Q.11 A dice is rolled 180 times using normal distribution find the probability that face 4 will turn up atleast 35 times. Given that area $(0 < z < 1) = 0.3413$

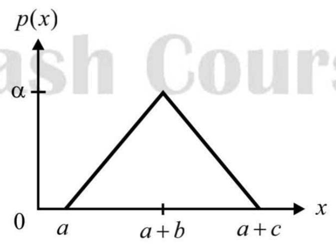
(A) 0.3413 (B) 0.5 (C) 0.15 (D) 0.513

Q.12 In a sample of 100 students, the mean of the marks (only integers) obtained by them in a test is 14 with its standard deviation of 2.5 (marks obtained can be fitted with a normal distribution). The percentage of students scoring 16 marks is

(A) 36 (B) 23 (C) 12 (D) 10

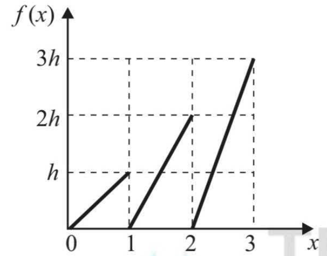
(Area under standard normal curve between $z = 0$ and $z = 0.6$ is 0.2257 and between $z = 0$ and $z = 1.0$ is 0.3413)

Q.13 Probability density function differential function $p(x)$ of a random variable x is as shown below. The value of α is



(A) $\dfrac{2}{c}$ (B) $\dfrac{1}{c}$ (C) $\dfrac{2}{(b+c)}$ (D) $\dfrac{1}{(b+c)}$

Q.14 The graph of a function $f(x)$ is shown in the figure



For $f(x)$ to be a valid probability density function, the value of h is

(A) 1/3 (B) 2/3 (C) 1 (D) 3

Q.15 Consider the continuous random variable with probability density function

$$f(t) = 1 + t \text{ for } -1 \le t \le 0$$
$$= 1 - t \text{ for } 0 \le t \le 1$$

The standard deviation of the random variable is

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{6}}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$

Q.16 Two independent random variables X and Y are uniformly distributed in the interval [–1, 1]. The probability that max $[X, Y]$ is less than $\frac{1}{2}$ is

(A) $\frac{3}{4}$ (B) $\frac{9}{16}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$

Q.17 Let X and Y are two random variables given that $E(X) = 10$, $Var(X) = 25$. Then find value of a and b such that both are greater than zero for the line $Y = aX - b$ with the help of $E(Y) = 0$ and $Var(Y) = 1$

(A) 1, 10 (B) $\frac{1}{5}, 2$ (C) $\frac{1}{3}, \frac{10}{3}$ (D) 1, 1

Q.18 If E denotes expectation, the variance of a random variable X is given by

(A) $E[X^2] - E^2[X]$ (B) $E[X^2] + E^2[X]$ (C) $E[X^2]$ (D) $E^2[X]$

Q.19 $P_x(x) = Me^{-2|x|} + Ne^{-3|x|}$ is the probability density function for the real random variable X over the entire x axis. M and N are both positive real numbers. The equation relating M and N is

(A) $M + \frac{2}{3}N = 1$ (B) $2M + \frac{1}{3}N = 1$ (C) $M + N = 1$ (D) $M + N = 3$

Q.20 A six-faced fair dice is rolled five times. Then probability (in %) of obtaining 'ONE' at least four times is

(A) 33.3 (B) 3.33 (C) 0.33 (D) 0.0033

Q.21 X and Y are two independent random variables with variances 1 and 2, respectively. Let $Z = X - Y$. The variance of Z is

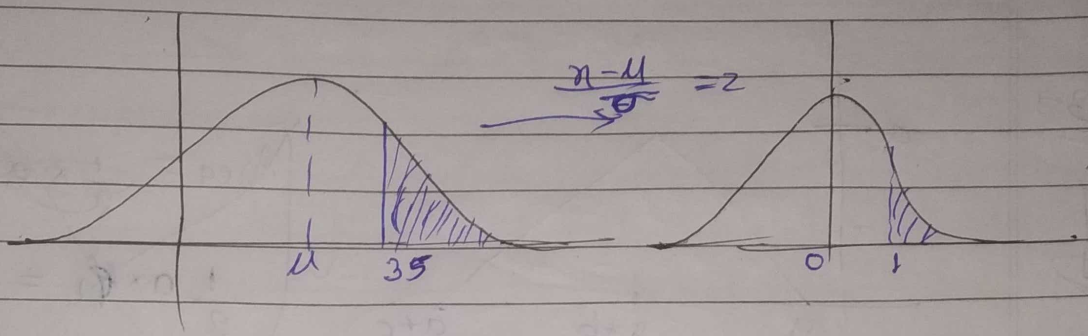
(A) 0 (B) 1 (C) 2 (D) 3

Q.22 An examination paper has 150 multiple-choice questions of one mark each, with each question having four choices. Each incorrect answer fetches – 0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all these students is

(A) 0 (B) 2550 (C) 7525 (D) 9375

11 ⇒ $n = 180$, Sample-space for dice $= \{1, 2, 3, 4, 5, 6\}$

(i) $P(4) = 1/6$, $\mu = np = 180 \times \frac{1}{6} = 30$



$\sigma = \sqrt{npq}$

$= \sqrt{180 \times \frac{1}{6} \times \frac{5}{6}}$

$= 5$

$, Z = \frac{35-30}{5} = 1$

$P(x \geq 35) = P(z \geq 1)$

$= 0.5 - \text{Area } (0 \,\&\, 1)$

$= 0.5 - 0.3413$

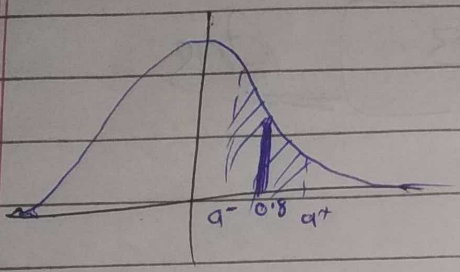
$= \boxed{0.15}$

12 ⇒ $n = 100, \quad m = 14, \quad \sigma = 2.5$

C

$$\frac{x - \mu}{\sigma} = z$$

$$\frac{16 - 14}{2.5} = 0.8 = z$$

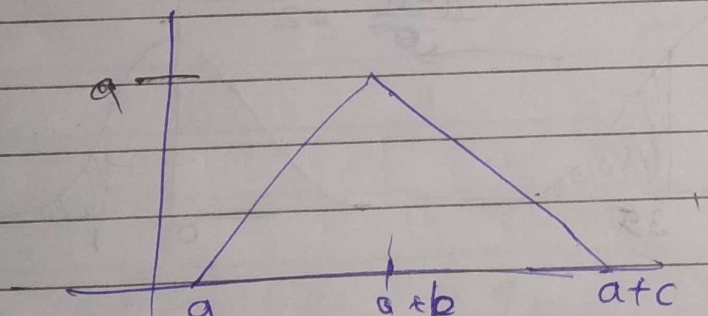


$$P(z=a) = P(0 \text{ to } a^+) - P(0 \text{ to } a^-)$$
$$= 0.3413 - 0.2257$$
$$= 0.1156 \times 100$$
$$= 11.56\%$$
$$= 12\%$$

Ans

13 ⇒

A



~~Area = $\frac{1}{2} \times a \times 1$~~

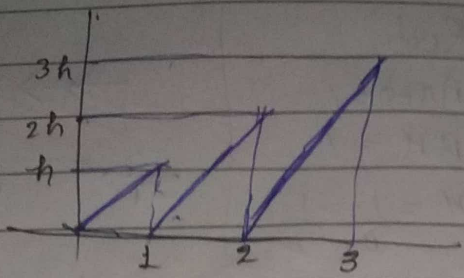
~~$\frac{1}{2} a \times h = 1$~~

~~$h =$~~

$$\text{Area} = 1$$
$$\frac{1}{2} BH = 1$$
$$H = \frac{2}{B} = \frac{2}{(a+c) - a}$$
$$\boxed{H = \frac{2}{c}}$$

14⇒

A



Area = 1

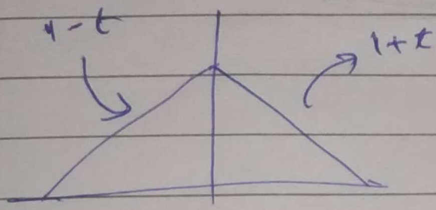
$\frac{1}{2}[h+2h+3h] = 1$

$h = 1/3$ Ans

15⇒ $f(t) = 1+t \quad , \quad -1 \leq t \leq 0$

$= 1-t \quad , \quad 0 \leq t \leq 1$

B



$SD = \sqrt{Var(x)}$

$= \sqrt{msv - (mean)^2}$

$msv = \int_{-\infty}^{\infty} t^2 f(t)\, dt.$

$= \int_{-1}^{0} [t^2(1+t)]\,dt + \int_{0}^{1} [t^2[1-t]]\,dt$

$= 1/6$

$SD = \sqrt{1/6 - 0}$

$SD = \frac{1}{\sqrt{6}}$ Ans

16 =

B

Green

$P(x \leq 1/2) = BH$

$= (1 + 1/2) \times \frac{1}{2}$

$= 3/2 \times 1/2$

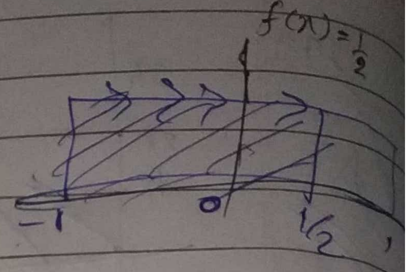
$= 3/4$

Red

Area = 1

BH = 1

$H = \frac{1}{B} = \frac{1}{2}$



$= P[(x \leq 1/2)(y \leq 1/2)]$

$= \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$ Ans

17 =

B

$Y = ax - b$

$E(Y) = E(ax) - b$

$0 = a \times 10 - b$

$b = 10a$ —— (1)

$Var(Y) = a^2 \, var(x)$

$1 = a^2 \times 25$

$a = \frac{1}{5}$

$b = 2$

18 ⇒ $var = MSV - (mean)^2$

(A)

$$= E(x^2) - E^2(x)$$ Ans

19 ⇒ $P_x(x) = Me^{-2|x|} + Ne^{-3|x|}$

(A)

$$= \int_{-\infty}^{\infty} P_x(x)\,dx = 1$$

$$2 \times \int_{0}^{\infty} P_x(x)\,dx = 1$$

$$\int_{0}^{\infty} (Me^{-2x} + N.e^{-3x})\,dx = \frac{1}{2}$$

$$\left[\frac{M.e^{-2x}}{-2} + \frac{N.e^{-3x}}{-3}\right]_{0}^{\infty} = \frac{1}{2}$$

$$\frac{M}{2} + \frac{N}{3} = \frac{1}{2}$$

$$\boxed{M + \frac{2N}{3} = 1}$$ Ans

20 ⟹ $p = 1/6 \rightarrow$ getting 1

$q = 5/6 \longrightarrow$ not getting 1

$n = 5$

(c)

$$P(x \geq 4) = {}^nC_4\, p^4 \cdot q^{n-4} + {}^nC_5 \cdot p^5 \cdot q^{n-5}$$

$$= {}^5C_4 \times (1/6)^4 (5/6)^1 + {}^5C_5 (1/6)^5$$

$$= 3.343 \times 10^{-3}$$

$$\doteq \boxed{0.33\ \%}$$ Ans

21 ⟹ $Var(x) = 1$, $Var(y) = 2$

(D)

$$Z = X - Y = X + (-y)$$

$$Var(z) = Var(x) + Var(-y)$$

$$= 1 + (-1)^2 \times 2$$

$$= 1 + 2$$

$$\boxed{Var(z) = 3}$$ Ans

22 ⇒

Mark	1	-1/4
Probability	1/4	3/4

$\boxed{D}$

$E(\text{Marks}) = \Sigma\, x \cdot P(x)$

$= 1/4 + (-1/4) \times 3/4$

$= 1/16$

E (Marks) for 1 student →

$= \frac{150}{16} = 9.375$

For 1000 students –

$= 9.375 \times 1000 = \boxed{9375}$ Ans