Chapter 2 – Probability

Q.22 In a housing society, half of the families have a single child per family while the remaining half have two children per family. The probability that a child picked at random, has a sibling is ___________.

Q.23 Parcels from sender S to receiver R pass sequentially through two post-offices. Each post-office has a probability $\frac{1}{5}$ of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-office is_________.

Q.24 The probabilities of occurrence of events F and G are $P(F) = 0.3$ and $P(G) = 0.4$, respectively. The probability that both events occur simultaneously is $P(F \cap G) = 0.2$. The probability of occurrence of at least one event $P(F \cup G)$ is _________.

Q.25 The probability that a student knows the correct answer to a multiple choice question is 2/3. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is 1/4. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is

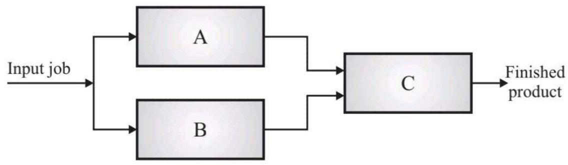
(A) $\frac{2}{3}$ (B) $\frac{3}{4}$

(C) $\frac{5}{6}$ (D) $\frac{8}{9}$

Q.26 Consider two identical bags B_1 and B_2 each containing 10 balls of identical shapes and sizes. Bag B_1 contains 7 Red and 3 Green balls, while bag B_2 contains 3 Red and 7 Green balls. A bag is picked at random and a ball is drawn from it, which was found to be Red. The probability that the Red ball came from bag B_1 (rounded off to one decimal place) is __________.

Q.27 The figure shows the schematic of a production process with machines A, B and C. An input job needs to be pre-processed either by A or by B before it is fed to C, from which the final finished product comes out. The probabilities of failure of the machines are given as,

$$P(A) = 0.15, \ P(B) = 0.05, \ P(C) = 0.1$$



Assuming independence of failures of the machines, the probability that a given job is successfully processed (up to the third decimal place) is _______.

Q.28 A screening test is carried out to detect a certain disease. It is found that 12 % of the positive reports and 15% of the negative reports are incorrect. Assuming that the probability of a person getting a positive report is 0.01, the probability that a person tested gets an incorrect report is

(A) 0.0027 (B) 0.0173

(C) 0.1497 (D) 0.2100

Q.29 The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

(A) 0.029 (B) 0.034

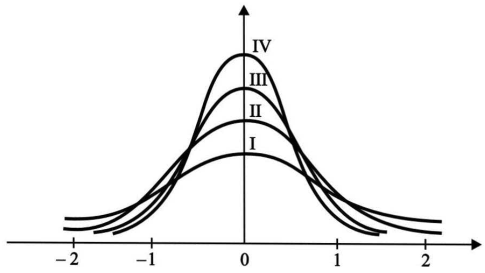
(C) 0.039 (D) 0.044

Q.30 A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is

(A) 0.0036 (B) 0.1937

(C) 0.2234 (D) 0.3874

Q.31 Among the four normal distribution with probability density functions as shown below, which one has the lowest variance?



(A) I (B) II

(C) III (D) IV

Q.32 Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

(A) $\sqrt{\mu}$ (B) μ^2

(C) μ (D) $\frac{1}{\mu}$

Q.33 Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25 respectively. Then $Y = \min(X_1, X_2)$ is

(A) Exponentially distributed with mean 1/6.

(B) Exponentially distributed with mean 2.

(C) Normally distributed with mean 3/4.

(D) Normally distributed with mean 1/6.

Q.34 If probability density function of a random variable x is

$$f(x) = \begin{cases} x^2, & -1 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Then, the percentage probability $P\left(\frac{-1}{3} \le x \le \frac{1}{3}\right)$ is

(A) 0.274 (B) 2.47

(C) 24.7 (D) 247

Q.35 The two sides of a *fair* coin are labelled as 0 and 1. The coin is tossed two times independently. Let M and N denote the labels corresponding to the outcomes of those tosses. For a random variable X, defined as $X = \min(M, N)$, the expected value $E[X]$ (rounded off to two decimal places) is ___________.

Q.36 In the following table, X is a discrete random variable and $P(x)$ is the probability density. The standard deviation of X is

x	1	2	3
$P(x)$	0.3	0.6	0.1

(A) 0.18 (B) 0.36

(C) 0.54 (D) 0.6

Q.37 A machine produces 0, 1 or 2 defective pieces in a day with associated probability of $\frac{1}{6}$, $\frac{2}{3}$ and $\frac{1}{6}$, respectively. The mean value and the variance of the number of defective pieces produced by the machine in a day respectively, are

(A) 1 and $\frac{1}{3}$ (B) $\frac{1}{3}$ and 1

(C) 1 and $\frac{4}{3}$ (D) $\frac{1}{3}$ and $\frac{4}{3}$

Q.38 Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is False?

(A) $E(XY) = E(X)\,E(Y)$

(B) $\mathrm{Cov}(X, Y) = 0$

(C) $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$

(D) $E(X^2Y^2) = (E(X))^2\,(E(Y))^2$

Q.39 The function $p(x)$ is given by $p(x) = A / x^{\mu}$ where A and μ are constants with $\mu > 1$ and $1 \le x < \infty$ and $p(x) = 0$ for $-\infty < x < 1$. For $p(x)$ to be probability density function, the value of A should be equal to

(A) $\mu - 1$ (B) $\mu + 1$

(C) $1/(\mu - 1)$ (D) $1/(\mu + 1)$

Q.40 If x is a random variable with the expected value of 5 and the variance of 1, then the expected value of x^2 is

(A) 36 (B) 26

(C) 25 (D) 24

(22) →

$$\text{Family} \begin{cases} \xrightarrow{f_1} \text{1 Child} \\ \xrightarrow{f_2} \text{2 Child (Sibling)} \end{cases}$$

$$P(\text{choose sibling}) = \frac{\text{Favourable}}{\text{Total}}$$

$$= \frac{2 \times n/2}{1 \times n/2 + 2 \times n/2}$$

$$= \frac{\cancel{n}}{3\cancel{n}/2}$$

$$P = 2/3$$ Ans

(23) →

$$\text{S} \text{ --- } \boxed{\text{PO-1}} \text{ --- } \boxed{\text{PO-2}} \text{ --- } \text{R}$$

PO-1: Lossy = 1/5, pass = 4/5

PO-2: Lossy = 1/5

$$P(\text{Lost}) = 1/5 + (4/5 \times 1/5) = 9/25$$

$$P\left(\frac{\text{PO-2}}{\text{Lost}}\right) = \frac{P[\text{PO-2} \cap \text{Lost}]}{P[\text{Lost}]} = \frac{4/25}{9/25}$$

$$P\left(\frac{\text{PO-2}}{\text{Lost}}\right) = 4/9$$ Ans

24 $\Rightarrow$ $P(F) = 0.3$, $P(G) = 0.4$, $P(F \cap G) = 0.2$

$P(F \cup G) = ?$

$= P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.2$

$= 0.5$ Ans

25 $\Rightarrow$ $P(KC) = 2/3$

$P(\overline{KC}) = 1/3$

D

$P(\overline{KC} \cap G) = 1/3 \times 1/4 = 1/12$

$P(G) = P(KC) + P(\overline{KC} \cap G)$

$= 2/3 + 1/12$

$= 9/12$

$P\left(\frac{KC}{G}\right) = \frac{P(KC \cap G)}{P(G)} = \frac{2/3}{9/12}$

$= \frac{8}{9}$ Ans

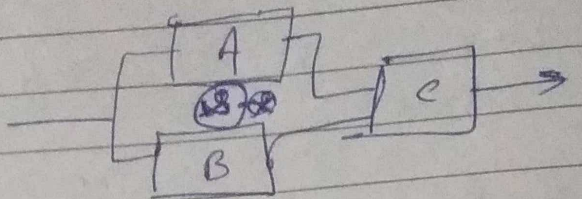
26 $\Rightarrow$

B_1	B_2
7 R, 3 G	3 R, 7 G
10	10

$P(R) = 1/2 * 7/10 + 1/2 \times 3/10 = 1/2$

$P\left(\frac{B_1}{R}\right) = \frac{P(B_1 \cap R)}{P(R)} = \frac{7/20}{1/2} = \frac{7}{10}$ Ans

27



$P(\text{Success}) = [1 - P(A)\cdot P(B)] \times [1 - P(C)]$

$= 0.9925 \times 0.9$

$= 0.89325$ Ans

28

2 Cases — { 12% incurred .. (+ve) ; 15% " (−ve) }

Positive, Negative

$\boxed{C}$

$\frac{P}{N+P} = 0.01 = \frac{1}{100}$

$100P = N + P$

$99P = N$

c1.

$12 + (15 \times 99)$

$= \frac{12}{100} \times \frac{1}{100} + \frac{15}{100} \times \frac{99}{100}$

$= 0.1497$

Ans

29

B

$P(r=0,1) = \frac{e^{-m}\cdot m^{r}}{r!}$

$\left\{ \frac{e^{-m}\cdot m^{r}}{r!} \cdot m = 5.2 \right.$

$P(r<2) = P(0) + P(1)$

$= e^{-5.2} + 5\cdot e^{-5.2}$

$= 6 \times e^{-5.3}$

$= 0.34$

Ans

30 → B

$$p(x=2) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

$$= {}^{10}C_{2} \times (1/10)^2 \times (9/10)^8$$

$$= 0.1937$$

Ans

31 → D

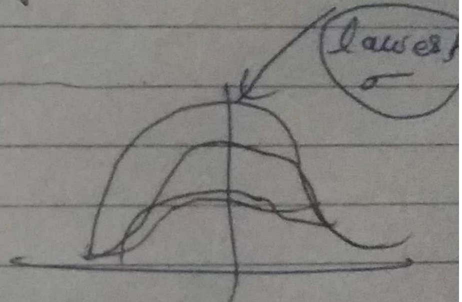
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$f(x)\Big]_{max} = \frac{1}{\sigma\sqrt{2\pi}}$$

peak $\propto \frac{1}{\sigma}$

$\sigma \downarrow$, peak $\uparrow$



IV Ans

32 → A

$$\sigma = \sqrt{4}$$

Ans

33 $\lambda = 1/2,\ 1/4$

A $f(x) = \frac{1}{\lambda} e^{-x/\lambda}$

Exponentially distributed with mean 16

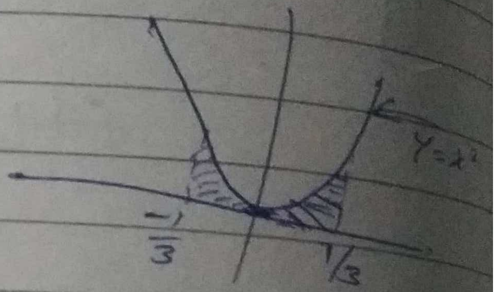
Ans

34 $P(x) = \int_{-1}^{1} x^2\,dx$

B $= K\left[\frac{x^3}{3}\right]_{-1}^{1}$

$= \frac{2K}{3} = 1$

$K = 3/2$



$P\left(-1/3 < x < 1/3\right) = \int_{-1/3}^{1/3} x^2\,dx$

$= 2.47$ Ans

35 ⇒

0, 0	0, 1	1, 0	1, 1	
0	0	0	1	→ X
1/4	1/4	1/4	1/4	→ p(x)

$$E(x) = \sum x \cdot p(x)$$
$$= 0 + 0 + 0 + 1 \times 1/4$$
$$= 1/4$$ Ans

36 ⇒

x	1	2	3
$p(x)$	0.3	0.6	0.1
x^2	1	2^2	3^2

$$SD = \sqrt{E(x^2) - (\text{mean})^2} \quad \text{mean} \to E(x)$$
=

$$E(x) = \sum x \cdot p(x) = 0.3 + 1.2 + 0.3 = 1.8$$
$$E(x^2) = \sum x^2 \cdot p(x^2) = 5.4$$

$$\cancel{SD = \sqrt{(0.3 + 2.4 + 0.9) - (0.3 + 1.2 + 0.3)}}$$

$$SD = \sqrt{5.4 - 1.8}$$
$$= \sqrt{3.6}$$
$$= 0.6$$ Ans

37 ⇒

x	0	1	2
$p(x)$	1/6	2/3	1/6

A

$$E(x) = \sum x \cdot p(x)$$
$$= 0 + 2/3 + 1/3$$
$$= 1$$

Varience = 1/3

38 ⇒ False

D

~~$E(x^2 \cdot y^2) = E x$~~

$$E(x^2 \cdot y^2) = [E(x)]^2 \cdot (E(y))^2$$

39 ⇒

A

$$\int_1^\infty \frac{A}{x^u}\,dx = 1$$

$$A \int_1^\infty x^{-u}\,dx = 1$$

$$A \cdot \left[\frac{x^{-u+1}}{-u+1}\right]_1^\infty = 1$$

$$A\left[\frac{1}{\infty} - \frac{1}{1-u}\right] = 1$$

$$A = u - 1$$ Ans

40 ⇒ $E(x) = 5$

$E(x^2) = msv$

B $Var(x) = 1$

$Var(x) = E(x^2) - (E(x))^2$

$1 = E(x^2) - 5^2$

$E(x^2) = 26$ Ans