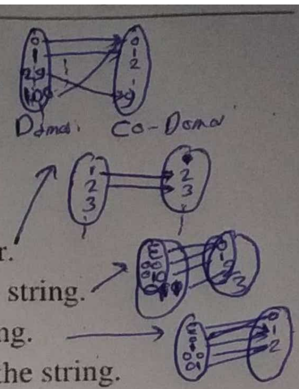
## FUNCTIONS



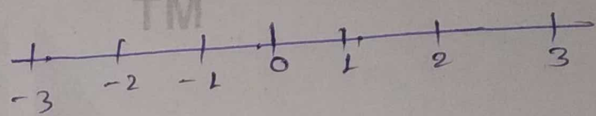

**Q 1.** Find the domain and range of these functions

a) The function that assigns to each nonnegative integer its last digit.
b) The function that assigns the next largest integer to a positive integer.
c) The function that assigns to a bit string the number of one bits in the string.
d) The function that assigns to a bit string the number of bits in the string.
e) The function that assigns to a bit string the longest string of ones in the string.

↳ Domain $= (0+1)^* 1 (0+1)^*$
Range $= 1^+$

**Q 2.** Find these values.

a) $\lfloor 1.1 \rfloor \longrightarrow 1$
b) $\lceil 1.1 \rceil \longrightarrow 2$
c) $\lfloor -0.1 \rfloor \longrightarrow -1$
d) $\lceil -0.1 \rceil \longrightarrow 0$
e) $\lceil 2.99 \rceil \longrightarrow 3$
f) $\lceil -2.99 \rceil \longrightarrow -2$
g) $\lfloor ½ + \lceil ½ \rceil \rfloor \longrightarrow 1$
h) $\lceil \lfloor ½ \rfloor + \lceil ½ \rceil + ½ \rceil \longrightarrow 2$




**Q 3.** Determine whether each of these functions { a, b, c, d } to itself is one-to-one.

a) $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$
b) $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$
c) $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$

**Q 4.** Determine whether $f : Z \times Z \rightarrow Z$ is onto if

a) $f(m, n) = m^2 - n^2$.
b) $f(m, n) = m + n + 1$.
c) $f(m, n) = |m| - |n|$.
d) $f(m, n) = m^2 - 4$.

**Q 5.** Determine whether each of these functions from Z to Z is one-to-one.

a) $f(n) = n - 1$
b) $f(n) = n^2+1 \longrightarrow n^2 \Rightarrow \pm 5$ (X)
c) $f(n) = n^3$
d) $f(n) = \lceil n/2 \rceil$ $\lceil 1/2 \rceil = 1$, $\lceil 2/2 \rceil = 1$ X

**Q 6.** Determine whether each of these function is a bijection from R to R.

a) $f(x) = -3x+4$
b) $f(x) = -3x^2 + 1.$
c) $f(x) = x^2 + 1$
d) $f(x) = x^3$

**Q 7.** Let S = {-1,0,2,4,7}. Find $f(s)$ if

a) $f(x) = 1.$ $\longrightarrow$ $f(S) = \{1\}$
b) $f(x) = 2x+1$ $\longrightarrow$ $f(S) = \{-1, 1, 5, 9, 15\}$
c) $f(x) = \lceil x/5 \rceil$ $\longrightarrow$ $f(S) = \{-1, 0, 1, 2\}$
d) $f(x) = \lfloor (x^2 + 1)/3 \rfloor$. $\longrightarrow$ $f(S) = \{0, 1, 5, 6\}$

**Q 8.** Let $f(x) = 2x$. What is

a) $f(Z)$? $\longrightarrow$ $\{\ldots -6, -4, -2, 0, 2, 4, \ldots\} = 2Z$
b) $f(N)$? $\longrightarrow$ $2N$
c) $f(R)$? $\longrightarrow$ $R$

## Functions

**Q.9** Find the inverse of the following

A. $f(x) = x^3 + 2$ $\Rightarrow (x-2)^{1/3}$

B. $f(x) = x^{1/3}$ $\Rightarrow x^3$

C. $f(x) = 1 - 2x^3$ $\Rightarrow \left(\frac{1-x}{2}\right)^{1/3}$

D. $f(x) = 3x - 2$ $\Rightarrow \left(\frac{x+2}{3}\right)$

E. $2/x$ $\Rightarrow 2/x$

F. $\frac{x+1}{x-1}$ $\Rightarrow \frac{x+1}{x-1}$

G. $\sqrt{x-1}$ $\Rightarrow x^2+1$

H. $(x+1)^{1/3}$ $\Rightarrow x^3-1$

**Q.10** Let f be the function from R to R defined by $f(x) = x^2$. Find

a) $f^{-1}(\{1\})$. $\rightarrow \{1\}$

b) $f^{-1}(\{x \mid 0 < x < 1\})$. $\rightarrow \{x \mid 0 < x < 1\}$

c) $f^{-1}(\{x \mid x > 4\})$. $\rightarrow \{x \mid x > 2\}$

**Q.11** Find $f \circ g$ and $g \circ f$, where $f(x) = x^2+1$ and $g(x) = x + 2$ are function from R to R.

$(x^2+4x+5)$ $(x^2+3)$

**Q.12** $f(x) = x^2$ and $g(x) = 2^x$. Find $f \circ f(x)$, $g \circ g(x)$, $f \circ g(x)$, $g \circ f(x)$. $2^{x^2}$

$x^4$ $2^{2^x}$ $2^{2x}$

$\boxed{f \circ g(✓) \quad g \circ f(X)}$

**Q.13** Let $f: R \to R$ be given by $f(x) = x^2$ and $g: R - \{2\} \to R$ be given by $g(x) = x/(x-2)$. Find $f \circ g$. Is $g \circ f$ defined? $x^2/(x-2)^2$

$x^2/(x^2-2)$ X

**Q.14** If $f(x) = -3x+2$, $g(x) = |x-4|$, then find $\boxed{f \circ g(-2) = -16}$

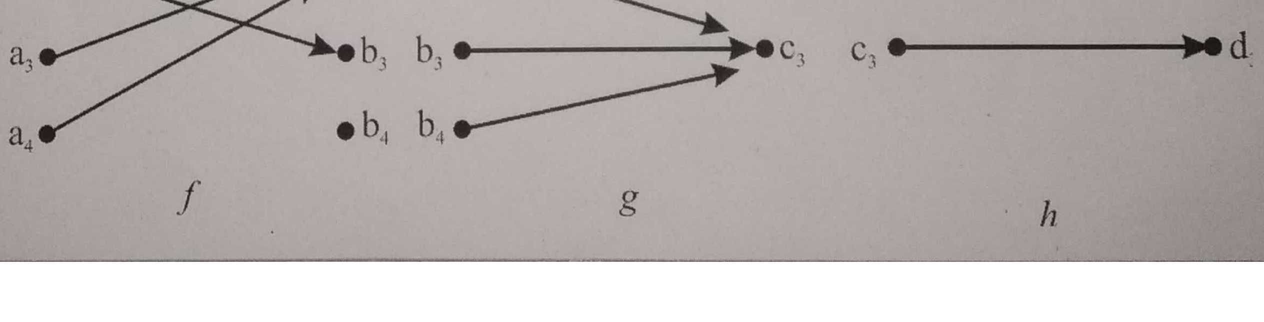
**Q.15** If $f(x) = (x-1)/(x+2)$, $g(x) = (x+1)/(x-2)$. Find $f \circ g(x)$. $= \dfrac{\left(\frac{x+1}{x-2}\right) - 1}{\left(\frac{x+1}{x-2}\right) + 2} = \frac{1}{x-1}$

**Q.16** If $f(x) = x^2 + 2$, $g(x) = \sqrt{(x-2)}$. Find $f \circ g(x)$ $= (x-2) + 2 = x$

**Q.17** Let $f$ be a functions from $A$ to $B$ be a function From $B$ to $C$.

(a) For the Functions $f$ and $g$ shown in the figure determine $g \circ f$

(b) For the Functions $f$, $g$ and $h$ shown in the figure, determine $h \circ (g \circ f)$ and $(h \circ g) \circ f$.



**Q.18** Let $f(x) = ax+b$ and $g(x) = cx+d$, where a,b,c and d are contants. Determine for which constants a,b,c, and d it is true that $f \circ g = g \circ f$.

$\boxed{\text{False}}$

$f \circ g = g \circ f$

$f \circ g \Rightarrow a(cx+d)+b = c(ax+b)+d$

$\cancel{acx} + ad + b = \cancel{acx} + bc + d$

$ad + b \neq bc + d$

## FUNCTIONS

19. Cardinality of the subset of a set is
    a) Always less than or equal to the cardinality of the set
    b) Always less than the cardinality of the set
    c) Always equal to the cardinality of the set
    d) None of these
20. Cardinality of the proper subset of a set is
    a) Always less than the cardinality of the set
    b) Always equal to the cardinality of the set
    c) May be less than or equal to the cardinality of the set
    d) None of these
21. Which of the following statements are true?
    T a) There may exist proper subset of a set S which has the same cardinality as the set S
    T b) There may exist proper superset of a set S which has the same cardinality as the set S
    F c) Cardinality of a set is never equal to its proper subset
    F d) Cardinality of a set is never equal to its proper superset
    F e) There may exist a set S such that power set of S has the same cardinality as the set S
    T f) Cardinality of power set of a set is never equal to the cardinality of the set itself.
22. If there exists injection A→B, then
    a) Cardinality of A is less than the cardinality of B
    b) Cardinality of A is less than or equal to the cardinality of B
    c) Cardinality of A is equal to the cardinality of B
    d) Nothing can be concluded about the cardinality of A and B
23. If there exists injection A→B, then
    a) A may be a subset of B
    b) B maybe a subset of A
    c) A maybe a proper subset of B
    d) B may be a proper subset of A
24. If there exists bijection A→B, then
    a) A and B must have the same cardinality
    b) A and B may have different cardinality
    c) $|A| \le |B|$ and $|A| \ge |B|$
    d) Nothing can be concluded about the cardinality of A and B
25. If there exists a bijection A→B, then
    a) A may be a subset of B
    b) B may be a subset of A
    c) A may be a proper subset of B
    d) B may be a proper subset of A
26. Let there exists injection f: A→B such that f(A) is a proper subset of B, then
    a) Cardinality of A may be less than the cardinality of B
    b) Cardinality of A must be less than the cardinality of B
    c) A and B may have equal cardinality
    d) Cardinalities of A and B must not be equal

27. Let there exists injection f: A→B such that f(A) is a proper subset of B, then

a) A and B both may be finite sets
b) A and B both may be infinite sets
c) A and B may be equivalent sets
d) A and B may be equal sets

28. Let there exists injection f:S→S such that f(S) is a proper subset of S, then

a) S may be a finite set
b) S must be a finite set
c) S may be an infinite set
d) S must be an infinite set

29. Which of the following statements are true?

F a) A countable set is always finite.
F b) A countable set is always infinite.
T c) A countable set may be finite.
T d) A countable set may be infinite
T e) A finite set is always countable.
F f) An infinite set is always countable.
T g) A finite set may be countable.
T h) An infinite set may be uncountable.
F i) An uncountable set is always finite
T j) An uncountable set is always infinite
F k) An uncountable set may be finite
T l) An uncountable set may be infinite
T m) Elements of a finite set can be always counted
F n) Elements of an infinite set can be always counted
T o) Elements of an infinite set may be counted
T p) Elements of every countable set can be counted
F q) Elements of every uncountable set can be counted
F r) Elements of an uncountable set may be counted

30. Cardinality of every countable set is

a) Less than the cardinality of natural numbers
b) Equal to the cardinality of natural numbers
c) Less than or equal to the cardinality of the natural numbers
d) Either one of (a), (b) or (c) may be true depending upon the set