Multi linear Regression (MLR)

Work process

The (MLR) for two variables x_1 and x_2 given as.

$$y = f(x_1, x_2)$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

β_0, β_1 and β_2 are coefficients for the MLR for two variables.

This can be generalized for 'n' variables.

Table 1:-

Ex

Product1 sales. x_1	Product2 Sales x_2	Weekly sales. y.
3	8	-3.7
4	5	3.5
5	7	2.5
6	3	11.5
2	1	5.7.

We have to find ,

$$b_0 = \bar{y} - b_1 \cdot \bar{x}_1 - b_2 \cdot \bar{x}_2$$

$$b_1 = \frac{\Sigma x_2^2 \, \Sigma x_1 y - \Sigma x_1 x_2 \, \Sigma x_2 y}{\Sigma x_1^2 \, \Sigma x_2^2 - (\Sigma x_1 x_2)^2}$$

$$b_2 = \frac{\Sigma x_1^2 \, \Sigma x_2 y - \Sigma x_1 x_2 \, \Sigma x_1 y}{\Sigma x_1^2 \, \Sigma x_2^2 - (\Sigma x_1 x_2)^2}$$

Table: 2 :

y	x_1	x_2	$(x_1)^2$	$(x_2)^2$	$(x_1 y)$	$(x_2 y)$	$x_1 \cdot x_2$
-3.7	3	8	9	64	-11.1	-29.6	24
3.5	4	5	16	25	14	17.5	20
2.5	5	7	25	49	12.5	17.5	35
11.5	6	3	36	9	6.9	34.5	18
5.7	2	1	4	1	11.4	5.7	2
19.5	20	24	$\Sigma = 90$	$\Sigma = 148$	Σ 95.8	45.6	99

In general, the regression sum is

$$\sum_R x_i^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}, \quad n \to \text{number of objects.}$$

if $i=1$ $$\boxed{\sum_R x_1^2 = \sum x_1^2 - \frac{(\sum x_1)^2}{n}} \rightarrow (1)$$

if $i=2$ $$\boxed{\sum_R x_2^2 = \sum x_2^2 - \frac{(\sum x_2)^2}{n}} \rightarrow (2)$$

cross product of features with target.

$$\sum_R x_i y = \sum x_i y - \frac{(\sum x_i)(\sum y)}{n}$$

if $i=1$ $$\boxed{\sum_R x_1 y = \sum x_1 y - \frac{(\sum x_1)(\sum y)}{n}} \rightarrow (3)$$

if $i=2$ $$\boxed{\sum_R x_2 y = \sum x_2 y - \frac{(\sum x_2)(\sum y)}{n}} \rightarrow (4)$$

So for two independent variables we have all the terms to find the co-efficients

Now finally

$$\boxed{\sum_R x_1 x_2 = \sum x_1 \cdot x_2 - \frac{(\sum x_1)(\sum x_2)}{n}} \rightarrow (5)$$

Now calculate ① ② ③ ④ and ⑤ by substituting the required terms from Table ②.

Eqn ①

$$\sum_R x_1^2 = \sum x_1^2 - \frac{(\sum x_1)^2}{n}$$

$$= 90 - \frac{(20)^2}{5}$$

eq ②

$$\sum_R x_2^2 = \sum x_2^2 - \frac{(\sum x_2)^2}{n}$$

$$= 148 - \frac{(24)^2}{5} = 32.8$$

eq ③

$$\sum_R x_1 y = 95.8 - \frac{(20)(19.5)}{5}$$

$$= 17.8$$

eq (4)

$$\sum_R x_2 y = 45.6 - \frac{(24)(19.5)}{5}$$

$$= -48$$

eqn (5)

$$\sum x_1 . x_2 = 99 - \frac{(20)(24)}{5}$$

$$= 3$$

Now

Let us find b_1 and b_2, that leads us to find b_0.

$$\boxed{b_1 = \frac{(32.8)(17.8) - (3)(-48)}{[(10)(32.8) - (3)^2]}}$$

$$b_1 = \frac{727.84}{319} = 2.28.$$

$$\boxed{b_1 = 2.28}$$

$$\boxed{b_2 = \frac{(10)(-48) - (3)(17.8)}{319}}$$

$$= \frac{-533.4}{319} = -1.67.$$

$$\boxed{b_2 = -1.67}$$

Now find.

$$b_0 = \bar{y} - b_1 . \bar{x}_1 - b_2 \bar{x}_2$$

$$\bar{y} = \frac{19.5}{5} = 3.9$$

$$\bar{x}_1 = \frac{20}{5} = 4$$

$$\bar{x}_2 = \frac{24}{5} = 4.8.$$

So $b_0 = (3.9) - (2.28)(4) - (-1.67)(4.8)$

$$= 3.9 - 9.12 + 8.016$$

$$\boxed{b_0 = 2.796}$$

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2$$

$$= 2.796 + 2.28 x_1 + (-1.67) x_2.$$

So Predict the decision based on the coefficients and check for error and calculate R^2.