Call Option Pricing using Binomial/Trinomial Models and Halton Sequences

Report submitted by

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Introduction

This project is focused on pricing European call options and American call options using different binomial and trinomial models and calculating their sensitivities (Greeks). The project is divided into three main parts:

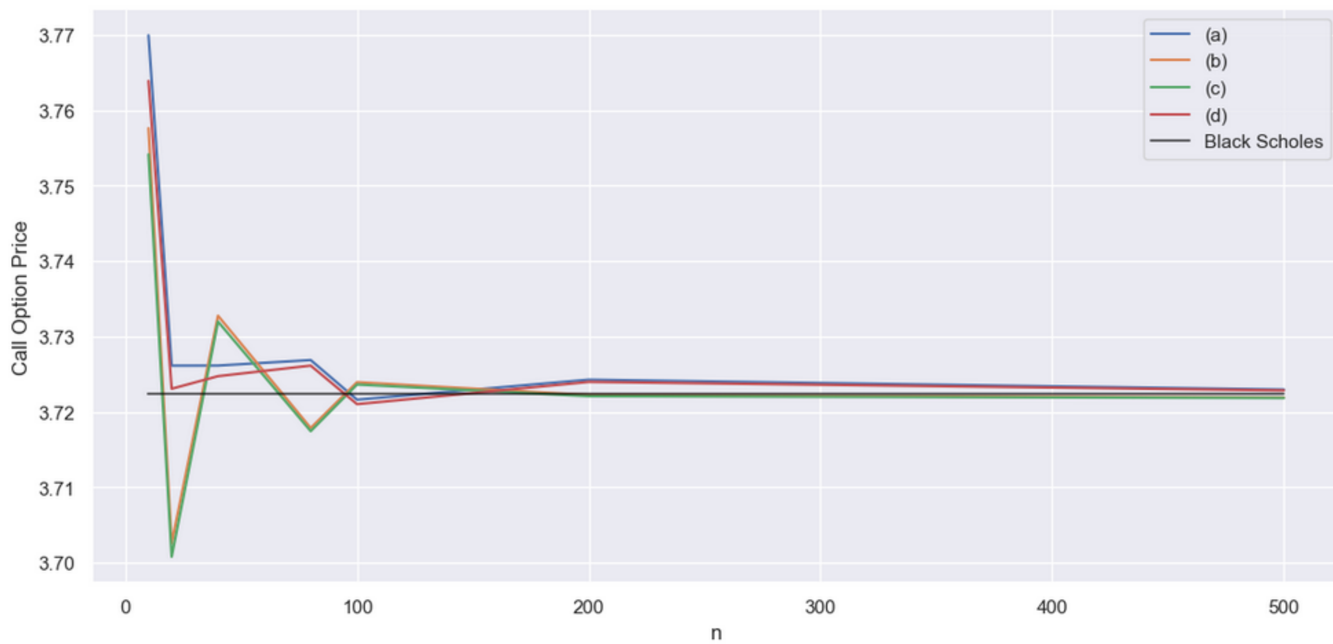
1. Pricing European call options using four different binomial models and comparing the results with the Black-Scholes model.
2. Calculating the sensitivities (Greeks) of American call options using one of the binomial models.
3. Pricing European call options using a trinomial model.
4. Stochastic process estimate using halton sequences and monte carlo simulation of call option.

1 Binomial Model

In this project, we implemented four different parameter choices for a binomial model for pricing European options. The binomial model calculates the price of an option by building a binomial tree with up and down states and probabilities. We compared the option prices computed by the binomial model with the Black-Scholes model, which is an analytical solution for European option pricing.

- Parameter choice (a) uses a non-recombining tree.
- Parameter choice (b) uses an additive model based on stock price growth and the interest rate.
- Parameter choice (c) uses a multiplicative model based on the geometric Brownian motion assumption.
- Parameter choice (d) uses a multiplicative model with an approximation for small time steps.

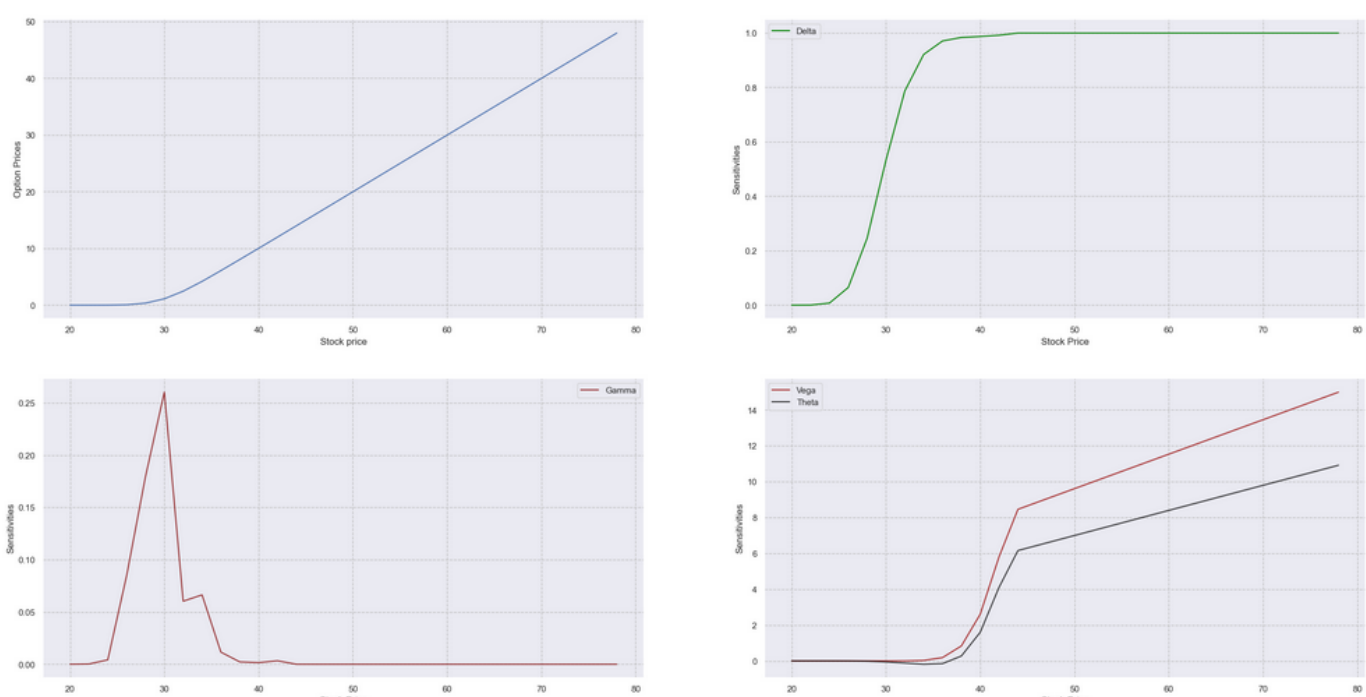
For each parameter choice, we computed the option prices for various values of the number of time steps, n, and compared the results with the Black-Scholes model. The results showed that all four parameter choices converge to the Black-Scholes price as the number of time steps increases.



2 Sensitivities of American Options

We then extended the binomial model to price American options by allowing for early exercise of the option. We also calculated the option sensitivities, or "Greeks", using finite differencing. The Greeks include Delta, Vega, Gamma, and Theta, which represent the sensitivity of the option price to changes in the underlying stock price, volatility, and time.

We computed the option prices and sensitivities for a range of stock prices and plotted the results. The plots show that Delta and Vega tend to be highest when the stock price is near the strike price, while Gamma and Theta are more sensitive to changes in the stock price when the option is out-of-the-money or in-the-money.

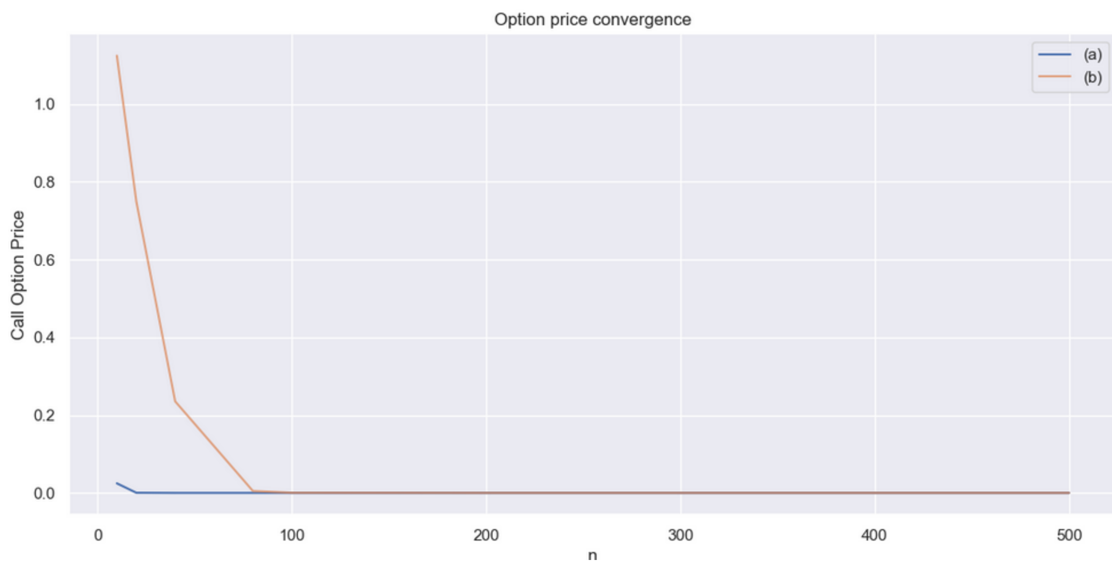


3 Trinomial Model

We implemented a trinomial model for pricing European options, which is an extension of the binomial model that includes a middle state in addition to the up and down states. We considered three different parameter choices for the trinomial model and compared the option prices with the Black-Scholes model.

- Parameter choice (a) uses a model based on the geometric Brownian motion assumption.
- Parameter choice (b) uses an model based on log stock price growth and the interest rate.

The results showed that convergence of the option prices for the log based growth model is not as sharp as the one with GBM assumption. Both the call option prices converge to 0 for given option input parameters. The trinomial model provides an alternative to the binomial model for pricing European options and may offer more accurate results for certain situations.



4 Halton's Low-Discrepancy Sequences Call Option Pricing Model

We have implemented the Halton's Low-Discrepancy Sequences Call Option Pricing model. The purpose of this model is to improve the accuracy of option pricing calculations by using low-discrepancy sequences generated by Halton's algorithm. Low-discrepancy sequences are more uniformly distributed across the problem domain compared to traditional random number generation techniques, which can lead to more accurate option pricing.

First, we defined the function, which generates an N-dimensional Halton sequence with the given bases. This function is used to generate the low-discrepancy sequences needed for our option pricing calculations. Next, we took user inputs for Halton bases b1 and b2, the number of points N, and the necessary parameters for the Black-Scholes model: initial stock price S0, option strike X, time to maturity T, interest rate r, and volatility sigma.We then generated the Halton sequence using the provided bases and N.

Using the Box-Muller transform, we converted the Halton sequence into normally distributed random variables z1 and z2. These variables were then used to simulate the stock prices at maturity, ST1 and ST2. With the simulated stock prices, we calculated the corresponding call option prices, C1 and C2, by considering the difference between the stock price at maturity and the option strike price, discounted by the interest rate over the time to maturity.

Finally, we calculated the average call option price by taking the mean of the call option prices for both sets of simulated stock prices, and presented the result to the user.

```
Enter Halton Base b1: 3
Enter Halton Base b2: 7
Enter Number of points N: 10000
Enter initial stock price S0: 32
Enter option strike X: 30
Enter time to maturity T: 0.5
Enter interest rate r: 0.05
Enter volatility : 0.24
```

```
Call Option price: 3.726747296389288
```

Conclusion

In this project, we implemented binomial and trinomial models for pricing European and American options and calculated option sensitivities. The results demonstrated the convergence of the models to the Black-Scholes price as the number of time steps increases and provided insights into the behavior of option sensitivities. These models can be useful tools for option pricing and risk management in financial markets.