Computational modeling of interest rates and bond derivatives

Report submitted by

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Question 1

In this question we performs calculation related to the pricing of bonds and options using the Vasicek interest rate model. Following are some important utility functions.

1. `vasicek_model`: This function takes in parameters T (time to maturity), dt (time increment), r0 (initial short rate), sigma (volatility of interest rates), k (speed of reversion to the mean), and r_mean (mean interest rate). The function then performs a Monte Carlo simulation based on the Vasicek model to compute the path of interest rates over the given time to maturity.

2. `bond_price_calc`: This function calculates the price of a bond at time t, given the path of interest rates, rt, the coupons, ct, and the face value of the bond. It does so by discounting the future cash flows (coupons and face value) to the present using the simulated path of interest rates.

3. `call_option_calc`: This function calculates the price of a European call option on a bond, given the path of interest rates, the strike price, and the bond's face value. The option's maturity date is given by T.

The calculations in this problem provide a good illustration of how the Vasicek model can be used in the pricing of bonds and options. The Monte Carlo simulation technique used here allows us to account for the stochastic nature of interest rates. It should be noted that in practice, the parameters of the Vasicek model would be estimated from market data.

Input/Output:

(a) ZC Bond $r0$=5% σ=10% κ=0.82 r_{mean} =0.05 $Face$=1000 T=0.5$years$
975.8895476571282
(b) Semi Annual Coupon Bond $r0$=5% σ=10% κ=0.82 r_{mean} =0.05 $Face$=1000 T=0.5$years$ C=30
1039.5504319073561
(c) Call option on ZC bond with K=980 $Expiry$=0.25
8.595094530228053
(d) Call option on Semi annual coupon bond with K=980 $Expiry$=0.25
94.97931619051538
(e) Call option on Semi annual coupon bond with K=980 $Expiry$=0.25 using explicit formula
97.35273129679005

Question 2

Following is the step-by-step description of the implementation.

1. CIR Model Function (cir_model): The function defines the stochastic process using the CIR model parameters (mean reversion rate `k`, long-term mean `r_mean`, initial rate `r0`, and standard deviation `sigma`). A Monte Carlo simulation is used to generate the path of interest rates over time `T` with time step `dt`.

2. Simulation of Bond Prices and Call Option Prices: The function call_option_calc (previously defined in the Vasicek model algorithm) is used to calculate call option prices on bonds for a series of Monte Carlo simulations. For each simulation, the CIR model is used to generate an interest rate path. The call option price is calculated using the interest rate path and averaged over all simulations to obtain the expected call option price.

3. Finite Difference Method for Call Option Pricing: A finite difference method is implemented to solve the partial differential equation for the call option price, taking into account the mean-reversion property of the interest rate. A grid is set up with the rate range from `r_min` to `r_max` and the time range from 0 to `T`. The terminal condition for the option price is set, and the tri-diagonal system of equations is solved iteratively backward in time. The boundary conditions are also included in the system.

4. Analytical Call Option Pricing using CIR Model: An analytical solution for the price of a call option under the CIR model is used to calculate the price of the call option. The bond price is calculated using the analytical bond pricing formula under the CIR model, and the call option price is then calculated as the discounted payoff. The prices are averaged over a number of Monte Carlo simulations to obtain the expected option price.

Inputs/Output:

(a) ZC Bond Call Option $r0$=5% σ=12% κ=0.92 $\bar{r}$=0.055 $Face$=1000 S=1yr K=980 T=0.5yr
0.3780133430746902

(b) CIR Call using IFD $r0$=5% σ=12% κ=0.92 $\bar{r}$=0.055 $Face$=1000 S=1yr K=980 T=0.5yr
0.4120747264274198

(c) Calculating Call option price using explicit formula for CIR bonds
0.44109590009085803

Question 3

1. G2++ Model Function (g2_model): The function simulates the G2++ two-factor interest rate model, which is an extension of the Hull-White model. It generates a path of interest rates over time `T` with time step `dt` based on two correlated Brownian motions. `a`, `b`, `sigma`, `eta` and `rho` are model parameters, `phi` is a constant shift, and `r0` is the initial short rate.

2. Monte Carlo Simulation of Put Option Prices: The code then performs a Monte Carlo simulation to compute the prices of European put options. For each simulation, the G2++ model is used to generate a path of short rates. The discount factor at times `T` and `S` and the value of the put option at time `T` are calculated based on the generated short rate path. Both standard and antithetic variates are used in the simulation to increase accuracy. The option prices are averaged over all simulations to obtain the estimated put option price.

3. Explicit Formula for Put Option Price: An explicit formula for the price of a European put option under the G2++ model is then used to compute the put option price. This formula involves the standard Black-Scholes-Merton formula with a volatility that depends on the G2++ model parameters and the bond prices at times `T` and `S`.

Inputs/Outputs:
T = 0.5 # Put option expiration
S = 1.0 # Bond maturity
r0 = 0.03 # Initial interest rate
phi = 0.03 # Constant term
a = 0.1
b = 0.3
sigma = 0.03
eta = 0.08
rho = 0.7
K = 950 # Strike price
F = 1000 # Face value
num_simulations = 10000

European Put Option price (Monte Carlo): 1.9253005671751882
European Put Option price (Explicit Formula): 1.9054888399475942