

# Density Clustering

# Clustering

## Problem description

- Given:  
A data set of  $N$  data items which are  $d$ -dimensional data feature vectors.
- Task:  
Determine a natural, useful partitioning of the data set into a number of clusters ( $k$ ) and noise.

# Density-Based Clustering

- A cluster is defined as a connected dense component which can grow in any direction that density leads.
- Density, connectivity and boundary
- Arbitrary shaped clusters and good scalability

| K-means Clustering   | DBScan Clustering   |
|--|---|
| Clusters formed are more or less spherical or convex in shape and must have same feature size.   | Clusters formed are arbitrary in shape and may not have same feature size.  |
| K-means clustering is sensitive to the number of clusters specified.   | Number of clusters need not be specified.   |
| K-means Clustering is more efficient for large datasets.   | DBScan Clustering can not efficiently handle high dimensional datasets.   |
| K-means Clustering does not work well with outliers and noisy datasets.  | DBScan clustering efficiently handles outliers and noisy datasets.  |
| In the domain of anomaly detection, this algorithm causes problems as anomalous points will be assigned to the same cluster as “normal” data points. | DBScan algorithm, on the other hand, locates regions of high density that are separated from one another by regions of low density. |
| It requires one parameter : Number of clusters (K)   | It requires two parameters : Radius(R) and Minimum Points(M)  |
| Varying densities of the data points doesn't affect K-means clustering algorithm.  | DBScan clustering does not work very well for sparse datasets or for data points with varying density.                              |

# Two Major Types of Density-Based Clustering Algorithms

- Connectivity based:  
DBSCAN, GDBSCAN, OPTICS and DBCLASD
- Density function based:  
DENCLUE

# DBSCAN [Ester et al.1996]

- Clusters are defined as Density-Connected Sets (wrt. Eps, MinPts)
- Density and connectivity are measured by local distribution of nearest neighbor
- Target low dimensional spatial data

# DBSCAN

- Definition 1: Eps-neighborhood of a point

$$N_{\text{Eps}}(p) = \{q \in D \mid \text{dist}(p,q) \leq \text{Eps}\}$$

- Definition 2: Core point

$$|N_{\text{Eps}}(q)| \geq \text{MinPts}$$

# DBSCAN

- Definition 3: Directly density-reachable

A point  $p$  is directly density-reachable from a point  $q$  wrt.  $Eps$ ,  $MinPts$  if

- 1)  $p \in N_{Eps}(q)$  and
- 2)  $|N_{Eps}(q)| \geq MinPts$  (core point condition).



# DBSCAN

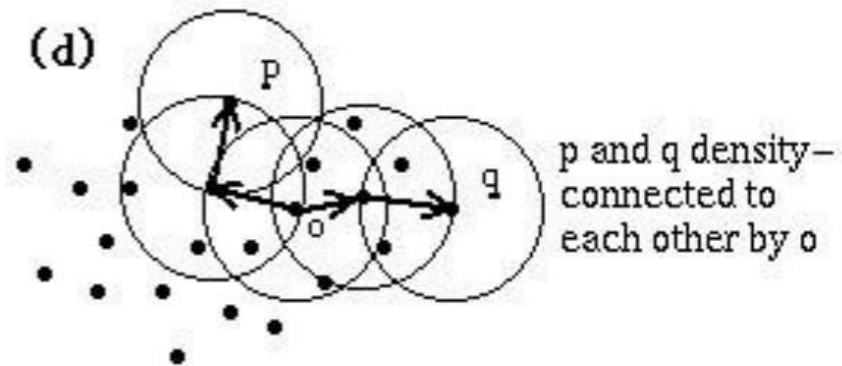
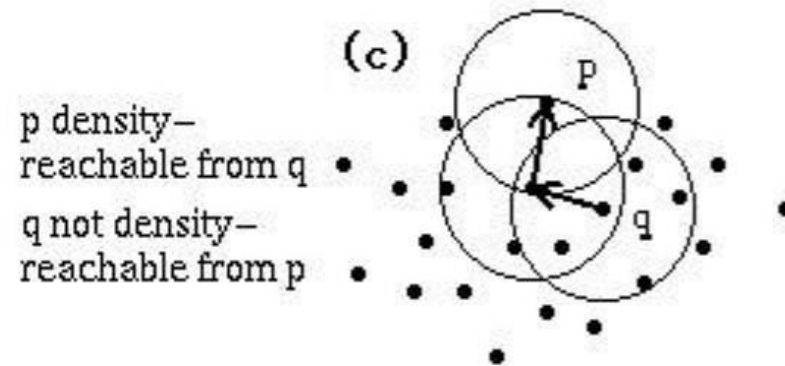
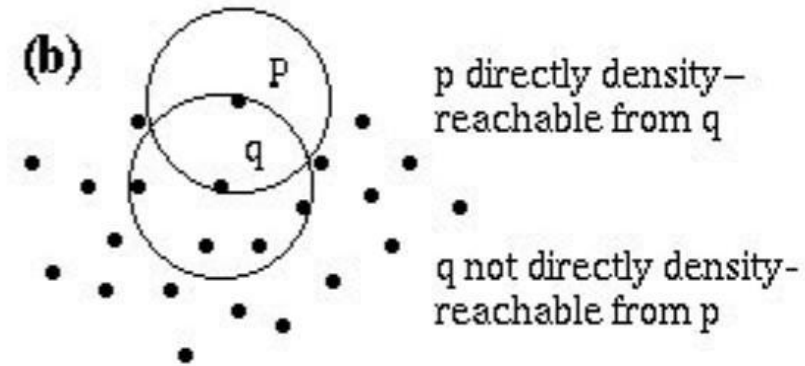
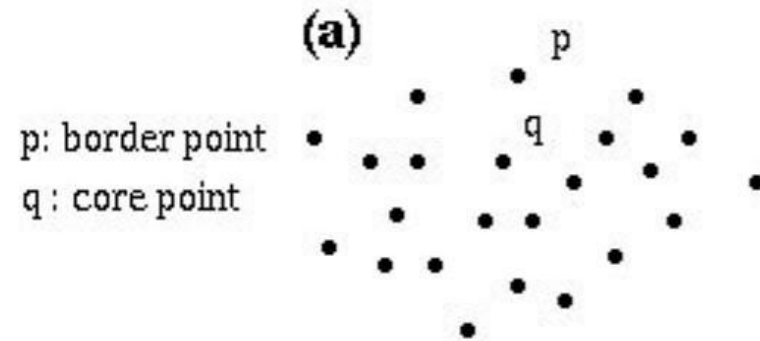
- Definition 4: Density-reachable

A point  $p$  is density-reachable from a point  $q$  wrt.  $Eps$  and  $MinPts$  if there is a chain of points  $p_1, \dots, p_n$ ,  $p_1 = q$ ,  $p_n = p$  such that  $p_{i+1}$  is directly density-reachable from  $p_i$

- Definition 5: Density-connected

A point  $p$  is density-connected to a point  $q$  wrt.  $Eps$  and  $MinPts$  if there is a point  $o$  such that both,  $p$  and  $q$  are density-reachable from  $o$  wrt.  $Eps$  and  $MinPts$ .

# DBSCAN

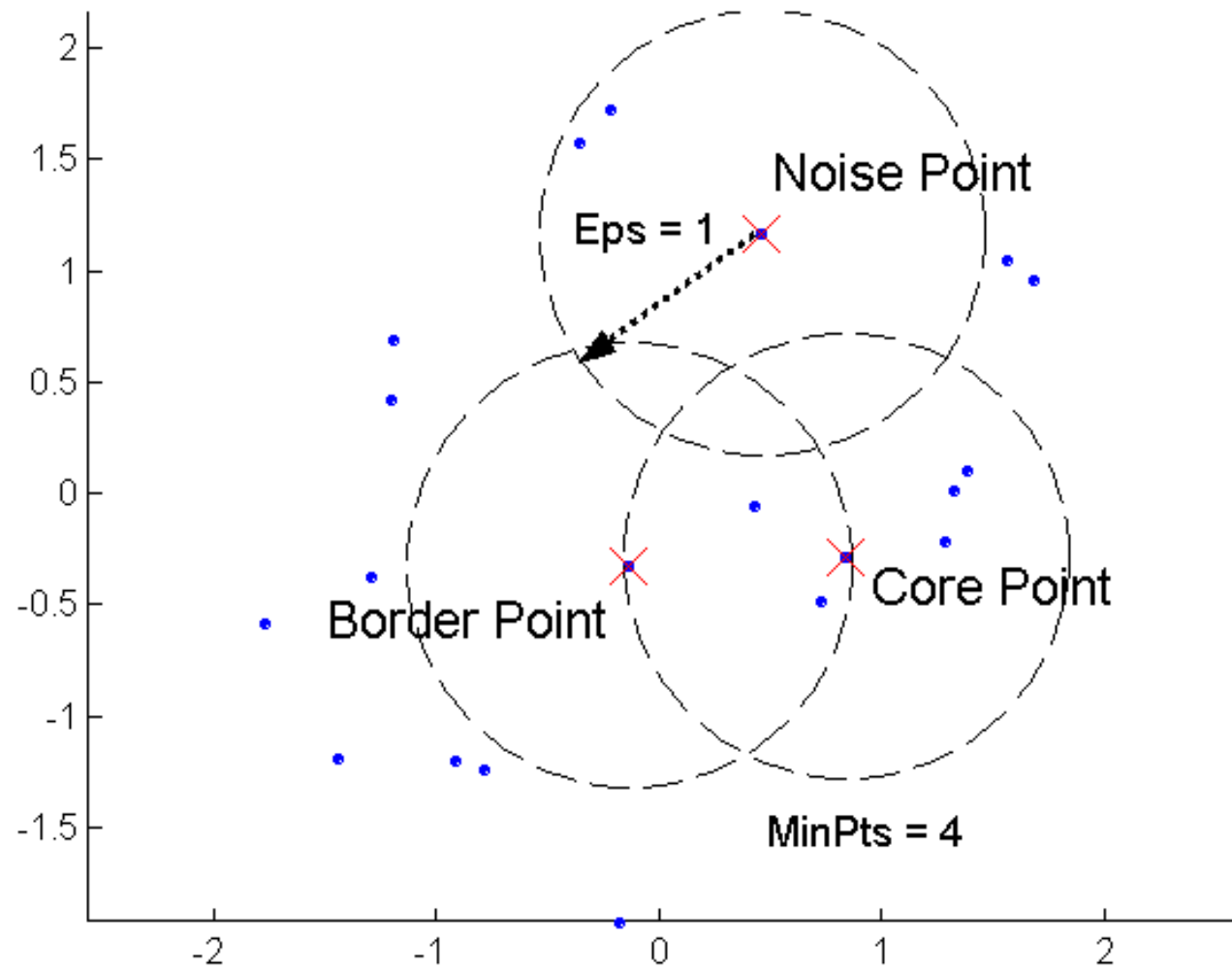


# DBSCAN

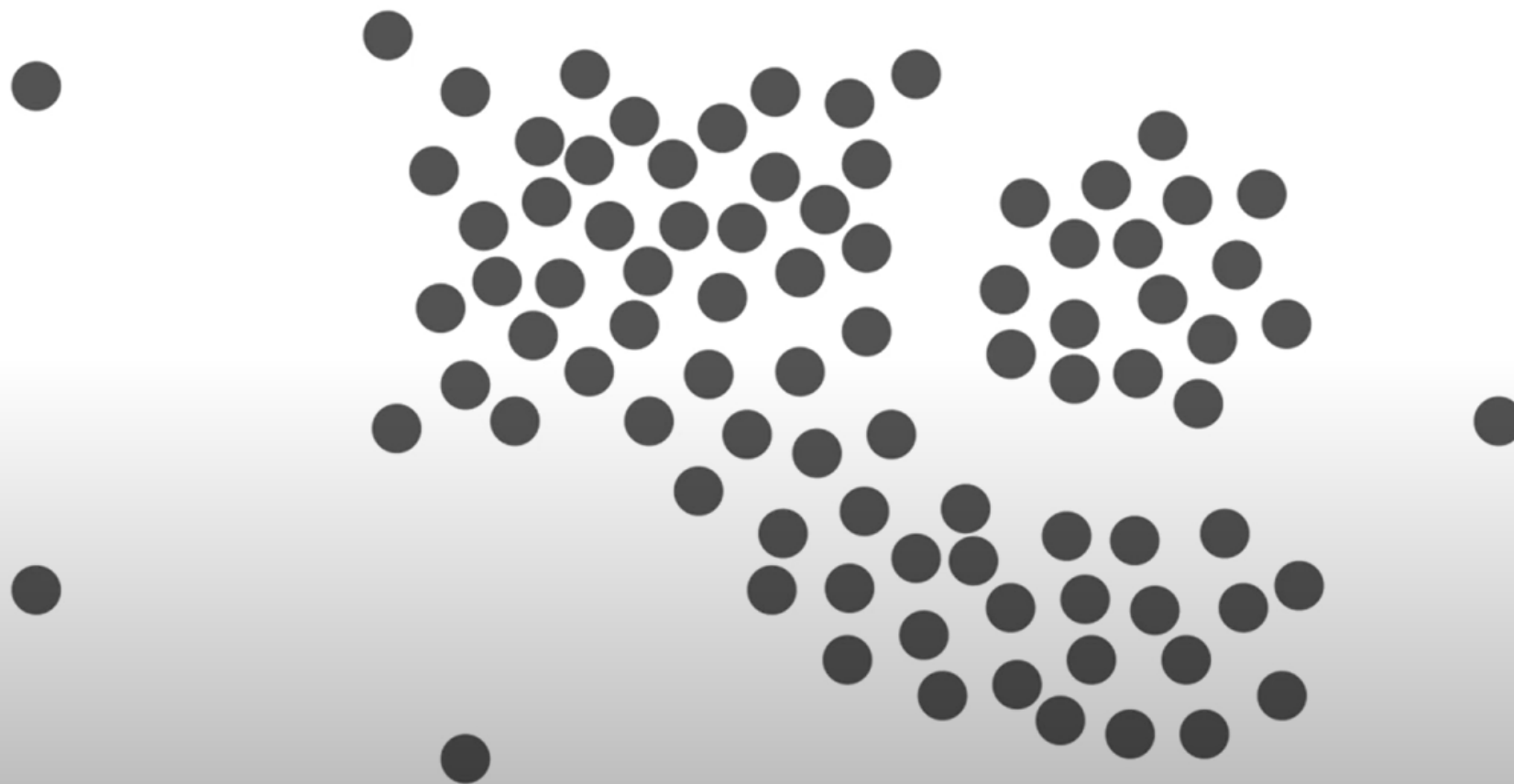
DBSCAN is a density-based algorithm.

- Density = number of points within a specified radius  $r$  (Eps)
- A point is a **core point** if it has more than a specified number of points (MinPts) within Eps
  - These are points that are at the interior of a cluster
- A **border point** has fewer than MinPts within Eps, but is in the neighborhood of a core point
- A **noise point** is any point that is not a core point or a border point.

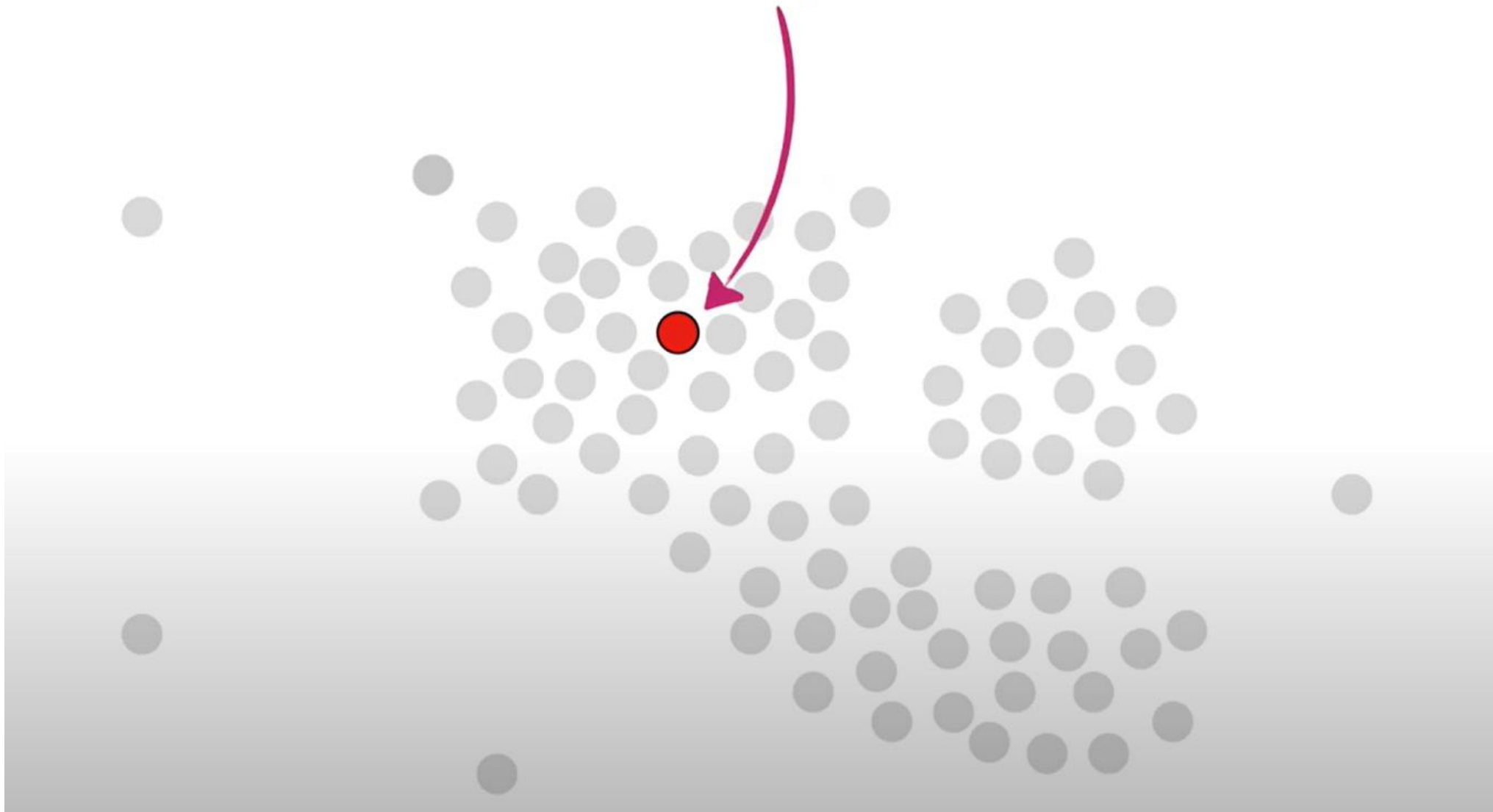
## DBSCAN: Core, Border, and Noise Points



...the first thing we can do is count the number of points close to each point.

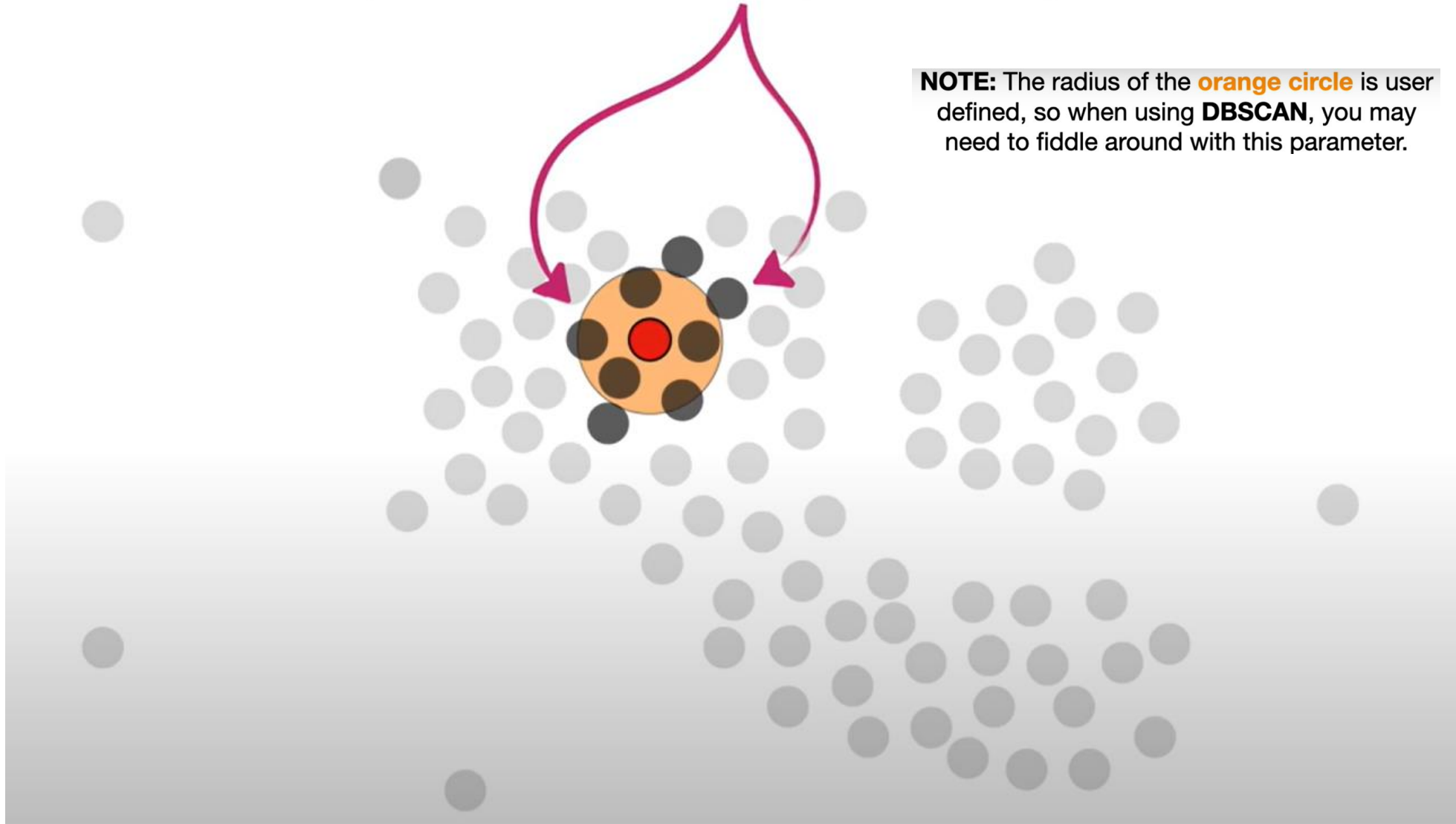


For example, if we start  
with this **red point**...

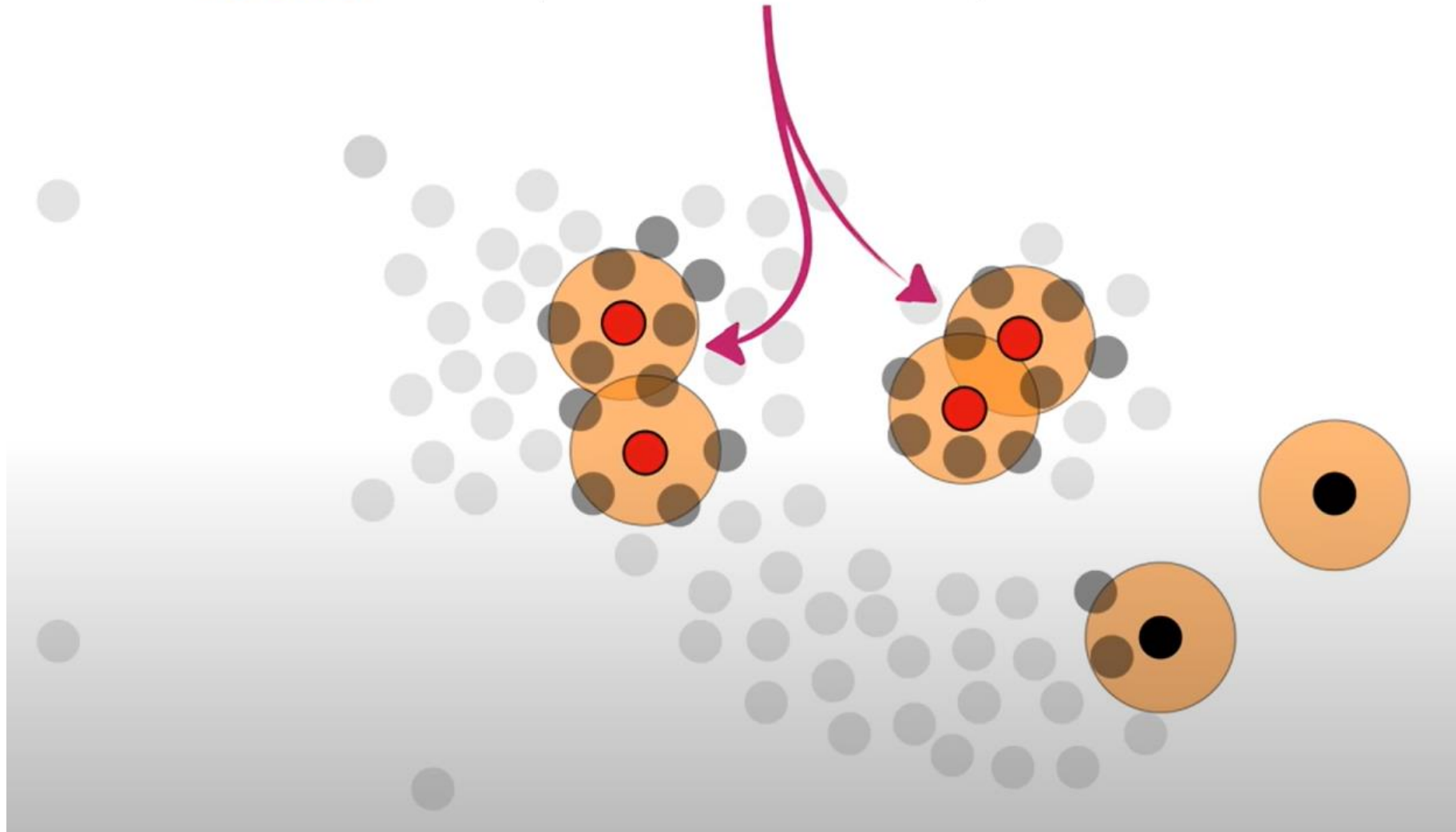


...then we see that the **orange circle** overlaps, at least partially, **8** other points.

**NOTE:** The radius of the **orange circle** is user defined, so when using **DBSCAN**, you may need to fiddle around with this parameter.

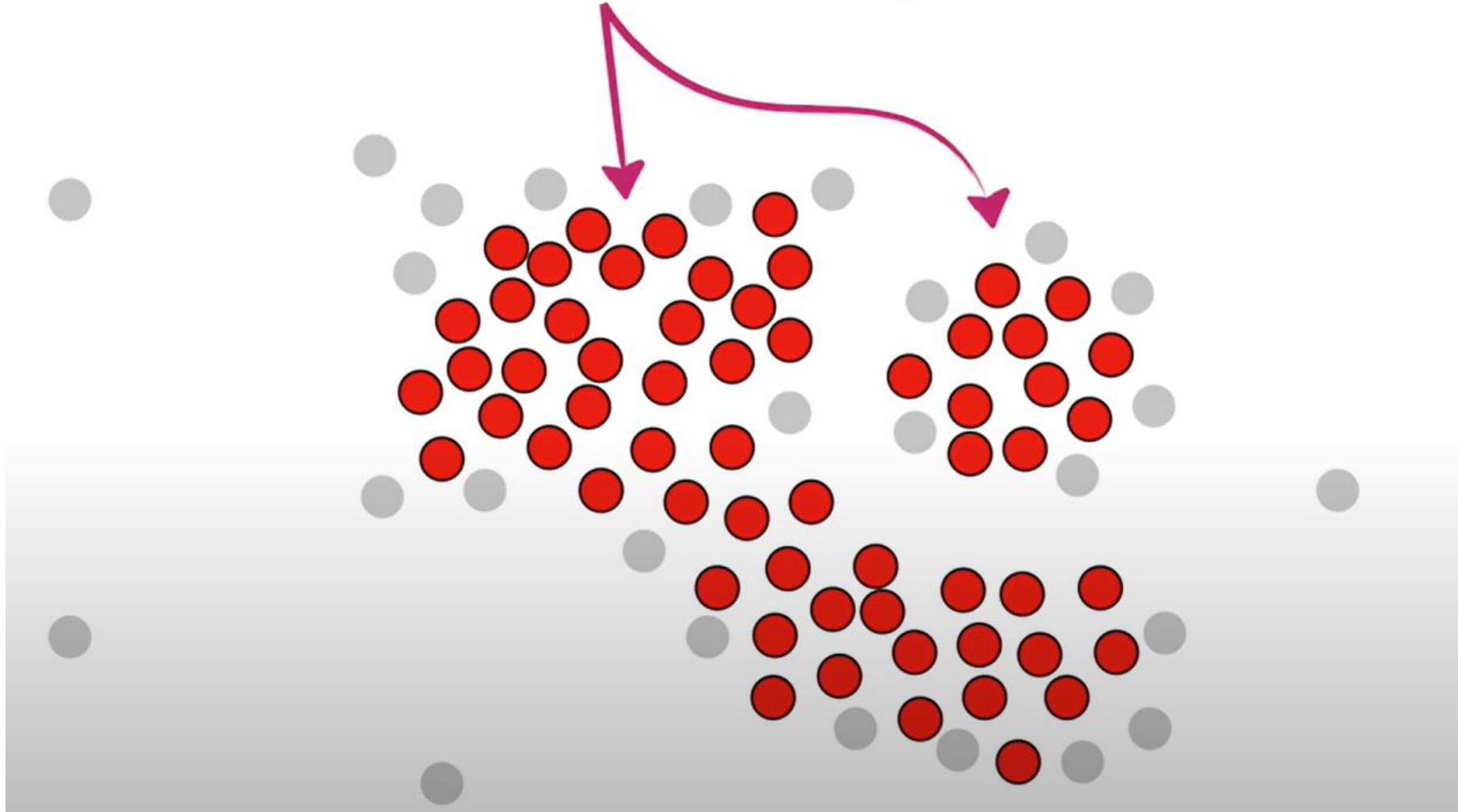


Anyway, these 4 points are some of the **Core Points**, because their **orange circles** overlap at least 4 other points...

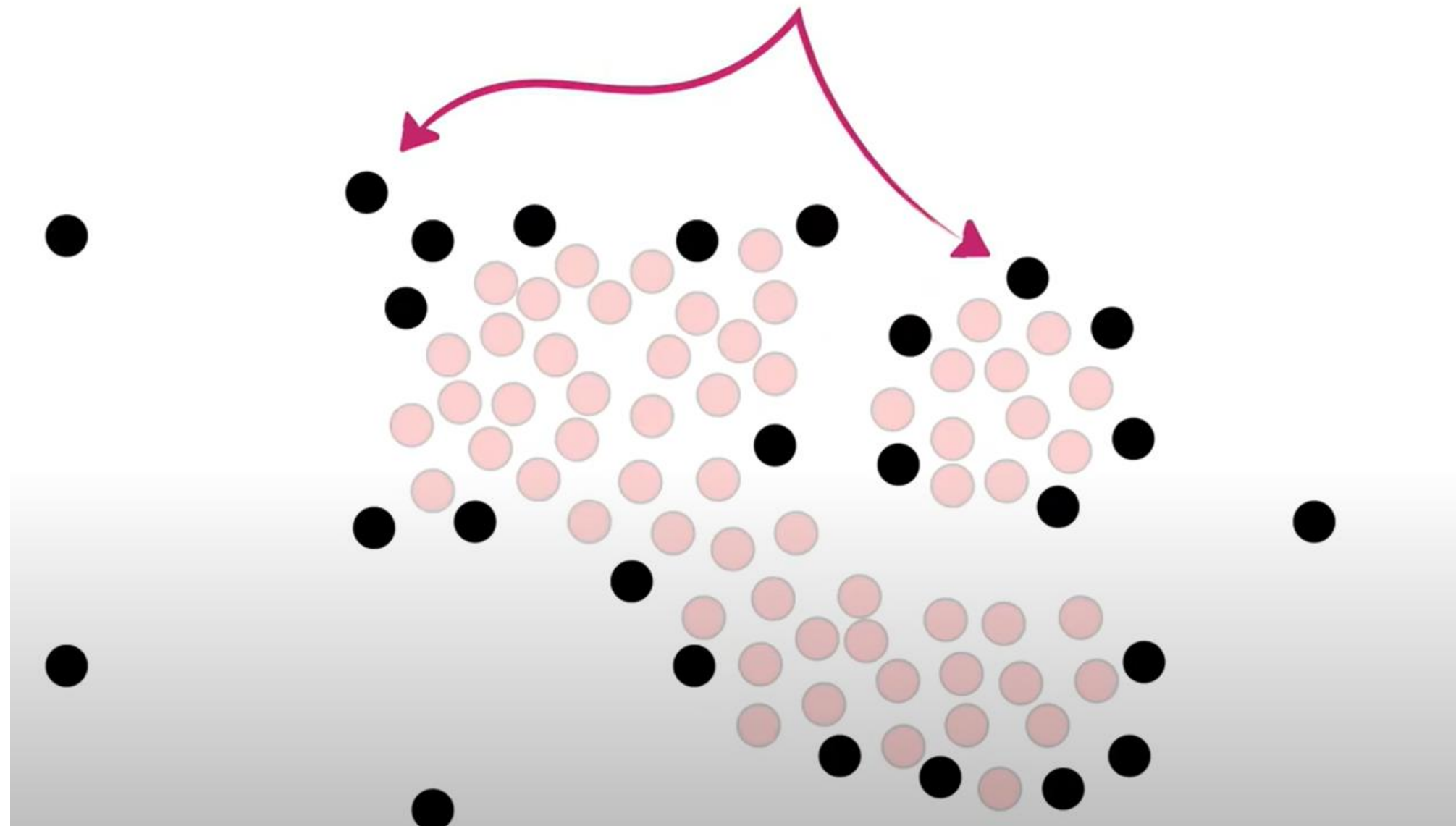




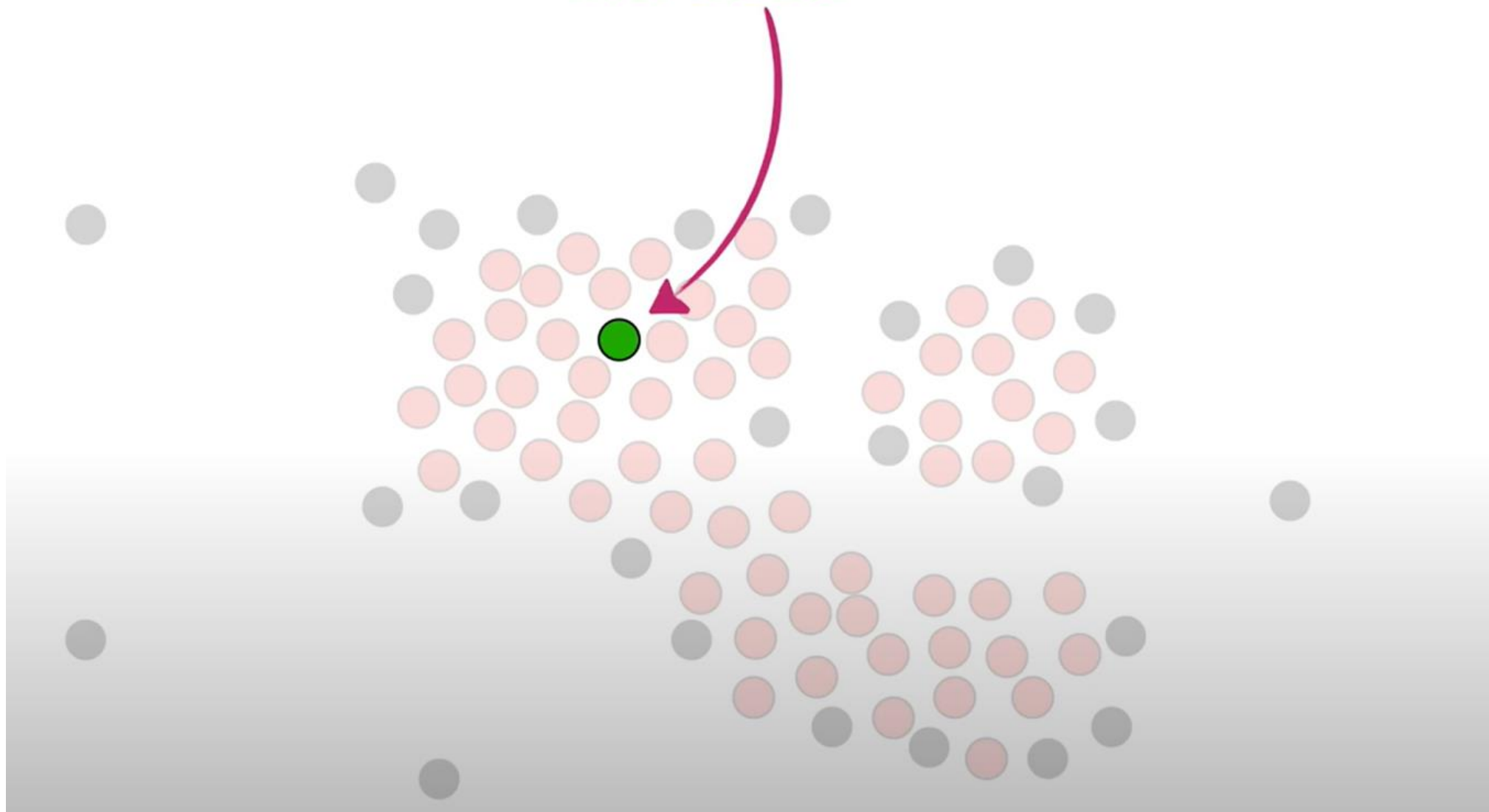
Ultimately, we can call all of these **red points** **Core Points** because they are all close to **4** or more other points...

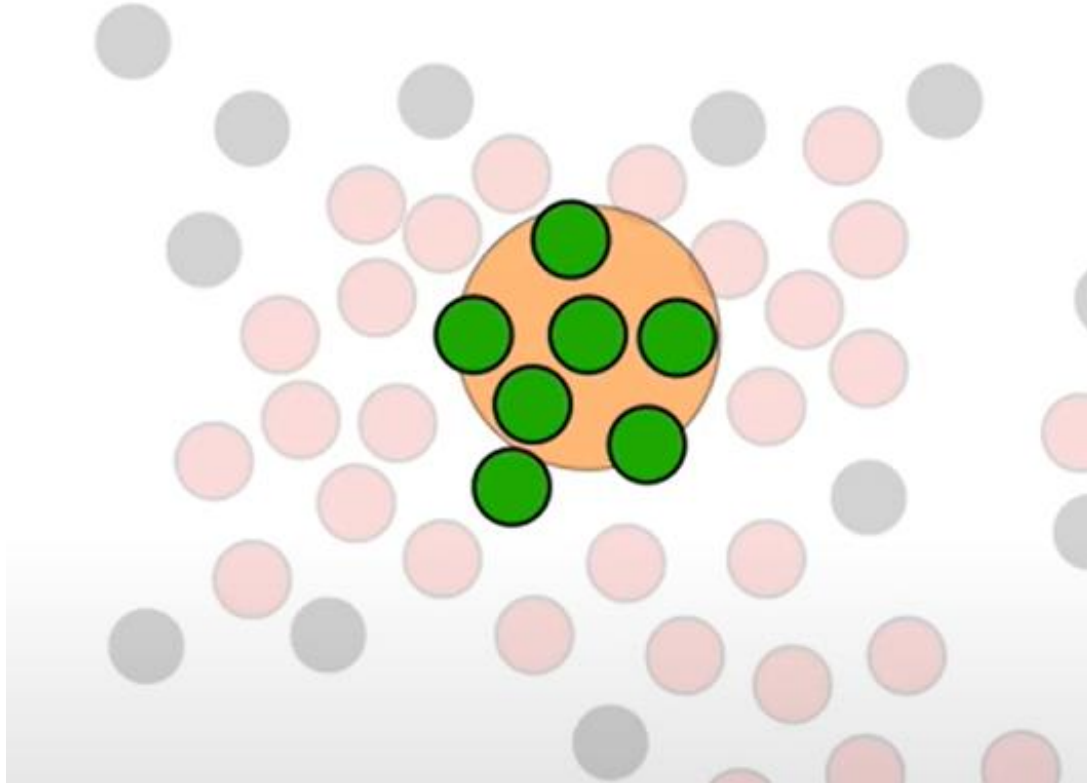


...and the remaining points  
are **Non-Core**.



...and assign it to  
the **first cluster**.

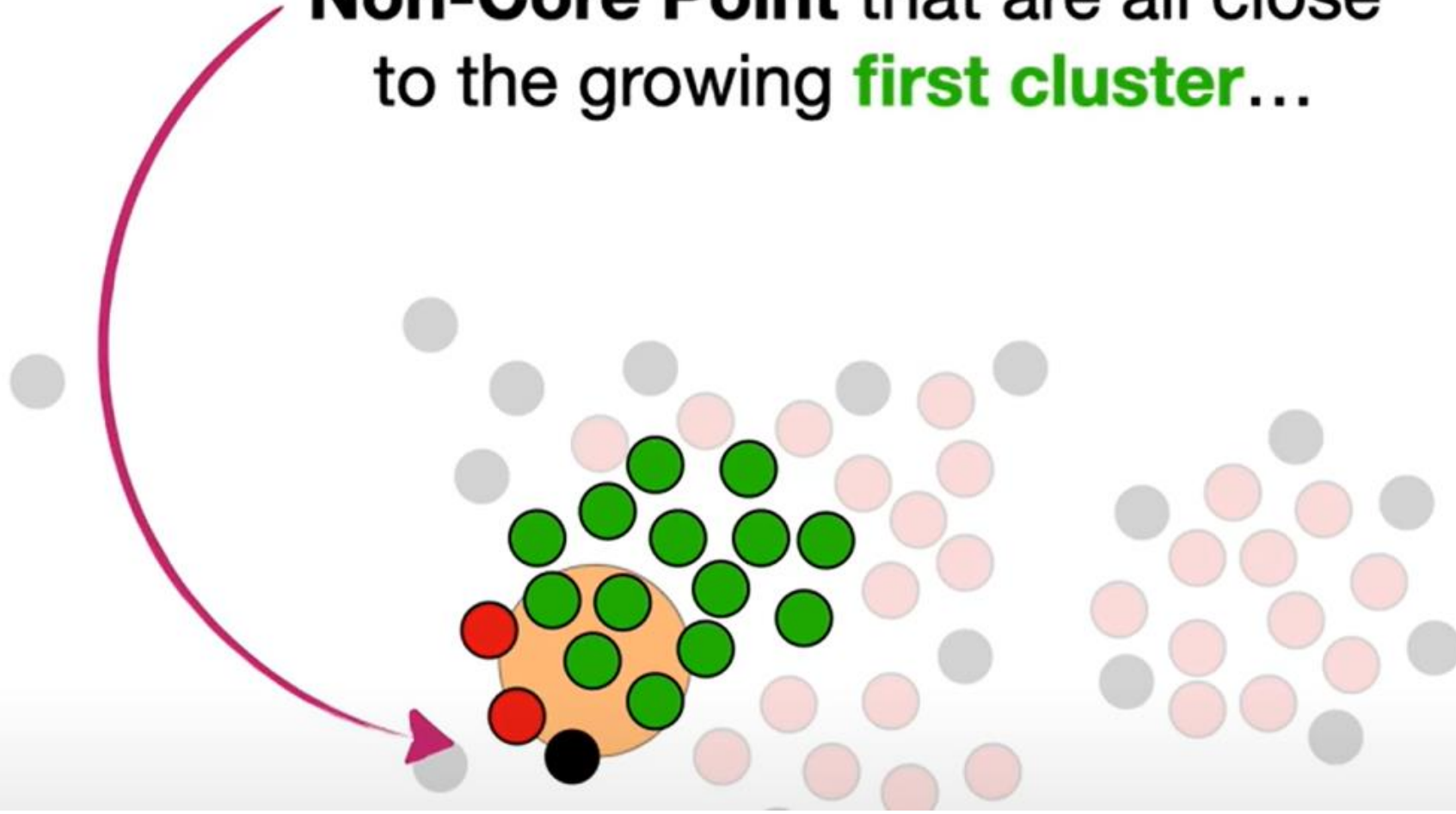




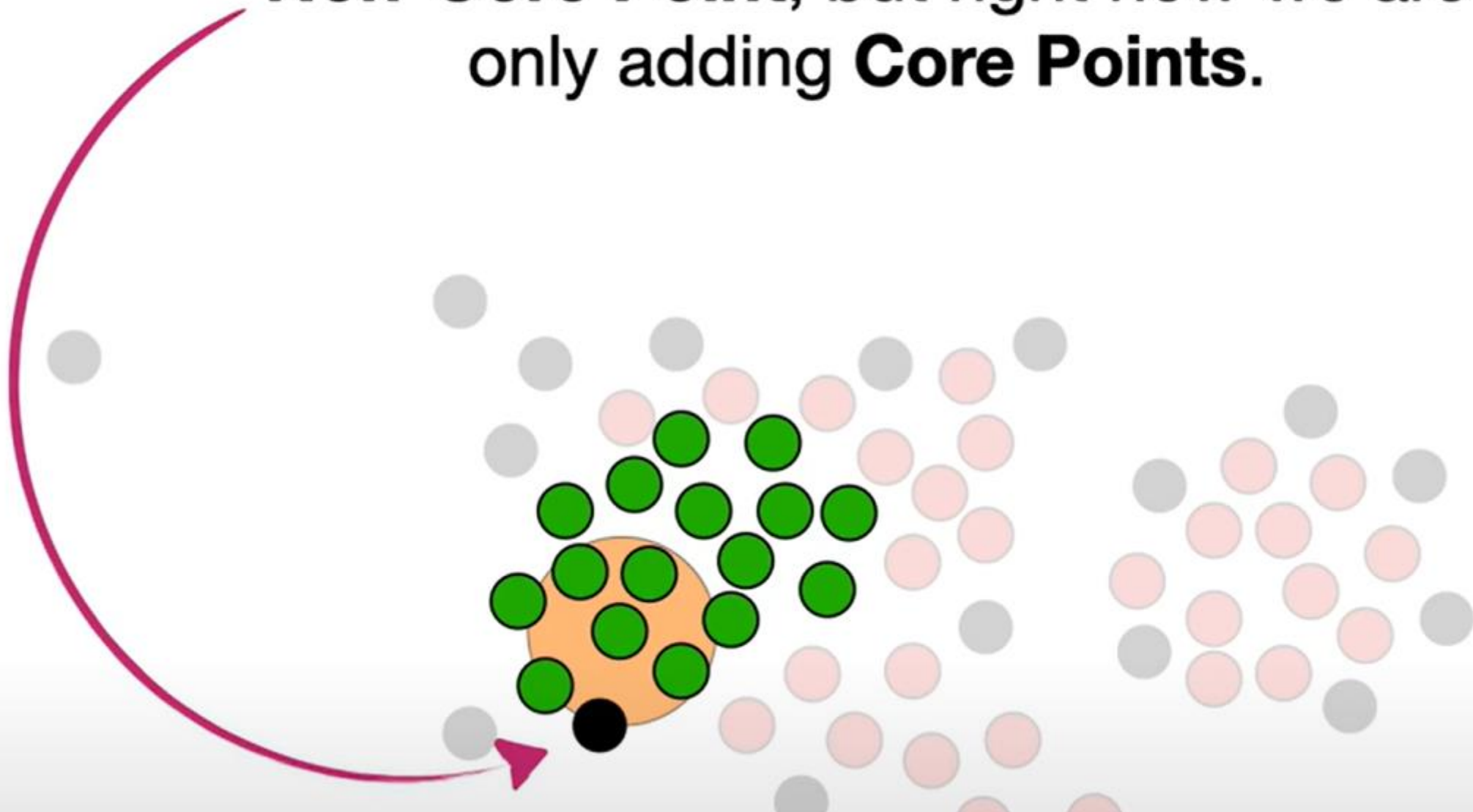
Then the **Core Points** that are close to the growing **first cluster** join it...



Here we see **2 Core Points** and **1 Non-Core Point** that are all close to the growing **first cluster**...

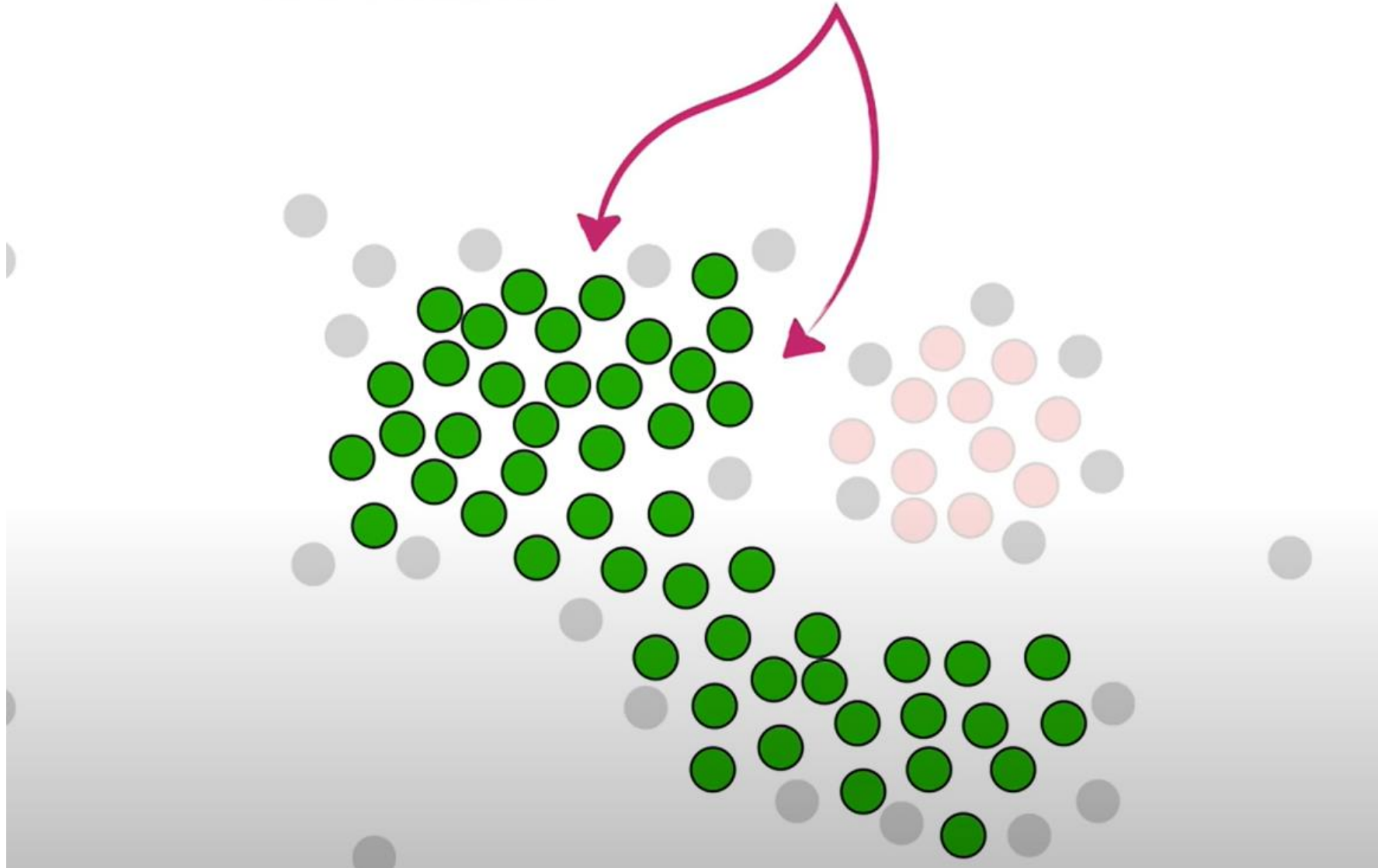


That said, eventually we will add this  
**Non-Core Point**, but right now we are  
only adding **Core Points**.

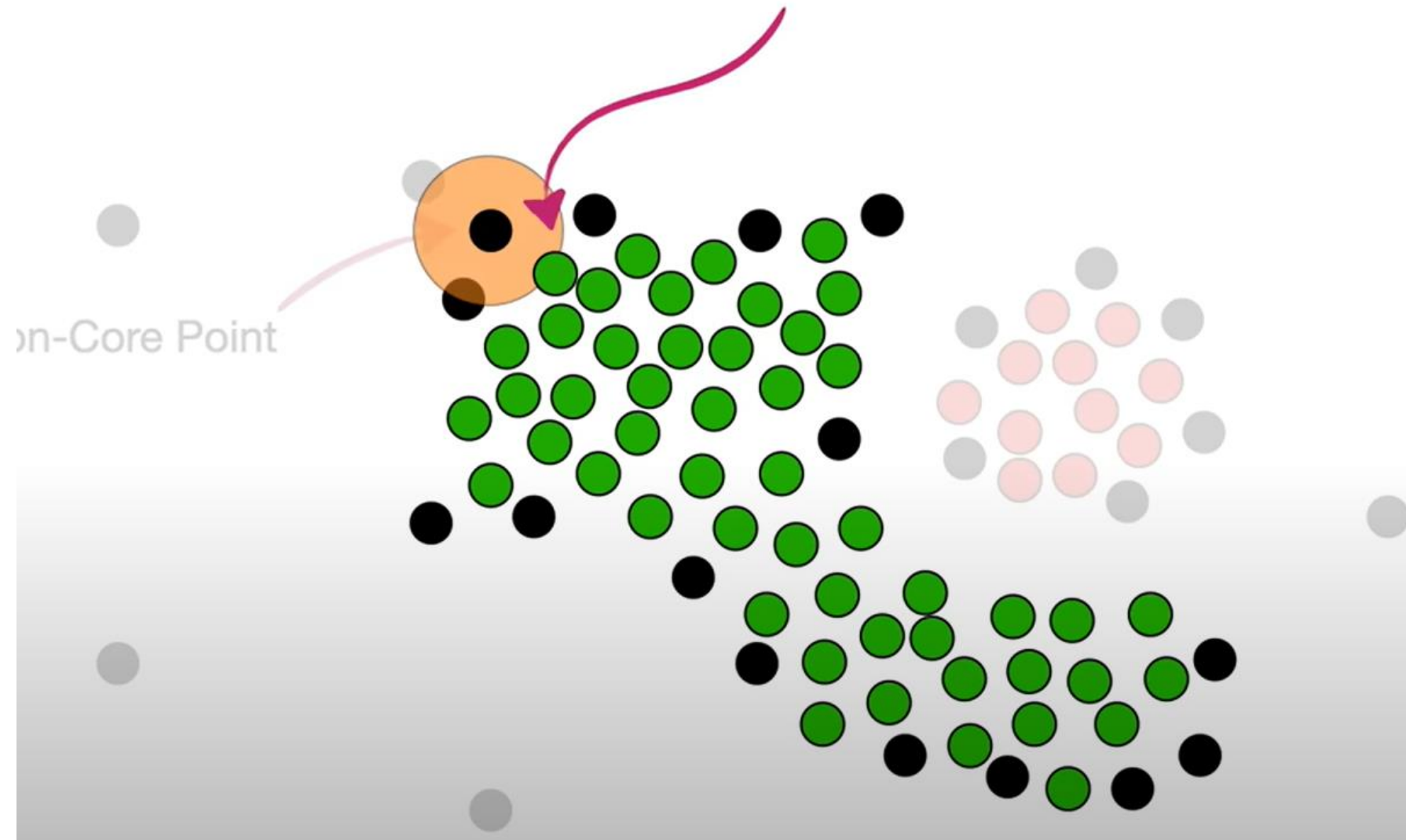




**NOTE:** At this point, every single point in the **first cluster** is a **Core Point**...

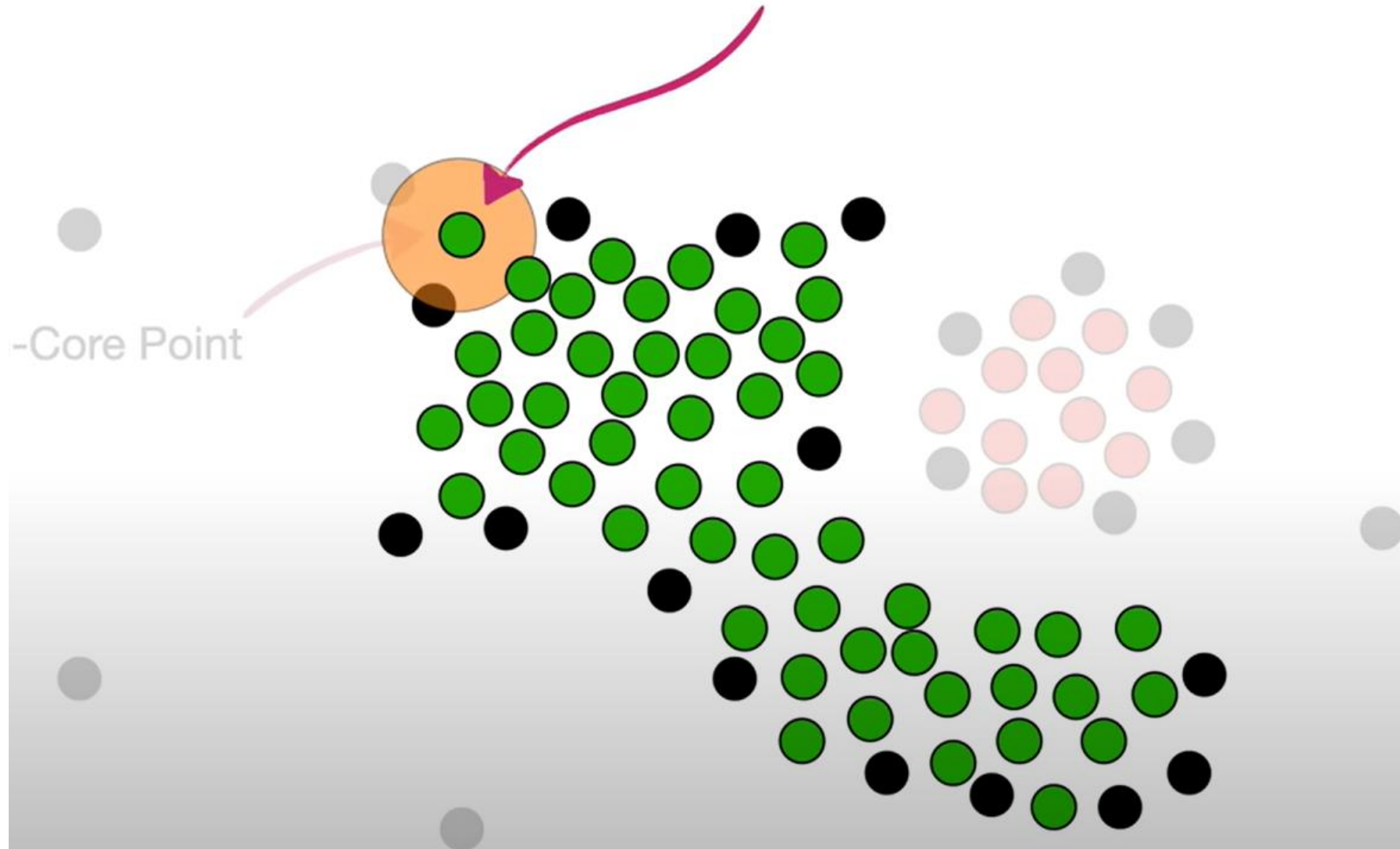


...is close to a **Core Point** in  
the **first cluster**...

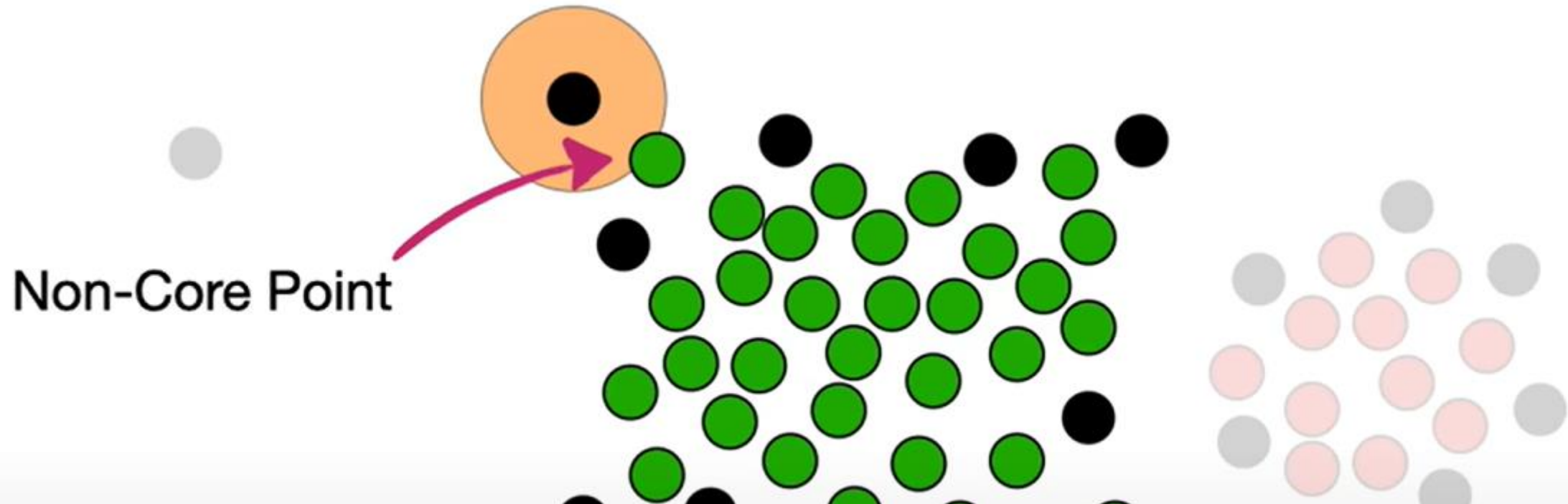




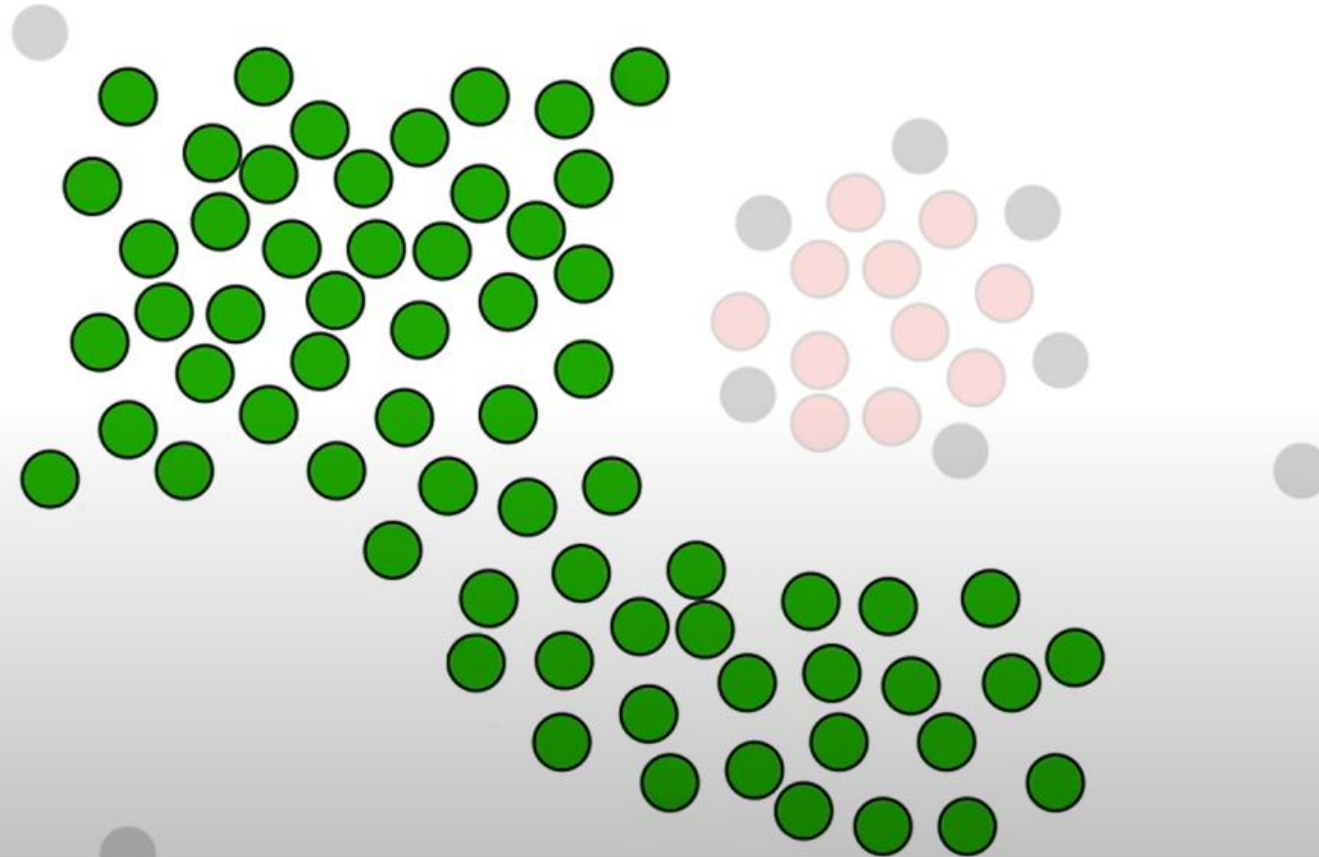
...so we add it to the  
**first cluster.**



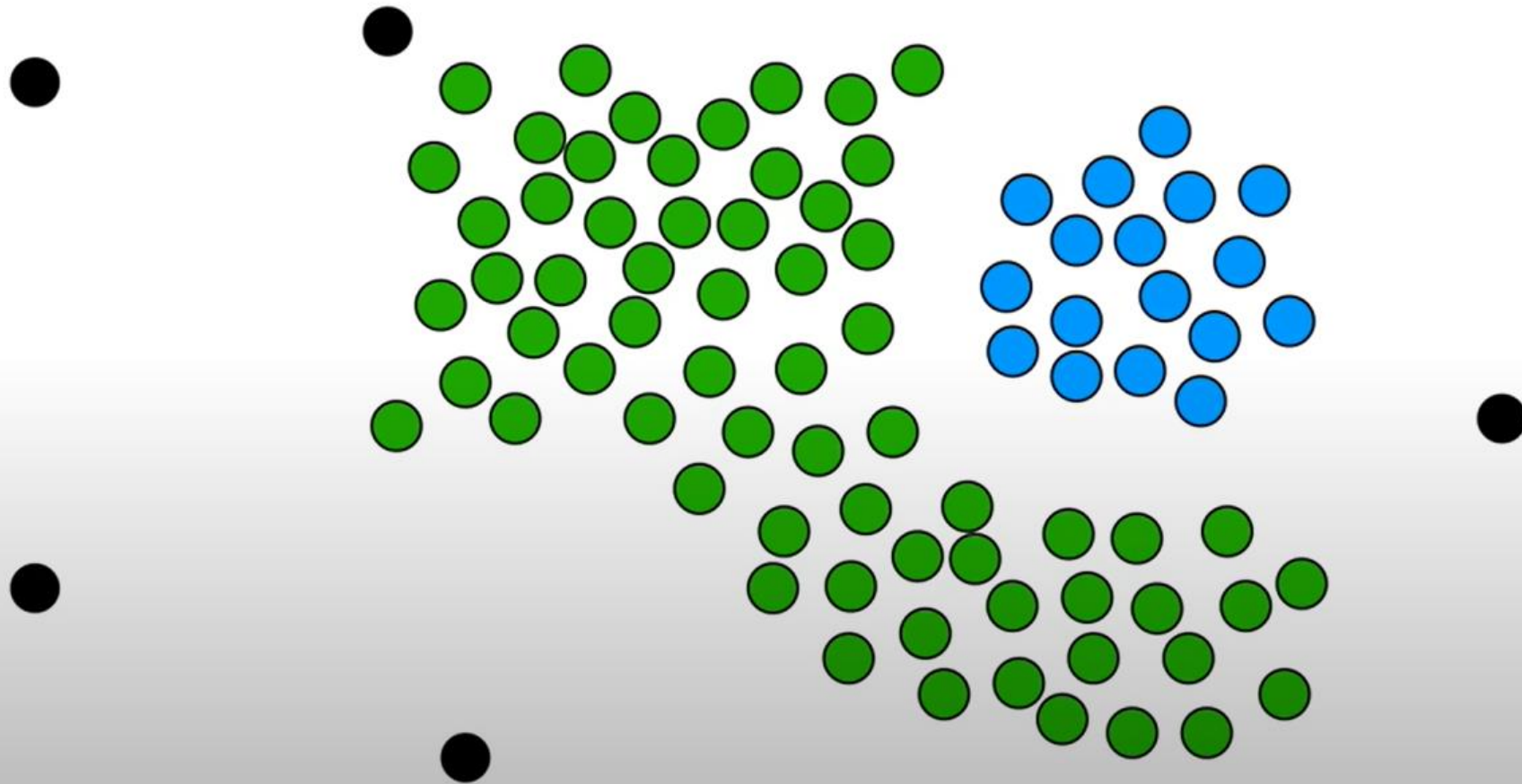
So, unlike **Core Points**, **Non-Core Points** can only join a cluster. They can not extend it further.



Now we add all of the **Non-Core Points** that are close **Core Points** in the **first cluster** to the **first cluster**.



Lastly, because all of **Core Points** have been assigned to a cluster, we're done making new clusters...



# Example

| Point | X   | y   |
|-------|-----|-----|
| A     | 0.5 | 1.2 |
| B     | 1.0 | 1.8 |
| C     | 1.2 | 1.5 |
| D     | 3.8 | 3.2 |
| E     | 3.9 | 3.8 |
| F     | 4.5 | 4.2 |
| G     | 2.0 | 2.2 |
| H     | 2.3 | 2.0 |

| Point | A    | B    | C    | D    | E    | F    | G    | H    |
|-------|------|------|------|------|------|------|------|------|
| A     | 0    | 0.78 | 0.76 | 3.76 | 4.36 | 5.02 | 1.86 | 2.05 |
| B     | 0.78 | 0    | 0.36 | 3.10 | 3.74 | 4.45 | 1.28 | 1.37 |
| C     | 0.76 | 0.36 | 0    | 2.78 | 3.41 | 4.08 | 0.96 | 1.06 |
| D     | 3.76 | 3.10 | 2.78 | 0    | 0.61 | 1.22 | 1.84 | 1.58 |
| E     | 4.36 | 3.74 | 3.41 | 0.61 | 0    | 0.72 | 2.50 | 2.27 |
| F     | 5.02 | 4.45 | 4.08 | 1.22 | 0.72 | 0    | 3.31 | 3.05 |
| G     | 1.86 | 1.28 | 0.96 | 1.84 | 2.50 | 3.31 | 0    | 0.36 |
| H     | 2.05 | 1.37 | 1.06 | 1.58 | 2.27 | 3.05 | 0.36 | 0    |

minPts=2 and eps=0.8

# Example

| Point | A    | B    | C    | D    | E    | F    | G    | H    |
|-------|------|------|------|------|------|------|------|------|
| A     | 0    | 0.78 | 0.76 | 3.76 | 4.36 | 5.02 | 1.86 | 2.05 |
| B     | 0.78 | 0    | 0.36 | 3.10 | 3.74 | 4.45 | 1.28 | 1.37 |
| C     | 0.76 | 0.36 | 0    | 2.78 | 3.41 | 4.08 | 0.96 | 1.06 |
| D     | 3.76 | 3.10 | 2.78 | 0    | 0.61 | 1.22 | 1.84 | 1.58 |
| E     | 4.36 | 3.74 | 3.41 | 0.61 | 0    | 0.72 | 2.50 | 2.27 |
| F     | 5.02 | 4.45 | 4.08 | 1.22 | 0.72 | 0    | 3.31 | 3.05 |
| G     | 1.86 | 1.28 | 0.96 | 1.84 | 2.50 | 3.31 | 0    | 0.36 |
| H     | 2.05 | 1.37 | 1.06 | 1.58 | 2.27 | 3.05 | 0.36 | 0    |

| Point |         |
|-------|---------|
| A     | A, B, C |
| B     | A, B, C |
| C     | A, B, C |
| D     | D, E    |
| E     | D, E, F |
| F     | E, F    |
| G     | G, H    |
| H     | G, H    |

minPts=2 and eps=0.8

# Example

| Point |         |
|-------|---------|
| A     | A, B, C |
| B     | A, B, C |
| C     | A, B, C |
| D     | D, E    |
| E     | D, E, F |
| F     | E, F    |
| G     | G, H    |
| H     | G, H    |

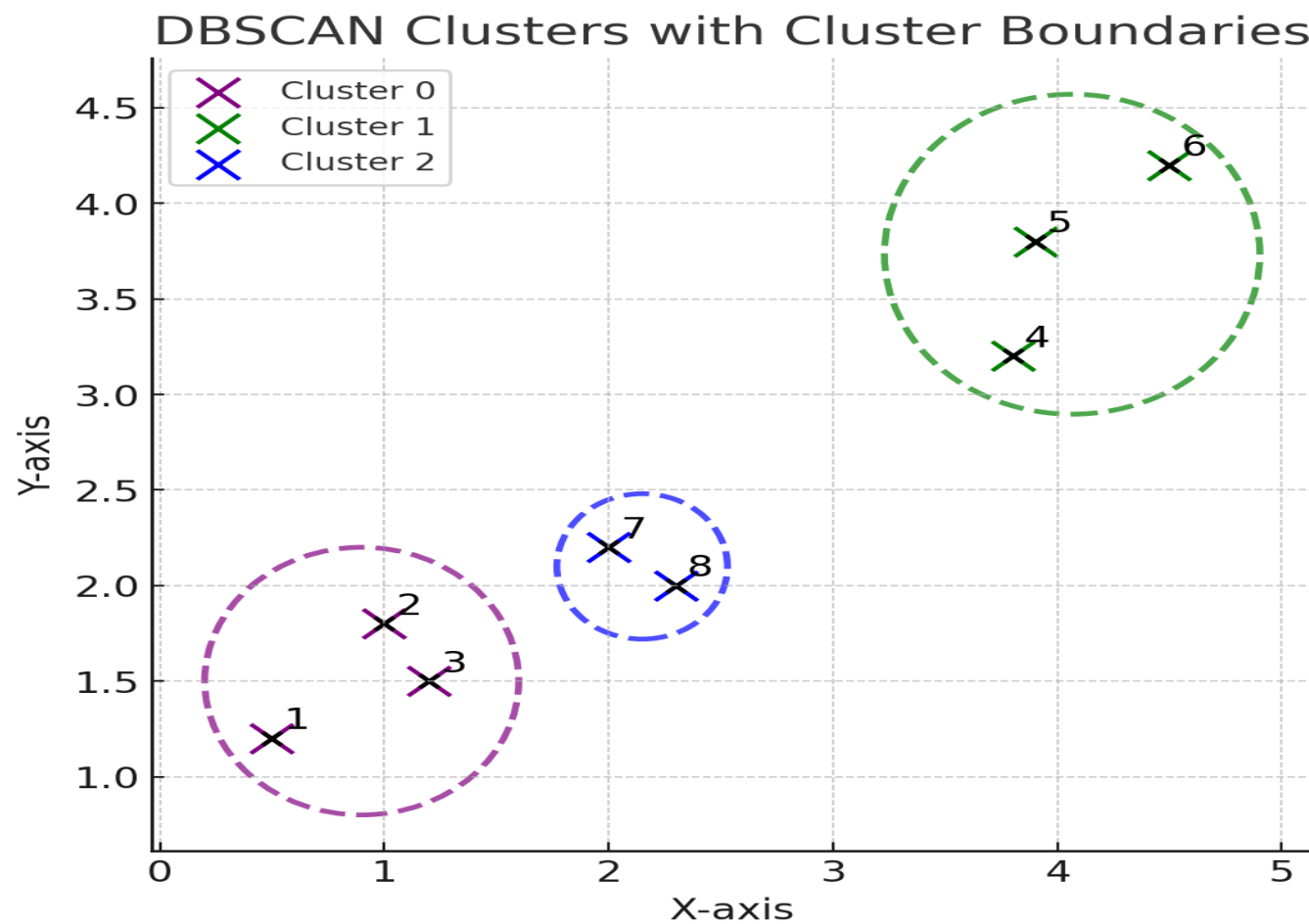
| Point | Status |  |
|-------|--------|--|
| A     | Core   |  |
| B     | Core   |  |
| C     | Core   |  |
| D     | Core   |  |
| E     | Core   |  |
| F     | Core   |  |
| G     | Core   |  |
| H     | Core   |  |

| Cluster | Points  |
|---------|---------|
| 1       | A, B, C |
| 2       | D, E, F |
| 3       | G, H    |

minPts=2 and eps=0.8

Core Points are A, B, C, D, E, F, G and H

# Example





# Example

| Point |         |
|-------|---------|
| A     | A, B, C |
| B     | A, B, C |
| C     | A, B, C |
| D     | D, E    |
| E     | D, E, F |
| F     | E, F    |
| G     | G, H    |
| H     | G, H    |

Core Points: A, B, C and E

Border Points: D and F

| Point | Status |         |
|-------|--------|---------|
| A     | Core   |         |
| B     | Core   |         |
| C     | Core   |         |
| D     | Noise  | Border  |
| E     | Core   |         |
| F     | Noise  | Border  |
| G     | Noise  | Outlier |
| H     | Noise  | Outlier |

Outliers/Noise Points: G and H

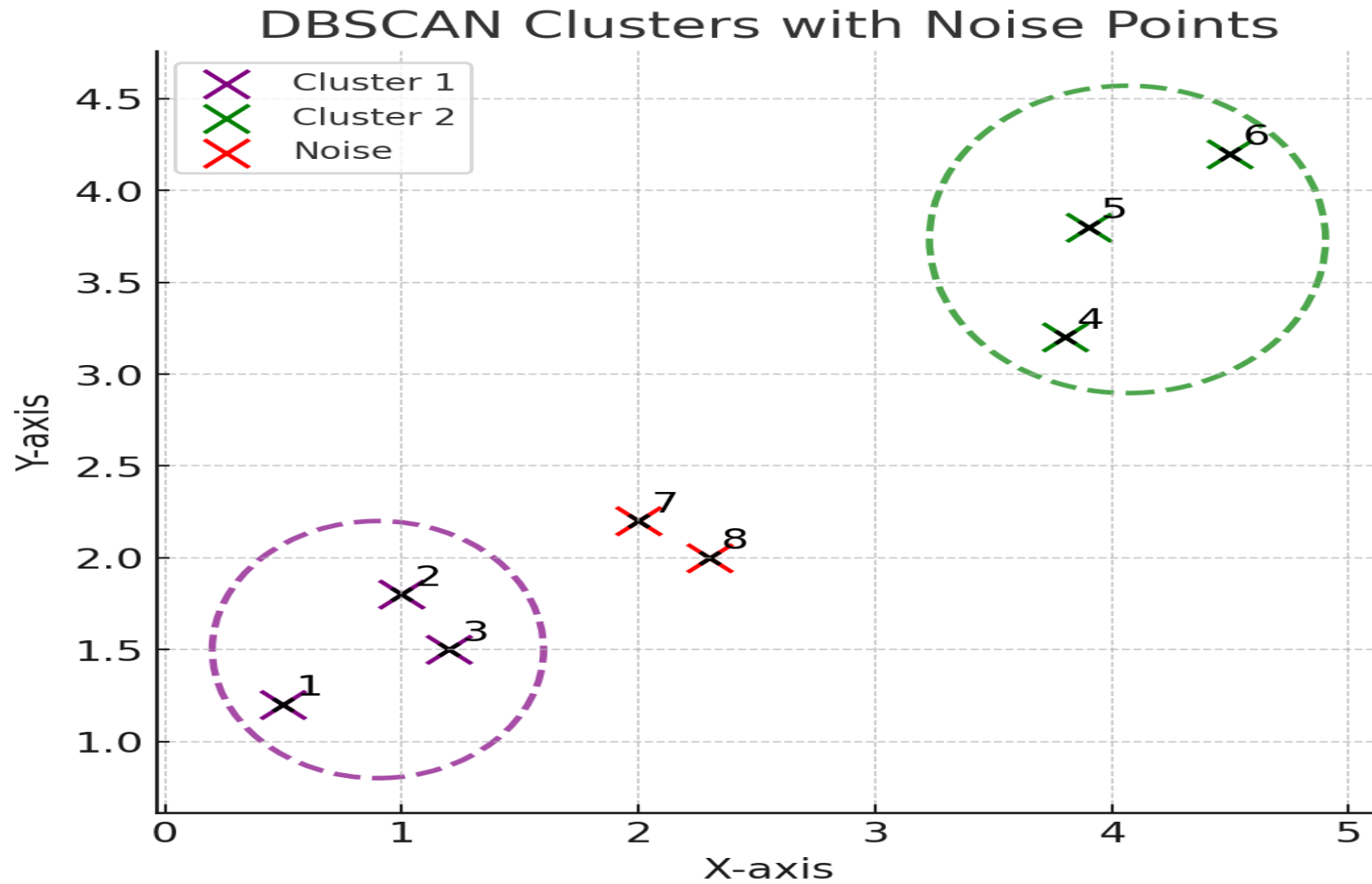
minPts=3 and eps=0.8

D is part of core point (E)

F is part of core point (E)

| Cluster | Points  |
|---------|---------|
| 1       | A, B, C |
| 2       | D, E, F |
| Outlier | G, H    |

# Example



# Practice Problem

- Apply the DBSCAN algorithm to the given data points and
- Create the clusters with
- $\text{minPts} = 4$  and
- $\text{epsilon} (\epsilon) = 1.9$ .

Data Points:

P1: (3, 7)

P2: (4, 6)

P3: (5, 5)

P4: (6, 4)

P5: (7, 3)

P6: (6, 2)

P7: (7, 2)

P8: (8, 4)

P9: (3, 3)

P10: (2, 6)

P11: (3, 5)

P12: (2, 4)