1. Let X is a Discrete Random Variable whose Probability distribution is given below.

X :	0	1	2	3
$P(X)$:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{24}$	$\frac{1}{8}$

Find the Expected Value of $h(x) = (x-1)^2$

Soln :-

$$E[(x-1)^2] = \sum_x (x-1)^2 p(x)$$

$$= (0-1)^2 \times \frac{1}{3} + (1-1)^2 \times \frac{1}{2} +$$

$$(2-1)^2 \times \frac{1}{24} + (3-1)^2 \times \frac{1}{8}$$

$$= \frac{1}{3} + 0 + \frac{1}{24} + \frac{4}{8} = \frac{1}{3} + \frac{1}{24} + \frac{1}{2}$$

$$= \frac{8+1+12}{24} = \frac{21}{24} \quad \frac{7}{8}$$

$E(x) = 7/8$ ..

If x is a R.V. whose pdf is given by

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find the expected value of $h(x) = e^{3x/4}$

Soln :-

$$E[e^{3x/4}] = \int_0^\infty e^{3x/4} \cdot e^{-x} \, dx$$

$$= \int_0^\infty e^{x\left[\frac{2}{4}-1\right]} \, dx$$

$$= \int_0^\infty e^{-x/4} \, dx$$

$$= \left[\frac{-e^{-x/4}}{1/4} \right]_0^\infty$$

$$= -4[0-1] = 4$$

3. The Joint Probability distribution of X and Y is as follows

$$f(x,y) = \frac{1}{30}(x+y), \quad \begin{matrix} x = 0, 1, 2, 3 \\ y = 0, 1, 2 \end{matrix}$$

Find $E[2X - Y]$.

$$E[2X - Y] = 2E[X] - E[Y]$$

Y \ X	0	1	2	3	$p(y)$
0	$f(0,0) = 0$	$f(1,0) = 1/30$	$2/30$	$3/30$	$6/30 = 1/5$
1	$4/30$	$2/30$	$3/30$	$4/30$	$10/30 = 1/3$
2	$2/30$	$3/30$	$4/30$	$5/30$	$14/30 = 7/15$
$p(x)$	$4/10$	$1/5$	$3/10$	$2/5$	1

x:	0	1	2	3
$p(x)$:	$\frac{3}{30}$	$\frac{6}{30}$	$\frac{9}{30}$	$\frac{12}{30}$

$$E(x) = \sum_{2} x\, p(x)$$

$$= \left(0 \times \frac{3}{30}\right) + \left(1 \times \frac{6}{30}\right) + \left(2 \times \frac{9}{30}\right) + \left(3 \times \frac{12}{30}\right)$$

$$= 0 + \frac{6}{30} + \frac{18}{30} + \frac{36}{30} = \frac{60}{30} = 2$$

y	0	1	2
$p(y)$:	$\frac{6}{30}$	$\frac{10}{30}$	$\frac{14}{30}$

$$E(Y) = \sum_{y} y\, p(y)$$

$$= 0 + \left(\frac{10}{30}\right) + \left(\frac{28}{30}\right)$$

$$= \frac{38}{30} = \frac{19}{15} //$$

$$E[2X - Z] = 2E(X) - E(Z)$$

$$= 2\left(\frac{2}{3}\right) - \frac{17}{15}$$

$$= 4 - \frac{17}{15}$$

$$= \frac{60 - 17}{15} = \frac{41}{15} //$$