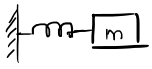
Mini Quiz 2

a) $\theta = \pi/3 \Rightarrow \Delta\lambda = \lambda_c(1-\cos\pi/3) = \lambda_c(1-1/2) = \lambda_c/2$

b) $\frac{3\lambda}{4} - \lambda' = \frac{\lambda_c}{2} \Rightarrow \lambda' = \frac{3\lambda_c}{4} - \frac{\lambda_c}{2} = \frac{\lambda_c}{2}$ //

Mini Quiz-3

$h\nu = KT$ — known.

Model vibrations as 

$$\Rightarrow \nu = \frac{KT}{h}$$

$$\sqrt{\frac{K}{m}} = \frac{KT}{h} \Rightarrow T \propto m^{-1/2}$$

$$\Rightarrow m_2 = 207\left(\frac{8}{10}\right)^2$$ //

Mini Quiz-4

$$v_p = \omega/k = Ak^{1/2} \qquad v_g = \frac{d\omega}{dk} = \frac{3}{2}Ak^{1/2}$$

Mini Quiz-5

$$g(\omega) = \frac{1}{\sqrt{2\pi}}\int f(t)e^{-i\omega t}dt = \frac{1}{\sqrt{2\pi}}\int_{-t_0}^{t_0} Ae^{-i\omega t}dt = \frac{A}{\sqrt{2\pi}}\left[\frac{e^{-i\omega t}}{i\omega}\right]_{-t_0}^{t_0}$$

$$= \frac{A}{\sqrt{2\pi}}\left[\frac{e^{i\omega t_0} - e^{-i\omega t}}{i\omega}\right] = \frac{A}{\sqrt{2\pi}}\left[\frac{2i\sin(\omega t)}{i\omega}\right]$$

$$= \frac{2A\sin(\omega t)}{\omega\sqrt{2\pi}}$$

Mini Quiz-6

$$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right) \Rightarrow \int \psi\psi^* dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\Rightarrow A^2 \int_0^L \frac{1-\cos\left(\frac{2n\pi x}{L}\right)}{2} dx = 1$$

$$\Rightarrow A^2\left[\frac{L-0}{2}\right] = 1 \Rightarrow A = \sqrt{\frac{2}{L}}\, e^{i\theta}$$ ↳ not needed, but eh..

Mini Quiz-7

$$\psi(x) = A\sin\left(\frac{5\pi x}{2L}\right)\cos\left(\frac{\pi x}{2L}\right) = \frac{A}{2}\sin\left(\frac{3\pi x}{2L}\right) + \frac{A}{2}\sin\left(\frac{2\pi x}{2L}\right)$$

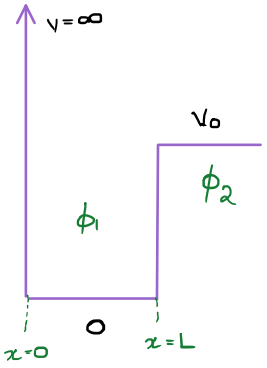
ψ_1 ψ_2

Energy can be superposed!

$$k^2 = \frac{2mE}{\hbar^2} \Rightarrow E = \frac{k^2\hbar^2}{2m} \Rightarrow E = \left(\frac{3\pi}{2L}\right)^2 \frac{\hbar^2}{2m} + \left(\frac{\pi}{L}\right)^2 \frac{\hbar^2}{2m}$$

$$\Rightarrow E = \frac{13\pi^2\hbar^2}{8mL} //$$

Mini Quiz-8



$$\phi_1(x) = A\sin(Kx) + B\cos(Kx)$$

$$\phi_2(x) = Ce^{-\alpha x}$$

$\Rightarrow B = 0$ by continuity at $x=0 \Rightarrow \phi_1(x) = A\sin(Kx)$

$(x=L) \Rightarrow A\sin(KL) = Ce^{-\alpha L}$

$AK\cos(KL) = -C\alpha e^{-\alpha L}$

$$\Rightarrow \boxed{\tan(KL) = -K/\alpha}$$

$$\tan(KL) = \frac{-K}{\alpha} \quad \text{where} \quad K = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{mV_0}{4\hbar^2}} = \frac{\sqrt{mV_0}}{2\hbar} //$$

$$\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \frac{\sqrt{3mV_0}}{2\hbar} //$$

$$\tan\left(\frac{\sqrt{mE}}{2\hbar} L\right) = \frac{-1}{\sqrt{3}} \Rightarrow \frac{\sqrt{mE}}{2\hbar} L = -\tan^{-1}\left(1/\sqrt{3}\right)$$

$$E = \frac{4\hbar^2}{mL^2}\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]^2$$