3. K - Maps

Q.1 The output expression for the Karnaugh map shown below is

[GATE 2016, IISc Bangalore]

A \ BC	00	01	11	10
0	1	0	0	1
1	1	1	1	1

(A) $A+\overline{B}$ (B) $A+\overline{C}$ (C) $\overline{A}+\overline{C}$ (D) $\overline{A}+C$

Q.2 The SOP (sum of products) form of a Boolean functions is $\sum(0,1,3,7,11)$, where inputs are *A*, *B*, *C*, *D*, (*A* is MSB, and *D* is LSB). The equivalent minimized expression of the function is

[GATE 2014, IIT Kharagpur]

(A) $(\overline{B}+C)(\overline{A}+C)(\overline{A}+\overline{B})(\overline{C}+D)$ (B) $(\overline{B}+C)(\overline{A}+C)(\overline{A}+\overline{C})(\overline{C}+D)$

(C) $(\overline{B}+C)(\overline{A}+C)(\overline{A}+\overline{C})(\overline{C}+\overline{D})$ (D) $(\overline{B}+C)(A+\overline{B})(\overline{A}+\overline{B})(\overline{C}+D)$

Q.3 Consider the following Boolean expression for

$F : (P, Q, R, S) = PQ + \overline{P}QR + \overline{P}Q\overline{R}S$

The minimal sum-of-products form of F is __________.

[GATE 2014, IIT Kharagpur]

(A) $PQ + QR + QS$ (B) $P + Q + R + S$ (C) $\overline{P}+\overline{Q}+\overline{R}+\overline{S}$ (D) $\overline{P}R + \overline{P}\overline{R}S + P$

Q.4 $f(A,B,C,D) = \Pi M$ (0, 1, 3, 4, 5, 7,9,11, 12,13,14,15) is a maxterm representation of Boolean function *f* (*A*, *B*, *C*, *D*) where *A* is the MSB and *D* is the LSB. The equivalent minimized representation of this function is

[GATE 2015, IIT Kanpur]

(A) $(A+\overline{C}+D)(\overline{A}+B+D)$ (B) $A\overline{C}D+\overline{A}BD$

(C) $\overline{A}C\overline{D}+A\overline{B}C\overline{D}+A\overline{B}\overline{C}\overline{D}$ (D) $(B+\overline{C}+D)(A+\overline{B}+\overline{C}+D)(\overline{A}+B+C+D)$

Q.5 The number of min-terms after minimizing the following Boolean expression is __________ .

$[D'+AB'+A'C+AC'D+A'C'D]'$

[GATE 2015, IIT Kanpur]

Q.6 In the Karnaugh map shown below, X denotes a don't care term. What is the minimal form of the function represented by the Karnaugh map?

[GATE 2008, IISc Bangalore]

cd \ ab	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

(A) $\overline{b}.\overline{d}+\overline{a}.\overline{d}$ (B) $\overline{a}.\overline{b}+\overline{b}.\overline{d}+\overline{a}.b.\overline{d}$ (C) $\overline{b}.\overline{d}+\overline{a}.b.\overline{d}$ (D) $\overline{a}.\overline{b}+\overline{b}.\overline{d}+\overline{a}.\overline{d}$

Q.7 Given, f (w, x, y, z) = Σ_m (0, 1, 2, 3, 7, 8, 10) + Σ_d (5, 6, 11, 15)

where *d* represents the don't-care condition in Karnaugh maps. Which of the following is a minimum product – of – sums (POS) form of f(w, x, y, z)?

[GATE 2017, IIT Roorkee]

(A) $f=(\overline{w}+\overline{z})(\overline{x}+z)$ (B) $f=(\overline{w}+z)(x+z)$ (C) $f=(w+z)(\overline{x}+z)$ (D) $f=(w+\overline{z})(\overline{x}+z)$

Q.8 The minimal sum-of-products expression for the logic function f represented by the given Karnaugh map is

[GATE 2009, IIT Roorkee]

RS \ PQ	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1	1	0
10	0	0	1	0

(A) $QS+P\overline{R}S+PQR+\overline{P}RS+\overline{P}Q\overline{R}$ (B) $\overline{Q}\,\overline{S}+\overline{P}R\overline{S}+\overline{P}\,\overline{Q}R+P\overline{R}\,\overline{S}+P\overline{Q}R$

(C) $\overline{P}R\overline{S}+\overline{P}\,\overline{Q}\,\overline{R}+P\overline{R}\,\overline{S}+P\overline{Q}R$ (D) $P\overline{R}S+PQR+\overline{P}RS+\overline{P}Q\overline{R}$

Q.9 Consider the Karnaugh map given below, where X represents "don't care" and blank represents 0

[GATE 2017, IIT Roorkee]

dc \ ba	**00**	**01**	**11**	**10**
00		x	x	
01	1			x
11	1			1
10		x	x	

Assume for all inputs (a, b, c, d), the respective complements $(\overline{a},\overline{b},\overline{c},\overline{d})$ are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is ______.

Q.10 Following is the K-map of a Boolean function of five variables *P, Q, R, S* and *X*. The minimum sum-of-product (SOP) expression for the function is

[GATE 2016, IISc Bangalore]

RS \ PQ	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$X=0$

RS \ PQ	00	01	11	10
00	0	1	1	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	0

$X=1$

(A) $\overline{P}\,\overline{Q}S\overline{X}+P\overline{Q}S\overline{X}+Q\overline{R}\,\overline{S}X+QR\overline{S}X$ (B) $\overline{Q}S\overline{X}+Q\overline{S}X$

(C) $\overline{Q}SX+Q\overline{S}\,\overline{X}$ (D) $\overline{Q}S+Q\overline{S}$

Q.11 Which are the essential prime implicants of the following Boolean function?

$f(a, b, c) = a'c + ac' + b'c$

[GATE 2004, IIT Delhi]

(A) $a'c$ and ac' (B) $a'c$ and $b'c$ (C) $a'c$ only (D) ac' and bc'

Q.12 Consider the Boolean function,

$F(w,x,y,z) = wy + xy + \overline{w}xyz + \overline{w}\overline{x}y + xz + \overline{x}\overline{y}\overline{z}$

Which one of the following is the complete set of essential prime implicants?

[GATE 2014, IIT Kharagpur]

(A) $w, y, xz, \overline{x}\overline{z}$ (B) w, y, xz (C) $y, \overline{x}\overline{y}\overline{z}$ (D) $y, xz, \overline{x}\overline{z}$

4. Number System

Q.1 The decimal equivalent of binary number 10110.11 is

(A) 16.75 (B) 20.75 (C) 16.50 (D) 22.75

Q.2 If $(2.3)_{base4} + (1.2)_{base4} = (y)_{base4}$; What is the value of y? ____________________.

Q.3 Given $(135)_{base\,x} + (144)_{base\,x} = (323)_{base\,x}$.

What is the value of base x ? __________.

Q.4 The base of the number system for the addition operation $24+14=41$ is_____.

[GATE 2011, IIT Madras]

Q.5 Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is _____________ .

[GATE 2011, IIT Madras]

Q.6 Consider the equation $(43)_x = (y3)_8$ where x and y are unknown. The number of possible solutions is __________.

[GATE 2015, IIT Kanpur]

Q.7 11001, 1001 and 111001 correspond to the 2's complement representation of which one of the following sets of number?

[GATE 2004, IIT Madras]

(A) 25, 9 and 57 respectively. (B) – 6, – 6 and – 6 respectively.

(C) – 7, – 7 and – 7 respectively. (D) – 25, – 9 and – 57 respectively.

Q.8 A number in 4-bit two's complement representation is $X_3 X_2 X_1 X_0$. This number when stored using 8 – bits will be

[GATE 1999, IIT Bombay]

(A) $0000X_3X_2X_1X_0$ (B) $1111X_3X_2X_1X_0$

(C) $X_3X_3X_3X_3X_3X_2X_1X_0$ (D) $\overline{X}_3\overline{X}_3\overline{X}_3\overline{X}_3X_3X_2X_1X_0$

Q.9 Two 2's complement numbers having sign bits 'x' and 'y' are added and the sign bit of the result is 'z'. Then, the occurrence of overflow is indicated by the Boolean function

[GATE 1998, IIT Delhi]

(A) xyz (B) $\overline{x}\overline{y}\overline{z}$ (C) $\overline{x}\overline{y}z + xy\overline{z}$ (D) $xy + yz + zx$

Q.10 When two 8-bit numbers $A_7..........A_0$ and $B_7..........B_0$ in 2's complement representation (with A_0 and B_0 as the least significant bits) are added using a ripple-carry adder, the sum bits obtained are $S_7..........S_0$ and the carry bits are $C_7..........C_0$. An overflow is said to have occurred if

[GATE 2017, IIT Roorkee]

(A) The carry bit C_7 is 1

(B) All the carry bits ($C_7..........C_0$) are 1

(C) $\left(A_7.B_7.\overline{S}_7+\overline{A}_7.\overline{B}_7.S_7\right)$ is 1

(D) $\left(A_0.B_0.\overline{S}_0+\overline{A}_0.\overline{B}_0.S_0\right)$ is 1

Q.11 Assuming all numbers in 2's complement form, which of the following additions will result in overflow?

(A) 1100 + 1100

(B) 0011 + 0111

(C) 1111 + 0111

(D) All of the above

K-Map

① ⇒ B

A \ BC	00	01	11	10
0	1	0	0	1
1	1	1	1	1

$\rightarrow$ $A + \bar{C}$ Ans

② ⇒ A

$\Sigma m(0, 1, 3, 7, 11)$

$\Pi M(2, 4, 5, 6, 8, 9, 10, 12, 13, 14, 15)$

AB \ CD	$C+D$	$\bar{C}+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	0	1	3	2: 0
$A+\bar{B}$	4: 0	5: 0	7	6: 0
$\bar{A}+\bar{B}$	12: 0	13: 0	15: 0	14: 0
$\bar{A}+B$	8: 0	9: 0	11	10: 0

$(\bar{B}+C)(\bar{A}+C)(\bar{A}+B)(\bar{C}+D)$ Ans

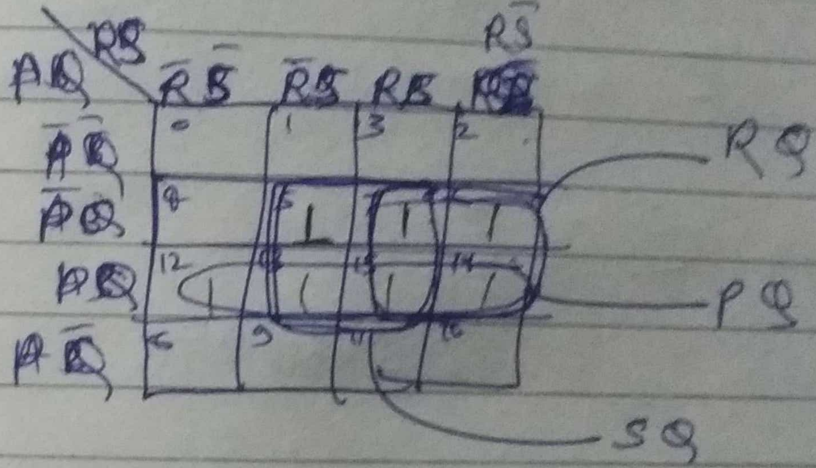
(3) (A)

$F = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$

11XX 011X 0101

1100 0110 0101
1101 0111
1110
1111

12, 13, 14, 15 6, 7 5



$PQ + QR + QS$ Ans

(4) – $F = \pi M(0, 1, 3, 5, 7, 9, 11, 12, 13, 14, 15)$ (C)

$F = \Sigma m(2, 4, 6, 8, 10)$

πM (A) (0) . 010, 011 → 2, 3

Σm (B) 101, 011 → 5, 3

Σm (C) 010, 100, 1000 → 2, 4, 8 ✓

πM (D) 010, 0110, 0111 → 2, 6, 7

5 ⇒ $F = \overline{(\bar{D} + A\bar{B} + \bar{A}C + A\bar{C}D + \bar{A}\bar{C}D)}$

1 $\quad \bar{F} = \bar{D} + A\bar{B} + \bar{A}C + A\bar{C}D + \bar{A}\bar{C}D$

$F =$ XXX0 $\quad$ 10XX $\quad$ 0X1X $\quad$ 1X01 $\quad$ 0X01

(0, 2, 4, 6, 8, 10, 12, 14) $\quad$ (8, 9, 10, 11) $\quad$ (2, 3, 6, 7) $\quad$ (9, 13) $\quad$ (1, 5)

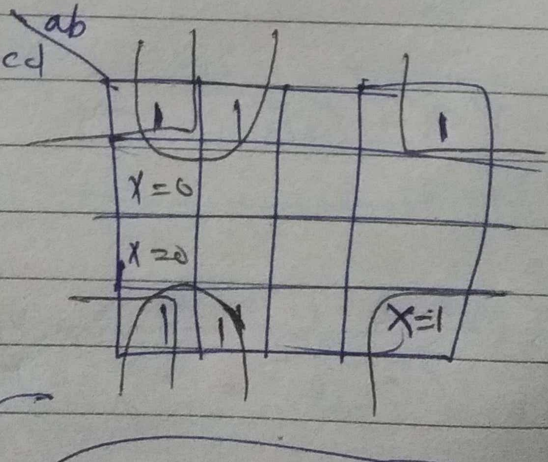
$\bar{F} = \Sigma m(0, 1, 2 \ldots \ldots 14)$

$F = \Sigma m(15)$

→ 1 Ans

6 ⇒

A



→ $\bar{b}\bar{d} + \bar{a}\bar{d}$ $\quad$ Ans

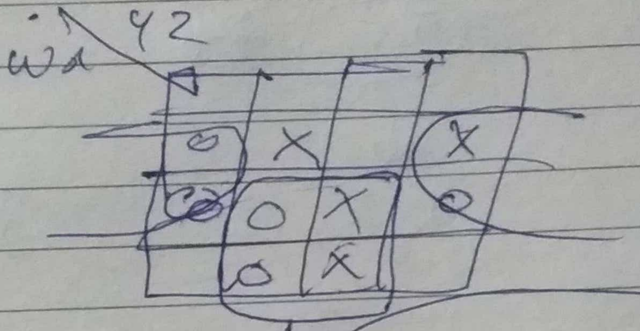
(7) ⇒

$F = \Sigma_m(0, 1, 2, 3, 7, 8, 10) + \Sigma_d(5, 6, 11, 15)$

(A)

~~$F = \Pi M(4, 5, 6, 9, 11, 12, 13, 14, 15) + \Sigma_d(5, 6, 11, 15)$~~

$F = \Pi M(4, 9, 12, 13, 14) + \Sigma_d(5, 6, 11, 15)$

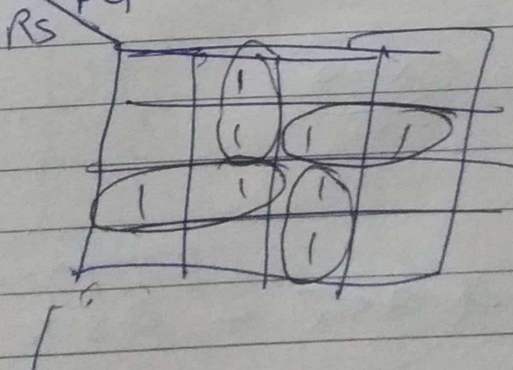


$\Rightarrow (\bar{w} + \bar{z})(\bar{x} + z)$

Ans

(8) ⇒

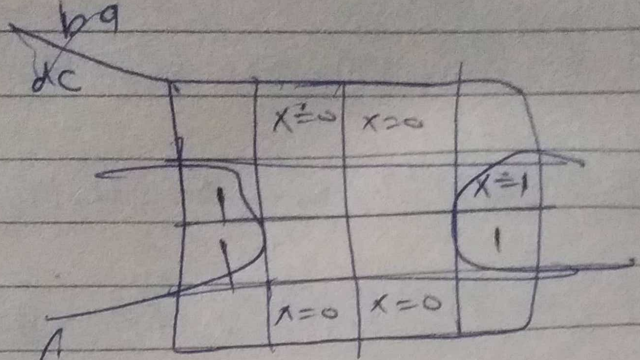
(D)



$\Rightarrow \bar{P}Q\bar{R} + P\bar{R}S + PQR + \bar{P}RS$

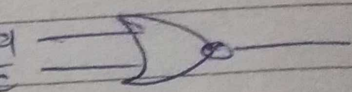
9 ⇒

T



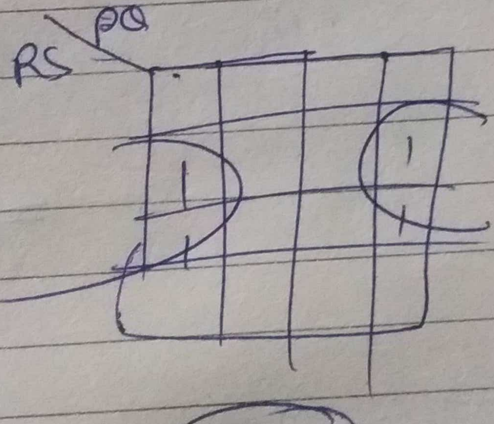
$\bar{a}\bar{c}$ Ans

$\overline{a+c} \Rightarrow$

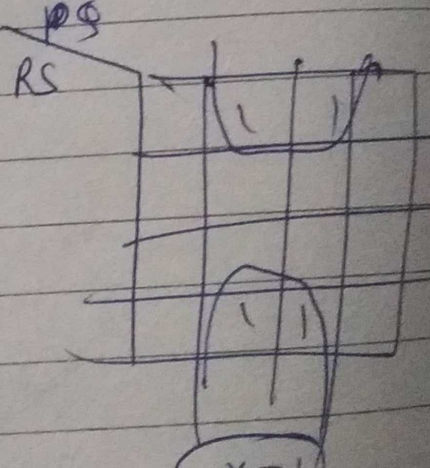


10 ⇒

B



x = 0



x = 1

$\bar{Q}S\bar{x} + Q\bar{S}x$ Ans

(11-8) $F = \bar{a}c + a\bar{c} + \bar{b}c$

(A)

$a \backslash bc$	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$		1	1	
a	1	1		1

EPI — $\bar{a}c$ — (1-3)

$\bar{b}c$ — (1-5)

$a\bar{b}$ — (4,5)

EPI — $a\bar{c}$ — (4,6)

→ Prime Implicants

(12-8) $F = wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$

(D)

$wx \backslash yz$				
	1		1	1
		1	1	1
		1	1	1
	1	1	1	1

EPI = y, xz, $\bar{x}\bar{z}$ Ans

Number System

(1) $(10110.11)_2 = (22.75)_{10}$ Ans

(2) $(2.3)_4 + (1.2)_4 = (y)_4$

$(2\times1 + 3\times 1/4) + (1\times1 + 2\times 1/4) = (y)_4$

$= 3 + 5/4$

$\Rightarrow 3 + \frac{1+4}{4}$

$4 + 1/4$

$= (10.1)_4$

$\boxed{y = 10.1}$ Ans

(3) $(135)_x + (144)_x = (323)_x$

$(x^2 + 3x + 5) + (x^2 + 4x + 4) = 3x^2 + 2x + 3$

$x^2 - 5x - 6 = 0$

$x = 6, \cancel{-1}$

$\boxed{x = 6}$ Ans

4 ⇒

$$(24)_x + (14)_x = (41)_x$$

$$(2x+4) + (x+4) = 4x+1$$

$$x = 7$$ **Ans**

5 ⇒ $\boxed{3}$ Sol^n

$$(123)_5 = (x8)_y \longrightarrow \text{Possible sol}^n$$

$$1 \times 5^2 + 2 \times 5 + 3 = xy + 8$$

$$xy = 30$$

$y > 8$

x	y
1	30
2	15
3	10
5	6
6	5
10	3
15	2
30	1

Ans

6 ⇒ $\boxed{5}$ Sol^n

$$(43)_x = (43)_8$$

$0 < y < 8$ $\quad x > 4$

$$4x + 3 = 8y + 3$$

$$x = 2y$$

x	y
5	10
6	1

x	y
6	3
8	4
10	5
12	6
14	7

7 ⇒

c

11001 $\xrightarrow{2's}$ 00110+1 = −7

1001 $\xrightarrow{2's}$ 0110+1 = −7

111001 $\xrightarrow{2's}$ 000110+1 = −7

8 ⇒

4-bit $(X_3 X_2 X_1 X_0)$ → 2's comp

A

↓

8bit

⇓

if signed → then MS → replicate

if unsigned → then MS → zero

maximum result unsigned

9 ⇒

c

sign bit − (x + y) = sign bit − (z)

2's comp

X = 0, Y = 0 ⟶ z = 0

X = 1, Y = 1 ⟶ z = 1, z = 0 (overflow)

$xy\bar{z} + \bar{x}\bar{y}z$ Ans

10 ⟹

C

C_7	C_6	overflow
0	0	No
0	1	yes
1	0	yes
1	1	No

C_7	C_6
A_7	1
B_7	1
S_7	0

$S_7 = 1$ otherwise

only $C_6 = 0$

$C_7 = 0$, $C_6 = 1$

$\longrightarrow \bar{A}_7 \bar{B}_7 S_7$

$$\left(A_7 B_7 \bar{S}_7 + \bar{A}_7 \bar{B}_7 S_7 \right) = 1$$ Ans

11 ⟹ Overflow result –

B

A 1100 + 1100

B 0011 + 0111

C 1111 + 0111