

λ_o = threshold wavelength

W_o = work function of metal

3.3.8. EXPLAINATION OF LAWS OF PHOTOELECTRIC EMISSION FROM EINSTEIN'S PHOTOELECTRIC EQUATION

Ist Law: Increasing the intensity of radiation means increasing the number of photons incident on the cathode. Now one photon ejects one electron. Therefore, as we increase the intensity of radiation, the number of emitted photoelectrons increases. Note that changing the frequency of radiation will nor have any effect on the value of photoelectric current as it is above ν_o (threshold frequency).

IInd Law: The energy of a photon ($E = h\nu$) depends upon the frequency of incident radiation and not upon its intensity. Now there is a minimum energy required to eject a photoelectron from a metal's surface. This is called ***work function*** (W_o) of the metal. Therefore, the incoming photon must have minimum frequency (i.e. threshold frequency) to cause photoelectric emission.

IIIrd Law: Increasing the frequency of incident radiation means increasing the energy (E = hν) of the incident photons. Hence greater the frequency of incident radiation, greater will be the kinetic energies of the emitted photoelectrons.

IVth Law: As soon as the photon of suitable frequency reaches the surface of the metal, an electron is emitted. This is because when an electron absorbs a photon and acquires the energy all at once, there need not be any time delay between the incidence of light and the emission of electron.

SAMPLE PROBLEMS:

1. A metal has threshold wavelength of 6000 Å. Calculate (i) threshold frequency (ii) the work function of metal in eV. Given h = 6.62 × 10⁻³⁴ Js and e = 1.6 × 10⁻¹⁹ C.

Sol. Here $\lambda_o = 6000$ Å $= 6000 \times 10^{-10}$ m; $h = 6.62 \times 10^{-34}$ Js; $e = 1.6 \times 10^{-19}$ (C)

(i) Threshold frequency, $\nu_o = \dfrac{c}{\lambda_o} = \dfrac{3.0\times10^{8}}{6000\times10^{-10}} = 5.0\times10^{14}$ Hz

(ii) Work function, $W_o = h\nu_o = 6.2 \times 10^{-34} \times 5 \times 10^{14}$

$$= 3.31 \times 10^{-19}\text{ J} = \frac{3.31\times10^{-19}}{1.6\times10^{-19}} = 2.069\text{ eV}$$

2. If radiation of wavelength 5000 Å is incident on a surface of work function 1.2 eV, find the value of stopping potential.

Sol. Here, $W_o = 1.2\text{ eV} = 1.2 \times 1.6 \times 10^{-19} = 1.92 \times 10^{-19}$ J

$\lambda = 5000$ Å $= 5000 \times 10^{-10}$ m

We know that $\dfrac{1}{2}mv_{\max}^2 = h\nu - W_o = \dfrac{hc}{\lambda} - W_o = \dfrac{6.62\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}} - 1.92\times10^{-19}$

$$= (3.97 \times 10^{-19}) - (1.92 \times 10^{-19}) = 2.05 \times 10^{-19}\text{ J}$$

If e is the charge on electron and V_o is stopping potential, then

$$eV_o = \frac{1}{2}mv_{\max}^2 = 2.05\times10^{-19}\,J$$

or $$V_o = \frac{2.05\times10^{-19}}{e} = \frac{2.05\times10^{-19}}{1.6\times10^{-19}} = 1.28\ \text{V}$$

3.4 DE-BROGLIE HYPOTHESIS

According to de-Broglie, ***every moving particle is associated with a wave which controls the particle in every respect***. The wave associated with a particle is called ***matter wave*** or ***de-Broglie wave***. De-Broglie proposed that the wavelength of a material particle would be related to its momentum in the same way as for a photon (i.e. $p = h/\lambda$). That is for a particle of mass 'm' moving with speed 'v', de-Broglie wavelength is given by

De-Broglie wavelength, $\lambda = \frac{h}{mv}$ where h = Plank's constant

The de-Broglie waves, though often called as ***matter waves,*** are not composed of matter.

Note: *The intensity of wave at a point represents the probability of the associated particle being there.*

3.4.1 DERIVATION OF DE-BROGLIE WAVELENGTH

According to quantum theory of radiation, energy of a photon is given by

$$E = h\nu \qquad \text{..... (i)}$$

If we consider a photon to be a particle of mass 'm', then the energy associated with it, according to Einstein's mass-energy relation is given by

$$E = mc^2 \qquad \text{...... (ii)}$$

where c is the velocity of photon.

From (i) and (ii), we get

$$h\nu = mc^2 \qquad \text{or} \qquad m = \frac{h\nu}{c^2} \qquad \text{..... (iii)}$$

Now, momentum, p = mass × velocity = m × c = $\frac{h\nu}{c^2}\times c$ [from (iii)]

$$= \frac{h\nu}{c} = \frac{h}{\lambda} \qquad \left(\because c = \nu\lambda \quad or \quad \frac{\nu}{c} = \frac{1}{\lambda}\right)$$

or $$\lambda = \frac{h}{p} \qquad \text{..... (iv)}$$

Eq. (v) is general one and applies equally to photons as well as other material particles. If a material particle of mass 'm' is moving with velocity 'v', then momentum of the particle

$$p = mv$$

Hence, de-Broglie wavelength is given by

$$\lambda = \frac{h}{mv} \qquad \text{...... (v)}$$

This is called ***de-Broglie relation***. It connects the momentum which is the characteristic of the particle with the wavelength, which is the characteristic of the wave.

From Eq. (vi), we may draw following conclusions:

(i) *Lighter the particle, greater is its de-Broglie wavelength.*

(ii) *The faster the particle moves, smaller is its de-Broglie wavelength.*

(iii) *The de-Broglie wavelength of a particle is independent of the nature of the particle.*

(iv) *The de-Broglie waves are not electromagnetic in nature as the em waves are produced by moving charge parcticles.*

***SAMPLE PROBLEMS*:**

1. Calculate the de-Broglie wavelength associated with an electron moving with a velocity 0.5c and rest mass of electron 9.1 × 10^{-31} kg.

Sol. Here, $m_o = 9.1 \times 10^{-31}$ kg; $h = 6.62 \times 10^{-34}$ J-s; $v = 0.5c$

or $$\frac{v}{c} = 0.5$$

Since electron is moving at a speed comparable to the velocity of light (c).

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{h}{mv_o}\sqrt{1-\frac{v^2}{c^2}} \qquad \left(\because m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \right)$$

$$= \frac{6.62\times10^{-34}\sqrt{1-(0.5)^2}}{9.1\times10^{-31}\times1.5\times10^{8}} = 4.2\times10^{-12}\ m\ = 0.042\ \text{Å}$$

2. Calculate the de-Broglie wavelength associated with 100 g of rifle bullet travelling with a speed of 400 ms⁻¹.

Sol. De-Broglie wavelength associated with the bullet is

$$\lambda = \frac{h}{mv}$$

Here h = 6.62×10^{-34} J-s; m = 100 g = 10^{-1} kg; v = 400 ms^{-1}

$$\therefore \qquad \lambda = \frac{6.62\times10^{-34}}{10^{-1}\times400} = 16.55 \times 10^{-36}\ \text{m}$$

3. Monochromatic light of frequency 6.0 × 10^{14} Hz is produced by a laser. The power emitted is 2.0 × 10^{-3} W. (a) What is the energy of a photon in the light beam? (B) how many per second, on the average, are emitted by the source?

Sol. (a) Each photon has an energy

$$E = h\nu = (6.63 \times 10^{-34}\ \text{Js})\ (6.0 \times 10^{14}\ \text{Hz}) = 3.98 \times 10^{-19}\ \text{J}$$

(b) Let the power emitted in the beam equals N times the energy of each photon. Then number of photons emitted per second

$$= \frac{Power\ emitted}{Energy\ of\ each\ Photon} = \frac{2.0\times10^{-3}\ W}{3.98\times10^{-19}\ J} = 5.0\times10^{15}\ \text{photons/sec}$$

4. The work function of cesium is 2.14 eV. Find (a) the threshold frequency for cesium, and (b) the wavelength of the incident light if the photocurrent is brought to zero by a stopping potential of 0.60 eV.

Sol. (a) We have, work function $W_o = h\nu_o$

$$\Rightarrow \qquad \nu_o = \frac{W_o}{h} = \frac{2.14\times1.6\times10^{-19}\ J}{6.63\times10^{-34}\ Js} = 5.16 \times 10^{14}\ \text{Hz.}$$

(b) We have $(K.E.)_{Max} = eV_o$

$$\text{or}\ \ h\nu - W_o = eV_o \qquad \Rightarrow \qquad eV_o = \frac{hc}{\lambda} - W_o$$

or $$\lambda = \frac{hc}{(eV_o + W_o)} = \frac{(6.63\times10^{-34}\,Js)\cdot(3\times10^{8}\,ms^{-1})}{(0.6\,eV + 2.14\,eV)} = \frac{19.89\times10^{-26}\,Jm}{(2.64\,eV)}$$

= 471 nm.

3.4.2 DE-BROGLIE WAVELENGTH OF AN ELECTRON

Suppose an electron at rest has been accelerated through a potential difference of V volts and gains a velocity v. If 'm' and 'e' are the mass and charge of electron respectively, then

Work done on electron = eV

K.E. gained by electron = $\frac{1}{2}mv^2$

$$\therefore \qquad \frac{1}{2}mv^2 = eV$$

or velocity of electron, $v = \sqrt{\frac{2eV}{m}}$

If λ is the de-Broglie wavelength associated with the electron, then

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2eV/m}}$$

or $$\lambda = \frac{h}{\sqrt{2emV}}$$

Substituting the values of h (= 6.62 × 10^{-34} J-s), e (= 1.6 × 10^{-19} C) and m (= 9.1 × 10^{-31} kg), we get

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times1.6\times10^{-19}\times9\times10^{-31}\times V}} = \frac{12.27\times10^{-10}}{\sqrt{V}}\ \text{m}$$

or $$\lambda = \frac{12.27}{\sqrt{V}}\,\text{Å} \qquad \text{..... (i)}$$

<u>Note:</u> Eq. (i) is a very handy formula for electron wavelength at low energies i.e. at low voltages.

SAMPLE PROBLEMS:

1. Determine the wavelength of an electron that has been accelerated through a potential difference of 100 volts.

Sol. De-Broglie wavelength is given by

$$\lambda = \frac{12.27}{\sqrt{V}}\,\mathring{A} = \frac{12.27}{\sqrt{100}}\,\mathring{A} = 1.227\ \mathring{A}$$

2. Calculate the de-Broglie wavelength of 500 eV proton. Given that mass of proton = 1.67 × 10⁻²⁷ kg.

Sol. Mass of proton, $m_p = 1.67 \times 10^{-27}$ kg

Energy of proton, $E = 500\ \text{eV} = 500 \times 1.6 \times 10^{-19} = 8 \times 10^{-17}$ J

De-Broglie wavelength of proton is given by

$$\lambda_p = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times8\times10^{-17}}} = 1.28\times10^{-12}\ \text{m}$$

3.4.3 DAVISSON AND GERMER EXPERIMENT

This experiment was demonstrated by ***Davisson*** and ***Germer*** in 1927 to establish the wave nature of an electron.

Fig 3.8 (a) shows the experimental arrangement. Electrons for heated cathode C are accelerated by a potential difference V between the cathode and anode (A) The narrow hole in the anode renders electrons into a fine beam and allows it to strike the nickel crystal N cut along cubical diagonal. The electrons are scattered in all directions by the atoms in the crystal. An electron detector is used to find the intensity of the scattered electron beam. For different values of scattering angle ϕ, the intensity of the scattered beam can be measured by rotating the detector at different positions on circular

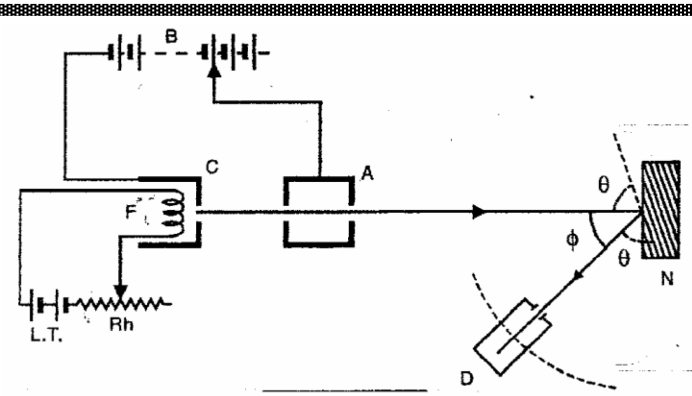

Fig. 3.8 (a)

scale.

The polar graphs are plotted between angle ϕ and the intensity of scattered electron beam at different accelerating voltages [Fig. 3.9 (b)]. As can be seen from the graphs that there occurs a strong peak corresponding to a sharp diffraction maximum in the electron distribution when accelerating voltage is 54 V and the scattering angle $\phi = 50^o$. The appearance of bump in a particular direction is due to diffraction of electrons from the regularly spaced atoms of nickel crystal by virtue of their wave nature.

From Fig. 3.9 (b), it follows that

$$\theta = \frac{1}{2}(180^o - \phi)$$

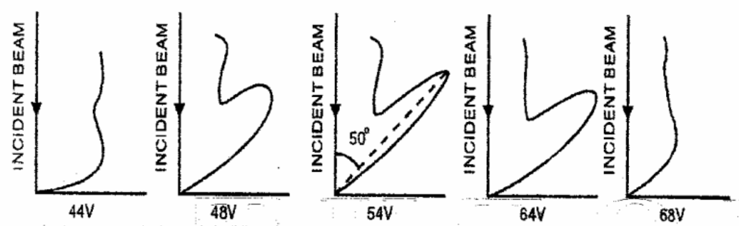

Fig. 3.9 (b)

For $\phi = 50^o$,

$$\theta = \frac{1}{2}(180^o - 50^o) = \frac{1}{2}(130^o) = 65^o$$

According to **Bragg's Law** for first order diffraction maxima, we have

$2d \sin \theta = 1 \times \lambda$

For Ni crystal, $d = 0.91$ Å

$\therefore \ \lambda = 2 \times 0.91 \times \sin 65^o = 1.66$ Å

According to de-Broglie hypothesis, the wavelength of the wave associated with electron is given by

$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67 \text{ Å}$$

This shows the remarkable agreement of two results. Thus, the experiment establishes wave nature of an electron in motion.