11/10/21

A salesman in a departmental store claims that at most 60 percent of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of the salesman? Use a level of significance of 0.05.

Given:

$P = 0.6$, $p = \frac{35}{50} = 0.7$

$Q = 1 - P = 0.4$, $\alpha = 0.05$

$H_0 : P = 0.6$

$H_a : P \leq 0.6$ [one tailed test]

Test statistic

$$Z = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.7 - 0.6}{\sqrt{\frac{(0.6)(0.4)}{50}}}$$

$$= \frac{0.100}{0.067} = 1.45$$

$\therefore |Z| = 1.45$ [calculated value]

T.V : $Z_{0.05}$ is 1.645

con: $|Z| < Z_{0.05}$ so accept H_0

95% confidence limit for P

$$\hat{p} - Z_{0.05}\sqrt{\frac{pq}{n}} \le P \le \hat{p} + Z_{0.05}\sqrt{\frac{pq}{n}}$$

$$0.7 - (1.645)(0.069) \le P \le 0.7 + (1.645)(0.069)$$

$$\Rightarrow 0.59 \le P \le 0.81$$ ✓

Test 2:

Test of significance for Difference of proportions:

Case(i):

$H_0: P_1 = P_2$

$H_a:$ (i) $P_1 \neq P_2$

(ii) $P_1 > P_2$

(iii) $P_1 < P_2$

To test the significant difference between the sample proportion p1 and p2.

2. Test statistic $z = \dfrac{p_1 - p_2}{\sqrt{pq\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$

where p_1=sample proportion 1;p_2=sample proportion 2; p= total proportion

where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ *and* $q = 1 - p$

Total proportion, $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$, $q = 1 - p$

Case(ii):

To test the significant difference between sample proportion(p_1) and Total proportion(p) where

Test statistic $z = \dfrac{p_1 - p}{\sqrt{\dfrac{n_2 pq}{n_1(n_1+n_2)}}}$ where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ *and* $q = 1 - p$

$H_0: P = P_1$ (or P_2)

H_a: (i) $P \neq P_1(P_2)$

(ii) $P > P_1(P_2)$

(iii) $P < P_1(P_2)$

Case(iii):

If the sample proportions are not known then

Test statistic $z = \dfrac{P_1 - P_2}{\sqrt{\dfrac{P_1 Q_1}{n_1} + \dfrac{P_2 Q_2}{n_2}}}$

$H_0: P_1 = P_2$

H_a: (i) $P_1 \neq P_2$

(ii) $P_1 > P_2$

(iii) $P_1 < P_2$

Where P_1=large population 1; P_2=large population 2

$Q_1 = 1 - P_1$, $Q_2 = 1 - P_2$

Case (i)

1. Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same at 5% level.

Given:

$n_1 = 400$, $p_1 \left[= \frac{x_1}{n_1}\right] = \frac{200}{400} = 0.5$

$n_2 = 600$, $p_2 \left[= \frac{x_2}{n_2}\right] = \frac{325}{600} = 0.54$

Total proportion, $p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{200 + 325}{400 + 600}$

$= 0.522$

$$q = 1 - p$$

$$= 1 - 0.522$$

$$q = 0.478$$

$H_0 : P_1 = P_2$

$H_a : P_1 \neq P_2$ (Two tailed test)

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.5 - 0.54}{\sqrt{0.25\left[\frac{1}{400} + \frac{1}{600}\right]}}$$

$$= \frac{-0.04}{0.032} = -1.234$$

$$|Z| = 1.234$$

T.V: $Z_{0.05}$ is 1.96

Con: $|Z| < Z_{0.05}$ So accept the

The proportions of men and women in favour of ~~the~~ proposal are same at 5% level

95% C.I. for $(p_1 - p_2)$

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{\alpha} \sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

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Case(2)

2. In a random sample of 400 students of the University of Teaching Department, it was found that 300 students failed in the examination. In another sample of 500 students of the affiliated colleges, the number of failures in the same examination was found to be 300. Find out whether the proportion of failures in the university teaching departments significantly greater than the proportion of failures in the university teaching departments and affiliated colleges taken together.

Sol:-

$n_1 = 400$, $p_1 = \frac{300}{400} = 0.75$

$n_2 = 500$ $p_2 = \frac{300}{500} = 0.6$

Total proportion :

$$p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{300 + 300}{400 + 500} = \frac{600}{900}$$

$$= 0.67$$

$$q = 1 - p = 1 - 0.67$$

$$\boxed{q = 0.23}$$

$H_0: P_1 = P$

$H_a: P_1 > P$ (one tailed test)

$$Z = \frac{p_1 - p}{\sqrt{\frac{p_2 p_2}{n_1 (n_1 + n_2)}}} = \frac{0.75 - 0.67}{\sqrt{\frac{(500)(0.224)}{400 \times 900}}}$$

$$= \frac{0.08}{\sqrt{0.0031}} = \frac{0.08}{0.056}$$

$Z = 1.43$

$|Z| = 1.43$

T.V : $Z_{0.05}$ is 1.645

Con: $|Z| < Z_{0.05}$ so accept H_0

i.e The proportion of failures in the university teaching department is same as total proportion.

Case (iii)

3. In two large populations, there are 30% and 25% respectively of fair haired people. Is their difference likely to be hidden in samples of 1200 and 900 respectively from the two populations?

Given :

$P_1 = 0.3$, $P_2 = 0.25$

$Q_1 = 0.7$ $Q_2 = 0.75$

$H_0 : P_1 = P_2$

$H_a : P_1 \neq P_2$ (Two tailed test)

Test statistic

$$Z = \frac{P_1 - P_2}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}}$$

$$= \frac{0.3 - 0.25}{\sqrt{0.000175 + 0.00021}} = \frac{0.05}{\sqrt{0.000385}}$$

$$= \frac{0.05}{0.0196}$$

$$Z = 2.55$$

$$\therefore \quad |Z| = 2.55$$

T.V : $Z_{0.05}$ is 1.96

Con: $|Z| > Z_{0.05}$ so Reject H_0.

Z-Test for Single Mean

Test 3

Test of significance of the difference between sample mean and population mean.

The test statistic $z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}}$.

$H_0 : \mu = \mu_0$

$H_a : \mu \neq \mu_0$

$\mu > \mu_0$

$\mu < \mu_0$

Note *1. If σ is not known, the sample S.D. 's' can be used in its place, as s is nearly equal to σ when n is large.*

2. 95% confidence limits for μ are given by $\dfrac{|\mu - \bar{X}|}{\sigma / \sqrt{n}} \leq 1.96$, *i.e.* $\left(\bar{X} - 1.96 \dfrac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \dfrac{\sigma}{\sqrt{n}}\right)$, *if σ is known. If σ is not known, then the 95% confidence interval is* $\left(\bar{X} - \dfrac{1.96\, s}{\sqrt{n}}, \bar{X} + \dfrac{1.96\, s}{\sqrt{n}}\right)$

$\left[\bar{x} \pm z_\alpha \, \sigma / \sqrt{n} \right]$

1) A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm, and the S.D. is 10 cm?

$n = 100$, $\bar{x} = 160$, $\mu = 165$, $\sigma = 10$

$H_0: \mu = 165$

$H_a: \mu \neq 165$ (Two tailed test)

Test statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{160 - 165}{10/\sqrt{100}}$$

$z = -5$

$|z| = 5$

T.V : $Z_{0.05}$ is 1.96

Con : $|z| > Z_{0.05}$ so Reject H_0.

95% C.I. for μ

$\bar{x} - z_{\alpha} \sigma/\sqrt{n} \leq \mu \leq \bar{x} + z_{\alpha} \sigma/\sqrt{n}$

$$160 - (1.96)(1) \le \mu \le 160 + 1.96$$

$$158.04 \le \mu \le 161.96$$