24MCA0242

Q.1 (a) The probability that Sam parks in a no-parking zone and gets a parking ticket is 0.06, and the probability that Sam cannot find a legal parking space and has to park in the no-parking zone is 0.20. On Tuesday, Sam arrives at school and has to park in a no-parking zone. Find the probability that he will get a parking ticket.

→ Sol^n;

Let A be the event that Sam parks in a no parking zone

B be the event that Sam gets a parking ticket.

then, $P(A \cap B) = 0.06$

$P(A) = 0.20$

Now, $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.06}{0.20}$$

$$\Rightarrow P(B/A) = 0.3.$$

∴ The probability of Sam getting a parking ticket is 0.3.

Ans.

(b) A game is played by drawing 4 cards from an ordinary deck and replacing each card after it is drawn. Find the probability that atleast 1 ace card is drawn.

→ Sol^n;

The probability of getting an ace card from a ordinary deck of 52 cards $= \frac{4}{52}$.

∴ Probability of not drawing an ace card $= 1 - \frac{4}{52}$

$= \frac{48}{52}$.

Now,

Probability of not drawing an ace card in 4 draws

$$= \frac{48}{52} \times \frac{48}{52} \times \frac{48}{52} \times \frac{48}{52} = \left(\frac{\cancel{48}^{12}}{\cancel{52}_{13}}\right)^4$$

$$= \left(\frac{12}{13}\right)^4$$

$\therefore$ Probability of getting atleast 1 ace card $= 1 - \left(\frac{12}{13}\right)^4$

$$= \frac{28561 - 20736}{28561}$$

$$= \frac{7825}{28561}$$

$$= 0.27.$$

$\therefore$ The probability of drawing atleast 1 ace card is 0.27.

Ans.

Q.2 A certain virus infects one in every 200 people. A test used to detect the virus in a person is positive 80% of the time if the person has the virus and 5% of the time if the person does not have the virus. (This 5% result is called a false positive.)

(a) Using Bayes' Theorem, if a person tests positive, determine the probability that the person is infected.

(b) Using Bayes' Theorem, if a person tests negative, determine the probability that the person is not infected.

→ Sol^n,

Let A be the event that a person is infected.
then, $\bar{A}$ be the event that a person is not infected.

Let B be the event that a person tested positive.
then $\bar{B}$ be the event that a person tested negative

$$P(A) = \frac{1}{200} = 0.005 \qquad P(\bar{A}) = 1 - \frac{1}{200} = \frac{199}{200} = 0.995$$

$$P(B/A) = 0.8$$

$$P(\bar{B}/A) = 1 - 0.8 = 0.2$$

$$P(B/\bar{A}) = 0.05$$

$$P(\bar{B}/\bar{A}) = 1 - 0.05 = 0.95$$

Now, $P(B) = P(A).P(B/A) + P(\bar{A}).P(B/\bar{A})$

$$= (0.005 \times 0.8) + (0.995 \times 0.05)$$

$$= 0.004 + 0.049$$

$$= 0.053$$

$$P(\bar{B}) = P(A).P(\bar{B}/A) + P(\bar{A}).P(\bar{B}/\bar{A})$$

$$= (0.005 \times 0.2) + (0.995 \times 0.95)$$

$$= 0.001 + 0.945$$

$$= 0.946$$

(a) $P(A/B) = \dfrac{P(A).P(B/A)}{P(B)}$

$$= \frac{0.8 \times 0.005}{0.053} = \frac{0.004}{0.053}$$

$$= 0.075$$

$\therefore$ The probability that a person is infected given that he tests positive is 0.075. Ans

(b) $$P(\bar{A}/\bar{B}) = \frac{P(\bar{A}) \cdot P(\bar{B}/\bar{A})}{P(\bar{B})}$$

$$= \frac{0.995 \times 0.95}{0.946} = \frac{0.945}{0.946}$$

$$= 0.99$$

$\therefore$ The probability that a person is not infected given that he tests negative is 0.99. Ans

Q.3 Suppose that the error in the reaction temperature, in °C, for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & \text{for } -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Verify that $f(x)$ is a density function.

(b) Find $P(0 < X < 1)$.

$\rightarrow$ (a) To verify $f(x)$ is a density function, we need to show that $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\therefore \int_{-1}^{2} \frac{x^2}{3}\, dx = \frac{1}{3}\int_{-1}^{2} x^2\, dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2} = \frac{1}{9}\left[2^3-(-1)^3\right]$$

$$= \frac{1}{9}[9] = 1.$$

$\therefore$ $f(x)$ is a density function. Verified

(b) $P(0 < X < 1) = \int_0^1 \frac{x^2}{3}\, dx = \frac{1}{3}\int_0^1 x^2\, dx$

$= \frac{1}{3}\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{9}\left[1^3 - 0^3\right]$

$= \frac{1}{9}$

$\therefore P(0 < X < 1) = \frac{1}{9}$ Ans

Q.4 The joint probability function of X and Y is given by —

$$f(x, y) = \frac{x+y}{21}, \quad x = 1, 2, 3 \text{ and } y = 1, 2.$$

i) Find all the conditional distributions

ii) Find the conditional distribution of X, when $Y = 1$.

iii) Find $P(X \leq 1)$, $P(Y < 3)$, $P(X \leq 2, Y < 3)$,
$P(X \leq 1 / Y \leq 3)$ and $P(X + Y \leq 4)$

→ Solⁿ:

Y \ X	1	2	3	P(Y)
1	$2/21$	$3/21$	$4/21$	$9/21$
2	$3/21$	$4/21$	$5/21$	$12/21$
P(X)	$5/21$	$7/21$	$9/21$	1

X	1	2	3
$P(X=x)$	$5/21$	$7/21$	$9/21$

Y	1	2
$P(Y=y)$	$9/21$	$12/21$

(i) $P(X=1/Y=1) = \dfrac{P(1,1)}{P(Y=1)} = \dfrac{2/21}{9/21} = \dfrac{2}{9}$

$P(X=1/Y=2) = \dfrac{P(1,2)}{P(Y=2)} = \dfrac{3/21}{12/21} = \dfrac{3}{12} = \dfrac{1}{4}$

$P(X=2/Y=1) = \dfrac{P(2,1)}{P(Y=1)} = \dfrac{3/21}{9/21} = \dfrac{3}{9} = \dfrac{1}{3}$

$P(X=2/Y=2) = \dfrac{P(2,2)}{P(Y=2)} = \dfrac{4/21}{12/21} = \dfrac{4}{12} = \dfrac{1}{3}$

$P(X=3/Y=1) = \dfrac{P(3,1)}{P(Y=1)} = \dfrac{4/21}{9/21} = \dfrac{4}{9}$

$P(X=3/Y=2) = \dfrac{P(3,2)}{P(Y=2)} = \dfrac{5/21}{12/21} = \dfrac{5}{12}$

$P(Y=1/X=1) = \dfrac{P(1,1)}{P(X=1)} = \dfrac{2/21}{5/21} = \dfrac{2}{5}$

$P(Y=1/X=2) = \dfrac{P(1,2)}{P(X=2)} = \dfrac{3/21}{7/21} = \dfrac{3}{7}$

$P(Y=1/X=3) = \dfrac{P(1,3)}{P(X=3)} = \dfrac{4/21}{9/21} = \dfrac{4}{9}$

$P(Y=2/X=1) = \dfrac{P(2,1)}{P(X=1)} = \dfrac{3/21}{5/21} = \dfrac{3}{5}$

$P(Y=2/X=2) = \dfrac{P(2,2)}{P(X=2)} = \dfrac{4/21}{7/21} = \dfrac{4}{7}$

$P(Y=2/X=3) = \dfrac{P(2,3)}{P(X=3)} = \dfrac{5/21}{9/21} = \dfrac{5}{9}$

Ans.

(ii) $P(X=1/Y=1) = \dfrac{P(1,1)}{P(Y=1)} = \dfrac{2/21}{9/21} = \dfrac{2}{9}$

$P(X=2/Y=1) = \dfrac{P(2,1)}{P(Y=1)} = \dfrac{3/21}{9/21} = \dfrac{3}{9} = \dfrac{1}{3}$

$P(X=3/Y=1) = \dfrac{P(3,1)}{P(Y=1)} = \dfrac{4/21}{9/21} = \dfrac{4}{9}$.

Ans.

(iii) $P(X \leq 1) = P(X=1) = \dfrac{5}{21}$

$P(Y<3) = P(Y=1) + P(Y=2)$

$= \dfrac{9}{21} + \dfrac{12}{21} = \dfrac{21}{21}$

$= 1.$

$P(X \leq 2, Y<3) = P(X=1, Y=1) + P(X=1, Y=2) + P(X=2, Y=1)$
$+ P(X=2, Y=2)$

$= \dfrac{2}{21} + \dfrac{3}{21} + \dfrac{3}{21} + \dfrac{4}{21}$

$= \dfrac{12}{21}$

$P(X \leq 1/Y \leq 3) = \dfrac{5/21}{1} = \dfrac{5}{21}$

$P(X+Y \leq 4) = P(1,1) + P(1,2) + P(2,1) + P(2,2) + P(3,1)$

$= \dfrac{2}{21} + \dfrac{3}{21} + \dfrac{3}{21} + \dfrac{4}{21} + \dfrac{4}{21}$

$= \dfrac{16}{21}$

Ans.

8.5 No tortilla chip aficionado likes soggy chips, so it is important to find characteristics of the production process that produce chips with an appealing texture. The following data on X = frying time (sec) and Y = moisture content (%) appeared in the article "Thermal and Physical Properties of Tortilla Chips as a Function of Frying Time." (J. of Food Processing and Preservation, 1995: 179–189).

X	5	10	15	20	25	30	45	60
Y	16.3	9.7	8.1	4.2	3.4	2.9	1.9	1.3

Find the correlation coefficient of X and Y.

→ Sol^n:

X	Y	$(X-\bar{X})$	$(Y-\bar{Y})$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
5	16.3	−21.25	10.32	451.56	106.50	−219.3
10	9.7	−16.25	3.72	264.06	13.84	−60.45
15	8.1	−11.25	2.12	126.56	4.49	−23.85
20	4.2	−6.25	−1.78	39.06	3.17	11.13
25	3.4	−1.25	−2.58	1.56	6.66	3.23
30	2.9	3.75	−3.08	14.06	9.49	−11.56
45	1.9	18.75	−4.08	351.56	16.65	−76.5
60	1.3	33.75	−4.68	1139.06	21.90	−157.95
$\Sigma X = 210$	$\Sigma Y = 47.8$	$\Sigma(X-\bar{X}) = 0$	$\Sigma(Y-\bar{Y}) = -0.04$	$\Sigma(X-\bar{X})^2 = 2387.49$	$\Sigma(Y-\bar{Y})^2 = 182.7$	$\Sigma(X-\bar{X})(Y-\bar{Y}) = -535.25$

$$\text{Mean } (\bar{x}) = \frac{\Sigma x}{n} = \frac{210}{8} = 26.25$$

$$\text{Mean } (\bar{y}) = \frac{\Sigma y}{n} = \frac{47.8}{8} = 5.975 \approx 5.98$$

$\therefore$ Correlation Coefficient, $r = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$

$$= \frac{-535.25}{\sqrt{2387.49}\sqrt{182.7}}$$

$$= \frac{-535.25}{48.86 \times 13.52}$$

$$= \frac{-535.25}{660.59}$$

$$= -0.81$$

$\therefore \quad r = -0.81$

Ans