College of Engineering, Pune

Graph Theory and Applications

Lab Project

Analysis of different heuristics for N-Puzzle

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Abstract

The N-Puzzle is known in finite versions such as the 8-puzzle (a 3x3 board) and the 15-puzzle (a 4x4 board), and with various names like "sliding block", "tile puzzle", etc. The N-Puzzle is a board game for a single player. It consists of $N^2 - 1$ numbered squared tiles in random order, and one blank space ("a missing tile"). The objective of the puzzle is to rearrange the tiles in order by making sliding moves that use the empty space, using the fewest moves. Moves of the puzzle are made by sliding an adjacent tile into the empty space.

Various algorithmic search approaches have been developed to computerize the solving of these puzzles, many of which utilize heuristic techniques. We try to explore a range of these search techniques and heuristic concepts, and also compare the performance of each through rigorous testing and analysis of their implementation with 8-Puzzle.

Heuristics for the N-Puzzle:

- **Manhattan Distance:** The Manhattan Distance is the distance between two points measured along axes at right angles. The name alludes to the grid layout of the streets of Manhattan, which causes the shortest path a car could take between two points in the city.

  For the 8-puzzle, if $x_i(s)$ and $y_i(s)$ are the x and y coordinates of tile i in state s, and if upper-line(x_i) and upper-line (y_i) are the x and y coordinates of tile i in the goal state, the heuristic is:
  $h(s) = \sum_{i=1}^{8}(|x_i(s) - \bar{x_i}| + |y_i(s) - \bar{y_i}|)$

- **Linear Conflict:** Two tiles t_j and t_k are in a linear conflict if t_j and t_k are in the same line, the goal positions of t_j and t_k are both in that line, t_j is to the right of t_k and goal position of t_j is to the left of the goal position of t_k.

  The linear conflict adds at least two moves to the Manhattan Distance of the two conflicting tiles, by forcing them to surround one another. Therefore the heuristic function will add a cost of 2 moves for each pair of conflicting tiles.

- **N-Max Swap:** This heuristic also known as Gaschnig's heuristic. It is an admissible heuristic, since it underestimates the distance function of the problem and gives a closer approximation of the distance.

  The heuristic function can be implemented by using 2 arrays:
  P – represents the current permutation.
  B – the location of element i in the permutation array.

  The algorithm: Iteratively swap P[B[n]] with P[B[B[n]]] (for the n-puzzle)

- **X-Y:** The X-Y heuristic decompose the problem into two one-dimensional problems where the "space" can swap with any tile in an adjacent row/column. The heuristic function adds the number of steps from the two sub-problems.

  The heuristic function: computes the minimum number of column-adjacent blank swaps to get all tiles in their destination column + the minimum number of row adjacent blank swaps to get all tiles in their destination row.