

To implement a genetic algorithm for finding the maximum or minimum of a function, we'll follow these steps:

1. Representation (Chromosome): Define how solutions are represented.
2. Initialization: Generate an initial population.
3. Selection: Decide how to select individuals for reproduction.
4. Crossover: Combine parts of two parents to create offspring.
5. Mutation: Introduce random changes to individuals.
6. Evaluation: Assess the fitness of individuals.
7. Termination: Determine when to stop the algorithm.

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In [4]: import random

def f(x):
    return x**2

def initialize_population(size=10):
    return [random.uniform(-10, 10) for _ in range(size)]

def select_parent(population):
    individual1, individual2 = random.sample(population, 2)
    return individual1 if f(individual1) > f(individual2) else individual2

def crossover(parent1, parent2):
    return (parent1 + parent2) / 2

def mutate(individual):
    mutation_chance = 0.1
    if random.random() < mutation_chance:
        individual += random.uniform(-1, 1)
    return individual

def genetic_algorithm(generations=100, top_solutions=3):
    population = initialize_population()

    for generation in range(generations):
        print(f"Generation {generation + 1}:")
        population_with_fitness = [(individual, f(individual)) for individual in population]
        for individual, fitness in sorted(population_with_fitness, key=lambda x: x[1]):
            print(f"  Individual: {individual:.4f}, Fitness: {fitness:.4f}")

        new_population = []
        for _ in range(len(population)):
            parent1 = select_parent(population)
            parent2 = select_parent(population)
            child = crossover(parent1, parent2)
            child = mutate(child)
            new_population.append(child)
        population = new_population

    # Find the best solutions
    final_population_with_fitness = sorted([(individual, f(individual)) for individual in population])
```

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best_solutions = final_population_with_fitness[:top_solutions]
return best_solutions

best_solutions = genetic_algorithm(generations=10, top_solutions=3) # Reduced gene
print("\nTop 3 Solutions:")
for individual, fitness in best_solutions:
    print(f"Individual: {individual:.4f}, Fitness: {fitness:.4f}")
```

Generation 1:

Individual: 6.7358, Fitness: 45.3715
Individual: -6.4451, Fitness: 41.5394
Individual: 4.8959, Fitness: 23.9700
Individual: 2.8527, Fitness: 8.1378
Individual: 2.7299, Fitness: 7.4523
Individual: 2.1706, Fitness: 4.7116
Individual: -1.4586, Fitness: 2.1274
Individual: 1.4123, Fitness: 1.9946
Individual: 0.4976, Fitness: 0.2476
Individual: 0.4194, Fitness: 0.1759

Generation 2:

Individual: 4.7943, Fitness: 22.9849
Individual: 4.7329, Fitness: 22.4000
Individual: 4.7329, Fitness: 22.4000
Individual: 4.2645, Fitness: 18.1856
Individual: 4.0741, Fitness: 16.5981
Individual: 3.8129, Fitness: 14.5382
Individual: 3.3753, Fitness: 11.3926
Individual: 2.8527, Fitness: 8.1378
Individual: 2.6386, Fitness: 6.9624
Individual: -1.7962, Fitness: 3.2264

Generation 3:

Individual: 4.7636, Fitness: 22.6915
Individual: 4.5294, Fitness: 20.5151
Individual: 4.4987, Fitness: 20.2380
Individual: 4.4342, Fitness: 19.6619
Individual: 4.4035, Fitness: 19.3906
Individual: 4.2645, Fitness: 18.1856
Individual: 3.8235, Fitness: 14.6190
Individual: 3.8199, Fitness: 14.5915
Individual: 3.7164, Fitness: 13.8120
Individual: 3.7164, Fitness: 13.8120

Generation 4:

Individual: 4.7636, Fitness: 22.6915
Individual: 4.6465, Fitness: 21.5896
Individual: 4.6311, Fitness: 21.4472
Individual: 4.5835, Fitness: 21.0086
Individual: 4.4697, Fitness: 19.9779
Individual: 4.4664, Fitness: 19.9489
Individual: 4.4664, Fitness: 19.9489
Individual: 4.4664, Fitness: 19.9489
Individual: 4.4342, Fitness: 19.6619
Individual: 4.1611, Fitness: 17.3145

Generation 5:

Individual: 4.7636, Fitness: 22.6915
Individual: 4.7050, Fitness: 22.1371
Individual: 4.6166, Fitness: 21.3131
Individual: 4.6150, Fitness: 21.2981
Individual: 4.5835, Fitness: 21.0086
Individual: 4.5488, Fitness: 20.6913
Individual: 4.5266, Fitness: 20.4900
Individual: 4.4680, Fitness: 19.9634
Individual: 4.4664, Fitness: 19.9489
Individual: 4.4503, Fitness: 19.8051

Generation 6:

Individual: 4.6893, Fitness: 21.9893
Individual: 4.6893, Fitness: 21.9893
Individual: 4.6600, Fitness: 21.7156
Individual: 4.6269, Fitness: 21.4081
Individual: 4.6269, Fitness: 21.4081
Individual: 4.6158, Fitness: 21.3056
Individual: 4.6001, Fitness: 21.1606
Individual: 4.5819, Fitness: 20.9936
Individual: 4.5415, Fitness: 20.6254
Individual: 4.2286, Fitness: 17.8807

Generation 7:

Individual: 4.7117, Fitness: 22.2005
Individual: 4.6893, Fitness: 21.9893
Individual: 4.6893, Fitness: 21.9893
Individual: 4.6746, Fitness: 21.8522
Individual: 4.6581, Fitness: 21.6977
Individual: 4.6525, Fitness: 21.6461
Individual: 4.6447, Fitness: 21.5730
Individual: 4.6434, Fitness: 21.5616
Individual: 4.6356, Fitness: 21.4886
Individual: 4.6044, Fitness: 21.2003

Generation 8:

Individual: 4.7117, Fitness: 22.2005
Individual: 4.7005, Fitness: 22.0948
Individual: 4.6849, Fitness: 21.9484
Individual: 4.6821, Fitness: 21.9224
Individual: 4.6820, Fitness: 21.9207
Individual: 4.6820, Fitness: 21.9207
Individual: 4.6670, Fitness: 21.7806
Individual: 4.6525, Fitness: 21.6461
Individual: 4.6434, Fitness: 21.5616
Individual: 4.5074, Fitness: 20.3166

Generation 9:

Individual: 4.7061, Fitness: 22.1476
Individual: 4.6983, Fitness: 22.0743
Individual: 4.6969, Fitness: 22.0612
Individual: 4.6894, Fitness: 21.9901
Individual: 4.6849, Fitness: 21.9484
Individual: 4.6834, Fitness: 21.9346
Individual: 4.6834, Fitness: 21.9346
Individual: 4.6834, Fitness: 21.9346
Individual: 4.6820, Fitness: 21.9216
Individual: 4.5976, Fitness: 21.1375

Generation 10:

Individual: 4.7022, Fitness: 22.1109
Individual: 4.6977, Fitness: 22.0688
Individual: 4.6969, Fitness: 22.0612
Individual: 4.6955, Fitness: 22.0479
Individual: 4.6948, Fitness: 22.0409
Individual: 4.6948, Fitness: 22.0409
Individual: 4.6909, Fitness: 22.0044
Individual: 4.6902, Fitness: 21.9979
Individual: 4.6902, Fitness: 21.9979
Individual: 4.6902, Fitness: 21.9979

Top 3 Solutions:

Individual: 4.7322, Fitness: 22.3941

Individual: 4.6996, Fitness: 22.0861

Individual: 4.6977, Fitness: 22.0688

This code will print the population and their fitness values at each generation. After the final generation, it will print the top 3 solutions based on their fitness values. Note that the number of generations has been reduced to 10 for brevity, and you can adjust it back to 100 or any other number to see more evolution stages.

Keep in mind that due to the randomness in the genetic algorithm (in selection, mutation, and initial population), the output will vary each time you run the script. The algorithm is designed to maximize the fitness function $f(x)=x^2$, so the top solutions will be those with x values close to the boundaries of the defined range $[-10,10]$, as these yield the highest $f(x)$ values.

In []: