Relations

Q.1 How many different elements does A×B have if A has m elements and B has n elements?

Q.2 Let A={1,2} Construct the set P(A) × A.

Q.3 Which of the following cardinalities are not possible for A×B? Given that |A|, |B| ≠ 1

a) 6
b) 2
c) 21

Q.4 The elements of A×B are

a) unordered pairs
b) ordered pairs
c) unordered
d) ordered

Q.5 If A×B = ϕ, then

a) Both A and B are empty sets
b) A and B both may be non-empty sets
c) Atleast one of A and B is an empty set
d) Atleast one of A and B is a non-empty set

Q.6 Which of the following statements are true?

a. (A×B)×C=A×(B×C)
b. A×B×C = (A×B)×C
c. |A×B×C| = |(A×B)×C|
d. A×B×C×D = (A×B)×(C×D)

Q.7 If A has m elements, B has n elements and C has p elements then find the cardinality of the following sets?

a) P(A×B)
b) P(A) × P(B)
c) A×B×C
d) P(A×B)×P(C)

Q.8 If A×B is a infinite set then

a) A and B both may be finite sets
b) A and B both are infinite sets
c) Atleast one of A and B is an infinite set
d) Atleast one of A and B is a finite set

Q.9 If A×B×C is a infinite set, then

a) A, B and C are infinite sets
b) A, B and C may be finite sets
c) Atleast one of A, B and C must be a infinite set
d) Atleast two of A, B and C must be a infinite set

Q.10 If A×B = B ×A, then

a) Atleast one of A and B may be empty set
b) Both A and B must be empty sets
c) A and B may be equal sets
d) A ⊆B and B ⊆A

Q.11 Let Z^+ and Z^- denote the set of positive integers and set of negative integers respectively

$Z^+ \times Z^- = \{(a,b) \mid a \in A \text{ and } b \in B\}$

then find the value of

$\Sigma(a+b) =$ 0

$Z^+ \times Z^- = \{1,2,\dots\} \times \{-1,-2,-3,\dots\}$

$= \{(1,-1),(1,-2),\dots$

$(2,-1),(2,-2),\dots\}$

Q.12 Let $A \times B = \{(a,\phi),(b,\phi),(c,\phi)\}$ then cardinality of A and B are respectively

a) 3 and 0

b) 4 and 1

c) 3 and 1

d) Can't be determined

$A = \{a,b,c\} \Rightarrow |A| = 3$

$B = \{\phi,\phi,\phi\} = \{\phi\} \Rightarrow |B| = 1$

Q.13 List the ordered pairs in the relation from

$A = \{0,1,2,3,4\}$ to $B = \{0,1,2,3\}$, where $(a,b) \in R$ if and only if

a) $a = b \Rightarrow \{(0,0),(1,1),(2,2),(3,3)\}$

b) $a + b = 4 \Rightarrow \{(1,3),(2,2),(3,1),(4,0)\}$

c) $2 < ab < 8 \Rightarrow \{(1,3),(2,2),(2,3),(3,1),(3,2),(3,1)\}$

d) $b \bmod a = 1 \Rightarrow \{(2,1),(2,3),(3,1),(4,1)\}$

e) $a = b \pmod 2 \Rightarrow \{(0,0),(0,2),(1,1),(1,3),(2,0),(2,2),(3,1),(3,3),(3,0),(4,2)\}$

f) $\gcd(a,b) = 1 \Rightarrow \{(0,1),(1,0),(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,1),(3,3)\}$

g) $\text{lcm}(a,b) = 2 \Rightarrow \{(1,2),(2,1),(2,2)\}$

Q.14 If A has m elements and B has n elements, then find the following, for the relations A to B

a) Total number of relations possible $\Rightarrow |A \times B| = 2^{mn}$

b) Maximum and minimum cardinality possible for a relation $\Rightarrow$ max $= |A \times B| = mn$, min $= |\phi| = 0$

c) Number of relations possible of cardinality p $\Rightarrow {}^{mn}C_p$

Q.15 How many relations are there on the set {a,b,c,d} that contain the pair (a,a)? $\Rightarrow 2^{16-1} = 2^{15}$

Q.16 $A = \{4,3,2\}$

$B = \{9,7\}$

$R_1 = \left\{\left(\frac{4e^2}{9},7\right),\left(\frac{b^2}{12},7\right),(2,7),(3,9)\right\}$

$R_2 = \left\{\left(\frac{a}{2},7\right),(c^2,d^2),(3,9),(2,7),(4,7)\right\}$

R_1 and R_2 are relations A to B such that $R_1 \subset R_2$ and $|R_2| = 5$

Find the possible integral values for a, b, c, d and e.

a = 6
b = 6
c = 2
d = 3
e = 3

$A \times B = \{(4,9),(4,7),(3,9),(3,7),(2,9),(2,7)\}$

$R_2 \subseteq A \times B \Rightarrow R_2 = \{(a/2,7),(c^2,d^2),(3,9),(2,7),(4,7)\}$

$(a/2,7) = (3,7) \Rightarrow a = 6$

$(c^2,d^2) = (4,9)$ or $(2,4) \Rightarrow c^2 = 4 \Rightarrow c = 2$, $d^2 = 9 \Rightarrow d = 3$

$R_1 \subset R_2$ $(3,7)$

$\left(\frac{a}{2},7\right),(4,7) = \left(\frac{4e^2}{9},7\right),\left(\frac{b^2}{12},7\right) \Rightarrow \left(\frac{4e^2}{9},7\right) = (4,7)$

$\left(\frac{b^2}{12},7\right) = (4,7)$ $(3,7) \Rightarrow b^2 = 36 \Rightarrow b = 6$

$e^2 = 9 \Rightarrow e = 3$

Q.17 X = {1,2,3}
Y={2,4,7,9}
R={(x,y) | xεX, yεY, x and y are coprime}
Represent the relation R in the following ways: --

a) Listing
b) Table method
c) Matrix
d) Directed graph
e) On X and Y axes(graph)

Q.18 Let X be the "less than" relation on the set of real numbers and Y be the "greater than" relation on the set of real numbers, that is X={(x,y) | x<y} and Y = {(x,y) | x>y}. Find the following:

a) $X \cup Y$
b) $X \cap Y$
c) X-Y
d) Y-X
e) $X \Delta Y$
f) $\bar{X}$
g) $\bar{Y}$
h) X^{-1}
i) Y^{-1}

Q.19 Which of the following statements are true?

a) $|R| = |R^{-1}|$
b) $|R| = |\bar{R}|$
c) $(R_1 \cup R_2)^{-1} = (R_1)^{-1} \cup (R_2)^{-1}$
d) $(R_1 \cap R_2)^{-1} = (R_1)^{-1} \cap (R_2)^{-1}$
e) $(R_1 - R_2)^{-1} = (R_1)^{-1} - (R_2)^{-1}$
f) $(R1 \Delta R2)^{-1} = (R_1)^{-1} \Delta (R_2)^{-1}$

Q.20 Let A = { 1,2, 3, 4, 5}. if R = { (a,b) | a ∈ A ∧ b ∈ A ∧ ((a –b) is an integral non zero multiple of 2)} and S = { (a,b) | a ∈ A ∧ b ∈ A ∧ ((a –b) is an integral non zero multiple of 3)}, (a)find R ∪ S and R ∩ S.

(b) If A = {1, 2, 3, ...}, what is R ∩ S for R and S as defined in (a)?

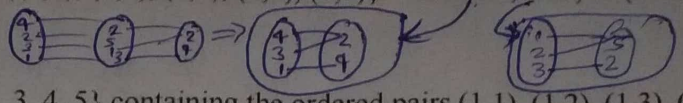
Q.21 Let R be the relation A to B and $\bar{R}$ be the complement of relation R. Given that $|R| = 7$ and $|\bar{R}| = 11$. Find the possible no. values of the ordered pair(|A|, |B|)

Q.22 Let R be a relation A to B such that every other relation R_0, A to B satisfies the condition $R \cup R_0 = R$, What is the cardinality of R?
Given that |A| =3 and |B| =7

Q.23 Let R be a relation A to B such that every other relation R_0, A to B satisfies the condition $R \cap R_0 = R$, What is the cardinality of R?
Given that |A| =3 and |B| =7

Q.24 Let R = {(1,2), (3,4), (2,2)} and S = {(4,2), (2,5), (3,1), (1,3), (1,3)}. Find R∘S, S∘R, R∘(S∘R), (R∘S)∘R, R∘R, S∘S and R∘R∘R.

R∘S



Q.25 Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1,1). (1,2), (1,3), (2,3), (2,4), (3,1), (3,4), (3,5), (4,2), (4,5), (5,1), (5,2), (5,4). Find

a) R^2 → R∘R

b) R^3 → R∘R∘R = R^2∘R

c) R^4 → R^3∘R

d) R^5 → R^4∘R

Q.26 Let P = { (1,2), (2,4), (3,3)} and Q = {(1,3), (2,4),(4,2)}. Find P∪Q, P∩Q, D(P), D(Q), D(P∪Q), R(P),R(Q) and R (P∩Q). Show that

$D(P \cup Q) = D(P) \cup D(Q)$

$R(P \cap Q) \subseteq R(P) \cap R(Q)$

P∪Q = {(1,2), (1,3) (2,4), (3,3), (4,2)}

P∩Q = {(2,4)}

D(P) = {1, 2, 3, 4}

D(Q) = {1, 2, 4}

D(P∪Q) = {1, 2, 3, 4}

R(P) = {2, 3, 4}

R(Q) = {2, 3, 4}

R(P∩Q) = {4}

Q.27 What are the range of the relations

$S = \{\langle x, x^2 \rangle \mid x \in N\}$ and $T = \{\langle x, 2x \rangle \mid x \in N\}$

Where N = {0, 1, 2, ...}?

R(S) = {x²}

R(T) = {2x}