Chapter 3 – Integral & Differential

Q.41 Consider points P and Q in the x-y plane, with $P=(1,\ 0)$ and $Q=(0,1)$. The line integral $2\int_P^Q (xdx+ydy)$ along the semicircle with the line segment PQ as its diameter

(A) is – 1.

(B) is 0.

(C) is 1.

(D) depends on the direction (Clockwise or anti-clockwise) of the semicircle.

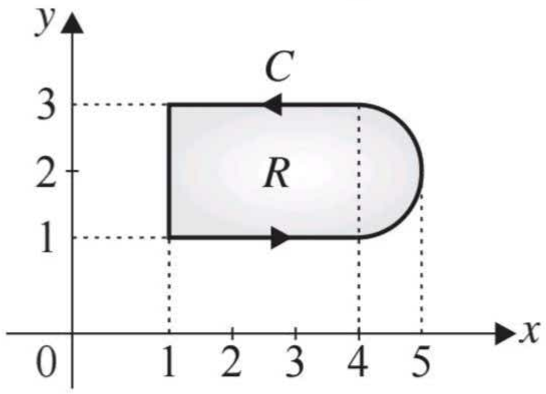
Q.42 Suppose C is the closed curve defined as the circle $x^2+y^2=1$ with C oriented anti-clockwise. The value of $\oint(xy^2dx+x^2y\,dy)$ over the curve C equals ________.

Q.43 Consider the line integral

$$\oint_c (x\,dy-y\,dx)$$

the integral being taken in a counterclockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below.

The region R is the area enclosed by the union of a 2 x 3 rectangle and a semi-circle of radius 1. The line integral evaluates to



(A) $8+\pi$ (B) $12+\pi$

(C) $16+2\pi$ (D) $6+\frac{\pi}{2}$

Q.44 The value of the line integral

$$\int_c (2xy^2dx + 2x^2ydy + dz)$$

along a path joining the origin (0, 0, 0) and the point (1, 1, 1) is

(A) 0 (B) 2

(C) 4 (D) 6

Q.45 The value of the integral

$$\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)} dx \text{ is}$$

(A) 3 (B) 0

(C) – 1 (D) – 2

Chapter 5 – Maxima & Minima

Q.57 Let $f(x)=3x^3-7x^2+5x+6$. The maximum value of $f(x)$ over the interval [0, 2] is _______ (upto one decimal place).

Q.58 The function $f(x)=x^3-6x^2+9x+25$

has

(A) a maxima at $x = 1$ and a minima at $x = 3$.

(B) a maxima at $x = 3$ and a minima at $x = 1$.

(C) no maxima, but minima at $x = 3$.

(D) a maxima at $x = 1$, but no minima.

Q.59 The continuous function $f(x, y)$ is said to have saddle point at (a, b) is, where the subscripts x, y etc. denote partial derivatives.

(A) $f_x(a, b)=f_y(a, b)=0$

$f_{xy}^2-f_{xx}f_{yy}<0$ at (a, b)

(B) $f_x(a, b)=0;\ f_y(a, b)=0$

$f_{xy}^2-f_{xx}f_{yy}>0$ at (a, b)

(C) $f_x(a, b)=0;\ f_y(a, b)=0$

f_{xx} and $f_{yy}<0$ at (a, b)

(D) $f_x(a, b)=f_y(a, b)=0$

$f_{xy}^2-f_{xx}f_{yy}=0$ at (a, b)

Q.60 A scalar valued function is defined as $f(x)=x^TAx+b^Tx+c$, where A is a symmetric positive definite matrix with dimension $n\times n$; b and x are vectors of dimension $n\times 1$. The minimum value of $f(x)$ will occur when x equals

(A) $(A^TA)^{-1}b$

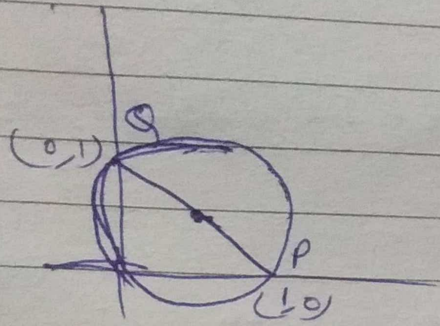
(B) $-(A^T A)^{-1} b$

(C) $-\left(\dfrac{A^{-1} b}{2}\right)$

(D) $\dfrac{A^{-1} b}{2}$

41 ⇒

B



$\therefore \int_P^Q (x\,dx + y\,dy)$

$\cancel{= 2 \cdot \iint \left(\frac{dy}{dx} - \frac{dx}{dy}\right) dx\,dy}$

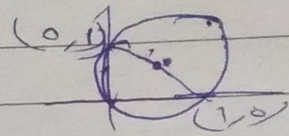
$$\therefore \iint_S \left(\frac{dy}{dx} - \frac{dx}{dy}\right) dx\,dy = \boxed{0} \text{ Ans}$$

42 ⇒

$$\oint_C (xy^2\,dx + x^2 y\,dy)$$

$$= \boxed{0} \text{ Ans}$$

$x^2 + y^2 = 1$



$1-(-1)$
$= 2$

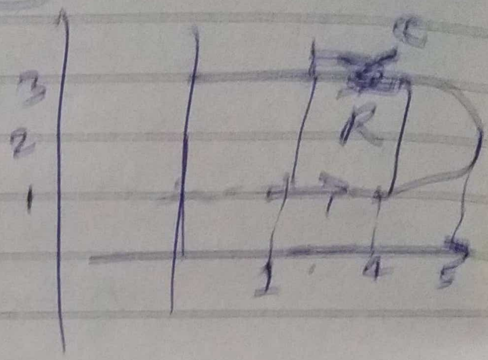
43 → B

$$\oint_c (x\,dy - y\,dx)$$

$$= 2 \times \text{Area}$$

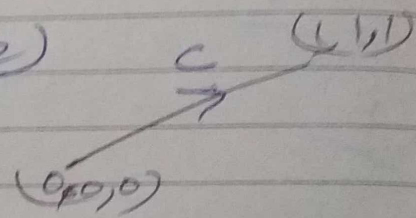
$$= 2 \times \left(2\times 3 + \frac{\pi 1^2}{2}\right)$$

$$= 12 + \pi$$ Ans



44 → B

$$\int (2xy^2\,dx + 2x^2y\,dx + dz)$$



$$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1} = t$$

$$\int_0^1 (2t^3 + 2t^3 + 1)\,dt$$

$$= \left[\cancel{4}\times\frac{t^4}{\cancel{4}} + t\right]_0^1$$

$$= 1 + 1$$

$$= 2$$ Ans

45 →

(B)

$$\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\,dx$$

$x-1 = t$

$dx = dt$

$x = 0$ to 2

$t = -1$ to 1

$$= \int_{-1}^{1} \frac{t^2 \sin t}{t^2 + \cos t}\,dt$$

→ odd

$= \boxed{0}$ Ans

57 →

$f(x) = 3x^3 - 7x^2 + 5x + 6$

$f'(x) = 9x^2 - 14x + 5 = 0$

$x = 5/9, 1$

Interval

→ $[0, 2]$ → $[0, 5/9, 1, 2]$

$\max[f(2)] = 3\times2^3 - 7\times2^2 + 5\times2 + 6$

$= 12$ Ans

58 ⇒ $f(x) = x^3 - 6x^2 + 9x + 25$

A

$f'(x) = 3x^2 - 12x + 9 = 0$

$x = 1, 3$

$f''(x) = 6x - 12$

$f''(1) = -6$ (maxima)

$f''(3) = 6$ (minima)

A ⇒ A maxima at $x=1$ & minima at $x=3$

Ans

59 ⇒ $rt - s^2 < 0$ → Stationary point

B

B — $f_x(a, b) = 0$, $f_y(a, b) = 0$

$f_{xy}^2 - f_{xx} f_{yy} > 0$ at (a, b)

$s^2 - rt$

$rt - s^2 < 0$

(60) $\Rightarrow$ $f(x) = x^T A x + b^T x + c$

C

$f(x) = Ax^2 + bx + c$

$f'(x) = 2Ax + b = 0$

$$x = \frac{-b}{2A}$$

$$x = \frac{-1}{2} A^{-1} b$$

C