SVM

Module-3

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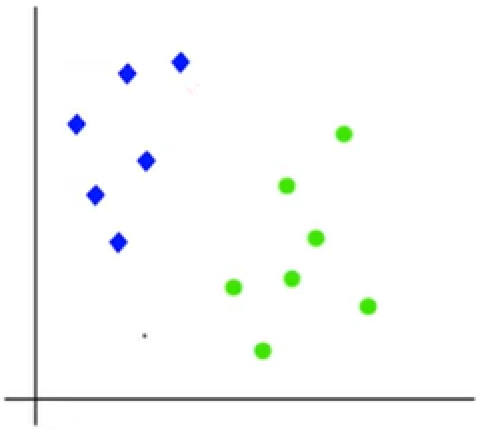
SCORE, VIT

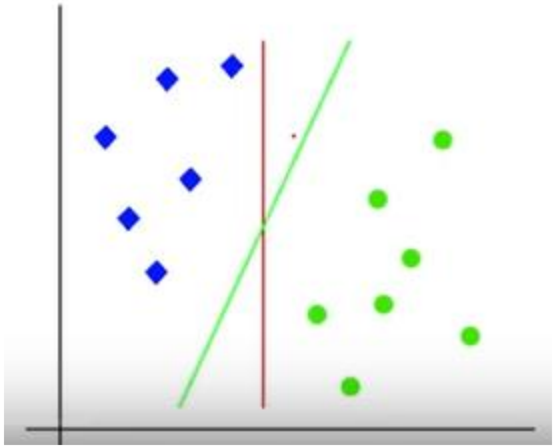
Introduction

- Supervised Learning
- Used for both classification and regression, mainly used for classification algorithms
- The goal of SVM is to create a best line or decision boundary that can segregate n-dimensional space into classes.
- Best decision boundary is called the hyperplane

- There can be multiple lines to segregate the classes in n-dimensional space but we need to find the best decision boundary to classify the data points

Need to choose the best line that separates the two classes





- The dimensions of the hyperplane depend on the features present in the dataset, which means if there are 2 features (as shown in image), then hyperplane will be a straight line.
  - And if there are 3 features, then hyperplane will be a 2-dimension plane.
- We always create a hyperplane that has a maximum margin, which means the maximum distance between the data points.

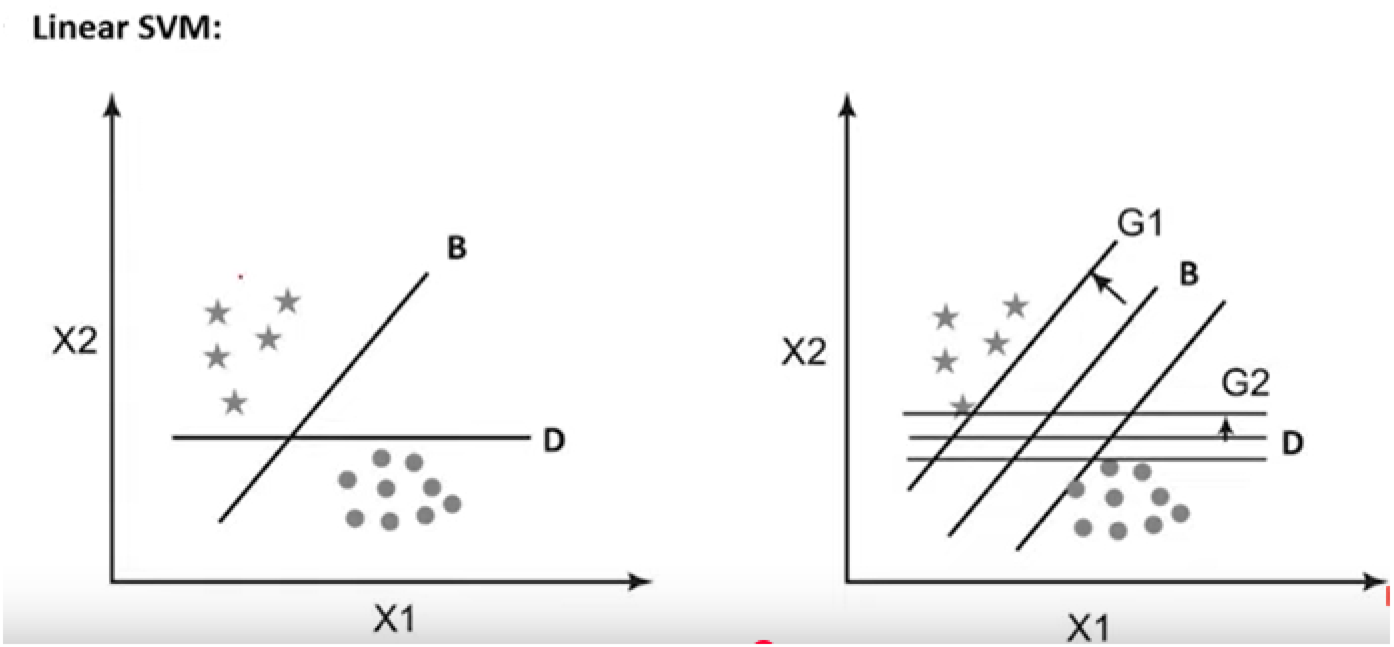
- The data points or vectors that are the closest to the hyperplane and which affect the position of the hyperplane are termed as Support Vector.
- Since these vectors support the hyperplane, hence called a Support vector.

Types of SVM

SVM can be of two types:

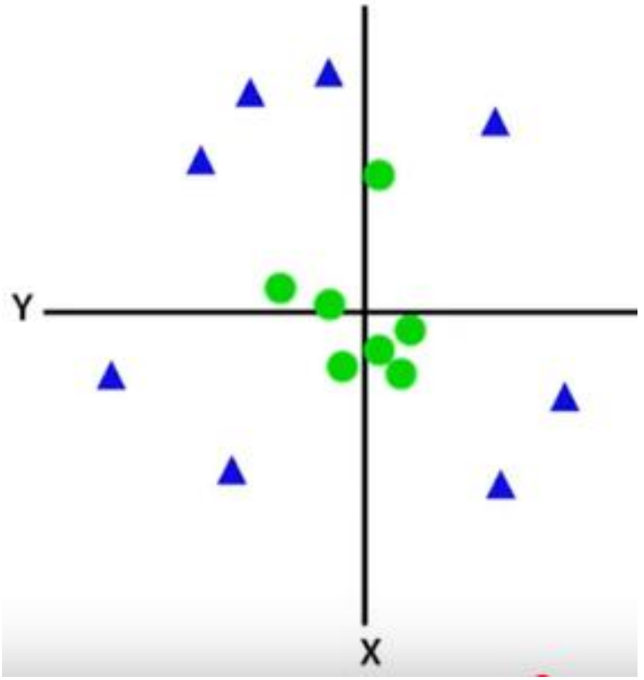
1. **Linear SVM:** Linear SVM is used for linearly separable data, which means if a dataset can be classified into two classes by using a single straight line, then such data is termed as linearly separable data, and classifier is used called as Linear SVM classifier.
2. **Non-linear SVM:** Non-Linear SVM is used for non-linearly separated data, which means if a dataset cannot be classified by using a straight line, then such data is termed as non-linear data and classifier used is called as Non-linear SVM classifier.

Linear SVM

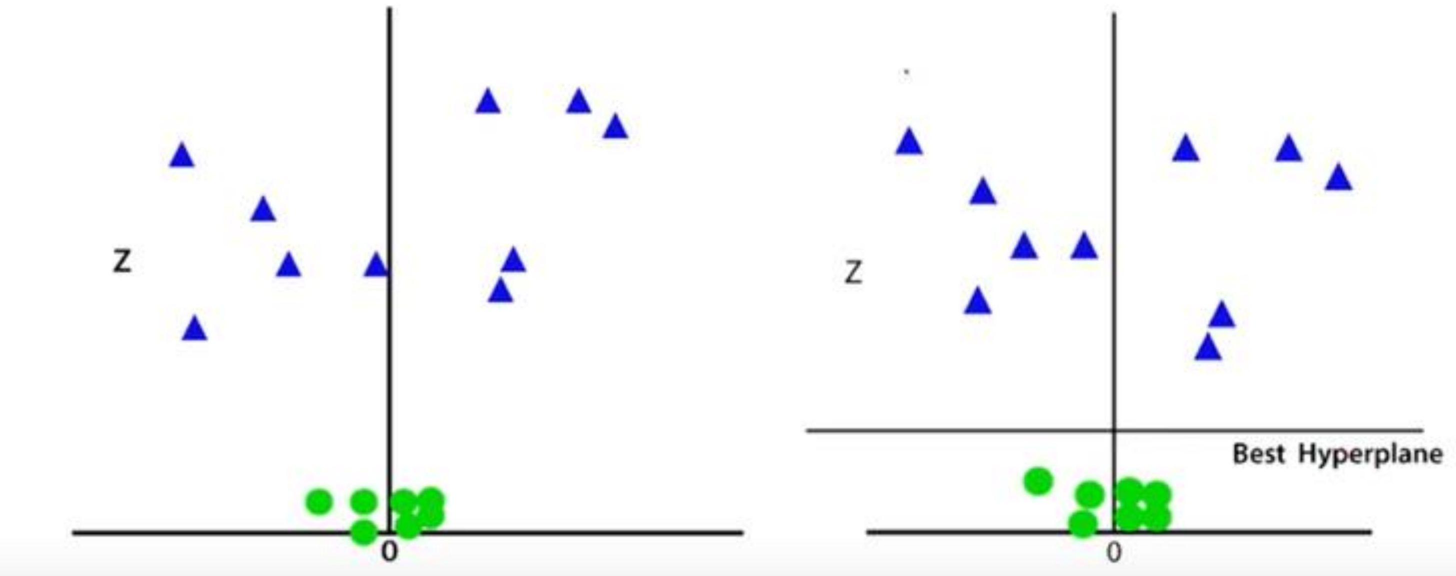


Non-Linear SVM

- If data is linearly arranged, then we can separate it by using a straight line, but for non-linear data, we cannot draw a single straight line.
- So to separate these data points, we need to add one more dimension.
- For linear data, we have used two dimensions x and y, so for non-linear data, we will add a third dimension z.
- It can be calculated as: $z=x^2+y^2$



Non-Linear SVM



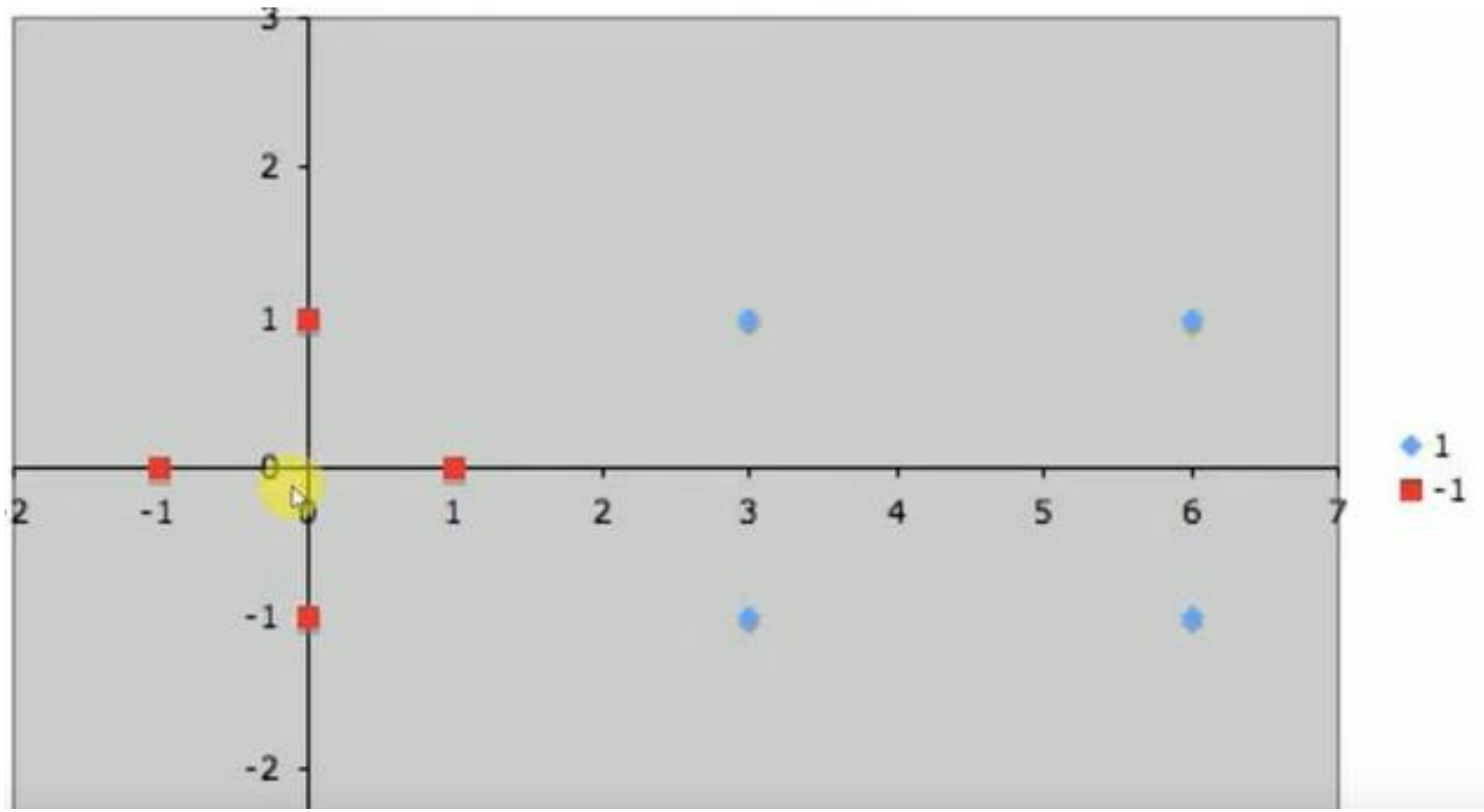
Linear SVM-Solved example

Suppose we are given the following positively labeled data points,

$$\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\}$$

and the following negatively labeled data points,

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$$

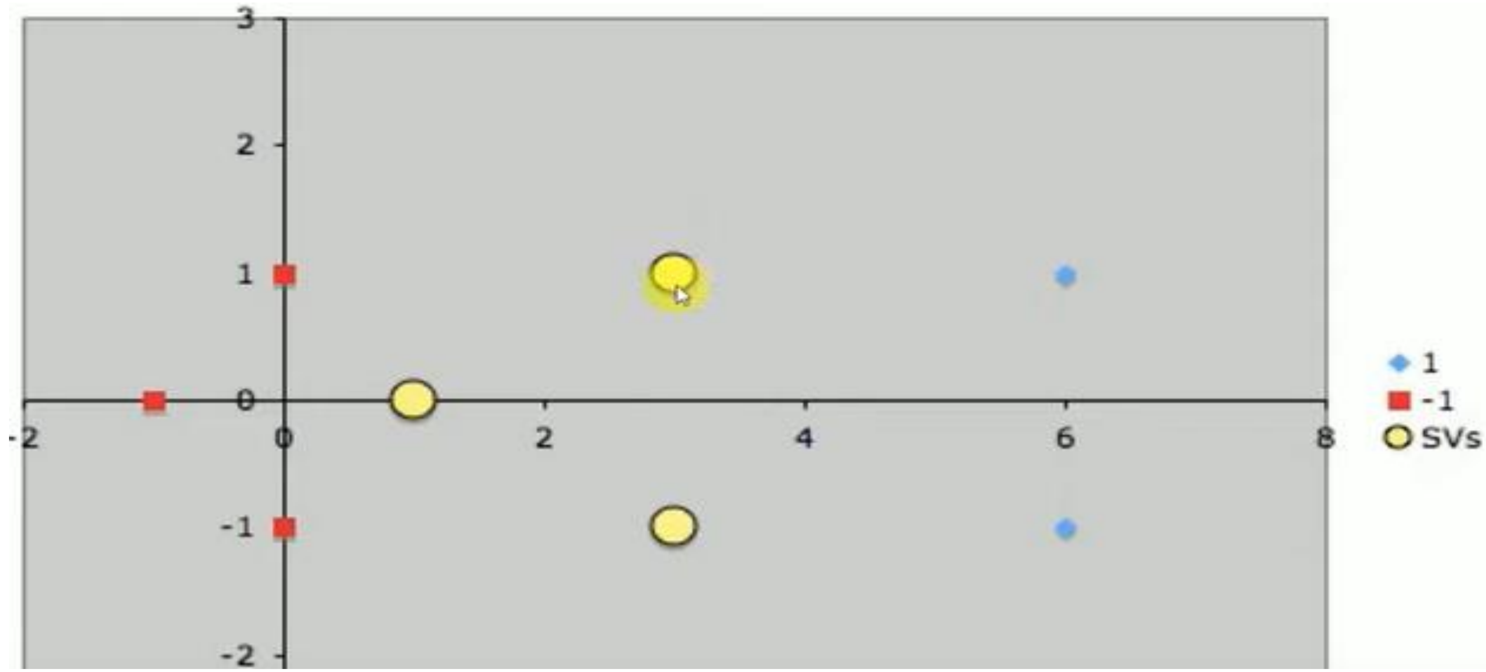


3
2
1
0
-1
-2
-2
-1
0
1
2
3
4
5
6
7
1
-1

Support vectors

- By inspection, it should be obvious that there are **three** support vectors,

$$\left\{ s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, s_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, s_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right\}$$



- Each vector is augmented with a 1 as a bias input

- So, $s_1 = \binom{1}{0}$, then $\widetilde{s_1} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

- Similarly,

- $s_2 = \binom{3}{1}$, then $\widetilde{s_2} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $s_3 = \binom{3}{-1}$, then $\widetilde{s_3} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

$$
\begin{aligned}
\alpha_1 \tilde{s_1} \cdot \tilde{s_1} + \alpha_2 \tilde{s_2} \cdot \tilde{s_1} + \alpha_3 \tilde{s_3} \cdot \tilde{s_1} &= -1 \\
\alpha_1 \tilde{s_1} \cdot \tilde{s_2} + \alpha_2 \tilde{s_2} \cdot \tilde{s_2} + \alpha_3 \tilde{s_3} \cdot \tilde{s_2} &= +1 \\
\alpha_1 \tilde{s_1} \cdot \tilde{s_3} + \alpha_2 \tilde{s_2} \cdot \tilde{s_3} + \alpha_3 \tilde{s_3} \cdot \tilde{s_3} &= +1
\end{aligned}
$$

$$\alpha_1(1+0+1) + \alpha_2(3+0+1) + \alpha_3(3+0+1) = -1$$

$$\alpha_1(3+0+1) + \alpha_2(9+1+1) + \alpha_3(9-1+1) = 1$$

$$\alpha_1(3+0+1) + \alpha_2(9-1+1) + \alpha_3(9+1+1) = 1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = 1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = 1$$

$$2\alpha_1 + 4\alpha_2 + 4\alpha_3 = -1$$

$$4\alpha_1 + 11\alpha_2 + 9\alpha_3 = 1$$

$$4\alpha_1 + 9\alpha_2 + 11\alpha_3 = 1$$

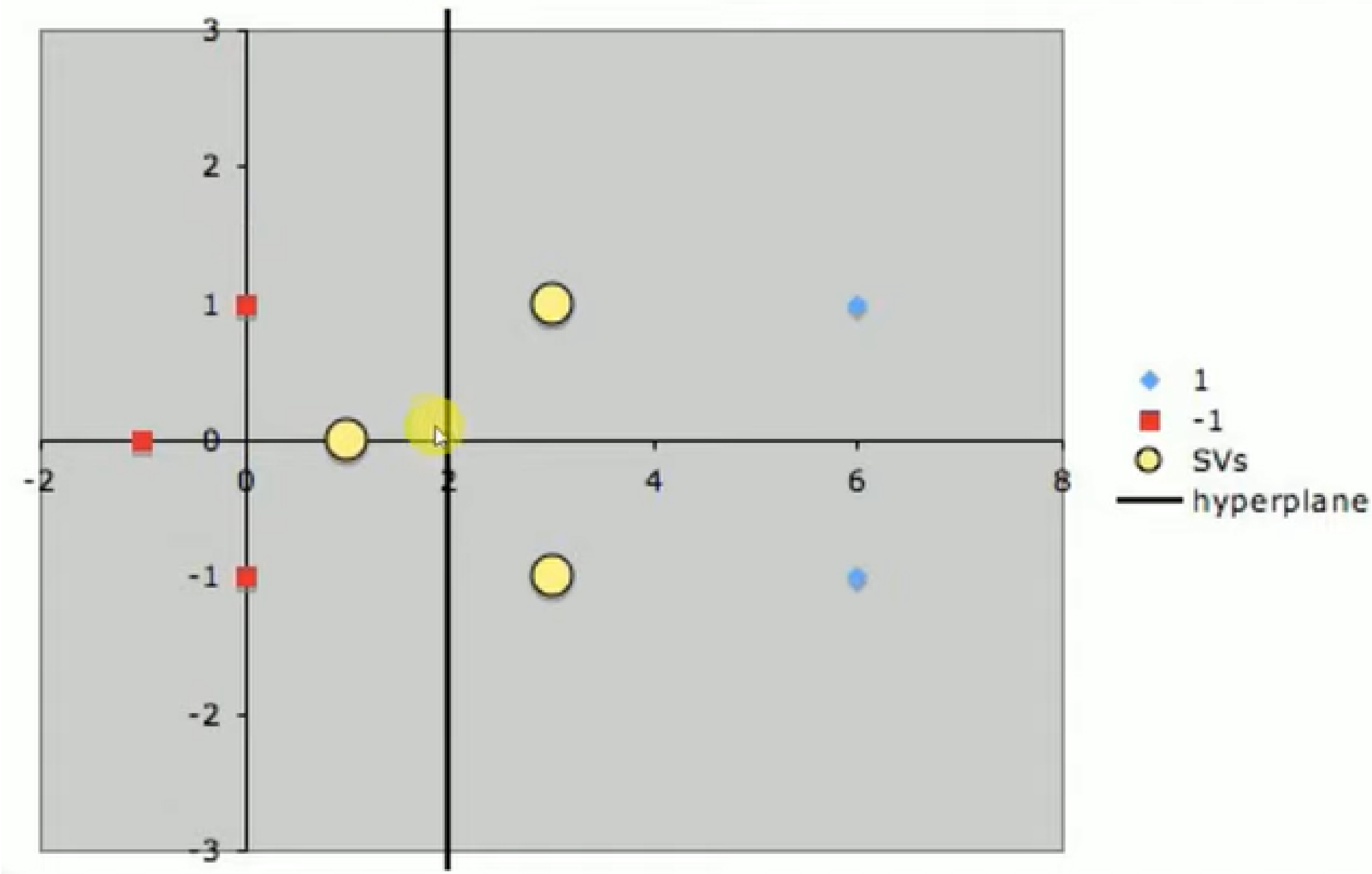
$$\alpha_1 = -3.5$$

$$\alpha_2 = 0.75$$

$$\alpha_3 = 0.75$$

$$
\begin{aligned}
\tilde{w} &= \sum_i \alpha_i \tilde{s}_i \\
&= -3.5\begin{pmatrix}1\\0\\1\end{pmatrix} + 0.75\begin{pmatrix}3\\1\\1\end{pmatrix} + 0.75\begin{pmatrix}3\\-1\\1\end{pmatrix} \\
&= \begin{pmatrix}1\\0\\-2\end{pmatrix}
\end{aligned}
$$

- Finally, remembering that our vectors are augmented with a bias.
- We can equate the last entry in $\widetilde{w}$ as the hyperplane offset b and write the separating
- Hyperplane equation $\boldsymbol{y} = \boldsymbol{wx} + \boldsymbol{b}$
- with $\boldsymbol{w} = \binom{1}{0}$ and $\boldsymbol{b} = -2$.



3
2
1
0
-1
-2
-3
-2
0
2
4
6
8
1
-1
SVs
hyperplane