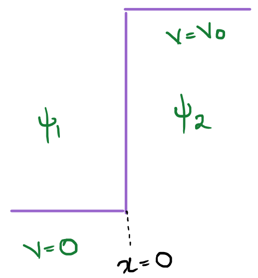
Tutorial Sheet-10

(1)



$E < V_0 \Rightarrow \psi_2(x) = Ce^{-\alpha x}$ where $\alpha = \sqrt{\dfrac{2m(V_0-E)}{\hbar^2}}$

$$\frac{P(x=x_0)}{P(x=0)} = \frac{C^2 e^{-2\alpha x_0}}{C^2} = \frac{1}{e} \Rightarrow e^{2\alpha x_0} = e$$

$$\Rightarrow \boxed{x_0 = 1/2\alpha}$$

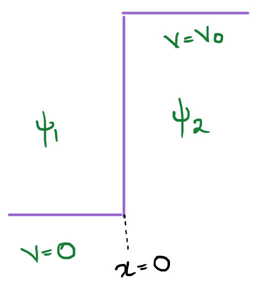
(b) $\Delta x = x_0 = 1/2\alpha \Rightarrow$ HUP $= \Delta x \Delta p \geq \dfrac{\hbar}{2} \Rightarrow \Delta p = \hbar\alpha = \sqrt{2m(V_0-E)}$

$$E = \frac{P^2}{2m} \Rightarrow \Delta E = \frac{2p\Delta p}{2m} = \frac{p}{m} \times \hbar\alpha = 2(V_0-E)$$

$$\Rightarrow E + \Delta E = 2V_0 - E \text{ and } E - \Delta E = 3E - 2V_0$$

$\Rightarrow$ E may exceed the value of V_0

(Q2)

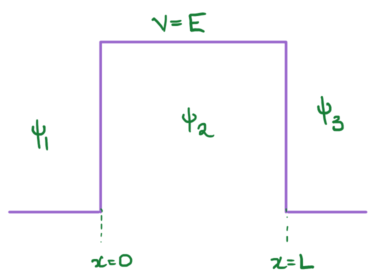


Same as 1a, with values.

$V_0 = 7eV,\ E = 3eV$

$$\frac{P(x=x_0)}{P(x=0)} = \frac{1}{2} \Rightarrow e^{-2\alpha x_0} = 1/2 \Rightarrow \boxed{x_0 = \frac{\ln 2}{2\alpha}}$$

(Q3) a)



$\psi_1 = Ae^{ik_1 x} + Be^{-ik_1 x}$

$\psi_2 = Cx + D$ — because $V=E$!

$\psi_3 = Ee^{ikx}$

$(x=0)$

$$A+B=D \quad \text{— ①}$$

$$(A-B)\,ik = C \Rightarrow A-B = -\frac{iC}{k} \quad \text{— ②}$$

$(x=L)$

$$CL+D = Ee^{ikL} \quad \text{— ③}$$

$$C = Eike^{ikL} \quad \text{— ④}$$

$$① + ② \Rightarrow 2A = D + \frac{C}{ik} \; ; \quad ④ \rightarrow ③ \Rightarrow D = (Ee^{ikL})(1-ikL)$$

$$\Rightarrow 2A = (Ee^{ikL})(1-ikL) + \frac{Eike^{ikL}}{ik} = (Ee^{ikL})(1-ikL+1)$$

$$\Rightarrow \frac{E}{A} = \frac{2e^{-ikL}}{2-ikL} \Rightarrow \boxed{\text{Transmission Coeff.} = \left|\frac{2e^{-ikL}}{2-ikL}\right|^2 = \frac{4}{4+k^2L^2}}$$

(b) For Transmission $= 1/2$; $8 = 4 + k^2L^2 \Rightarrow kL = 2 \Rightarrow \boxed{L = \lambda/\pi}$

(Q4) For the claim to be correct; the wavefunction needs to be continuous and differentiable.

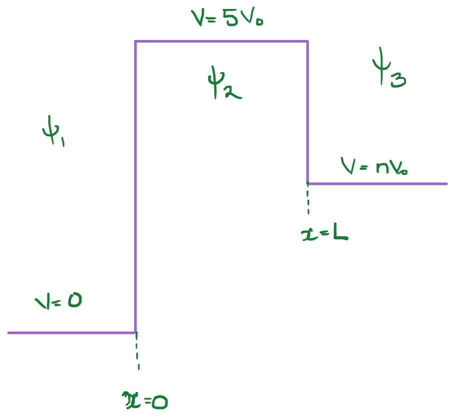
Cont. :- $Ae^{-k_1L} = Be^{-k_2L}$

Diff :- $-Ak_1e^{-k_1L} = -Bk_2e^{-k_2L}$

$\Rightarrow k_1 = k_2$ which is NOT true!

Therefore, the claim is false.

(Q5) * Correction in the question, $d = \frac{\pi\hbar}{\sqrt{8mV_0}}$ *



$\psi_1 = Ae^{iK_1x} + Be^{-iK_1x}$ $\qquad K_1 = \sqrt{\frac{2m(9V_0)}{\hbar}}$

$\psi_2 = Ce^{iK_2x} + De^{-iK_2x}$ $\qquad K_2 = \sqrt{\frac{2m(4V_0)}{\hbar}}$

$\psi_3 = Ee^{iK_3x}$ $\qquad K_3 = \sqrt{\frac{2m(9-n)V_0}{\hbar}}$

$(x=0)$

$A + B = C + D$ — ①

$K_1(A-B) = K_2(C-D)$ — ②

$(x=L)$

$Ce^{iK_2d} + De^{-iK_2d} = Ee^{iK_3d}$ ——— ③

$K_2\left(Ce^{iK_2d} - De^{-iK_2d}\right) = K_3Ee^{iK_3d}$ — ④

** But $d = \frac{\pi\hbar}{\sqrt{8mV_0}} \Rightarrow K_2d = \pi \Rightarrow \left(e^{\pm iK_2d} = 1\right)^{**}$ d is needed here!

$\Rightarrow \quad A + B = C + D \qquad , \qquad C + D = Ee^{iK_3d}$

$K_1(A-B) = K_2(C-D) \qquad K_2(C-D) = K_3Ee^{iK_3d}$

$$\frac{A+B}{A-B} = \frac{K_1}{K_2}\left(\frac{C+D}{C-D}\right) , \quad \frac{C+D}{C-D} = \frac{K_2}{K_3}$$

Transmission = $3/4 \Rightarrow$ Ref = $1/4$

$\Rightarrow A/B = \pm 2$

$$\Rightarrow \left(\frac{A+B}{A-B}\right)\left(\frac{2}{3}\right) = \frac{2}{\sqrt{9-n}}$$

$\Rightarrow 9 - n = 81$ or 1

$$\boxed{n = -72 \text{ or } 8}$$

(b) For $n = 8$; $\frac{C+D}{C-D} = \frac{2}{1}$; $\frac{B}{A} = \frac{1}{2}$ $\Rightarrow$ $C = \frac{9A}{8}$ and $D = \frac{3A}{8}$

(c) $B = A\left(\frac{K_1 - K_3}{K_1 + K_3}\right) \Rightarrow$ Phase change $= \operatorname{Im}\left(\frac{K_1 - K_3}{K_1 + K_3}\right)$

$$= \operatorname{Im}\left(\frac{3 - \sqrt{9-n}}{3 + \sqrt{9-n}}\right) = 0 \text{ at } \underline{n=9}$$

$$\neq 0 \text{ at } \underline{\underline{n = 72}}$$