In [1]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import datetime
import importlib
import statsmodels.api as sm
import seaborn as sns; sns.set_theme(color_codes=True)
```

In [2]:
```python
data = pd.read_csv('funds_price_history_stacked.csv')
data.drop(columns='Unnamed: 4', inplace=True)
data = data.rename(columns={'Value':'Price'})
data['Date'] = pd.to_datetime(data['Date'])
data
```

Out[2]:

	MStarID	FundName	Date	Price
0	F0000113ES	Morningstar US 1-5Y Tsy&Gv Bd TR USD	2022-01-02	207.53400
1	F0000113ES	Morningstar US 1-5Y Tsy&Gv Bd TR USD	2022-01-01	207.53400
2	F0000113ES	Morningstar US 1-5Y Tsy&Gv Bd TR USD	2021-12-31	207.53400
3	F0000113ES	Morningstar US 1-5Y Tsy&Gv Bd TR USD	2021-12-30	207.47379
4	F0000113ES	Morningstar US 1-5Y Tsy&Gv Bd TR USD	2021-12-29	207.38144
...	...	...	...	...
43212	F00000O00R	iShares Short Maturity Bond ETF	2013-09-29	50.01000
43213	F00000O00R	iShares Short Maturity Bond ETF	2013-09-28	50.01000
43214	F00000O00R	iShares Short Maturity Bond ETF	2013-09-27	50.01000
43215	F00000O00R	iShares Short Maturity Bond ETF	2013-09-26	50.00000
43216	F00000O00R	iShares Short Maturity Bond ETF	2013-09-25	50.00000

43217 rows × 4 columns

1. Finding the correct benchmark

Using correlation analysis of monthly data

In [3]:
```python
def extract_daily(df):
    df = df.sort_values(by='Date')
    df['Returns'] = (df['Price'] - df['Price'].shift(1)) / df['Price'].shift(1)
    return df

def extract_monthly(df):
    df = df.sort_values(by='Date')
    df['Returns'] = (df['Price'] - df['Price'].shift(1)) / df['Price'].shift(1)
    df['Returns'] = df['Returns'].fillna(0) + 1
    df.set_index('Date', inplace=True)
    df = df.resample('M').apply({
        'MStarID': 'last',
        'FundName': 'last',
        'Price': 'last',
        'Returns': 'prod'
    })
    df['Returns'] = df['Returns'] - 1
    df.reset_index(inplace=True)
```

```
    return df

def extract_weekly(df):
    df = df.sort_values(by='Date')
    df['Returns'] = (df['Price'] - df['Price'].shift(1)) / df['Price'].shift(1)
    df['Returns'] = df['Returns'].fillna(0) + 1
    df.set_index('Date', inplace=True)
    df = df.resample('W-MON').apply({
        'MStarID': 'last',
        'FundName': 'last',
        'Price': 'last',
        'Returns': 'prod'
    })
    df['Returns'] = df['Returns'] - 1
    df.reset_index(inplace=True)
    return df
```

In [4]:

```
groups = data.groupby('MStarID')
etf = groups.get_group('F00000O00R').copy()
etf = extract_monthly(etf)
etf.rename(columns={'Price': 'ETF_Price', 'Returns': 'ETF_Returns'}, inplace=True)
etf.drop(columns=['FundName', 'MStarID'], inplace=True)
etf
```

Out[4]:

	Date	ETF_Price	ETF_Returns
0	2013-09-30	50.01000	0.000200
1	2013-10-31	50.07000	0.001200
2	2013-11-30	50.14456	0.001489
3	2013-12-31	50.16370	0.000382
4	2014-01-31	50.23380	0.001397
...	...	...	...
96	2021-09-30	56.22083	0.000169
97	2021-10-31	56.17577	-0.000801
98	2021-11-30	56.14134	-0.000613
99	2021-12-31	56.13732	-0.000072
100	2022-01-31	56.13732	0.000000

101 rows × 3 columns

In [5]:

```
for fund_id, group in groups:
    print(fund_id, end= ' ')
    monthly_data = pd.merge(etf, extract_monthly(group), on=['Date'], how='inner')
    print(f"Price Correlation: {monthly_data['Price'].corr(monthly_data['ETF_Price'])}",
    print(f"Return Correlation: {monthly_data['Returns'].corr(monthly_data['ETF_Returns'
```

```
F00000O00R Price Correlation: 1.0 Return Correlation: 0.9999999999999998
F0000113ES Price Correlation: 0.9519169859418954 Return Correlation: -0.2020403310791361
F0000113ET Price Correlation: 0.9861859672467136 Return Correlation: 0.8449379391269307
F0000113EU Price Correlation: 0.9700671701072754 Return Correlation: 0.18202294213057907
F0000113F2 Price Correlation: 0.974618861421109 Return Correlation: -0.06956009323569091
F0000113FW Price Correlation: 0.9749374215983598 Return Correlation: 0.18157153907109352
```

Both, the highest price correlation and highest returns correlation of the ETF is for F0000113ET

Running regression on daily prices

In [6]:
```python
groups = data.groupby('MStarID')
etf = groups.get_group('F00000O00R').copy()
etf.drop(columns=['MStarID', 'FundName'], inplace=True)
etf.rename(columns={'Price': 'ETF_Price'}, inplace=True)
etf = etf.sort_values(by='Date')
etf
```

Out[6]:

	Date	ETF_Price
43216	2013-09-25	50.00000
43215	2013-09-26	50.00000
43214	2013-09-27	50.01000
43213	2013-09-28	50.01000
43212	2013-09-29	50.01000
...	...	...
40199	2021-12-29	56.12608
40198	2021-12-30	56.13732
40197	2021-12-31	56.13732
40196	2022-01-01	56.13732
40195	2022-01-02	56.13732

3022 rows × 2 columns

In [7]:
```python
groups = data.groupby('MStarID')
etf = groups.get_group('F00000O00R').sort_values(by='Date')
etf.drop(columns=['MStarID', 'FundName'], inplace=True)
etf.rename(columns={'Price': 'ETF_Price'}, inplace=True)
for fund_id, group in groups:
    print("\n\n")
    print(f'{fund_id} {group["FundName"].values[0]}')
    df = pd.merge(group.sort_values(by='Date'), etf, on='Date', how='inner')
    X = sm.add_constant(df['Price'])
    Y = df['ETF_Price']
    model = sm.OLS(Y, X).fit()
    print(model.summary())
```

```
F00000O00R iShares Short Maturity Bond ETF
                            OLS Regression Results
==============================================================================
Dep. Variable:              ETF_Price   R-squared:                       1.000
Model:                            OLS   Adj. R-squared:                  1.000
Method:                 Least Squares   F-statistic:                 1.293e+29
Date:                Fri, 17 Nov 2023   Prob (F-statistic):               0.00
Time:                        20:46:34   Log-Likelihood:                 82696.
No. Observations:                3022   AIC:                        -1.654e+05
Df Residuals:                    3020   BIC:                        -1.654e+05
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       5.065e-13   1.47e-13      3.447      0.001    2.18e-13    7.95e-13
Price          1.0000   2.78e-15    3.6e+14      0.000       1.000       1.000
==============================================================================
```

```
Omnibus:                     7577.590   Durbin-Watson:                   0.000
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              285.263
Skew:                           0.388   Prob(JB):                     1.14e-62
Kurtosis:                       1.710   Cond. No.                     1.35e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 1.35e+03. This might indicate that there are
strong multicollinearity or other numerical problems.



F0000113ES Morningstar US 1-5Y Tsy&Gv Bd TR USD
                            OLS Regression Results
==============================================================================
Dep. Variable:              ETF_Price   R-squared:                       0.902
Model:                            OLS   Adj. R-squared:                  0.902
Method:                 Least Squares   F-statistic:                 2.784e+04
Date:                Fri, 17 Nov 2023   Prob (F-statistic):               0.00
Time:                        20:46:34   Log-Likelihood:                -2970.0
No. Observations:                3022   AIC:                             5944.
Df Residuals:                    3020   BIC:                             5956.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         11.9037      0.245     48.500      0.000      11.422      12.385
Price          0.2111      0.001    166.839      0.000       0.209       0.214
==============================================================================
Omnibus:                       79.008   Durbin-Watson:                   0.003
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              165.695
Skew:                          -0.137   Prob(JB):                     1.05e-36
Kurtosis:                       4.114   Cond. No.                     4.05e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 4.05e+03. This might indicate that there are
strong multicollinearity or other numerical problems.



F0000113ET Morningstar US 1-5Y Corp Bd TR USD
                            OLS Regression Results
==============================================================================
Dep. Variable:              ETF_Price   R-squared:                       0.971
Model:                            OLS   Adj. R-squared:                  0.971
Method:                 Least Squares   F-statistic:                 1.018e+05
Date:                Fri, 17 Nov 2023   Prob (F-statistic):               0.00
Time:                        20:46:34   Log-Likelihood:                -1122.4
No. Observations:                3022   AIC:                             2249.
Df Residuals:                    3020   BIC:                             2261.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         24.6837      0.088    279.298      0.000      24.510      24.857
Price          0.1214      0.000    319.031      0.000       0.121       0.122
==============================================================================
Omnibus:                      311.932   Durbin-Watson:                   0.004
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              359.628
```

```
Skew:                           0.809   Prob(JB):                     8.09e-79
Kurtosis:                       2.512   Cond. No.                     3.21e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 3.21e+03. This might indicate that there are
strong multicollinearity or other numerical problems.



F0000113EU Morningstar US 1-5Y Core Bd TR USD
                            OLS Regression Results
==============================================================================
Dep. Variable:              ETF_Price   R-squared:                       0.939
Model:                            OLS   Adj. R-squared:                  0.939
Method:                 Least Squares   F-statistic:                 4.657e+04
Date:                Fri, 17 Nov 2023   Prob (F-statistic):               0.00
Time:                        20:46:35   Log-Likelihood:                -2253.1
No. Observations:                3022   AIC:                             4510.
Df Residuals:                    3020   BIC:                             4522.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         15.9511      0.171     93.265      0.000      15.616      16.286
Price          0.1772      0.001    215.802      0.000       0.176       0.179
==============================================================================
Omnibus:                       78.434   Durbin-Watson:                   0.003
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              167.632
Skew:                          -0.124   Prob(JB):                     3.97e-37
Kurtosis:                       4.127   Cond. No.                     3.84e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 3.84e+03. This might indicate that there are
strong multicollinearity or other numerical problems.



F0000113F2 Morningstar US Cash T-bill TR USD
                            OLS Regression Results
==============================================================================
Dep. Variable:              ETF_Price   R-squared:                       0.948
Model:                            OLS   Adj. R-squared:                  0.948
Method:                 Least Squares   F-statistic:                 5.453e+04
Date:                Fri, 17 Nov 2023   Prob (F-statistic):               0.00
Time:                        20:46:35   Log-Likelihood:                -2028.1
No. Observations:                3022   AIC:                             4060.
Df Residuals:                    3020   BIC:                             4072.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        -33.3448      0.369    -90.359      0.000     -34.068     -32.621
Price          0.6384      0.003    233.517      0.000       0.633       0.644
==============================================================================
Omnibus:                     1053.433   Durbin-Watson:                   0.003
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5772.048
Skew:                          -1.558   Prob(JB):                         0.00
Kurtosis:                       9.010   Cond. No.                     5.78e+03
```

```
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 5.78e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

```
F0000113FW Morningstar US 1-3Y Core Bd TR Hdg USD
                            OLS Regression Results
==============================================================================
Dep. Variable:              ETF_Price   R-squared:                       0.949
Model:                            OLS   Adj. R-squared:                  0.949
Method:                 Least Squares   F-statistic:                 5.615e+04
Date:                Fri, 17 Nov 2023   Prob (F-statistic):               0.00
Time:                        20:46:35   Log-Likelihood:                -1986.3
No. Observations:                3022   AIC:                             3977.
Df Residuals:                    3020   BIC:                             3989.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          6.8510      0.194     35.291      0.000       6.470       7.232
Price          0.2475      0.001    236.951      0.000       0.245       0.250
==============================================================================
Omnibus:                      220.550   Durbin-Watson:                   0.003
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              703.466
Skew:                          -0.345   Prob(JB):                    1.75e-153
Kurtosis:                       5.261   Cond. No.                     4.25e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 4.25e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Rationale for benchmark choice

- The correlations of monthly returns and prices both are maximum for the iShares ETF and F0000113ET Morningstar US 1-5Y Corp Bd TR USD
- Using monthly returns reduces noise and short-term volatility, providing a clearer view of long-term trends and correlations.
- Running OLS gives us statistically significant coefficients for all benchmarks vs the ETF. This implies that there is clear relationship between ETF and all benchmarks.
- However even with OLS F0000113ET Morningstar US 1-5Y Corp Bd TR USD is the best benchmark out of all the options. The R-squared is maximum (0.97) for F0000113ET which implies that it explains 97% of the variance of NEAR returns.

Construction of a composite benchmark

- We can take this up a notch and construct a composite benchmark consisting of all the index
- For simplicity lets keep this composite benchmark static. If its dynamic we would have to adjust for forward bias and introduce appropriate weekly/monthly rebalancing
- We can use monthly, daily or weekly returns to determine the composite. Lets go with weekly
- We run a multivariate regression of NEAR weekly returns against the weekly returns of all indexes

- The composite can be constructed by wieghing each index in the ratio of their respective coefficients

In [8]:
```
groups = data.groupby('MStarID')
merged_df = pd.DataFrame()
for fund_id, group in groups:
    df = group.sort_values(by='Date')
    df = extract_weekly(df)
    df.rename(columns={'Price': f'{fund_id}_Price', 'Returns': f'{fund_id}_Returns'}, in
    df.drop(columns=['FundName', 'MStarID'], inplace=True)
    if len(merged_df) == 0:
        merged_df = df
    else:
        merged_df = pd.merge(df, merged_df, on='Date', how='inner')
merged_df
```

Out[8]:

	Date	F0000113FW_Price	F0000113FW_Returns	F0000113F2_Price	F0000113F2_Returns	F0000113EU_Price	F
0	2013-09-30	175.56505	0.000576	132.01625	-0.000005	192.54321	
1	2013-10-07	175.54692	-0.000103	132.01379	-0.000019	192.56351	
2	2013-10-14	175.57928	0.000184	131.99925	-0.000110	192.62204	
3	2013-10-21	175.73897	0.000910	132.01912	0.000151	192.91776	
4	2013-10-28	175.82875	0.000511	132.02019	0.000008	193.19379	
...	...	...	...	...	...	...	
427	2021-12-06	197.97490	-0.001924	139.68875	0.000024	224.84357	
428	2021-12-13	197.95644	-0.000093	139.69025	0.000011	224.85707	
429	2021-12-20	198.06935	0.000570	139.69147	0.000009	225.09491	
430	2021-12-27	197.83632	-0.001177	139.69217	0.000005	224.74313	
431	2022-01-03	197.93813	0.000515	139.69391	0.000012	224.85744	

432 rows × 13 columns

It is fair to assume that there is going to be high multicollinearity among all the merged df return series hence we will use ridge regression instead of OLS because OLS will give unstable results

In [9]:
```
Y = merged_df['F00000OOR_Returns']
X = merged_df[['F0000113FW_Returns', 'F0000113F2_Returns', 'F0000113EU_Returns', 'F00001
X = sm.add_constant(X)
alpha = 3.8
model = sm.OLS(Y, X)
ridge_results = model.fit_regularized(alpha=alpha, L1_wt=0.0)
weights = np.array(ridge_results.params[1:])
weights_normalized = weights / weights.sum()
weights_df = pd.DataFrame({
    'MStarID': ['F0000113FW', 'F0000113F2', 'F0000113EU', 'F0000113ET', 'F0000113ES'],
```

```
    'CompositeWt': weights_normalized
})
weights_df
```

Out[9]:

	MStarID	CompositeWt
0	F0000113FW	0.141633
1	F0000113F2	0.001522
2	F0000113EU	0.172599
3	F0000113ET	0.658767
4	F0000113ES	0.025479

In [10]:

```
groups = data.groupby('MStarID')
composite_daily = pd.DataFrame(columns=['Date', 'BMK_Price', 'BMK_Returns'])
composite_daily['Date'] = groups.get_group('F00000O00R').sort_values(by='Date')['Date']
composite_daily.fillna(0, inplace=True)

composite_monthly = pd.DataFrame(columns=['Date', 'BMK_Price', 'BMK_Returns'])
composite_monthly['Date'] = extract_monthly(groups.get_group('F00000O00R')).sort_values(
composite_monthly.fillna(0, inplace=True)

composite_weekly = pd.DataFrame(columns=['Date', 'BMK_Price', 'BMK_Returns'])
composite_weekly['Date'] = extract_weekly(groups.get_group('F00000O00R')).sort_values(by
composite_weekly.fillna(0, inplace=True)

for fund_id, group in groups:
    if fund_id != 'F00000O00R':
        daily_df = pd.merge(extract_daily(group), weights_df, on='MStarID', how='inner')
        weekly_df = pd.merge(extract_weekly(group), weights_df, on='MStarID', how='inner
        monthly_df = pd.merge(extract_monthly(group), weights_df, on='MStarID', how='inn

        composite_daily = pd.merge(composite_daily, daily_df, on='Date', how='inner')
        composite_daily['BMK_Price'] = composite_daily['BMK_Price'] + (composite_daily['
        composite_daily['BMK_Returns'] = composite_daily['BMK_Returns'] + (composite_dai
        composite_daily = composite_daily[['Date', 'BMK_Price', 'BMK_Returns']]

        composite_weekly = pd.merge(composite_weekly, weekly_df, on='Date', how='inner')
        composite_weekly['BMK_Price'] = composite_weekly['BMK_Price'] + (composite_weekl
        composite_weekly['BMK_Returns'] = composite_weekly['BMK_Returns'] + (composite_w
        composite_weekly = composite_weekly[['Date', 'BMK_Price', 'BMK_Returns']]

        composite_monthly = pd.merge(composite_monthly, monthly_df, on='Date', how='inne
        composite_monthly['BMK_Price'] = composite_monthly['BMK_Price'] + (composite_mon
        composite_monthly['BMK_Returns'] = composite_monthly['BMK_Returns'] + (composite
        composite_monthly = composite_monthly[['Date', 'BMK_Price', 'BMK_Returns']]

composite_daily
```

Out[10]:

	Date	BMK_Price	BMK_Returns
0	2013-09-25	199.865678	0.000607
1	2013-09-26	199.801514	-0.000317
2	2013-09-27	199.908784	0.000529
3	2013-09-28	199.908784	0.000000
4	2013-09-29	199.908784	0.000000
...	...	...	...
3017	2021-12-29	243.066290	-0.000347

3018	2021-12-30	243.169391	0.000412
3019	2021-12-31	243.238018	0.000278
3020	2022-01-01	243.238018	0.000000
3021	2022-01-02	243.238018	0.000000

3022 rows × 3 columns

2. Graph the performance of ETF vs Benchmark

In [11]:

```
groups = data.groupby('MStarID')
etf = groups.get_group('F00000O0R').copy()
etf = extract_monthly(etf)
etf.rename(columns={'Price': 'ETF_Price', 'Returns': 'ETF_Returns'}, inplace=True)
etf.drop(columns=['FundName', 'MStarID'], inplace=True)

bmk = composite_monthly.copy()

merged_df = pd.merge(bmk, etf, on='Date', how='inner')
merged_df['BMK_CumRet'] = (1 + merged_df['BMK_Returns']).cumprod() - 1
merged_df['ETF_CumRet'] = (1 + merged_df['ETF_Returns']).cumprod() - 1

fig, axes = plt.subplots(3, 2, figsize=(20, 13))

axes[0,0].set_title('Cumulative Returns')
axes[0,0].plot(merged_df['Date'], merged_df['ETF_CumRet'], label="iShares ETF")
axes[0,0].plot(merged_df['Date'], merged_df['BMK_CumRet'], label='Composite Benchmark')
axes[0,0].legend()

axes[0,1].set_title('Log Cumulative Returns')
axes[0,1].plot(merged_df['Date'], np.log(1+merged_df['ETF_CumRet']), label="iShares ETF"
axes[0,1].plot(merged_df['Date'], np.log(1+merged_df['BMK_CumRet']), label='Composite Be
axes[0,1].legend()

axes[1,0].set_title('Price')
axes[1,0].plot(merged_df['Date'], merged_df['ETF_Price'], label="iShares ETF")
axes[1,0].plot(merged_df['Date'], merged_df['BMK_Price'], label='Composite Benchmark')
axes[1,0].legend()

axes[1,1].set_title('Log Price')
axes[1,1].plot(merged_df['Date'], np.log(merged_df['ETF_Price']), label="iShares ETF")
axes[1,1].plot(merged_df['Date'], np.log(merged_df['BMK_Price']), label='Composite Bench
axes[1,1].legend()

axes[2,0].set_title('Returns')
axes[2,0].plot(merged_df['Date'], merged_df['ETF_Returns'], label="iShares ETF")
axes[2,0].plot(merged_df['Date'], merged_df['BMK_Returns'], label='Composite Benchmark')
axes[2,0].legend()

axes[2,1].set_title('Log Returns')
axes[2,1].plot(merged_df['Date'], np.log(1 + merged_df['ETF_Returns']), label="iShares E
axes[2,1].plot(merged_df['Date'], np.log(1 + merged_df['BMK_Returns']), label='Composite
axes[2,1].legend()

plt.tight_layout()
```

3. Comparative Statistics

(a) Annualized Return and volatility

In [12]:

```
bmk_id = 'F0000113ET'
etf_id = 'F00000O0OR'

groups = data.groupby('MStarID')
# bmk = extract_daily(groups.get_group(bmk_id))
# bmk.drop(columns=['MStarID', 'FundName'], inplace=True)
# bmk.rename(columns={'Price': 'BMK_Price', 'Returns':'BMK_Returns'}, inplace=True)
bmk = composite_daily.copy()

etf = extract_daily(groups.get_group(etf_id))
etf.drop(columns=['MStarID', 'FundName'], inplace=True)
etf.rename(columns={'Price': 'ETF_Price', 'Returns':'ETF_Returns'}, inplace=True)

merged_df = pd.merge(bmk, etf, on='Date', how='inner').sort_values('Date')
merged_df.fillna(0, inplace=True)
merged_df
```

Out[12]:

	Date	BMK_Price	BMK_Returns	ETF_Price	ETF_Returns
0	2013-09-25	199.865678	0.000607	50.00000	0.0000
1	2013-09-26	199.801514	-0.000317	50.00000	0.0000
2	2013-09-27	199.908784	0.000529	50.01000	0.0002
3	2013-09-28	199.908784	0.000000	50.01000	0.0000
4	2013-09-29	199.908784	0.000000	50.01000	0.0000
...	...	...	...	...	...

3017	2021-12-29	243.066290	-0.000347	56.12608	0.0000
3018	2021-12-30	243.169391	0.000412	56.13732	0.0002
3019	2021-12-31	243.238018	0.000278	56.13732	0.0000
3020	2022-01-01	243.238018	0.000000	56.13732	0.0000
3021	2022-01-02	243.238018	0.000000	56.13732	0.0000

3022 rows × 5 columns

In [13]:

```
def calc_annualized_return(ret_series):
    tot_returns = (1 + ret_series).prod() - 1
    return tot_returns

def calc_annualized_vol(ret_series):
    vol = ret_series.std()
    vol = vol * np.sqrt(252)
    return vol

tuple_list = []
merged_df['Year'] = merged_df['Date'].dt.year
groups = merged_df.groupby('Year')
for year, group in groups:
    if year < 2022:
        ann_ret_bmk = calc_annualized_return(group['BMK_Returns'])
        ann_ret_etf = calc_annualized_return(group['ETF_Returns'])
        ann_vol_bmk = calc_annualized_vol(group['BMK_Returns'])
        ann_vol_etf = calc_annualized_vol(group['ETF_Returns'])
        tuple_list.append((year, ann_ret_bmk, ann_ret_etf, ann_vol_bmk, ann_vol_etf))

tuple_list.append(('Total', calc_annualized_return(merged_df['BMK_Returns']), calc_annua
output_df = pd.DataFrame(tuple_list, columns=['Period', 'Returns Benchmark (APR %)', 'Re
output_df['Returns Benchmark (APR %)'] = output_df['Returns Benchmark (APR %)'] * 100
output_df['Returns ETF (APR %)'] = output_df['Returns ETF (APR %)'] * 100
output_df
```

Out[13]:

	Period	Returns Benchmark (APR %)	Returns ETF (APR %)	Annualized Volatility Benchmark	Annualized Volatility ETF
0	2013	0.584732	0.327400	0.008704	0.002715
1	2014	1.792873	0.753653	0.010019	0.002391
2	2015	1.168512	0.691270	0.012699	0.002074
3	2016	2.398306	1.417830	0.010829	0.001716
4	2017	2.082706	1.550646	0.009037	0.001760
5	2018	1.158348	1.711769	0.009180	0.002073
6	2019	6.197948	3.465565	0.012808	0.002435
7	2020	5.020518	1.433725	0.022216	0.020822
8	2021	-0.625204	0.337453	0.009191	0.001838
9	Total	21.397013	12.274640	0.012587	0.007506

In [14]:

```
fig, axes = plt.subplots(1, 2, figsize=(20, 13))

axes[0].set_title('Annualized Returns APR')
bar_width = 0.35
x = np.arange(len(output_df['Period']))
```

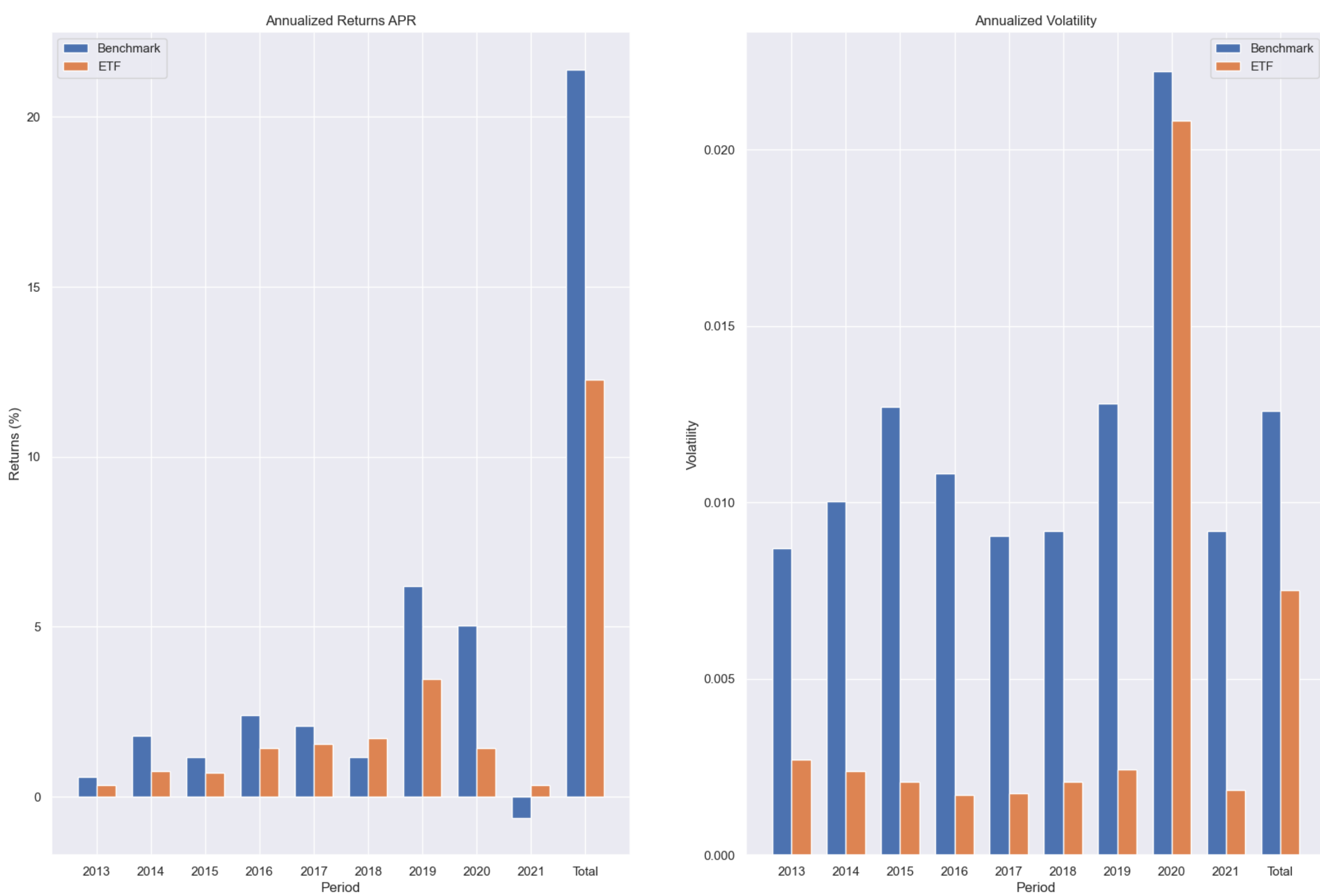
```
axes[0].bar(x - bar_width/2, output_df['Returns Benchmark (APR %)'], bar_width, label='B
axes[0].bar(x + bar_width/2, output_df['Returns ETF (APR %)'], bar_width, label='ETF')

# Add labels, title, and legend
axes[0].set_xlabel('Period')
axes[0].set_ylabel('Returns (%)')
axes[0].set_xticks(x)
axes[0].set_xticklabels(output_df['Period'])
axes[0].legend()


axes[1].set_title('Annualized Volatility')
bar_width = 0.35
x = np.arange(len(output_df['Period']))
axes[1].bar(x - bar_width/2, output_df['Annualized Volatility Benchmark'], bar_width, la
axes[1].bar(x + bar_width/2, output_df['Annualized Volatility ETF'], bar_width, label='E

# Add labels, title, and legend
axes[1].set_xlabel('Period')
axes[1].set_ylabel('Volatility')
axes[1].set_xticks(x)
axes[1].set_xticklabels(output_df['Period'])
axes[1].legend()

plt.show()
```



(b) Maximum Drawdown

In [15]:
```
def calc_dd(price_ser):
    prices = price_ser.values
    max_price = prices[0]
    max_drawdown = 0
    for price in prices:
        if price > max_price:
```

```
            max_price = price
        dd = abs(max_price - price)/max_price
        max_drawdown = max(dd, max_drawdown)
    return max_drawdown * 100

tuple_list = []
merged_df['Year'] = merged_df['Date'].dt.year
groups = merged_df.groupby('Year')
for year, group in groups:
    if year < 2022:
        bmk_dd = calc_dd(group['BMK_Price'])
        etf_dd = calc_dd(group['ETF_Price'])
        tuple_list.append((year, bmk_dd, etf_dd))
tuple_list.append(('Total', calc_dd(merged_df['BMK_Price']), calc_dd(merged_df['ETF_Pric
output = pd.DataFrame(tuple_list, columns=['Period', 'Max Drawdown (%) BMK', 'Max Drawdo
output_df = pd.merge(output_df, output, on='Period', how='inner')
output
```

Out[15]:

	Period	Max Drawdown (%) BMK	Max Drawdown (%) ETF
0	2013	0.405278	0.059952
1	2014	0.907218	0.122811
2	2015	0.808648	0.060017
3	2016	1.446282	0.040009
4	2017	0.600259	0.039861
5	2018	0.968301	0.137559
6	2019	0.806163	0.059708
7	2020	5.936630	5.640517
8	2021	1.229388	0.227202
9	Total	5.936630	5.640517

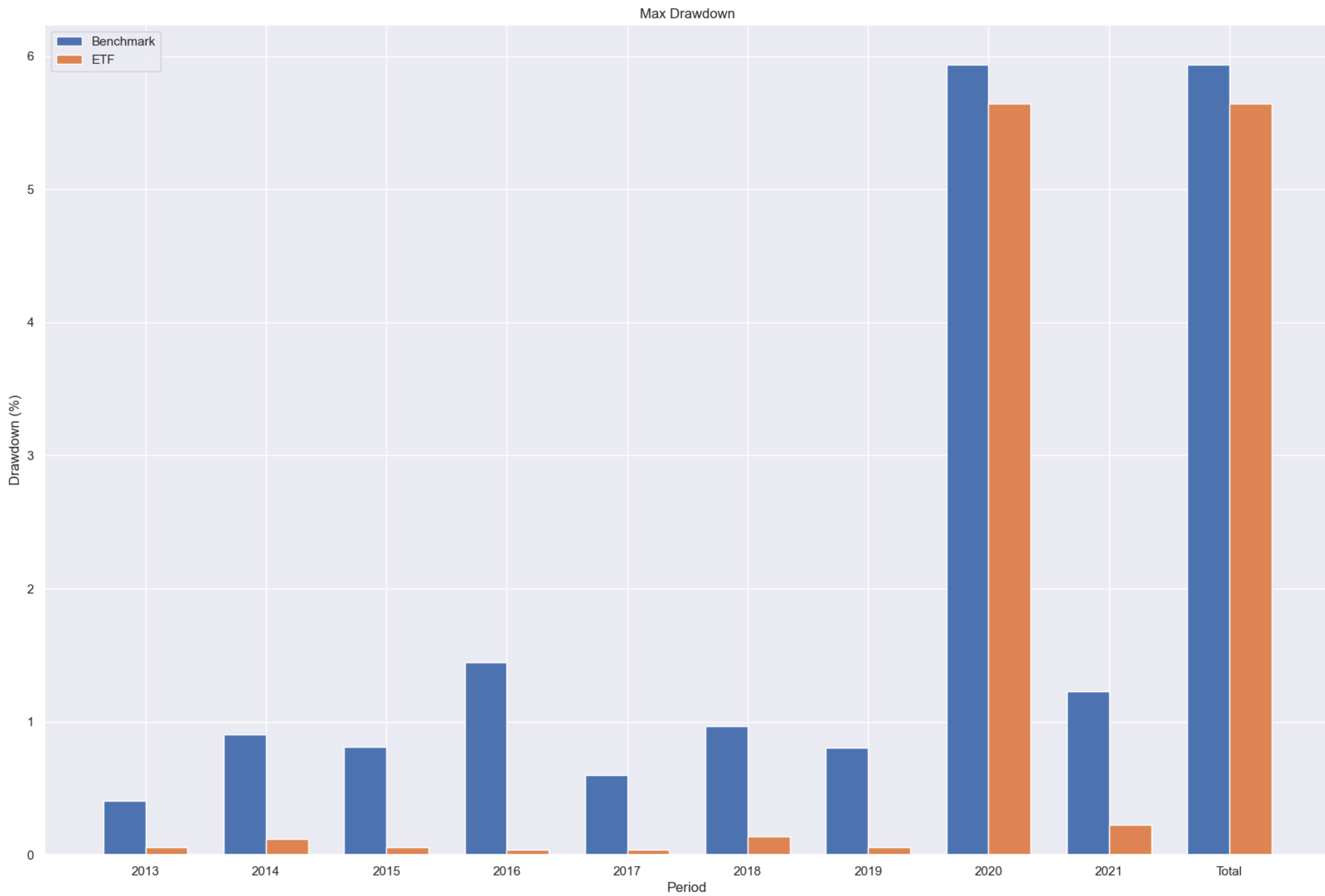
In [16]:

```
plt.figure(figsize=(20,13))
plt.title('Max Drawdown')
bar_width = 0.35
x = np.arange(len(output_df['Period']))
plt.bar(x - bar_width/2, output_df['Max Drawdown (%) BMK'], bar_width, label='Benchmark'
plt.bar(x + bar_width/2, output_df['Max Drawdown (%) ETF'], bar_width, label='ETF')

# Add labels, title, and legend
plt.xlabel('Period')
plt.ylabel('Drawdown (%)')
plt.xticks(x, output_df['Period'])
plt.legend()
```

Out[16]:

```
<matplotlib.legend.Legend at 0x1ca1585d190>
```



(c) Performance comparison using Sharpe Ratio of weekly returns

In [17]:
```
def calc_sharpe_ratio(ret_series):
    sr = ret_series.mean() / ret_series.std()
    return sr

tuple_list = []
etf = extract_weekly(data.loc[data['MStarID'] == etf_id])
etf.drop(columns=['MStarID', 'FundName'], inplace=True)
etf.rename(columns={'Price': 'ETF_Price', 'Returns': 'ETF_Returns'}, inplace=True)

bmk = composite_weekly.copy()

tuple_list = []
merged_df = pd.merge(etf, bmk, on='Date', how='inner')
merged_df['Year'] = merged_df['Date'].dt.year
groups = merged_df.groupby('Year')
for year, group in groups:
    if year < 2022:
        bmk_sr = calc_sharpe_ratio(group['BMK_Returns'])
        etf_sr = calc_sharpe_ratio(group['ETF_Returns'])
        tuple_list.append((year, bmk_sr, etf_sr))
tuple_list.append(('Total', calc_sharpe_ratio(merged_df['BMK_Returns']), calc_sharpe_rat
output = pd.DataFrame(tuple_list, columns=['Period', 'Sharpe Ratio BMK', 'Sharpe Ratio E
output_df = pd.merge(output_df, output, on='Period', how='inner')
output_df
```

Out[17]:

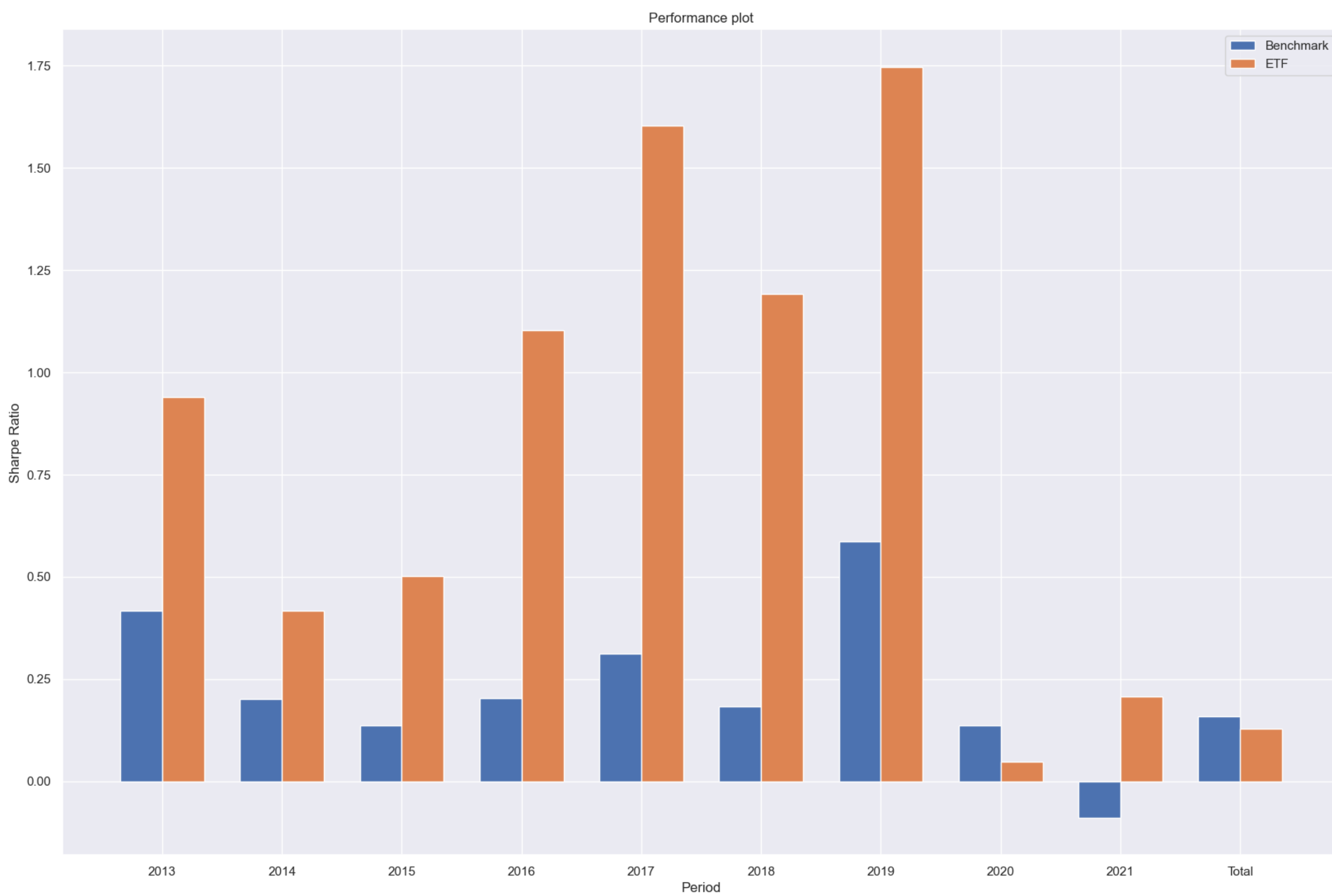
	Period	Returns Benchmark (APR %)	Returns ETF (APR %)	Annualized Volatility Benchmark	Annualized Volatility ETF	Max Drawdown (%) BMK	Max Drawdown (%) ETF	Sharpe Ratio BMK	Sharpe Ratio ETF
0	2013	0.584732	0.327400	0.008704	0.002715	0.405278	0.059952	0.416471	0.939431
1	2014	1.792873	0.753653	0.010019	0.002391	0.907218	0.122811	0.201464	0.417694

2	2015	1.168512	0.691270	0.012699	0.002074	0.808648	0.060017	0.136689	0.501545
3	2016	2.398306	1.417830	0.010829	0.001716	1.446282	0.040009	0.203841	1.103308
4	2017	2.082706	1.550646	0.009037	0.001760	0.600259	0.039861	0.312706	1.603914
5	2018	1.158348	1.711769	0.009180	0.002073	0.968301	0.137559	0.182085	1.191740
6	2019	6.197948	3.465565	0.012808	0.002435	0.806163	0.059708	0.586463	1.747219
7	2020	5.020518	1.433725	0.022216	0.020822	5.936630	5.640517	0.136890	0.048418
8	2021	-0.625204	0.337453	0.009191	0.001838	1.229388	0.227202	-0.089158	0.206600
9	Total	21.397013	12.274640	0.012587	0.007506	5.936630	5.640517	0.158521	0.128866

In [18]:
```
plt.figure(figsize=(20,13))
plt.title('Performance plot')
bar_width = 0.35
x = np.arange(len(output_df['Period']))
plt.bar(x - bar_width/2, output_df['Sharpe Ratio BMK'], bar_width, label='Benchmark')
plt.bar(x + bar_width/2, output_df['Sharpe Ratio ETF'], bar_width, label='ETF')

# Add labels, title, and legend
plt.xlabel('Period')
plt.ylabel('Sharpe Ratio')
plt.xticks(x, output_df['Period'])
plt.legend()
```

Out[18]: <matplotlib.legend.Legend at 0x1ca158fb640>



In [19]:
```
output_df.to_csv('performance_metrics.csv', index=False)
```