19/7/24

Conditional Probability

The conditional probability of an event B, assuming that the event A has already happened is denoted by $P(B/A)$ and defined as

$$P(B/A) = \frac{P(A\cap B)}{P(A)}, \quad P(A) \neq 0$$

Similarly,

$$P(A/B) = \frac{P(A\cap B)}{P(B)}, \quad P(B) \neq 0.$$

Ex: If $P(A)=0.6$, $P(B)=0.5$, $P(A\cap B)=0.2$. find (i) $P(A/B)$ (ii) $P(\bar{A}/B)$ (iii) $P(A/\bar{B})$.

Soln:

(i) $$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5}$$

(ii) $$P(\bar{A}/B) = \frac{P(\bar{A}\cap B)}{P(B)}$$

$$= \frac{P(B) - P(A\cap B)}{P(B)} = \frac{0.3}{0.5} = 3/5$$

(iii) $$P(A/\bar{B}) = \frac{P(A) - P(A\cap B)}{1-P(B)} = \frac{0.4}{0.5} = \frac{4}{5}$$

Note:

$P(A/B) + P(\bar{A}/B) = 1$

2) A die is rolled. If it shows an odd number, then find the probability of getting 5.

Soln:-

Sample space $S = \{1, 2 \ldots 6\}$

Let A - Event of die shows an odd number

B - Event of getting 5.

Then $A = \{1, 3, 5\}$

$B = \{5\}$

$A \cap B = \{5\}$

$\therefore P(A) = 3/6 = 1/2$

$P(A \cap B) = 1/6$

Now P(getting 5 / die shows an odd number) $= P(B/A)$

$= \dfrac{P(A \cap B)}{P(A)}$

$= 1/6 \,/\, 3/6$

$= 1/3$

Multiplication theorem on Probability

The Prob. of the Simultaneous happing of 2 events A and B is given by,

$$P(A \cap B) = P(A/B)\, P(B) \quad \text{(or)}$$

$$P(A \cap B) = P(B/A)\, P(A)$$

Independent Events:

Events are said to be Independent if occurrence (or) non-occurrence of any one of the event does no[t] affect the Prob. of occurrence or non-occurrence o[f] the other events.

Two events A and B are said to be Independen[t] iff $P(A \cap B) = P(A) \cdot P(B)$

Note:-

1. The above defn. is exactly equivalent to

$$P(A/B) = P(A), \quad \text{if } P(B) > 0$$

$$P(B/A) = P(B) \quad \text{if } P(A) > 0$$

2. The events $A_1, A_2 \ldots A_n$ are mutually Independent

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \ldots P(A_n)$$

3. If A and B are Independent then
   (i) $\bar{A}$ and $\bar{B}$ are Inde.
   (ii) A and $\bar{B}$ "
   (iii) $\bar{A}$ and B are also Indep.

Ex 1. Two cards are drawn from a pack of 52 cards in succession. Find the probability that both are Jack when the first drawn card is (i) replaced (ii) not replaced.

Soln:

Let A be the event of drawing a Jack in the 1st draw.

Let B be the event of drawing a Jack in the second draw.

Case (i) card is replaced.

$$n(A) = 4, \quad n(B) = 4, \quad n(S) = 52$$

Clearly the event A will not affect the probability of the occurrence of event B and

$\therefore$ A and B are Independent.

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{\cancel{4}}{\cancel{52}_{13}} \cdot \frac{\cancel{4}}{\cancel{52}_{13}} = \frac{1}{169}$$

Case (ii) card is not replaced

Here the first event A affects the probability of occurrence of the second event B

Thus, A and B are not Independent.

They are dependent events.

$\therefore P(A \cap B) = P(A) \cdot P(B/A)$

$$= \frac{\cancel{4}}{\cancel{52}_{13}} \cdot \frac{\cancel{3}}{\cancel{51}_{17}}$$

$$= 1/221.$$

2) A Coin is tossed twice. Events E and F are defined as follows, E = Head on 1st toss, F = Head on second toss. Find (i) $P(E \cup F)$, (ii) $P(E/F)$, (iii) $P(\bar{E}/F)$, (iv) Are the events E and F independent?

Soln:

The Sample Space, $S = \{(H,H), (H,T), (T,H), (T,T)\}$

$E = \{(H,H), (H,T)\}$

$F = \{(H,H), (T,H)\}$

$E \cap F = \{(H,H)\}$

$E \cup F = \{(H,H), (T,H), (H,T)\}$

(i) $P(E \cup F) = \frac{n(E \cup F)}{n(S)} = 3/4$

(or)

$= P(E) + P(F) - P(E \cap F)$

$= \frac{2}{4} + \frac{2}{4} - 1/4 = 1 - \frac{1}{4} = 3/4$

data <- matrix(c( ), ~~nrow~~, byrow)

~~chisq.test(data)~~

(ii) $P(E/F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{(1/4)}{(2/4)} = 1/2$

(iii) $P(\bar{E}/F) = \dfrac{P(\bar{E} \cap F)}{P(F)} = \dfrac{P(F) - P(E \cap F)}{P(F)}$

$= \dfrac{\frac{2}{4} - \frac{1}{4}}{2/4} = \dfrac{1/4}{2/4} = 1/2.$

(ii)

To P.T: $P(E \cap F) = P(E) \cdot P(F)$.

<u>L.H.S</u> $P(E \cap F) = 1/4$ — (1)

R.H.S $P(E) \cdot P(F) = \frac{2}{4} \cdot \frac{2}{4} = 1/4$ — (2)

(1) = (2)

$\Rightarrow$ E and F are Independent.

<u>Note</u>:

Suppose A and B are 2 events s.t. $P(A) \neq 0$ and $P(B) \neq 0$. (i) If A and B are mutually exclusive they cannot be Independent.

(ii) If A and B are Independent they cannot be mutually exclusive.

1. If A and B are 2 Independent events &
$P(A) = 0.4$ and $P(A \cup B) = 0.9$. find $P(B)$

Soln:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A).P(B) \quad [\text{A \& B are Indep}$$

$$0.9 = 0.4 + P(B)[1 - 0.4]$$

$$0.9 - 0.4 = 0.6\, P(B)$$

$$\frac{0.5}{0.6} = P(B)$$

$$\Rightarrow \boxed{P(B) = 5/6}$$

2) An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probability of hitting the plane in the 1st, 2nd, 3rd and 4th shot are respectively 0.2, 0.4, 0.2 and 0.1. Find the probability that the gun hits the plane.

Soln:-
Let H_1, H_2, H_3 and H_4 be the events of hitting the plane by the anti-aircraft gun in the 1st, 2nd, 3rd & 4th shot resp.

Let H - event that anti-aircraft gun hits the plane.
$\therefore \bar{H}$ - event that the plane is not

Shot down. Given that

$P(H_1) = 0.2, \quad P(H_2) = 0.4, \quad P(H_3) = 0.2, \quad P(H_4) = 0.1$

$P(\overline{H_1}) = 1 - 0.2 = 0.8$

$P(\overline{H_2}) = 1 - 0.4 = 0.6$

$P(\overline{H_3}) = 1 - 0.2 = 0.8$

$P(\overline{H_4}) = 1 - 0.1 = 0.9$

The prob. that the gun hits the plane is

$$P(H) = 1 - P(\overline{H})$$
$$= 1 - P(\overline{H_1 \cup H_2 \cup H_3 \cup H_4})$$
$$= 1 - P(\overline{H_1}) \cap P(\overline{H_2}) \cap P(\overline{H_3}) \cap P(\overline{H_4})$$
$$= 1 - P(\overline{H_1}) \cdot P(\overline{H_2}) \cdot P(\overline{H_3}) \cdot P(\overline{H_4})$$
$$= 1 - (0.8)(0.6)(0.8)(0.9)$$
$$= 1 - 0.3456$$

$$\boxed{P(H) = 0.6544}$$

2) X Speaks truth in 70 Percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

Soln:

A - event of X speaks the truth

B - " " Y speaks "

$\therefore \bar{A}$ - event of X not speaking the truth

$\bar{B}$ - " Y "

Let C be the event that they contradict each other.

Given that, $P(A) = 0.7 \Rightarrow P(\bar{A}) = 1 - 0.7 = 0.3$

$P(B) = 0.90 \Rightarrow P(\bar{B}) = 1 - 0.9 = 0.1$

C = (A speaks the truth and B does not or B speaks " and A ")

$= [(A \cap \bar{B})$ (or) $(\bar{A} \cap B)]$

$\because (A \cap \bar{B})$ and $(\bar{A} \cap B)$ are mutually exclusive.

$P(C) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$

$= (0.7)(0.1) + (0.3)(0.9)$

$= 0.07 + 0.270 = 0.34$