# Wind turbine positioning optimization of wind farm using greedy algorithm

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**4 authors:**


Kai Chen
UNSW Sydney



Mengxuan Song
Tongji University



Zhongyang He
Pennsylvania State University



Xing Zhang
Tsinghua University

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# Wind turbine positioning optimization of wind farm using greedy algorithm

K. Chen, M. X. Song, Z. Y. He, and X. Zhang[a)]
*Key Laboratory for Thermal Science and Power Engineering of Ministry of Education, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, People's Republic of China*

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In this paper, the greedy algorithm is used to solve the wind turbine positioning optimization problem. Various models are employed to describe the problem, including the linear wake model, the power-law power curve model with power control mechanisms, Weibull distribution, and the profit function. The incremental calculation method is developed to consider the influence of the adding turbine on other turbines in the wind farm and accelerate the wind power assessment process. The repeated adjustment strategy is used to improve the optimized result. Three cases with simple models and a case with realistic models are used to test the present method. The results show that the greedy algorithm with repeated adjustment can obtain a better result than bionic algorithm and genetic algorithm in less computational time. The proposed greedy algorithm is an effective solution strategy for wind turbine positioning optimization. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4800194]

## I. INTRODUCTION

Nowadays, the depletion of conventional fossil resources has awakened the attention to the renewable energies from all over the world. Wind energy becomes one of the most important alternative energies due to the advantages of rich resources, widely distributed and environment-friendly.[1] When the wind flows over the wind turbine, wake flow is generated due to the extraction of the wind power and the disturbance of the turbine rotor, which will reduce the power output of the turbine downstream. Therefore, the positions of turbines should be designed to reduce the wake effect and extract more power when constructing wind farm.

Much research has been done on wind turbine positioning optimization problem. Mosetti, Poloni, and Diviacco[2] first introduced genetic algorithm to solve the wind turbine positioning problem in 1994. In Mosetti's study, binary coding genetic algorithm was used and the target was minimizing the install cost per unit power output. The feasibility of genetic algorithm for wind turbine positioning optimization was validated through various wind cases. Based on Mosetti's study, Grady, Hussaini, and Abdullah[3] obtained better solutions through introducing some heuristic criteria and using a larger population and more generations. Wan *et al.*[4] fixed the number of turbines and maximized the total power output using real-coded genetic algorithm. The results showed that the total power outputs were significantly increased compared to the results by binary-coded genetic algorithm. Another commonly used optimization algorithm is greedy algorithm. In 2004, Ozturk and Norman used greedy algorithm to solve the wind turbine positioning problem. In the study, the adding, removing, and moving operators were employed to obtain a better wind turbine layout.[5] Song *et al.* developed the bionic algorithm with repeated adjustment to optimize wind turbine positioning problem. In bionic algorithm, each turbine intends to be located in the position where its own power output is maximized.

[a)]Electronic address: x-zhang@tsinghua.edu.cn.

The solution was improved through repeatedly adjusting the positions of the wind turbines.[6] Zhang *et al.* revealed the submodular property of the wind turbine positioning problem based on the linear turbine wake model. Based on this property, the lazy algorithm was used to accelerate the process of greedy algorithm.[7] Saavedra-Moreno *et al.* used the greedy heuristic algorithm to construct reasonable initial solutions and obtained good solution by using genetic algorithm.[8] Besides, other optimization methods have also been applied to optimize the wind turbine positioning problem, including Monte Carlo method,[9] simulated annealing method,[10] particle swarm optimization,[11] and pattern search approach.[12]

Compared to the evolutionary algorithm based on the population and randomly searching, greedy algorithm is based on a single turbine layout, which needs less computational time. Furthermore, there is no randomness for greedy algorithm. The same optimized layout is obtained for the same condition. Therefore, greedy algorithm is expected to have a better performance in turbine positioning problem than evolutionary algorithms. For bionic algorithm, each turbine intends to be relocated to the position where its own power output is maximized. The influence of the adding turbine to other existing turbines is ignored. Furthermore, for the relocation stage, it cannot guarantee obtaining a better layout for each movement. Therefore, bionic algorithm with repeated adjustment is difficult to converge. In the present study, the greedy algorithm is used to solve the wind turbine positioning problem. The incremental calculation method is developed to consider the influence of the adding turbine to other existing turbines and accelerate the wind power assessment process. The repeated adjustment strategy is used to improve the optimized result. The effectiveness and the applicability of the proposed method is illustrated through three numerical cases in previous study,[3] including uniform wind direction with a wind speed, multi-directional wind with uniform speed and multi-directional wind with variable speeds. A case with realistic models is considered to further test the performance of the proposed greedy algorithm for wind turbine positioning optimization problem.

The remainder of the paper is organized as follows. Section II presents the turbine wake and power generator models. Section III introduces the optimization methodology. Section IV discusses the computational results of four test cases. Section V presents the conclusions.

## II. COMPUTATIONAL MODELS

### A. Wake model

In this paper, the linear wake model in Mosetti's study[2] is introduced to calculate the turbine wake flow. The wake region is considered as a conical area, shown in Figure 1. The velocity is uniform in the wake region and equal to the incoming velocity outside the wake region. The velocity inside the wake region is expressed as

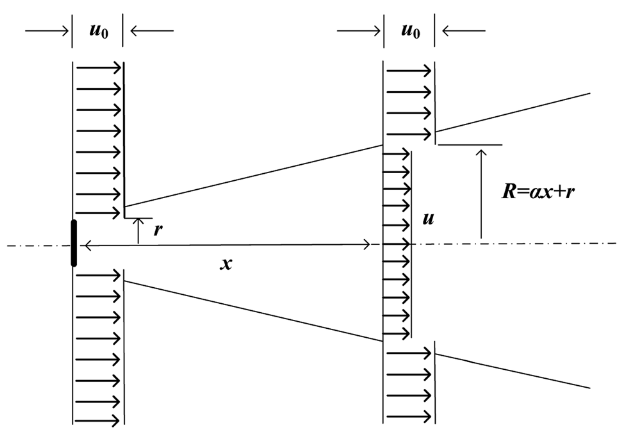



FIG. 1. Schematic of the wake model.

$$u = u_0\left[1 - \frac{2a}{\left(1 + \alpha\frac{x}{r_1}\right)^2}\right], \tag{1}$$

$$a = \frac{1 - \sqrt{1 - C_T}}{2}, \tag{2}$$

$$r_1 = r\sqrt{\frac{1-a}{1-2a}}, \tag{3}$$

$$\alpha = \frac{0.5}{\ln(h/z_0)}, \tag{4}$$

where $u_0$ is the local wind speed without placing the turbine and $x$ is the distance downstream the rotor. $r_1$ is the downstream rotor radius, $a$ is the axial induction factor, and $\alpha$ is the entrainment constant. $C_T$ is the trust coefficient, $r$ is the rotor radius, $h$ is the tower height of the wind turbine, and $z_0$ is the surface roughness of the site.

The wake region at a specified section in the crosswind direction is described by the wake influenced radius $R$, expressed as

$$R = \alpha x + r, \tag{5}$$

where $R$ increases linearly with $x$. In a wind farm with multiple turbines inside, the wake flows of the turbines will impact others' power output. Therefore, the wake interference effect should be taken into account. The square of the velocity deficit of a mixed wake is assumed to equal to the sum of the square of the velocity deficits for each wake at the calculated downwind position, so the velocity of the $i$th turbine is calculated by the following expression:[13]

$$u_i = u_{0i} - \sqrt{\sum_{j=1}^{N}\left[(u_{0j} - u_{ij})^2\right]}, \tag{6}$$

where $u_{0i}$ and $u_{0j}$ are the local velocities at the $i$th and the $j$th turbines' positions before placing the turbines. They equal to the incoming wind speed $u_0$ over flat terrain. $u_{ij}$ is the $i$th turbine speed in the wake region of the $j$th turbine and $N$ is the number of wind turbines.

### B. Power output

The power extracted from wind mainly depends on the local wind speed, including direction, intensity, and probability. In Grady's study,[3] the power output is calculated by

$$P_{\text{curve}}(u) = 0.3 \times u^3, \tag{7}$$

where $u$ is the local wind speed of the turbine. However, in reality, the power output of the turbine remains constant due to the power control mechanisms, which can protect the turbine when the wind speed is high. The power output with power control mechanisms can be modeled by the following expression:[14]

$$P_{\text{curve}}(u) = P_r \times \begin{cases} 0, & u < u_c \quad \text{or} \quad u > u_f \\ P_{\text{asc}}, & u_c \le u \le u_r \\ 1, & u_r \le u \le u_f, \end{cases} \tag{8}$$

where $u_c$ is the cut-in speed, $u_f$ is the cut-out speed, and $u_r$ is the rated speed. $P_{\text{curve}}$ is the real power output, $P_r$ is the rated output power of the turbine, and $P_{\text{asc}}$ is the turbine output as the percentage of $P_r$.

The total power output of wind farm can be modeled as

$$\begin{aligned} P_{\text{tot}} &= \sum_{i=1}^{M}\left[p_i\int_0^{\infty} P_{\text{layout}_i}(u_i)f_i(u)\mathrm{d}u\right] \\ &= \sum_{i=1}^{M}\left[p_i\int_0^{\infty}\left(\sum_{j=1}^{N}P_{\text{curve}}(u_{ij})\right)f_i(u)\mathrm{d}u\right], \end{aligned} \tag{9}$$

where $M$ is the number of wind directions, $p_i$ is the probability of the $i$th wind direction, and $P_{\text{layout}_i}(u_i)$ is the power output of the wind farm for $u_i$ in the $i$th wind direction. $N$ is the number of wind turbines, $u_{ij}$ is the local speed of the $j$th turbine in the $i$th wind direction, and $P_{\text{curve}}(u_{ij})$ is the power output for $u_{ij}$ through the power curve. $f_i(u)$ is the probability density function (PDF) of the wind speed in the $i$th direction. The 2-parameter Weibull distribution is commonly used to model the wind speed characteristics, shown as[15]

$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{(k-1)} e^{-\left(\frac{u}{c}\right)^k}, \tag{10}$$

where $k$ is the shape parameter and $c$ is the scale parameter.

## C. Objective

In previous study, the total power output by Eq. (9) is a common objective for wind turbine positioning optimization.[4] Another index to evaluate the performance of the wind farm is the levelized cost of energy ($LCOE$), shown as[16]

$$LCOE = \frac{C_C \cdot FCR + C_{\text{O\&M}}}{AEP}, \tag{11}$$

where the unit of $LCOE$ is dollars per kWh. $C_C$ is the total cost of the wind farm, including the turbine, support structure, and transmission costs, in dollars. Fixed charge rate ($FCR$) is the fix charge rate, including debt and equity costs, taxes, and insurance. $C_{\text{O\&M}}$ is the annual operation and maintenance cost, in dollars. $AEP$ is the annual energy production of the wind farm, in kWh per year. However, usually the maximum profit is the target when constructing the wind farm. Therefore, in this paper, the profit function is considered as the objective, modeled by the following expression:

$$\text{PROFIT} = AEP \cdot a_p - (C_C \cdot FCR + C_{\text{O\&M}}), \tag{12}$$

where the unit of the profit function is dollars per year. $a_p$ is the profit of 1 kWh electricity, in dollars per kWh. Maximizing Eq. (12) obtains the maximum profit of wind farm.

## D. Distance factor and efficiency

The wind turbines should be located far enough from each other to avoid damaging others in case of falling down when constructing wind farm. The distance factor is defined to describe the distances among wind turbines for a layout, shown as

$$\begin{aligned} d_F &= \min_{1\le i,j\le N, i\ne j}\left(\frac{D_{ij}}{R_{ij}}\right) \\ D_{ij} &= \sqrt{(x_i-x_j)^2+(y_i-y_j)^2} \\ R_{ij} &= h_i + h_j + r_i + r_j, \end{aligned} \tag{13}$$

where $R_{ij}$ is the sum of the heights of two turbines, including the hub height and the rotor radius. $D_{ij}$ is the horizontal distance between two turbines. When $d_F$ is greater than 1, the turbines will not damage others when falling down. $d_F \ge 1$ is defined as the safe distance condition.

The efficiency of the wind farm is defined as

$$\eta = \frac{P_{\mathrm{tot}}}{P_0}, \tag{14}$$

where $P_{\mathrm{tot}}$ is the power output expressed in Eq. (9) and $P_0$ is the total power output when all the turbines operate at the local speed without wake effect. As $\eta$ increases, $P_{\mathrm{tot}}$ increases and the total wake effect of the wind farm decreases. When $\eta$ equals to 1, each turbine is completely not affected by the wake flow of other turbines.

## III. WIND TURBINE POSITIONING OPTIMIZATION

### A. Greedy algorithm with repeated adjustment

In the present study, flat terrain is considered. The velocity of the empty field is treated as uniform. The greedy algorithm with repeated adjustment is used to solve the wind turbine positioning optimization problem. The algorithm contains two stages. Stage 1 starts from the empty layout. Turbines are added into the wind farm one by one according to the evaluation function value. Evaluation function depends on the objective, which is desirable for optimality. The detailed steps for stage 1 are listed as follows:

1. The wind farm is meshed to place wind turbines: In the present study, the wind farm is meshed by square grids, the same as previous studies.[2–4]
2. Add a turbine at the grid with the maximum wind speed.
3. Try each empty grid with a turbine and calculate the evaluation value.
4. Add a turbine at the grid with the maximum evaluation value and record the order of the turbines adding in the wind farm.
5. If the specified number of turbines is reached, stage 1 is completed. Otherwise, return to step 4 and continue the procedure.

In stage 1, the positions of the turbines are optimum for each adding turbine operation. However, this strategy cannot guarantee obtaining a global optimal solution for the last turbine layout. Therefore, stage 2 is necessary for further optimization of the layout. The steps of stage 2 are shown below.

1. Remove one turbine according to order of the turbines in the adding process.
2. Try each empty grid with a turbine and calculate the evaluation value.
3. Add a turbine back to the layout at the grid with the maximum evaluation value.
4. A cycle is defined when all the turbines go through the relocating procedure. If the layout does not change for the whole cycle, stage 2 is completed. Otherwise, return to step 1 and continue the procedure.

When the procedure is finished, the optimized layout is obtained. The wake flow is recalculated when a turbine is added or removed. Therefore, the wake flow effect is already contained in the evaluation function. For bionic algorithm, the evaluation function is the local speed at the grid before placing the adding turbine, which does not contain the influence of the adding turbine to other turbines in the wind farm. Bionic algorithm cannot guarantee obtaining better solution for each relocating operation in stage 2. Therefore, it needs more steps to finish stage 2 and the final optimized result does not match the objective exactly. For the present greedy algorithm, the evaluation function is the objective value, such as the total power output or the profit function, which contains the influence of the adding turbine to other existing turbines. For stage 1, the solution of each step in present study is better than the one by bionic algorithm. For stage 2, the objective value in present study does not reduce for each relocating operator. Therefore, it needs minimal steps to finish stage 2 and can obtain a better result than bionic algorithm.

### B. Incremental calculation method for wind power assessment

Wind power output determines the profit of the wind farm and wind power assessment is an important part of wind turbine positioning optimization. The total computational time can be reduced if the wind power assessment process is accelerated. In the present study, the increment calculation method is used. When adding the $k$th turbine into the wind farm, consider Eq. (6), the local speed of the $i$th turbine can be expressed as

$$u_i = \begin{cases} u_0\left(1-\sqrt{\left(\sum_{j=1}^{k-1, j\neq i}[(1-u_{ij}/u_0)^2]\right)+(1-u_{ik}/u_0)^2}\right) & i=1,2\cdots,k-1 \\ u_0\left(1-\sqrt{\left(\sum_{j=1}^{k-1}[(1-u_{kj}/u_0)^2]\right)}\right) & i=k, \end{cases} \tag{15}$$

where $(1-u_{ij}/u_0)$ is the speed deficit of $i$th turbine from the wake flow of $j$th turbine. The term $(1-u_{ik}/u_0)$ represents the influence of the $k$th turbine on the local speed of $i$th turbine. The term $(1-u_{kj}/u_0)$ represents the influence of the $j$th turbine on the local speed of $k$th turbine. In present study, these two influences are both considered. Note that the local wind speed of one turbine depends on other turbines' wake flow. The wake effect of each turbine in the speed deficit is independent of each other. From Eq. (1), $(1-u_{ij}/u_0)$ only depends on the distance between the $i$th turbine and the $j$th turbine. Therefore, when adding a turbine into the wind farm, the wake flows of original turbines do not need to be calculated again. That is, $(1-u_{ij}/u_0)$ has been calculated by the previous process. It only needs to calculate $(1-u_{ik}/u_0)$ and $(1-u_{kj}/u_0)$ when adding the $k$th turbine. This incremental calculation method can accelerate the wind power assessment process. When ignoring the term of $(1-u_{ik}/u_0)$ and the objective is chosen as the total power output, the greedy algorithm described in Sec. III A becomes the bionic algorithm developed by Song *et al.*[6] The comparison of proposed greedy algorithm and bionic algorithm is done in the following discussion.

### C. Modification for the objective

The objective has no constraint on the distances among the wind turbines. Therefore, the distances among the turbines may be too close for the optimized solution. In this study, a modification operator is defined to adjust the distances among the turbines during the optimization process. The modified objective for optimization is expressed as

$$E = \sigma \cdot \text{Object}, \tag{16}$$

where

$$\sigma = \prod_{1\le i,j\le N, i\neq j} \delta_{ij}, \tag{17}$$

$$\delta_{ij} = \begin{cases} 0 & D_{ij} < \lambda R_{ij} \\ 1 & D_{ij} \ge \lambda R_{ij} \end{cases}, \tag{18}$$

where $E$ is the modified objective for optimization, which is desirable for optimality. $\lambda$ is the adjust coefficient, $D_{ij}$ and $R_{ij}$ are defined in Eq. (13). When $D_{ij} < \lambda R_{ij}$, $E$ equals to 0. When $D_{ij} \ge \lambda R_{ij}$, $E$ equals to the objective. The modified operator makes the wind turbines move to the positions satisfying the distance condition ($D_{ij} \ge \lambda R_{ij}$). A large number of numerical experiments have been done. The results indicate that the distances among the wind turbines are able to be controlled through $\lambda$. $\lambda$ should be greater than 1 to guarantee the safe distance condition satisfied.

## IV. NUMERICAL STUDY

### A. Numerical procedure

Three test cases with simple models and a case with realistic models are introduced to verify the effectiveness of greedy algorithm with repeated adjustment. The wind farm domain is chosen as a square area with the size of 2000 m × 2000 m. The ground roughness of the site is chosen to be 0.3 m.[3] The definition of wind directions is shown in Figure 2(a). The top, bottom, left, and right are the north, south, west, and east, respectively. The wind from west to east is defined as 0° and the one from south to north is defined as 90°.

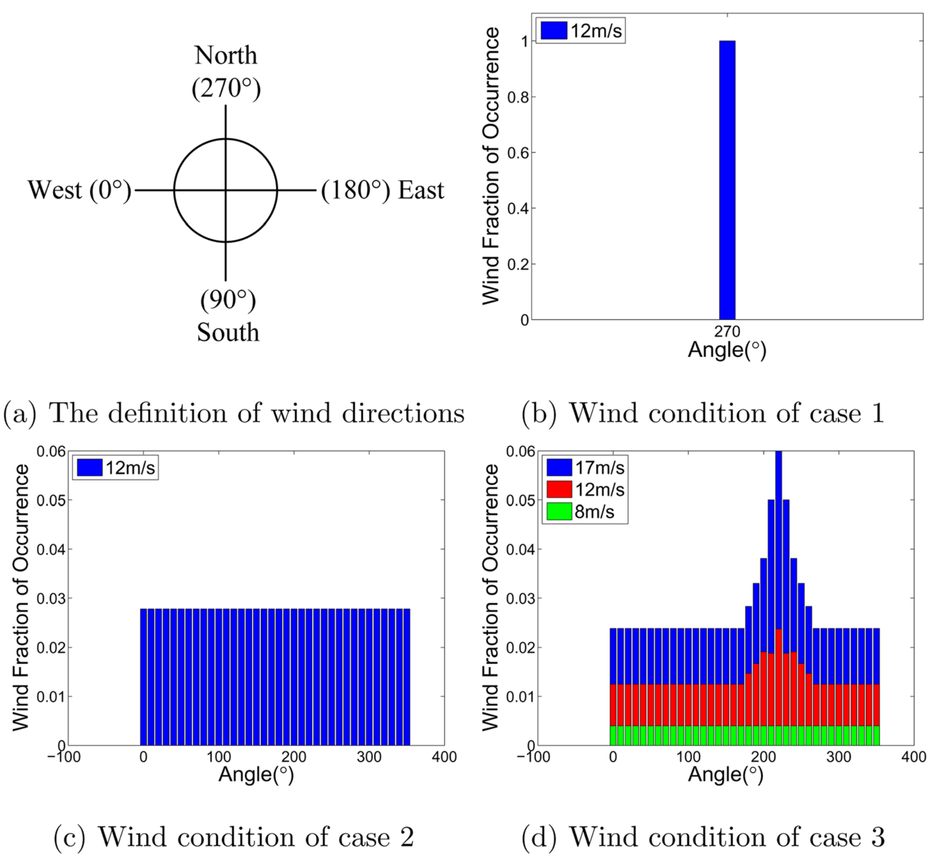
## B. Test cases with simple models

In this section, three cases in Grady's study[3] are studied, based on the power curve with power law as Eq. (7) and the target of maximizing the power output of the wind farm. The wind turbine properties used are shown in Table I. The cases are optimized by the greedy algorithm with stage 1 (denoted by Greedy-1) and the greedy algorithm with stages 1 and 2 (denoted by Greedy-2). The optimized results are compared with the ones by Grady (binary-coded genetic algorithm),[3] Zhang (Greedy algorithm)[7] and Wan (real-coded genetic algorithm).[4] The three cases are listed as follows:

- Case 1: Uniform wind direction (from north to south) with wind speed at 12 m/s. The wind condition is shown in Figure 2(b).
- Case 2: Multi-directional (36 directions with intervals of 10°) wind with uniform speed at 12 m/s. The wind condition is shown in Figure 2(c).
- Case 3: Multi-directional (36 directions with intervals of 10°) wind with speeds at 8, 12, and 17 m/s. The wind condition is shown in Figure 2(d).

### 1. *Case 1*

Table II shows the comparison of the results for Case 1. In this case, 30 turbines are designed to place in the domain. For 10 × 10 grids, the total power outputs of Greedy-1 and




(a) The definition of wind directions (b) Wind condition of case 1

(c) Wind condition of case 2 (d) Wind condition of case 3

FIG. 2. Wind conditions of the three test cases.

TABLE I. Wind turbine properties for the three test cases.[3]

| Property | Value |
|---|---|
| Hub height ($z$) | 60 m |
| Rotor diameter ($D$) | 40 m |
| Trust coefficient ($C_T$) | 0.88 |

Greedy-2 are the same as the results of Grady and Zhang. For the study of Wan, though the power output is high, the distance factor is only 0.39, which does not satisfy the safe distance condition. For $30 \times 30$ grids, Greedy-1 and Greedy-2 obtain very good results. The turbine wake effect is avoided completely and the efficiency is 100%, while the safe distance condition is still satisfied. The number of turbine wake calculation times ($N_w$) for each method is also presented in Table II, which is used to evaluate the computational cost of the algorithm. The results show that $N_w$ of greedy algorithm is much less than the one of genetic algorithm. The strategy of Greedy-1 is similar to the one of Zhang's study, but $N_w$ is less due to the incremental calculation method. For $10 \times 10$ grids, $N_w$ of Greedy-1 is about 1/10 of the one in Zhang's study. $N_w$ of Greedy-2 is several times more than Greedy-1, which is relative to the number of relocation cycles in stage 2. For Case 1, Greedy-1 has obtained a good enough result.

The optimized layouts using various methods are shown in Figure 3. Figure 3(a) shows Grady's solution. As the wind is from north to south, the turbines are located from west to east across the wind direction. Along the wind direction, the turbines are located as far as possible to reduce the wake effect. Figure 3(b) shows Wan's solution. In this layout, the turbines can be located in any positions in the domain. As Wan took Grady's result as the initial layout, the turbines are around the positions of turbines in Grady's layout. The turbines are placed staggered to reduce the wake effect. However, there is no constraint on the distance among the turbines. Some turbines are placed too close, which does not satisfy the safe distance condition. Figures 3(c) and 3(d) show the optimized solutions of present study. The result for $10 \times 10$ grids is the same as Grady's solution. For $30 \times 30$ grids, the turbines are placed staggered and compact to avoid the wake flow. Therefore, the turbine layout of present study can extract more power.

### *2. Case 2*

Figure 4 shows the optimized layouts by various methods for Case 2. Table III is the comparison of solution characteristics of each layout. In Case 2, 39 turbines are designed to place in the domain. The power output of Greedy-1 for $10 \times 10$ grids increases by 1.8% compared to Grady's study. The power output of Greedy-1 is almost the same as the one by Zhang for the same number of grids and $N_w$ is about 1/12 of the one in Zhang's study. Stage 2 does not improve the optimized result much when the grids are coarse. When the number of grids increases, the allowed positions for the turbines increase and the wake effect can be reduced

TABLE II. Comparison of results for Case 1.

| | Grady (GA-Binary) | Zhang (Greedy) | Wan (GA-Real) | Present study (Greedy-1) | | Present study (Greedy-2) | |
|---|---|---|---|---|---|---|---|
| Mesh | $10 \times 10$ | $10 \times 10$ | … | $10 \times 10$ | $30 \times 30$ | $10 \times 10$ | $30 \times 30$ |
| Total power output (kW) | 14311.9 | 14311.9 | 15262.2 | 14311.9 | 15520.0 | 14311.9 | 15520.0 |
| Efficiency (%) | 92.0 | 92.0 | 98.1 | 92.0 | 100.0 | 92.0 | 100.0 |
| Number of turbines | 30 | 30 | 30 | 30 | 30 | 30 | 30 |
| Adjust coefficient | … | … | … | 1.25 | 1.25 | 1.25 | 1.25 |
| Distance factor | 1.25 | 1.25 | 0.39 | 1.25 | 1.25 | 1.25 | 1.25 |
| $N_w$ | $3.6 \times 10^6$ | $3.5 \times 10^5$ | $3.3 \times 10^8$ | $3.5 \times 10^4$ | $3.8 \times 10^5$ | $1.1 \times 10^5$ | $1.1 \times 10^6$ |

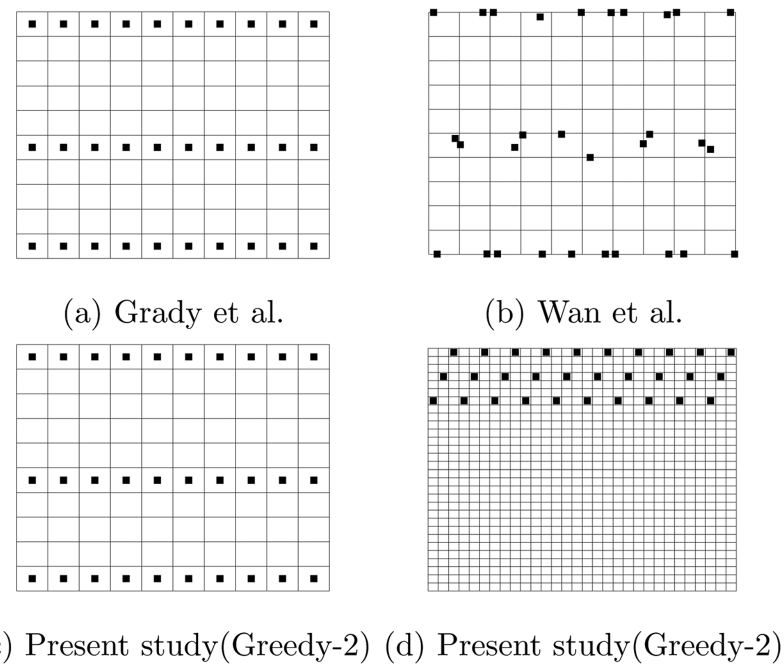



FIG. 3. Optimized layouts for Case 1.

effectively. The power output of Greedy-2 with $39 \times 39$ grids increases by 1.5% compared to Wan's study, and $N_{\mathrm{w}}$ of present study is less than 1/100 of the one in Wan's study. The repeated adjustment improves the optimized results for $39 \times 39$ grids. The total power output of Greedy-2 increases by 0.5% compared to Greedy-1. On the other hand, for Case 2, no directions dominate and the optimal layout should be centrosymmetric. The optimized layout of present study is similar to Wan's solution, but the turbines are more likely to be located at the edge of the domain, which enlarges the distances among the turbines and reduce the wake effect.

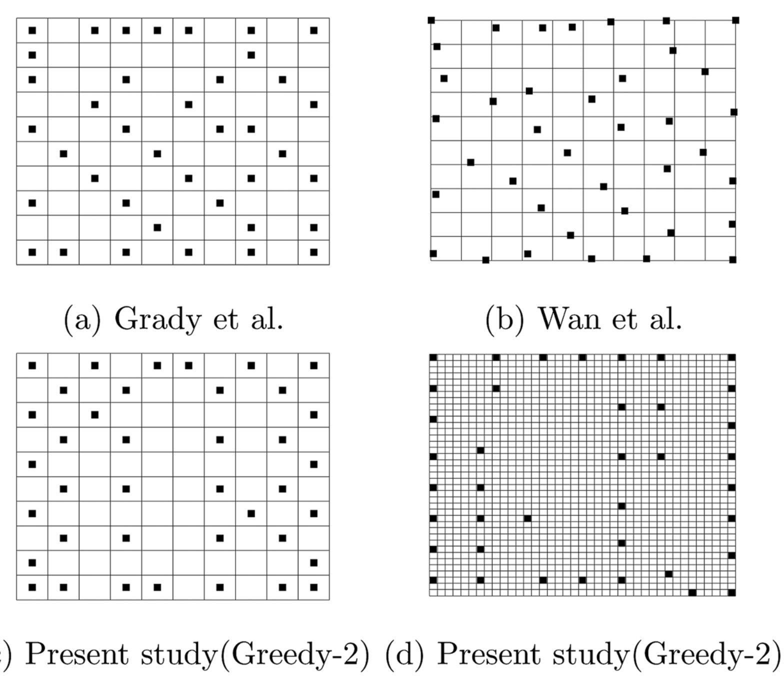



FIG. 4. Optimized layouts for Case 2.

TABLE III. Comparison of results for Case 2.

| | Grady (GA-Binary) | Zhang (Greedy) | Wan (GA-Real) | Present study (Greedy-1) | | Present study (Greedy-2) | |
|---|---|---|---|---|---|---|---|
| Mesh | $10 \times 10$ | $10 \times 10$ | … | $10 \times 10$ | $39 \times 39$ | $10 \times 10$ | $39 \times 39$ |
| Total power output (kW) | 17241.8 | 17549.6 | 18133.6 | 17549.2 | 18314.4 | 17555.7 | 18409.9 |
| Efficiency (%) | 85.3 | 86.8 | 89.7 | 86.8 | 90.6 | 86.9 | 91.1 |
| Number of turbines | 39 | 39 | 39 | 39 | 39 | 39 | 39 |
| Adjust coefficient | … | … | … | 1.25 | 1.25 | 1.25 | 1.25 |
| Distance factor | 1.25 | 1.25 | 1.21 | 1.25 | 1.28 | 1.25 | 1.36 |
| $N_w$ | $4.8 \times 10^{10}$ | $2.5 \times 10^{7}$ | $2.0 \times 10^{10}$ | $2.0 \times 10^{6}$ | $4.0 \times 10^{7}$ | $1.0 \times 10^{7}$ | $2.0 \times 10^{8}$ |

### 3. Case 3

For Case 3, the angles from 180° to 270° have higher wind speeds. Therefore, the turbines are more likely to lie across over these directions. The optimized layouts of various methods for Case 3 are shown in Figure 5. Table IV is the comparison of solution characteristics for each layout. In Case 3, 39 turbines are designed to place in the domain. As the wind characteristics of Case 3 are obtained from Figure 4 in Grady's paper, there are errors in the probabilities of Case 3 in our paper when compared with the studies of others. In order to compare with the previous studies, the optimized layouts of Grady's, Zhang's, and Wan's are evaluated using the wind characteristics of Case 3 in the present study. Similar to Case 2, the power output of Greedy-1 with $10 \times 10$ grids is increased a little compared to the one by Zhang and $N_w$ is about 1/12 of the one in Zhang's study. The power output of Greedy-1 for $10 \times 10$ grids increases by 1.6% compared to Grady's study. The power output of Greedy-2 for $39 \times 39$ grids increases by 1.4% compared to Wan's study. For $10 \times 10$ grids, the repeated adjustment operation does not improve the total power output. While for $39 \times 39$ grids, the total power output of Greedy-2 increases by 0.7%.

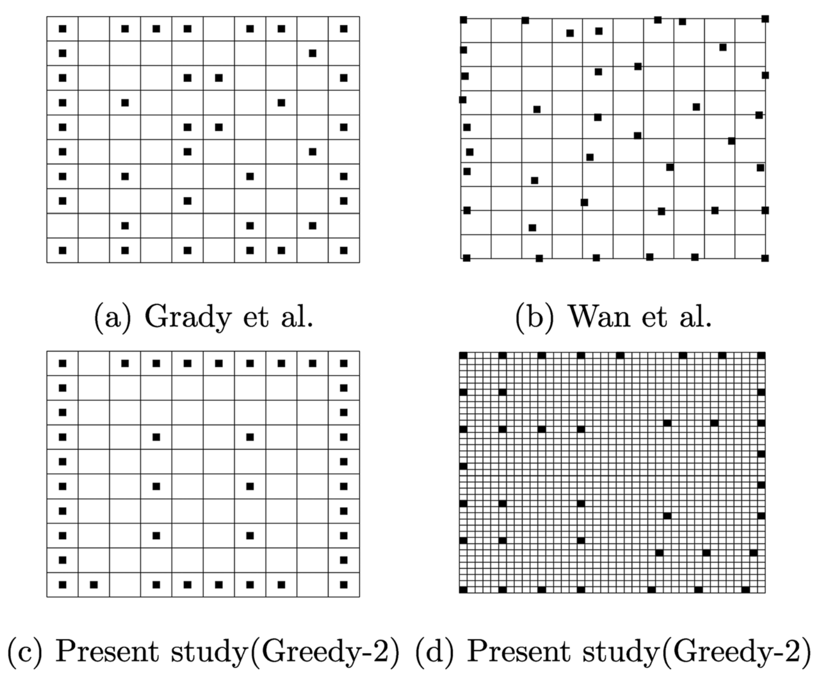


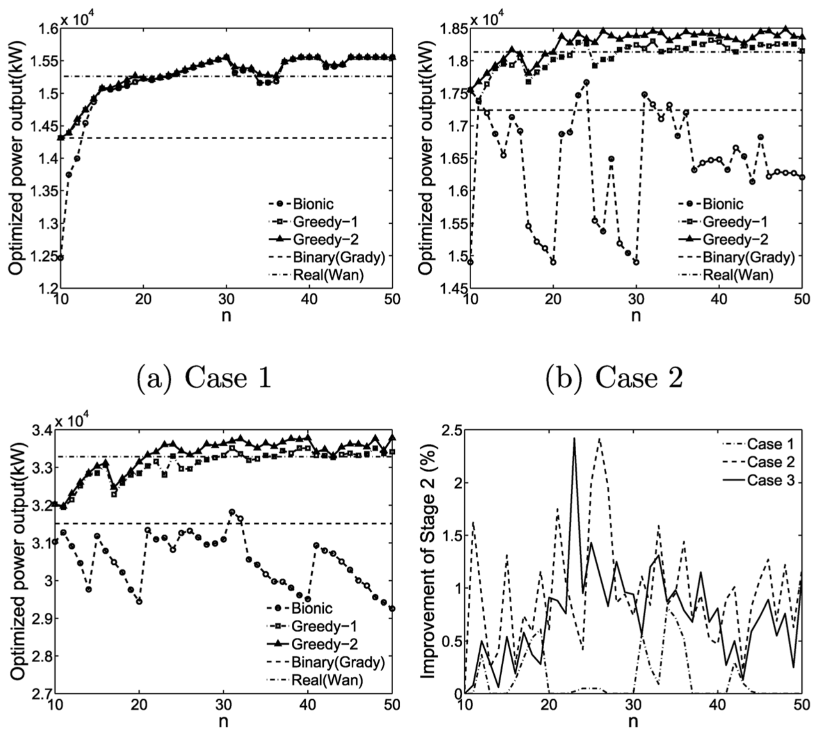

FIG. 5. Optimized layouts for Case 3.

TABLE IV. Comparison of results for Case 3.

| | Grady (GA-binary) | Zhang (Greedy) | Wan (GA-real) | Present study (Greedy-1) | | Present study (Greedy-2) | |
|---|---|---|---|---|---|---|---|
| Mesh | $10 \times 10$ | $10 \times 10$ | … | $10 \times 10$ | $39 \times 39$ | $10 \times 10$ | $39 \times 39$ |
| Total power output (kW) | 31511.7 | 31943.4 | 33282.9 | 32026.6 | 33511.4 | 32026.6 | 33739.4 |
| Efficiency (%) | 85.5 | 86.7 | 90.4 | 86.9 | 90.9 | 86.9 | 91.5 |
| Number of turbines | 39 | 39 | 39 | 39 | 39 | 39 | 39 |
| Adjust coefficient | … | … | … | 1.25 | 1.25 | 1.25 | 1.25 |
| Distance factor | 1.25 | 1.25 | 1.02 | 1.25 | 1.60 | 1.25 | 1.60 |
| $N_w$ | $1.4 \times 10^{11}$ | $7.5 \times 10^7$ | $6.0 \times 10^{10}$ | $6.0 \times 10^6$ | $1.2 \times 10^8$ | $1.8 \times 10^7$ | $1.1 \times 10^9$ |

### *4. Effect of the number of grids*

The number of grids limits the allowed positions of the wind turbines and affects the optimized result. In this section, the effect of the number of grids is studied. Figures 6(a)–6(c) show the optimized power outputs for various numbers of grids ($n \times n$) using bionic algorithm (Bionic), greedy algorithm with stage 1 (Greedy-1), and Greedy algorithm with stages 1 and 2 (Greedy-2). The results of Grady and Wan are also presented. Figure 6(d) shows the improvement of stage 2 for the three test cases. For Case 1, bionic algorithm has a similar performance as present study and the optimized power outputs are larger than the one of Wan's study when $n$ is large. The results of Greedy-1 and Greedy-2 are almost the same and stage 2 does not improve the optimized result much. The maximum improvement of the considered $n$s is only 0.8%. For Case 2 and Case 3, the performance of bionic algorithm becomes bad. The optimized




(a) Case 1 (b) Case 2

(c) Case 3 (d) Improvement of Stage 2

FIG. 6. The optimized results for various optimization methods.

results in most $n$s are worse than the ones by binary-coded genetic algorithm. Bionic algorithm cannot obtain good optimized results for the multi-directional wind cases on flat terrain. For Case 2 and Case 3, the improvement of stage 2 for greedy algorithm is obvious. The maximum improvement is 2.4% and the average improvement is about 1%. For the three cases, when $n$ is larger than 20, the optimized power outputs of present study are larger than the results of real-coded algorithm (Wan's study). As the grids limit the positions of the turbines, the optimized results do not increase with $n$ monotonously.

### C. Case 4: A case with realistic models

In this section, a case with realistic models is considered, based on Weibull distribution, the power curve with power control mechanisms as Eq. (8) and the objective as Eq. (12). Wind condition with 16 wind directions is considered, satisfying a Weibull distribution in each direction. The parameters of the Weibull distributions are generated randomly, shown in Figure 7. The wind turbine properties used in this case are listed in Table V.[17] Through 3-order fitting, the power curve model is expressed as

$$P(u) = \begin{cases} 0, & u < 3\text{m/s} \quad \text{or} \quad u \geq 25\text{m/s} \\ 0.68 \times u^3, & 3\text{m/s} \leq u \leq 13\text{m/s} \\ 1500, & 13\text{m/s} \leq u < 25\text{m/s}. \end{cases} \tag{19}$$

The parameters of PROFIT in Eq. (12) are listed in Table VI. Each wind turbine in the wind farm is connected to the connection point through the transmission wire. The connection point is set in the middle of the western boundary of the wind farm. The target is increasing the power output and reducing the transmission cost.

Table VII shows the optimized results of the present greedy algorithm and real-coded genetic algorithm in the wind farm with 19 turbines. The parameters of the real-coded genetic algorithm are summarized in Table VIII. The adjust coefficient is chosen as 1.05. In the present study, the domain is meshed into $19 \times 19$. The result indicates that the proposed greedy algorithm can obtain a better solution than real-coded genetic algorithm. The PROFIT value of present study is \$1 264 700, increasing by 8.7% compared to the result \$1 164 000 by real-coded genetic algorithm. The distance factor of present study is 1.4, increasing by 33.3%. The efficiency of present study is 93.2%, increasing by 3.0%. Furthermore, the computational time of present study is about 15 min, only 1/7 of the time using real-coded genetic algorithm. The transmission cost of present study is a litter larger, indicating that the distances of the

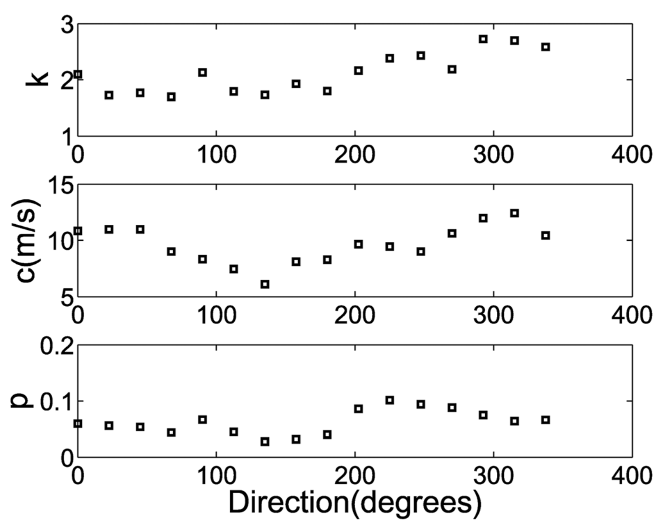



FIG. 7. Wind data for Case 4.

TABLE V. Wind turbine properties used in Case 4.[17]

| Property | Value |
|---|---|
| Hub height ($z$) | 80 m |
| Rotor diameter ($D$) | 77 m |
| Trust coefficient ($C_T$) | 0.88 |
| Cut-in speed ($u_c$) | 3 m/s |
| Rated speed ($u_r$) | 13 m/s |
| Cut-out speed ($u_f$) | 25 m/s |
| Rated power output ($P_r$) | 1500 kW |

TABLE VI. Parameters of PROFIT.[17]

| Parameter | Value |
|---|---|
| Profit for electricity (/kWh) | $0.06 |
| Turbine cost | $700 000 each |
| Transmission cost (/km) | $620 000 |
| Supporting structure cost | $600 000 each |
| *FCR* | 10% |
| Annual O and M cost ($C_{O\&M}/C_C$) | 2% |

TABLE VII. Comparison of the optimized results of Case 4.

| | Real | Present study |
|---|---|---|
| Number of variables | 19 | $19 \times 19$ |
| PROFIT | $1 164 000 | $1 264 700 |
| Efficiency (%) | 90.5 | 93.2 |
| Transmission cost | $9 302 100 | $9 748 100 |
| Number of turbines | 19 | 19 |
| Adjust coefficient | 1.05 | 1.05 |
| Distance factor | 1.05 | 1.40 |
| Computational time | 1 h 45 min | 15 min |

TABLE VIII. Parameters of the real-coded genetic algorithm.

| Parameter | Value |
|---|---|
| Number of genes | 19 |
| Population size | 200 |
| Number of generations | 3000 |
| Selection operator | Tournament |
| Crossover operator | Uniform |
| Crossover probability | 1.0 |
| Mutation operator | Simple |
| Mutation probability | 0.01 |
| Number of elitists | 3 |

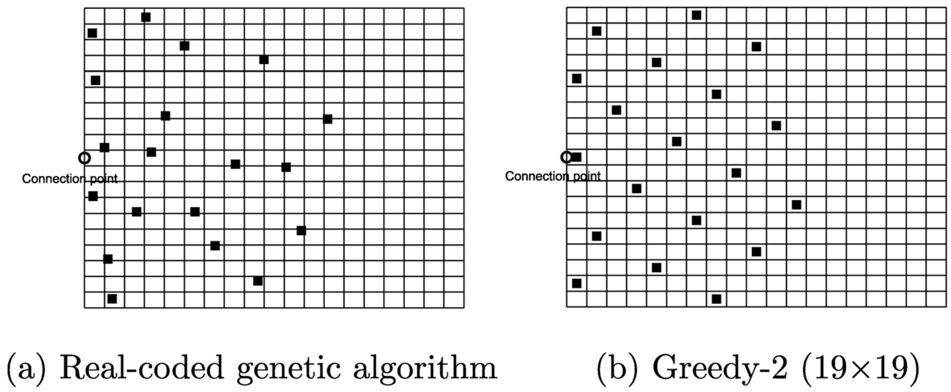



FIG. 8. Optimized layouts for Case 4.

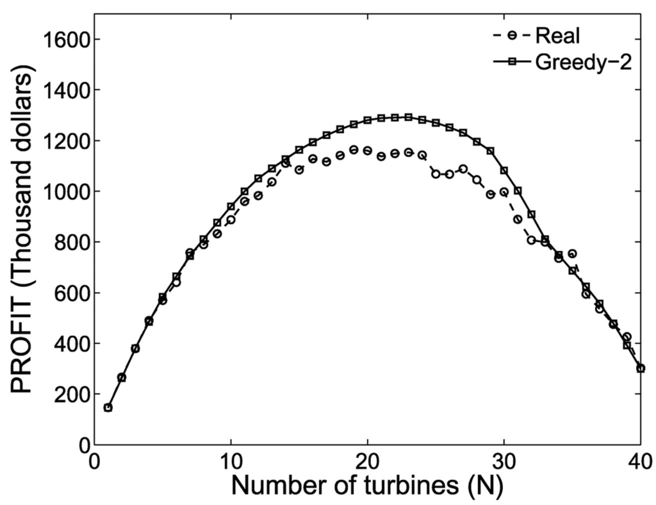



FIG. 9. Optimized results of Case 4 for various numbers of turbines.

turbines to the connection point is larger. However, this will reduce the wake effect. The increasing power output overweighs the increasing transmission cost.

Figure 8 shows the optimized layouts of the two methods. As PROFIT includes the transmission costs, the turbines are likely to locate near the connection point to reduce the transmission costs. Figure 9 shows the optimized PROFIT for various numbers of turbines ($N$). For Greedy-2, as $N$ increases, the number of the grids should increase to provide more positions to the turbines. Then the wake effect can be reduced effectively. Therefore, the number of grids is chosen as $19 \times 19$ when $N \leq 33$ and is chosen as $30 \times 30$ when $N \geq 34$. When the number of turbines is less than 10 or larger than 30, the optimized results by both algorithm are almost the same. When the number of turbines is between 10 and 30, the optimized profit by Greedy-2 is better than the ones by real-coded genetic algorithm. For Greedy-2, the maximum of the curve is at the point of $N = 23$, valued \$1 293 000. That is, when the number of turbines to be placed in the wind farm equals to 23, the profit function obtains the maximum value.

## V. CONCLUSIONS

In this paper, the greedy algorithm has been introduced to solve the wind turbine positioning problem in wind farm, based on the linear wake model, the power-law power curve model, Weibull distribution, and the profit model. When assessing the power output of wind farm, the incremental calculation method is used to consider both the wake effect from other turbines and the power deficit on other turbines, which can accelerate the wind power assessment process. The repeated adjustment strategies are introduced to improve the optimized result. The effectiveness of the methodology is demonstrated on four test cases, including three cases with simple models and one case with realistic models. The results indicate that the proposed greedy algorithm using incremental calculation method can obtain better solutions than bionic algorithm and genetic algorithm in less computational time. The optimized results can be improved through the repeated adjustment, especially for the multi-directional wind cases. As a conclusion, greedy algorithm with repeated adjustment is an effective solution strategy for the wind turbine positioning problem.

## ACKNOWLEDGMENTS

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