# Options Pricing and Risk Analysis

*Report submitted by*

*Akash Ringe*
*606079174*
*Masters in Financial Engineering*
*UCLA Anderson School of Management*

## Overview

This project explores the pricing of Asian options and call options, and it further evaluates their potential risk by using a Monte Carlo simulation approach. The pricing and risk analysis is especially useful for derivatives pricing, risk management, and investment decisions. The project is divided into two parts:

1. **Asian Options Pricing**: We use Monte Carlo simulations to evaluate the price of arithmetic and geometric Asian options. We also investigate the difference between these two types of Asian options and how this difference changes with respect to the strike price.
2. **Call Options Pricing and Risk Analysis**: We develop a complex pricing model for an exotic call option on a collateral loan that integrates jump-diffusion processes and credit risk factors. We simulate multiple scenarios to estimate the potential default risk and expected time of option exercise.

## Problem 1 - Pricing Arithemetic and Geometric Asian Options

1. Define the initial parameters:
    - Risk-free rate
    - Strike price
    - Underlying asset's price
    - Underlying asset's volatility
    - Time to maturity
2. Define the Monte Carlo simulation:
    - For each simulation, generate a path of the underlying asset's price using geometric Brownian motion.
    - Calculate the average price for both arithmetic and geometric Asian options using given formula.
3. For each simulation:
    - Calculate the payoff of the arithmetic and geometric Asian options.
    - Discount the payoffs to present value using the risk-free rate.
4. Calculate the average of the discounted payoffs across all simulations to obtain the option prices.

5. Calculaate the difference between the arithmetic Asian option price and the geometric Asian option price for different strike prices and volatilities.

Inputs: r=0.05 sigma=0.25 S0=50 T=1 X=30 m=20 x=2.5

Outputs:

| | Strike | AC price | GC price | AC-GC | A/G price |
|---|---|---|---|---|---|
| **0** | 30.0 | 20.249573 | 20.258587 | -0.009014 | 0.168987 |
| **1** | 32.5 | 17.900042 | 17.981510 | -0.081468 | 0.203831 |
| **2** | 35.0 | 15.515606 | 15.426364 | 0.089241 | 0.054313 |
| **3** | 37.5 | 13.150060 | 13.035956 | 0.114104 | 0.104551 |
| **4** | 40.0 | 10.859687 | 10.727787 | 0.131900 | 0.267773 |
| **5** | 42.5 | 8.578851 | 8.539357 | 0.039494 | 0.090195 |
| **6** | 45.0 | 6.685563 | 6.677025 | 0.008539 | 0.124450 |
| **7** | 47.5 | 4.946273 | 4.783212 | 0.163061 | 0.198847 |
| **8** | 50.0 | 3.420930 | 3.436882 | -0.015952 | 0.113390 |
| **9** | 52.5 | 2.318921 | 2.208071 | 0.110850 | 0.036724 |
| **10** | 55.0 | 1.522637 | 1.494048 | 0.028589 | 0.047266 |
| **11** | 57.5 | 0.947240 | 0.896720 | 0.050521 | 0.227557 |
| **12** | 60.0 | 0.557181 | 0.528670 | 0.028511 | 0.114076 |
| **13** | 62.5 | 0.333998 | 0.303362 | 0.030636 | 0.040430 |
| **14** | 65.0 | 0.192543 | 0.185089 | 0.007454 | 0.219648 |
| **15** | 67.5 | 0.096478 | 0.086636 | 0.009842 | 0.078890 |
| **16** | 70.0 | 0.058865 | 0.040008 | 0.018857 | 0.001769 |
| **17** | 72.5 | 0.036719 | 0.030675 | 0.006045 | 0.103288 |
| **18** | 75.0 | 0.015564 | 0.005983 | 0.009581 | -0.071930 |
| **19** | 77.5 | 0.006638 | 0.005554 | 0.001083 | 0.200854 |

# Problem 2

1. Define the initial parameters:
   - Risk-free rate
   - Strike price
   - Collateral value and its volatility
   - Credit quality of the borrower
   - Parameters for the jump-diffusion process (lambda1 and lambda2)
2. Define the Monte Carlo simulation:
   - For each simulation, generate a path of the collateral value using a jump-diffusion model.
3. For each simulation:
   - Evaluate the borrower's default probability.
   - Calculate the payoff of the call option considering the default risk.
   - Discount the payoff to present value using the risk-free rate.
4. Calculate the average of the discounted payoffs across all simulations to obtain the price of the call option.
5. Output the default probability and expected exercise time of the call option for different lambda1, lambda2, and maturity parameters

Inputs: V0 = 20000 L0 = 22000 mu = -0.1 sigma = 0.2 gamma = -0.4 lambda1 = 0.2
T = 5 r0 = 0.02 delta = 0.25 lambda2 = 0.4 alpha = 0.7 recovery = 0.95

Outputs:

**For** $\lambda_1 = 0.2 \quad \lambda_2 = 0.4 \quad T = 5$

```
option, prob, exercise_time = calc_option_value(0.
print(f"Option Value: {option}")
print(f"Probablity of default: {prob}")
print(f"Expected Option Exercise time: {exercise_t
```

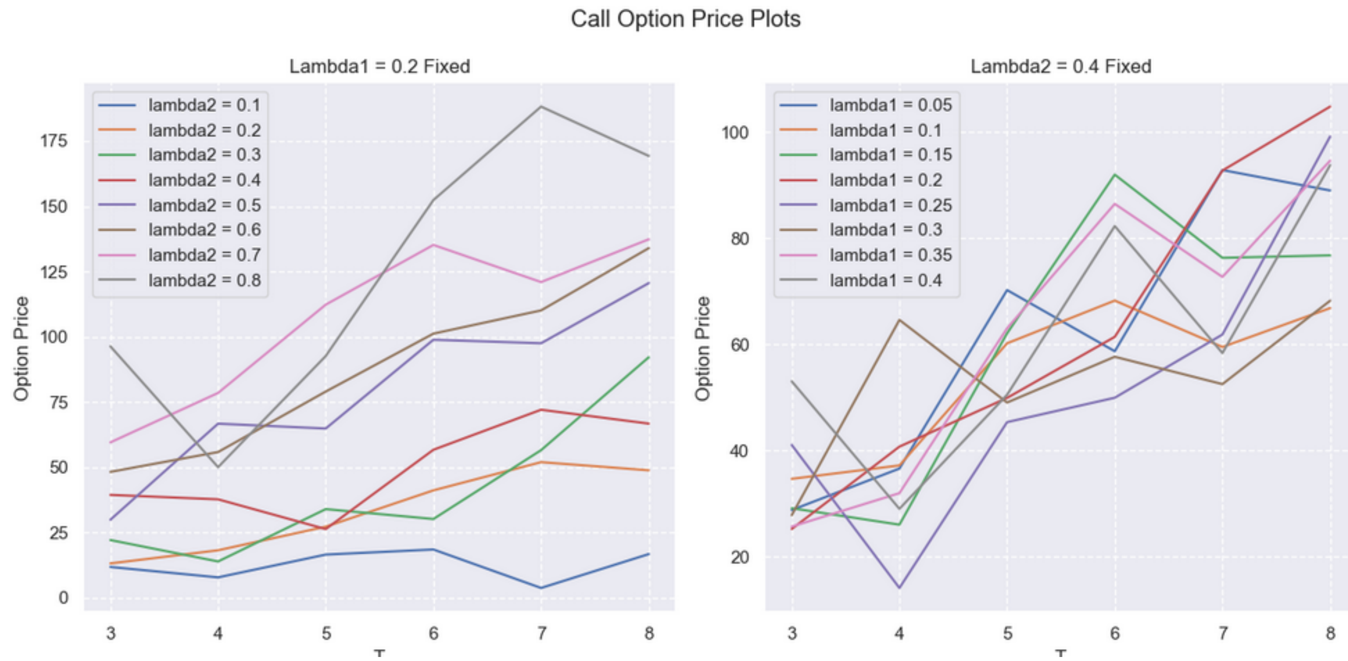
```
Option Value: 54.75903441031755
Probablity of default: 0.01559
Expected Option Exercise time: 2.499679281590763
```

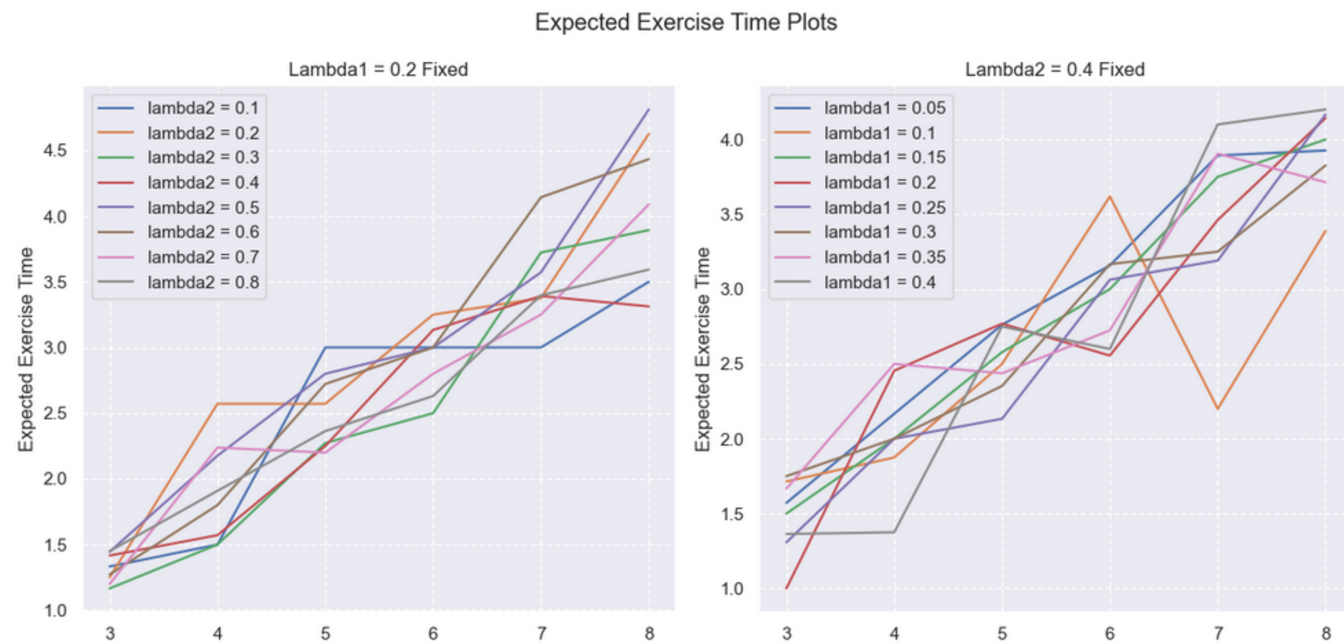
For lambda1 = 0.2 fixed

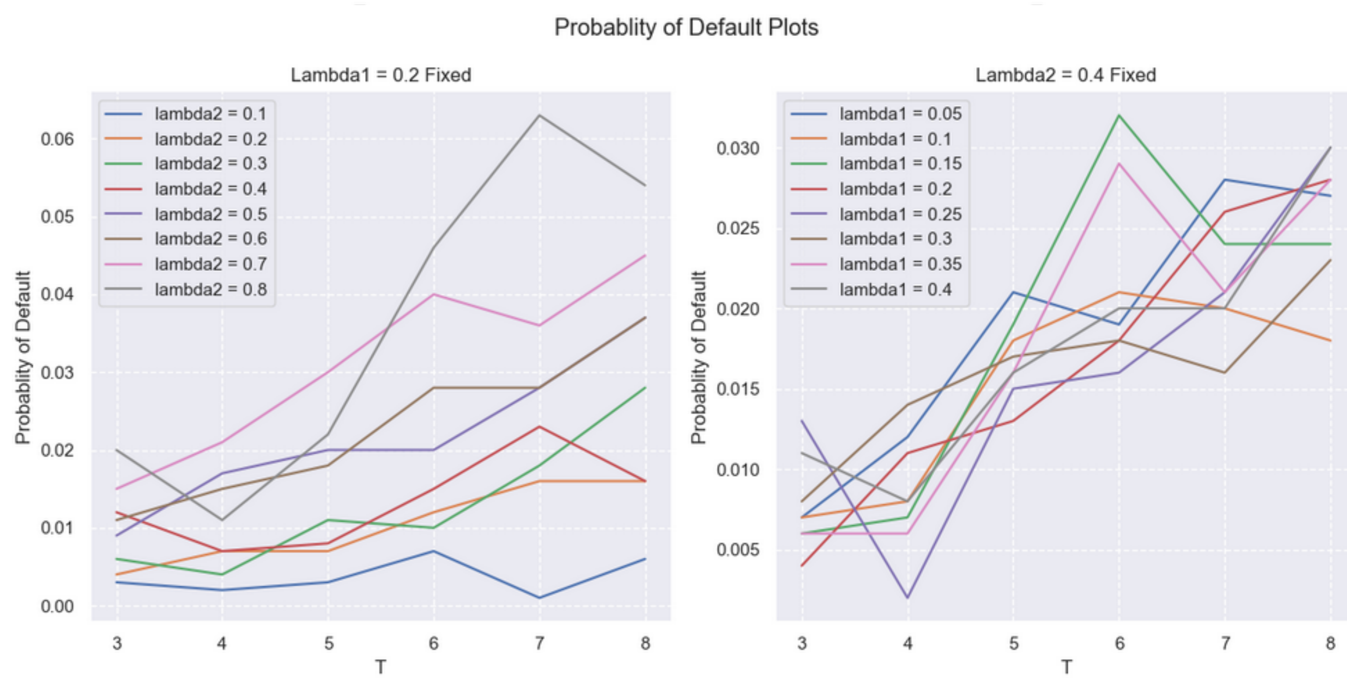
| | Lambda2 | T | Option Value | Probablity of Default | Expected Exercise Time |
|---|---|---|---|---|---|
| 0 | 0.1 | 3 | 11.739792 | 0.003 | 1.333333 |
| 1 | 0.2 | 3 | 13.094392 | 0.004 | 1.250000 |
| 2 | 0.3 | 3 | 22.054972 | 0.006 | 1.166667 |
| 3 | 0.4 | 3 | 39.358081 | 0.012 | 1.416667 |
| 4 | 0.5 | 3 | 29.840262 | 0.009 | 1.444444 |
| 5 | 0.6 | 3 | 48.179344 | 0.011 | 1.272727 |
| 6 | 0.7 | 3 | 59.488783 | 0.015 | 1.200000 |
| 7 | 0.8 | 3 | 96.300708 | 0.020 | 1.450000 |
| 8 | 0.1 | 4 | 7.748929 | 0.002 | 1.500000 |
| 9 | 0.2 | 4 | 18.128613 | 0.007 | 2.571429 |
| 10 | 0.3 | 4 | 13.847434 | 0.004 | 1.500000 |
| 11 | 0.4 | 4 | 37.663423 | 0.007 | 1.571429 |
| 12 | 0.5 | 4 | 66.660106 | 0.017 | 2.176471 |
| 13 | 0.6 | 4 | 55.777485 | 0.015 | 1.800000 |
| 14 | 0.7 | 4 | 78.431245 | 0.021 | 2.238095 |
| 15 | 0.8 | 4 | 49.989940 | 0.011 | 1.909091 |
| 16 | 0.1 | 5 | 16.481959 | 0.003 | 3.000000 |
| 17 | 0.2 | 5 | 27.177351 | 0.007 | 2.571429 |
| 18 | 0.3 | 5 | 33.907688 | 0.011 | 2.272727 |
| 19 | 0.4 | 5 | 26.255069 | 0.008 | 2.250000 |
| 20 | 0.5 | 5 | 64.794897 | 0.020 | 2.800000 |
| 21 | 0.6 | 5 | 78.966046 | 0.018 | 2.722222 |
| 22 | 0.7 | 5 | 112.276142 | 0.030 | 2.200000 |
| 23 | 0.8 | 5 | 92.580932 | 0.022 | 2.363636 |
| 24 | 0.1 | 6 | 18.432511 | 0.007 | 3.000000 |
| 25 | 0.2 | 6 | 41.096370 | 0.012 | 3.250000 |
| 26 | 0.3 | 6 | 30.120747 | 0.010 | 2.500000 |
| 27 | 0.4 | 6 | 56.657636 | 0.015 | 3.133333 |
| 28 | 0.5 | 6 | 98.825555 | 0.020 | 3.000000 |
| 29 | 0.6 | 6 | 101.163085 | 0.028 | 3.000000 |
| 30 | 0.7 | 6 | 135.158721 | 0.040 | 2.800000 |
| 31 | 0.8 | 6 | 152.356229 | 0.046 | 2.630435 |
| 32 | 0.1 | 7 | 3.686904 | 0.001 | 3.000000 |
| 33 | 0.2 | 7 | 51.911621 | 0.016 | 3.375000 |
| 34 | 0.3 | 7 | 56.508141 | 0.018 | 3.722222 |
| 35 | 0.4 | 7 | 71.996864 | 0.023 | 3.391304 |
| 36 | 0.5 | 7 | 97.473255 | 0.028 | 3.571429 |
| 37 | 0.6 | 7 | 110.061799 | 0.028 | 4.142857 |
| 38 | 0.7 | 7 | 120.926814 | 0.036 | 3.250000 |
| 39 | 0.8 | 7 | 188.172079 | 0.063 | 3.396825 |
| 40 | 0.1 | 8 | 16.672064 | 0.006 | 3.500000 |
| 41 | 0.2 | 8 | 48.788242 | 0.016 | 4.625000 |
| 42 | 0.3 | 8 | 92.112872 | 0.028 | 3.892857 |
| 43 | 0.4 | 8 | 66.684974 | 0.016 | 3.312500 |
| 44 | 0.5 | 8 | 120.531487 | 0.037 | 4.810811 |
| 45 | 0.6 | 8 | 133.935068 | 0.037 | 4.432432 |
| 46 | 0.7 | 8 | 137.315426 | 0.045 | 4.088889 |
| 47 | 0.8 | 8 | 169.257770 | 0.054 | 3.592593 |

For lambda2 = 0.4 fixed

| | Lambda1 | T | Option Value | Probablity of Default | Expected Exercise Time |
|---|---|---|---|---|---|
| 0 | 0.05 | 3 | 28.786654 | 0.007 | 1.571429 |
| 1 | 0.10 | 3 | 34.662412 | 0.007 | 1.714286 |
| 2 | 0.15 | 3 | 29.129425 | 0.006 | 1.500000 |
| 3 | 0.20 | 3 | 25.217242 | 0.004 | 1.000000 |
| 4 | 0.25 | 3 | 41.044778 | 0.013 | 1.307692 |
| 5 | 0.30 | 3 | 27.816774 | 0.008 | 1.750000 |
| 6 | 0.35 | 3 | 25.674848 | 0.006 | 1.666667 |
| 7 | 0.40 | 3 | 53.031500 | 0.011 | 1.363636 |
| 8 | 0.05 | 4 | 36.598873 | 0.012 | 2.166667 |
| 9 | 0.10 | 4 | 37.185353 | 0.008 | 1.875000 |
| 10 | 0.15 | 4 | 26.031747 | 0.007 | 2.000000 |
| 11 | 0.20 | 4 | 40.725825 | 0.011 | 2.454545 |
| 12 | 0.25 | 4 | 14.111335 | 0.002 | 2.000000 |
| 13 | 0.30 | 4 | 64.589823 | 0.014 | 2.000000 |
| 14 | 0.35 | 4 | 31.968979 | 0.006 | 2.500000 |
| 15 | 0.40 | 4 | 29.017663 | 0.008 | 1.375000 |
| 16 | 0.05 | 5 | 70.210347 | 0.021 | 2.761905 |
| 17 | 0.10 | 5 | 60.187346 | 0.018 | 2.500000 |
| 18 | 0.15 | 5 | 62.072928 | 0.019 | 2.578947 |
| 19 | 0.20 | 5 | 49.903920 | 0.013 | 2.769231 |
| 20 | 0.25 | 5 | 45.339486 | 0.015 | 2.133333 |
| 21 | 0.30 | 5 | 49.033683 | 0.017 | 2.352941 |
| 22 | 0.35 | 5 | 44.941039 | 0.014 | 2.357143 |
| 23 | 0.40 | 5 | 58.575167 | 0.016 | 2.562500 |
| 24 | 0.05 | 6 | 49.678262 | 0.014 | 3.071429 |
| 25 | 0.10 | 6 | 83.949449 | 0.023 | 2.956522 |
| 26 | 0.15 | 6 | 64.117591 | 0.018 | 2.666667 |
| 27 | 0.20 | 6 | 72.116908 | 0.019 | 2.684211 |
| 28 | 0.25 | 6 | 53.509886 | 0.018 | 2.777778 |
| 29 | 0.30 | 6 | 62.500184 | 0.015 | 3.466667 |
| 30 | 0.35 | 6 | 71.896442 | 0.020 | 2.600000 |
| 31 | 0.40 | 6 | 81.552875 | 0.025 | 3.200000 |
| 32 | 0.05 | 7 | 72.322983 | 0.020 | 3.800000 |
| 33 | 0.10 | 7 | 91.980152 | 0.027 | 3.777778 |
| 34 | 0.15 | 7 | 63.148526 | 0.021 | 3.809524 |
| 35 | 0.20 | 7 | 86.243374 | 0.024 | 3.458333 |
| 36 | 0.25 | 7 | 91.223579 | 0.028 | 3.357143 |
| 37 | 0.30 | 7 | 72.906679 | 0.022 | 3.636364 |
| 38 | 0.35 | 7 | 78.284250 | 0.024 | 3.333333 |
| 39 | 0.40 | 7 | 65.399968 | 0.020 | 3.250000 |
| 40 | 0.05 | 8 | 72.805444 | 0.027 | 4.222222 |
| 41 | 0.10 | 8 | 67.511130 | 0.021 | 3.761905 |
| 42 | 0.15 | 8 | 74.458553 | 0.024 | 3.666667 |
| 43 | 0.20 | 8 | 57.819989 | 0.018 | 4.166667 |
| 44 | 0.25 | 8 | 119.402180 | 0.035 | 3.771429 |
| 45 | 0.30 | 8 | 78.171651 | 0.025 | 4.360000 |
| 46 | 0.35 | 8 | 76.808876 | 0.027 | 4.074074 |
| 47 | 0.40 | 8 | 90.965563 | 0.026 | 4.153846 |


Call Option Price Plots
Lambda1 = 0.2 Fixed
lambda2 = 0.1
lambda2 = 0.2
lambda2 = 0.3
lambda2 = 0.4
lambda2 = 0.5
lambda2 = 0.6
lambda2 = 0.7
lambda2 = 0.8
Option Price
0
25
50
75
100
125
150
175
3
4
5
6
7
8
Lambda2 = 0.4 Fixed
lambda1 = 0.05
lambda1 = 0.1
lambda1 = 0.15
lambda1 = 0.2
lambda1 = 0.25
lambda1 = 0.3
lambda1 = 0.35
lambda1 = 0.4
Option Price
20
40
60
80
100
3
4
5
6
7
8


Expected Exercise Time Plots
Lambda1 = 0.2 Fixed
lambda2 = 0.1
lambda2 = 0.2
lambda2 = 0.3
lambda2 = 0.4
lambda2 = 0.5
lambda2 = 0.6
lambda2 = 0.7
lambda2 = 0.8
Expected Exercise Time
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
3
4
5
6
7
8
Lambda2 = 0.4 Fixed
lambda1 = 0.05
lambda1 = 0.1
lambda1 = 0.15
lambda1 = 0.2
lambda1 = 0.25
lambda1 = 0.3
lambda1 = 0.35
lambda1 = 0.4
Expected Exercise Time
1.0
1.5
2.0
2.5
3.0
3.5
4.0
3
4
5
6
7
8


Probablity of Default Plots
Lambda1 = 0.2 Fixed
lambda2 = 0.1
lambda2 = 0.2
lambda2 = 0.3
lambda2 = 0.4
lambda2 = 0.5
lambda2 = 0.6
lambda2 = 0.7
lambda2 = 0.8
Probablity of Default
0.00
0.01
0.02
0.03
0.04
0.05
0.06
3
4
5
6
7
8
T
Lambda2 = 0.4 Fixed
lambda1 = 0.05
lambda1 = 0.1
lambda1 = 0.15
lambda1 = 0.2
lambda1 = 0.25
lambda1 = 0.3
lambda1 = 0.35
lambda1 = 0.4
Probablity of Default
0.005
0.010
0.015
0.020
0.025
0.030
3
4
5
6
7
8
T