Assignment on Chain Rule

01. Given that, $f(z) = \ln(1+z)$, where $z = x^T x$, $x \in \mathbb{R}^d$

If $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$, then $x^T = \begin{bmatrix} x_1 & x_2 & \cdots & x_d \end{bmatrix}$

$$\therefore x^T x = \left[x_1^2 + x_2^2 + \dots + x_d^2\right]$$

Applying the chain rule, we find,

$$\frac{d(f)}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}\left(\ln(1+z)\right) \cdot \frac{d}{dx}\left(x^T x\right)$$

$$= \frac{1}{1+z} \cdot \frac{d(z)}{dz} \cdot \frac{d}{dx}\left(x_1^2 + x_2^2 + \dots + x_d^2\right)$$

$$= \frac{1}{1+z} \cdot \cancel{2}\left(2x_1^{\cancel{2}} + 2x_2^{\cancel{2}} + \dots + 2x_d^{\cancel{2}}\right)$$

$$= \frac{1}{1+z} \cdot 2\left(x_1 + x_2 + \dots + x_d\right)$$

$$\therefore \frac{d(f)}{dx} = \frac{2}{1+z} \sum_{i=1}^{d} x_i \ .$$

Answer

02. Given that, $f(z) = e^{-z/2}$, where $z = g(y) = y^T S^{-1} y$

$y = h(x) = x - \mu$

Using the chain rule,

$$\frac{d(f)}{dx} = \frac{d(f)}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Here, $\frac{d(f)}{dz} = -\frac{1}{2} e^{-z/2}$

$$\frac{dz}{dy} = 2S^{-1}y$$

and $\frac{dy}{dx} = I$, where I is ~~the~~ an identity matrix

Thus, $\frac{d(f)}{dx} = -\frac{1}{2} e^{-z/2} \cdot 2S^{-1}y \cdot I$

$$= \cancel{-\frac{e^{-z/2}}{2}}$$

$$\therefore \frac{d(f)}{dx} = -\frac{1}{2} e^{-\frac{\{(x-\mu)^T S^{-1} (x-\mu)\}}{2}} \cdot 2S^{-1}(x-\mu)$$

Answer: