**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

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| 1. **Manual Calculation**   Since, the car servicing begin after 10 min the average time increases ( = 45min + 10 mins = 55 mins  Probability of maximum time for car to be ready = P (Z= 60,=45,) = = P = P(Z=0.625) = 0.7324  Probability that the service Manager can’t meet his commitment = 1 – 0.7324 = 0.2676 |
| 1. **By Python** |
| **Answer: Option B** (Probability of service Manager can’t meet his commitment = 0.2625 = 26.59%) |

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

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| * As data is normally distributed and standard deviation (*σ*) = 6, * By the Empirical rule ,68% of data lies between *μ* ± *σ* (38 ± 6) or (32,44), * **Number of Employees between 38 and 44 = (68% \* 400) /2 = 136 Employees** * **Number of Employees Above 44 = (400 /2) – 136 = 64 Employees** * Therefore, the less Employees (72 Employees are less) at processing center are older than 44 than between 32 and 44.   **Answer: False** |

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

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| P (<30) = P (Z30, µ=38,)  Z30=  Z30= -1.33  P ( = 0.0918 = 9.18%  **Probable Number of Employees below 30 Years = 9.18%\*400 = 36.72 37 Employees**  Therefore, A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.  **Answer: True** |

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

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| * IF X1 is Normally Distributed, 2X1 is also Normally Distributed, 2X1 Is just Larger Scale of the X1.   If X ~ N (μ, σ2),  then 2X1 ~N (2μ, 2σ2)   * If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables (X1 ~ N (μ, σ2) and *X*2 ~ *N* (μ, σ2)),   X1 + X2 ~ N (μ1 + μ2, σ12 + σ2 2) |

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

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| X ~ N (µ=100, = 20)  Confidence Level = CL= 0.99 = 99%  Alpha =α = 1 – 0.99 = 0.1 = 1%  **Confidence Interval = CI =?**  CI = µ ± (Z α/2\*)  CI = 100 ± (Z0.05 \* 20)  CI = 100 ± (2.575 \* 20)  CI = 100 ± 51.5  **CI = (48.5, 151.5)** |  |
| **Answer: Option D** (Random variables that the probability of the random variable taking a value between them is 0.99 = (48.48, 151.51)) | |

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

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| Diivision1 = Profit1 ~ N (5, 32) = N (Ẋ1=5, S1 2= 32)  Division2 = Profit2 ~ N (7, 42) = N (Ẋ2=7, S22 =42) |
| µ = Company = (Profit1 + Profit2) = Mean Profit of Diivision1 + Division2 = 5 + 7 = 12  Mean Profit of Company in rupees = 12 \* 45 = 540 Million |
| 1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.   Variance of the Company Distribution = = 32 + 42 = 9 + 16 = 25 = 52  Standard Deviation of the Company Distribution = = = 5  Confidence Level = CL = 0.95  Confidence Interval = CI = µ ± Zα/2= 0.025( = 540 ± 1.96(= (99,981) In Millions |
| 1. Specify the 5th percentile of profit (in Rupees) for the company   To calculate 5th percentile from Z table Zα/2= 0.05 = -1.645  5th percentile = µ - Zα/2= 0.05(= 540 – 1.645(225) = 169.87 Million |
| 1. Which of the two divisions has a larger probability of making a loss in a given year?   Probability of Company 1 Making Loss = P (X<0)  Zx1=0  Probability at x < 0 = P(Zx1=0) = 0.0485 = 4.85%  Probability of Company 2 Making Loss = P (X<0)  Zx2=0  Probability at x < 0 = P(Zx2=0) = 0.0401 = 4.01%  Therefore, Company 1 has High Probability of Making a loss in a given year |