Summary of Gödel's incomplete theorem

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Gödel's incompleteness theorems are two theorems of mathematical logic that are concerned with the limits of provability in formal axiomatic theories. These results, published by Kurt Gödel in 1931, are important both in mathematical logic and in the philosophy of mathematics. The theorems are widely, but not universally, interpreted as showing that Hilbert's program to find a complete and consistent set of axioms for all mathematics is impossible.

- First Incompleteness Theorem:-Any consistent formal system F within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of F which can neither be proved nor disproved in F.

Proof sketch for the first theorem :-

1.Proof by contradiction, it includes Arithmetization of syntax, provability, Diagonalization 2.Proof via Berry's paradox 3.Computer verified proofs

- Second Incompleteness Theorem:-

For each formal system F containing basic arithmetic, it is possible to canonically define a formula Cons(F) expressing the consistency of F. This formula expresses the property that ”there does not exist a natural number coding a formal derivation within the system F whose conclusion is a syntactic contradiction.” The syntactic contradiction is often taken to be ”0=1”, in which case Cons(F) states ”there is no natural number that codes a derivation of ’0=1’ from the axioms of F.”Gödel's second incompleteness theorem shows that, under general assumptions, this canonical consistency statement Cons(F) will not be provable in nF. G¨odel's second incompleteness theorem shows that, under general assumptions, this canonical consistency statement Cons(F) will not be provable in F.