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You can download the sources of this presentation here:

github.com/severin-lemaignan/lecture-hri-data-analysis



**UWE
Bristol**

University
of the
West of
England



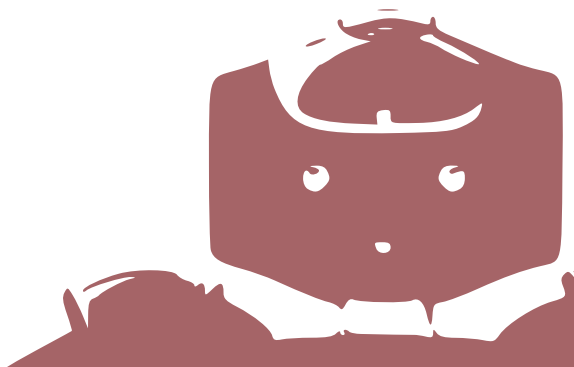
University of
BRISTOL

Data Analysis for HRI

Séverin Lemaignan

Bristol Robotics Lab

University of the West of England



IN THIS LECTURE

- Two questions to answer:

IN THIS LECTURE

- Two questions to answer:
Are my groups different?

IN THIS LECTURE

- Two questions to answer:
 - Are my groups different?
 - Does a specific variable explain the difference?

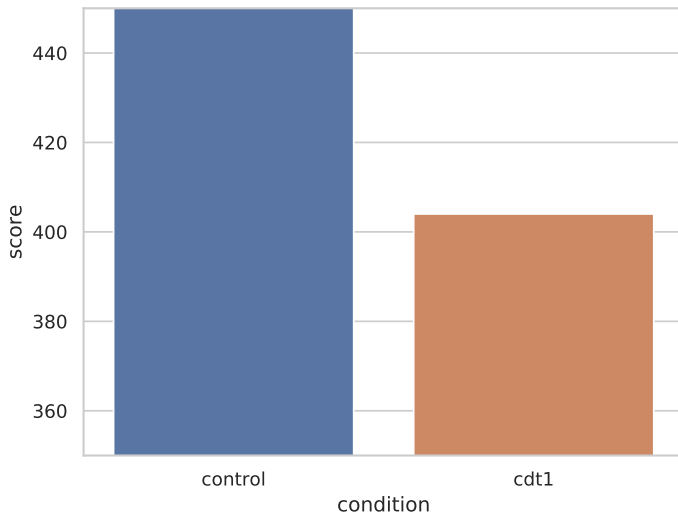
IN THIS LECTURE

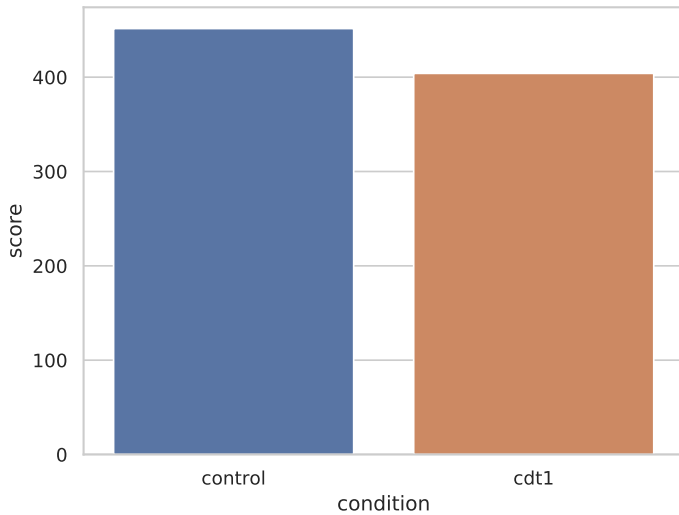
- Two questions to answer:
 - Are my groups different?
 - Does a specific variable explain the difference?
- Hands-on data analysis with Python!

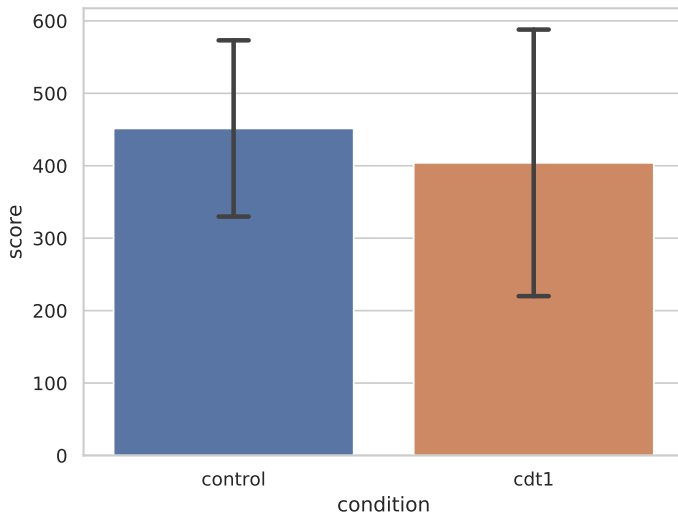
ARE MY TWO GROUPS DIFFERENT?

A DATASET

pptID	age	condition	score	heartrate
1	22	control	643	76
2	26	cdt1	234	72
3	24	control	356	73
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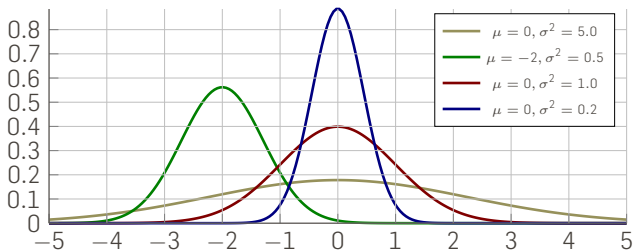


Is there a difference?

- Is the distribution the same?
- How big the difference? → **effect size**
- Could chance explain that difference?

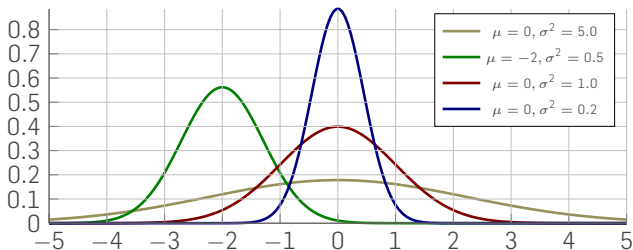
IS THE DISTRIBUTION THE SAME?

Data often (but not always!) follows a **normal** (or Gaussian) distribution. Two parameters: **mean** μ and **variance** σ^2 .



IS THE DISTRIBUTION THE SAME?

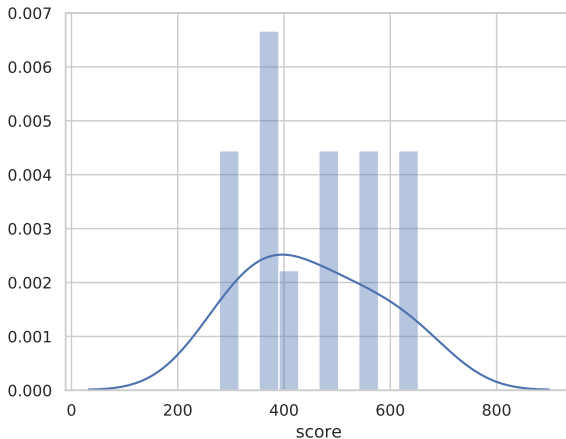
Data often (but not always!) follows a **normal** (or Gaussian) distribution. Two parameters: **mean** μ and **variance** σ^2 .



Many statistical tests only work if the underlying data follows a normal distribution – so-called **parametric tests**.

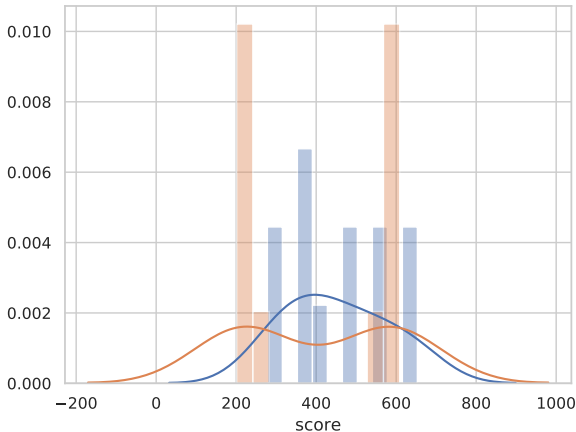
*You need to check that your data is normally distributed first!
(for instance, by plotting it)*

COMPARE DISTRIBUTIONS (HISTOGRAMS, DENSITY)

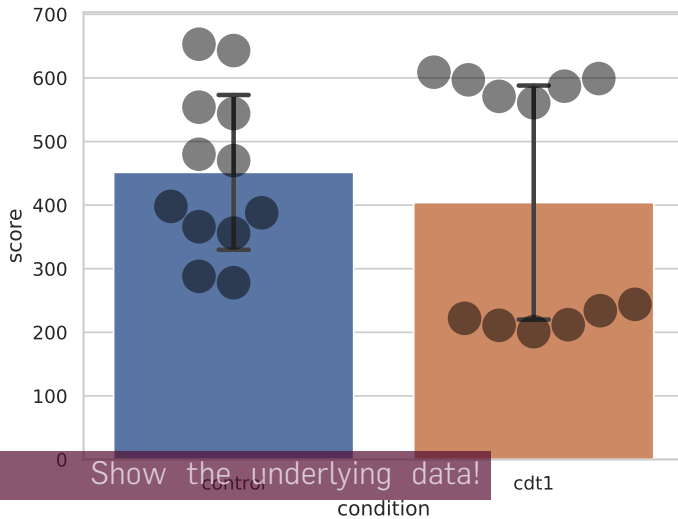


Control group

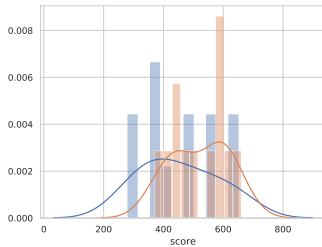
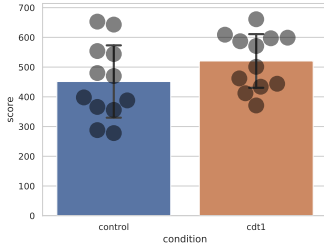
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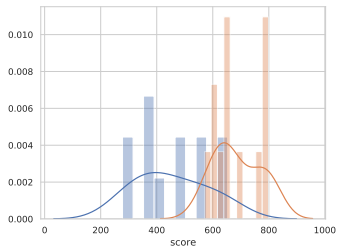
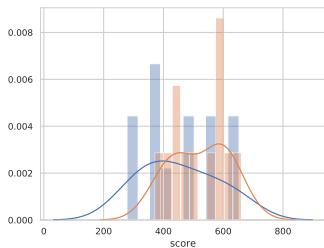
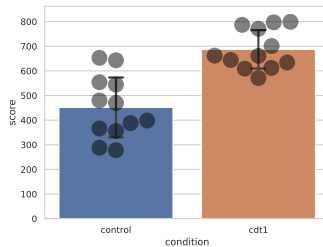
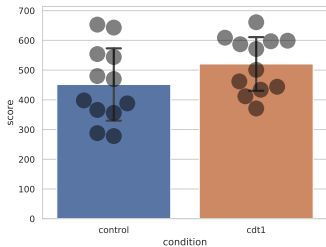
Control + condition group → beware the bimodal distribution!



TWO ADDITIONAL DATASETS



TWO ADDITIONAL DATASETS

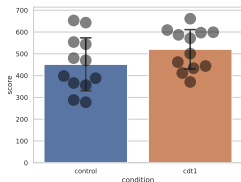


Are my two groups different?
○○○○○○○○○○●○○○○○○○○○○

Does one variable explain the difference?
○○○○○○○○

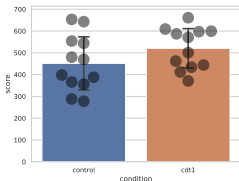
In practice
○○○○

HOW BIG IS THE DIFFERENCE?

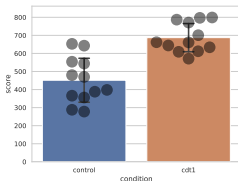


	mean	std
cdt1	516.5	85.3
control	451.5	127.1
$\mu_1 - \mu_2$	69.2	

HOW BIG IS THE DIFFERENCE?

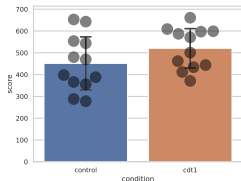


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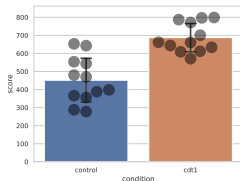


	mean	std
cdt1	687.3	81.5
control	451.5	127.1
$\mu_1 - \mu_2$	235.8	

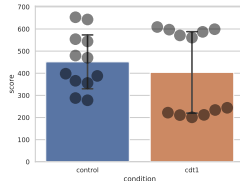
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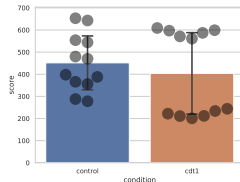
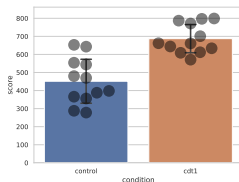
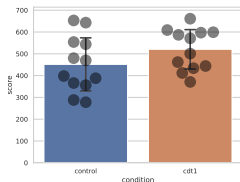


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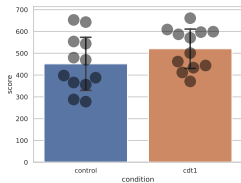
	mean	std
cdt1	404.0	192.2
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$\mu_1 - \mu_2$	47.5	

HOW BIG IS THE DIFFERENCE?

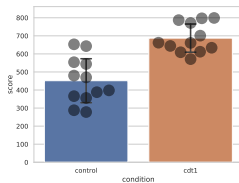


does not account for the variance in the dataset

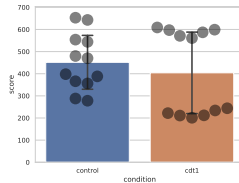
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$\mu_1 - \mu_2$	69.2	
$\frac{\mu_1 - \mu_2}{\sigma}$	0.62	

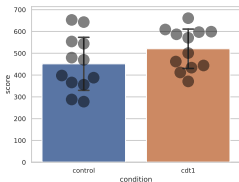


	mean	std
cdt1	687.3	81.5
control	451.5	127.1
$\mu_1 - \mu_2$	235.8	
$\frac{\mu_1 - \mu_2}{\sigma}$	2.21	

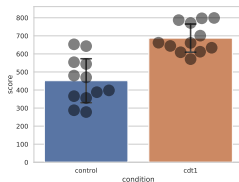


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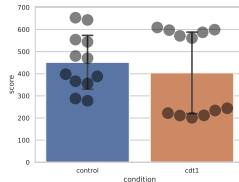
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A common measure of effect size: **Cohen's d** = $\frac{\mu_1 - \mu_2}{\sigma}$

→ Interactive visualisation and interpretation of Cohen's d

DIFFERENCE DUE TO CHANCE?

A statistical hypothesis test makes an assumption about the outcome, called the **null hypothesis**.

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⇒ **Meaning of a low *p*-value?**

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⇒ **Meaning of a low *p*-value?**

To interpret *p*, you need to choose a *significance level* α .
For instance, 10% (0.1), 5% (0.05), 2% (0.02)...

$$p = 0.05$$

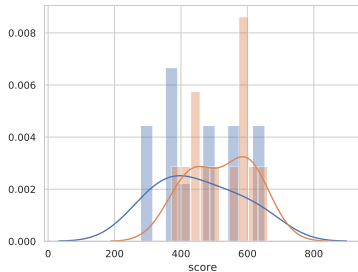
'There's only 5% of chance of observing these distributions if my null hypothesis is true (ie, no difference between my groups).'

HOW TO CALCULATE P ?

- If parametric data, **Student's t -test**

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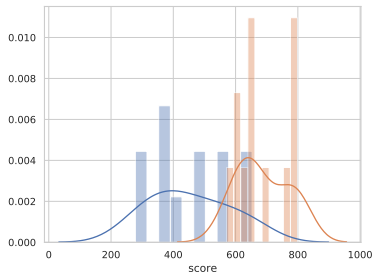
- If parametric data, **Student's t -test**



t statistic	-1.51
p	0.155

HOW TO CALCULATE P ?

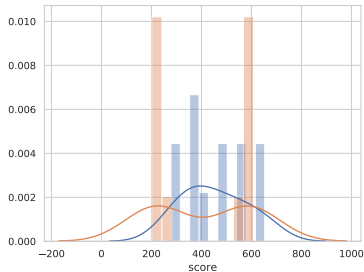
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t statistic	-5.41
p	< 0.001

HOW TO CALCULATE P ?

- If parametric data, **Student's t -test**



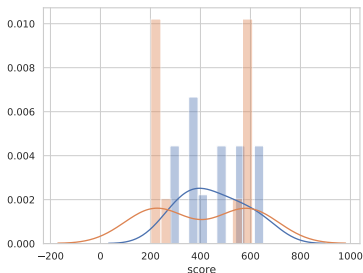
t statistic	0.71
p	0.48

HOW TO CALCULATE P ?

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U statistic	46.0
p	0.07

See [Wikipedia page](#) for examples and interpretation of U

IMPACT OF N ?

What is the impact of the sample size n on p ?

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BE CAREFUL WITH "STATISTICALLY SIGNIFICANT"!

gender	iq
male	76.51
male	76.53
female	76.66
female	76.65
female	76.64
female	76.63
male	76.54
female	76.64
male	76.51
female	76.60
female	76.63
male	76.52
female	76.64
male	76.51
female	76.60
female	76.63

BE CAREFUL WITH "STATISTICALLY SIGNIFICANT"!

t statistic	12.52
p	< 0.001
Mean female	76.64
Mean male	76.54

$$M_{female} > M_{male}, p < 0.001$$

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Cohen's d

$$d = \frac{\mu_1 - \mu_2}{\sigma} = 4.12 \Rightarrow \text{high, because } \sigma \text{ very low}$$

STATISTICAL POWER ANALYSIS

Statistical power

The statistical power of a hypothesis test is the probability of detecting an effect, if there is a true effect present to detect.

or:

Statistical power

The statistical power of the test is the probability that the test correctly rejects a *false* null hypothesis.

STATISTICAL POWER ANALYSIS

Types of errors

- **Type I error:** Reject the null hypothesis when there is in fact no significant effect (*too optimistic!*)
- **Type II error:** Not reject the null hypothesis when there is a significant effect (*too pessimistic!*)

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$$\text{Power} = 1 - \text{Type II Error}$$

STATISTICAL POWER ANALYSIS

A puzzle with four pieces:

- **Effect size**
- **Sample size**
- **Significance** (chance of Type I error – found inexistant effect)
- **Statistical power** ($1 -$ chance of Type II error – missed the effect)

EXAMPLE: POWER ANALYSIS OF STUDENT'S *T*-TEST

- **Effect size:** Cohen's $d > 0.8$
- **Significance:** 5%
- **Statistical power:** 80%
- **Sample size?**

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A good read on statistical power analysis:

A Gentle Introduction to Statistical Power and Power Analysis in
Python

ARE MY GROUPS DIFFERENT? SUMMARY

- 2 groups, independent measures, normal distribution:
Independent t -test

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Always report an **effect size** (for instance, **Cohen's d**)

Keep a close eye on your data distributions (**plot them**)

DOES ONE VARIABLE EXPLAIN THE
DIFFERENCE?

OUR DATASET

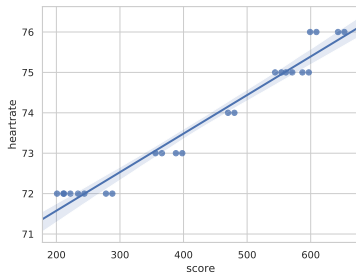
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ASSOCIATION

What is the degree of association between two variables?

→ main tool: correlation

PEARSON CORRELATION

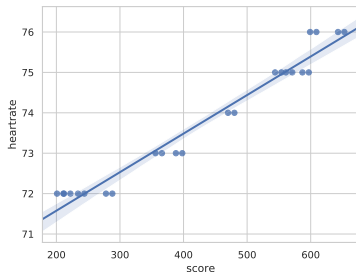


Pearson's correlation

ρ 0.98

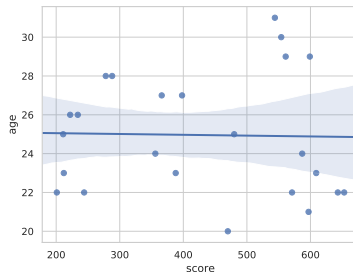
p < 0.001

PEARSON CORRELATION



Pearson's correlation

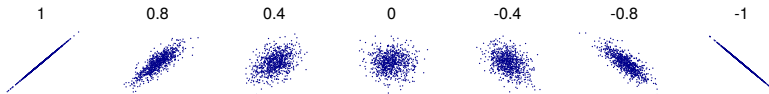
ρ	0.98
p	< 0.001



Pearson's correlation

ρ	-0.022
p	0.92

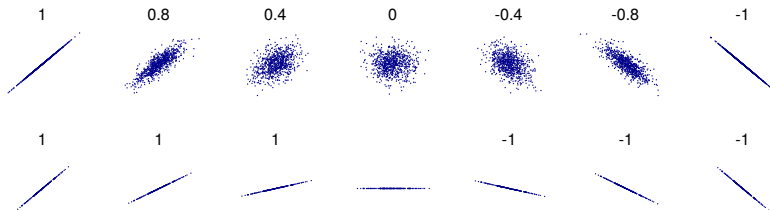
INTERPRETATION OF ρ



ρ reflects the degree of linearity and direction

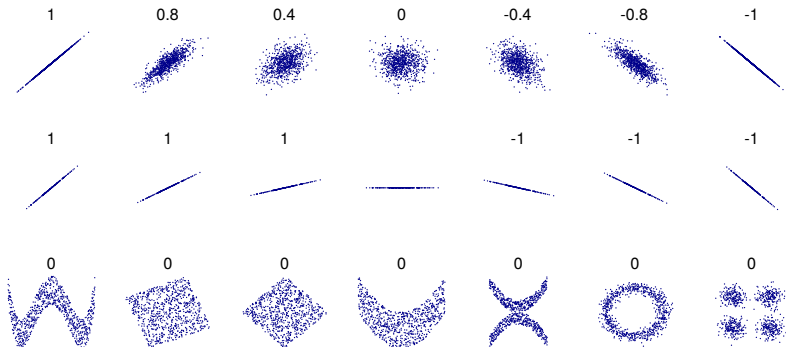
Source: *Wikipedia*

INTERPRETATION OF ρ



ρ does not reflect the slope of the regression line

INTERPRETATION OF ρ

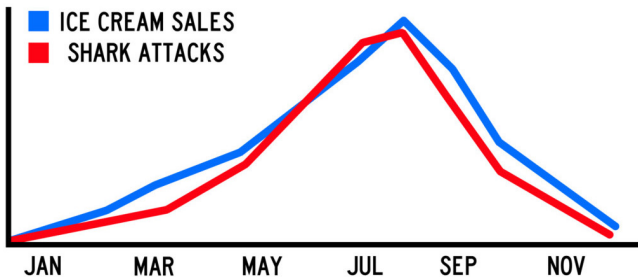


ρ does not capture non-linear interactions

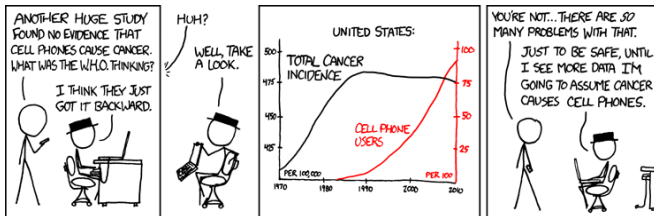
OTHER MEASURES OF ASSOCIATION

- Non-parametric ordinal data: **Spearman rank correlation**
- Association between categorical data (for instance, relationship between 'gender' and 'preferred style of cuisine'): **Pearson's Chi-Square** χ^2

CORRELATION IS NOT CAUSATION

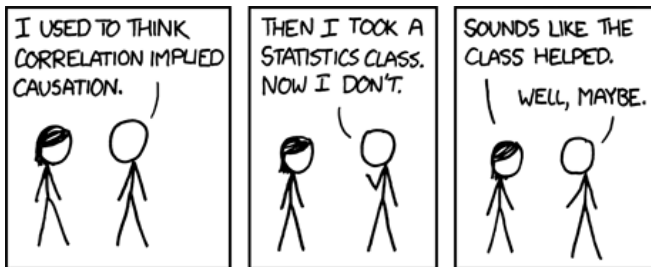


CORRELATION IS NOT CAUSATION



Source: XKCD

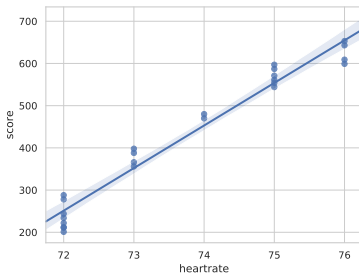
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Be careful when writing something like:



“the significant positive correlation between the heart rate and the score shows that you need to have a fast heart to win”

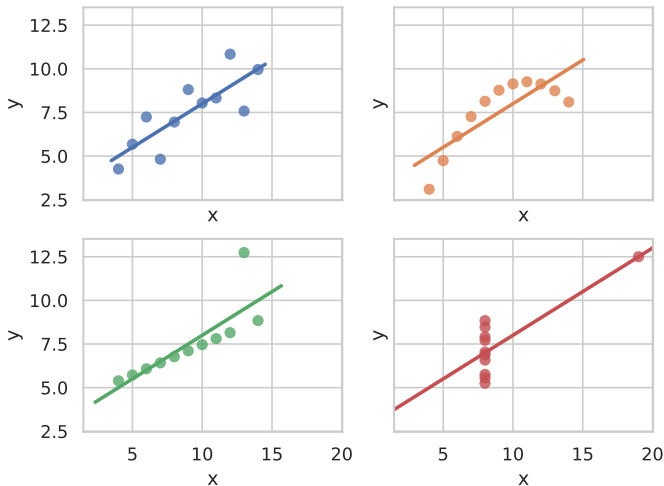
TO CONCLUDE: ANSCOMBE'S QUARTET

I		II		III		IV	
<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

TO CONCLUDE: ANSCOMBE'S QUARTET

Property	Value
Mean of x	9
Sample variance of x	11
Mean of y	7.50
Sample variance of y	4.125
Correlation between x and y	0.816
Linear regression line	$y = 3.00 + 0.500x$
Coefficient of determination of the linear regression	0.67

TO CONCLUDE: ANSCOMBE'S QUARTET



IN PRACTICE

THE TOOLS

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Jupyter notebooks are a great way of creating an interactive, easy-to-follow, data analysis.

(SIDE NOTE ON PYTHON FOR DATA ANALYSIS)

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- `anaconda` (and a few other): Python distribution for scientific computing

Let's give it a go!