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Total probability of an event

If $A_1, A_2 \ldots A_n$ are mutually exclusive and exhaustive events and B is any event in S then P(B) is called the total probability of event B ~~is called~~ and

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + \ldots + P(A_n) \cdot P(B/A_n)$$

$$= \sum_{i=1}^{n} P(A_i) P(B/A_i)$$

1. Urn-I contains 8 red and 4 blue balls and urn-II contains 5 red and 10 blue balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are red.

	Red balls	Blue balls	Total
urn-1	8	4	12
urn-2	5	10	15
Total	13	14	27

Sol :-

Let A_1 - event of selecting urn-I.

A_2 - event of selecting urn-2.

Let B be the event of selecting 2 red balls.

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We have to find the total Prob. of event B. i.e. $P(B)$.
Clearly, A_1 and A_2 are mutually exclusive and exhaustive events.
We have;

$P(A_1) = \frac{1}{2}$, $P(B/A_1) = \frac{8C_2}{12C_2} = \frac{14}{33}$

$P(A_2) = \frac{1}{2}$, $P(B/A_2) = \frac{5C_2}{15C_2} = \frac{2}{21}$

We know,

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)$$

$$= \frac{1}{2} \cdot \frac{14}{33} + \frac{1}{2} \cdot \frac{2}{21}$$

$$= \frac{20}{77}.$$

2) A factory has two machines I and II. Machine - I produces 40% of items of the output and machine - II produces 60% of the items. Further 4% of items produced by machine - I are defective and 5% produced by machine - II are defective. If an item is

drawn at random, find the Probability that is a defective item.

Soln :-

Let A_1 - event that the items are produced by M.I.

A_2 - event that items are produced by M. II.

B - The event of drawing a defective item.

Clearly, A_1 and A_2 are mutually exclusive and exhaustive events.

$\therefore P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)$

We have

$P(A_1) = 0.40$, $P(A_2) = 0.6$

$P(B/A_1) = 0.04$, $P(B/A_2) = 0.05$

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)$$

$$= (0.4)(0.04) + (0.6)(0.05)$$

$$= 0.046$$

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Baye's Theorem

If $A_1, A_2 \dots A_n$ are mutually exclusive and exhaustive events $\ni$: $P(A_i) > 0$, $i = 1, 2, \dots n$ and B is any event in which $P(B) > 0$, then

$$P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{P(A_1)\,P(B/A_1) + \dots + P(A_n)\,P(B/A_n)}$$

Ex 1. A factory has two machines I and II. Machine I produces 40% of items and Machine II produces 60% of the items. Further 4% of items produced by machine I are defective and 5% produced by machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by machine II.

Soln: A_1 - event produced by M-I
A_2 - " M. II
B - drawing a defective item

To find $P(A_2/B)$.

By Baye's thm.

$$P(A_2/B) = \frac{P(A_2) \cdot P(B/A_2)}{P(A_1)P(B/A_1) + P(A_2) \cdot P(B/A_2)}$$

$$= \frac{(0.6)(0.05)}{(0.4)(0.04) + (0.6)(0.05)}$$

$$= 15/23$$

2) A Construction Company employs 2 executive engineers. Engineer-I does the work for 60% of jobs of the Company. Eng-2 works for 40% of jobs of the Company. It is known from the past experience that the probability of an error when engineer-I does the work is 0.03, Whereas the probability of an error in the work of Eng-2 is 0.04. Suppose a serious error occurs in the work, which engineer

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Would you guess did the work?

Sol^n: Let A_1 & A_2 be the events of job done by Eng-I and Eng-II. Let B be the event that the error occurs at work.
To find $P(A_1/B)$ and $P(A_2/B)$.

Given: $P(A_1) = 0.60$, $P(B/A_1) = 0.03$
$P(B_1) = 0.40$, $P(B/A_2) = 0.04$

A_1 & A_2 are mutually exclusive and exhaustive events

$$\therefore P(A_1/B) = \frac{P(A_1).P(B/A_1)}{P(A_1)P(B/A_1) + P(A_2).P(B/A_2)}$$

$$= \frac{(0.60)(0.03)}{(0.60)(0.03) + (0.40)(0.04)}$$

$$= 9/17$$

$$P(A_2/B) = 8/17.$$

$\because P(A_1/B) > P(A_2/B)$, the chance of error done by Eng - I is greater than Eng - II.

$\therefore$ one may guess that the serious error would have been be done by Eng. 2