

# Discrete Mathematics

## (Set Theory)

### Section 1

Q.1 Which of the following are not sets?

- i. Set of good students ✓
- ii. Set of students who scored above 60 percent in annual exams ✓
- iii. Set of teachers whose above 50 percent student passed in the final exams
- iv. Set of teachers who taught very good last year ✓
- v. Set of all the tall teachers in Hyderabad ✓
- vi. Set of all the beautiful girls in IIT Delhi ✓
- vii. Set of all the decent boys in IIT Delhi ✓
- viii. Set of all honest politicians in India ✓
- ix. Set of all known planets ✓
- x. Set of all Intelligent students with IQ above 160 ✓

Q.2 Determine whether each of these pair of sets are equal

- i.  $\{3,3,3,7,3,7,3,5,5\}, \{3,7,5,5,5\}$
- ii.  $\{1,1,\{1\}\}, \{1\}$
- iii.  $\{\}, \{x \mid x \text{ is the man living on sun}\}$
- iv.  $\{1,\{1\},\{1,\{1\}\}\}, \{1,\{1,\{1\}\}\}$
- v.  $\{x \mid x \text{ is a prime number}\}$   
 $\{x \mid x \text{ is a odd prime number}\}$

Q.3 Represent the following sets in the Roster form

- i.  $\{x \mid x \text{ is a real number such that } x^2 = 16\}$   $\rightarrow \{-4, 4\}$
- ii.  $\{x \mid x \text{ is a non-negative integer less than 13}\}$   $\rightarrow \{0, 1, 2, 3, 4, 5, \dots, 12\}$
- iii.  $\{x \mid x \text{ is an integer such that } x^2 = 7\}$   $\rightarrow \{\}$
- iv.  $\{x \mid 3 < x < 7, x \in \mathbb{Z}\}$   $\rightarrow \{4, 5, 6\}$
- v.  $\{x \mid 2 < x^2 < 26, 2x + 3 = 11\}$   $\rightarrow \{4\}$
- vi.  $\{x \mid x^2 = 36, 2x + 3 = 9\}$   $\rightarrow \{3\}$
- vii.  $\{x \mid x = 2n, n \in \mathbb{N}, x \text{ is prime}\}$   $\rightarrow \{2\}$
- viii.  $\{x \mid x = \{n\}, n \in \mathbb{N}, n < 8\}$   $\rightarrow \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}\}$
- ix.  $\{x \mid x = \{2n+1\}, n \in \mathbb{N}, n < 5\}$   $\rightarrow \{\{1\}, \{3\}, \{5\}, \{7\}\}$
- x.  $\{x \mid x = \{ \{n\}, \{n+1\} \}, n \in \mathbb{N}, n < 4\}$   $\rightarrow \{\{\{1\}, \{2\}\}, \{\{2\}, \{3\}\}, \{\{3\}, \{4\}\}\}$

Q.4 Represent the following sets in the property method

- $\{0, 3, 6, 9, 12\} \rightarrow \{x \mid x=3n, n \in \mathbb{Z}, 0 \leq n \leq 4\}$
- $\{-4, -3, -2, -1, 0, 1, 2, 3\} \rightarrow \{x \mid -5 < x < 4, x \in \mathbb{Z}\}$
- $\{2, 3, 5, 7, 11, 13, \dots\} \rightarrow \{x \mid x \text{ is +ve prime no.}\}$
- $\{7, 11, 15, 19, \dots\} \rightarrow \{x \mid x=4n+3, n \in \mathbb{N}\}$
- $\{1, 2, 4, 8, 16, \dots\} \rightarrow \{x \mid x=2^{n+1}, n \in \mathbb{N}\}$

Q.5 Which of the following sets cannot be represented by property method directly?

- $\{\text{Cow, buffalo, goat, } \dots\}$
- $\{\text{Ramesh, Pen, 7, z}\}$
- $\{\text{Snow, cellphone, sun, chair}\}$

Q.6 For each of the following sets, determine whether 2 and  $\{2\}$  is an element of that set

- $\{x \in \mathbb{R} \mid x \text{ is an integer greater than 1}\} \rightarrow \{2, 3, 4, \dots\} \rightarrow 2(\checkmark), \{2\}(X)$
- $\{x \in \mathbb{R} \mid x \text{ is the square of an integer}\} \rightarrow \{0, 1, 4, 9, 16, \dots\} \rightarrow 2(X), \{2\}(X)$
- $\{2, \{2\}\} \rightarrow 2(\checkmark), \{2\}(\checkmark)$
- $\{\{2\}, \{\{2\}\}\} \rightarrow 2(X), \{2\}(\checkmark)$
- $\{\{2\}, \{2, \{2\}\}\} \rightarrow 2(X), \{2\}(\checkmark)$
- $\{\{\{2\}\}\} \rightarrow 2(X), \{2\}(X)$

Q.7 What is the cardinality of the following sets?

- $\{r\} \rightarrow 1$
- $\{\{r\}\} \rightarrow 1$
- $\{r, \{r\}\} \rightarrow 2$
- $\{r, \{r\}, \{r, \{r\}\}\} \rightarrow 3$
- $\emptyset \rightarrow 0$
- $\{\emptyset\} \rightarrow 1$
- $\{\emptyset, \emptyset\} \rightarrow 1$
- $\{\emptyset, \{\emptyset\}, \{\emptyset\}\} \rightarrow 2$
- $\{\emptyset, \{\emptyset\}, \{\emptyset, \emptyset, \{\emptyset\}\}\} \rightarrow 3$
- $\{x \mid x = \{\{n-1\}, \{n\}\}, n \in \mathbb{N}, n < 5\} \rightarrow 4$

$n = 1, 2, 3, 4$   
 $\{\emptyset, \{0\}\}, \{\emptyset, \{1\}\}, \{\emptyset, \{2\}\}, \{\emptyset, \{3\}\}$   
 $\{\{0\}, \{1\}\}, \{\{1\}, \{2\}\}, \{\{2\}, \{3\}\}$   
 $\{\emptyset, \{0, \{1\}\}\}, \{\emptyset, \{1, \{2\}\}\}, \{\emptyset, \{2, \{3\}\}\}$

Q.8 Identify the equal and equivalent sets from the following

- $\{x \mid x \neq 2n, x \neq 2n+1, n \in \mathbb{N}, x \in \mathbb{N}\} \rightarrow \{4\} \rightarrow 0$
- $\{\emptyset\} \rightarrow 1$
- $\emptyset \rightarrow 0$
- $\{\{\emptyset\}\} \rightarrow 1$
- $\{\{\emptyset\}, \{\emptyset\}\} \rightarrow \{\emptyset, \emptyset\} \rightarrow 1$
- $\{2, 4, 6, 8, 2, 4\} \rightarrow \{2, 4, 6, 8\} \rightarrow 4$
- $\{2, 6, 4, 8\} \rightarrow 1$
- $\{8, 6, 6, 4, 2\} \rightarrow \{2, 4, 6, 8\} \rightarrow 4$
- $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\{\emptyset\}\}\}, \{\emptyset, \{\{\emptyset\}, \{\emptyset\}\}\}\} \rightarrow 4$
- $\{x \mid x \in \mathbb{N}\} \rightarrow 1$

Equal set

0  $\rightarrow$  (I) (II)  
 1  $\rightarrow$  (III) (IV) (V) (VI) (VII) (X)  
 4  $\rightarrow$  (VIII) (IX)

Equivalent Sets

0  $\rightarrow$  (I) (III)  
 1  $\rightarrow$  (IV) (V)  
 4  $\rightarrow$  (VI) (VIII)



Q.9 Which of the following are the subsets of  $\{2, 4, 6, 8\}$ ?

- ☒ i.  $\{2, 4, 6, 8\}$
- ☒ ii.  $\{2, 6, 4\}$
- ☒ iii.  $\{2, 4\}$
- ☒ iv.  $\{6, 4, 4, 4\}$
- ☒ v.  $\{2, 8\}$
- ☐ vi.  $\{2, 4, 8, \{\phi\}\}$
- ☒ vii.  $\{6, 2, 8, \phi\}$
- ☐ viii.  $\{\{2\}, \{4\}, \{6\}, \{8\}\}$
- ☐ ix.  $\{\{2, 4, 6, 8\}\}$
- ☐ x.  $\{\{8, 6\}\}$

Q.10 Determine whether each of these statements is true or false.

- ☒ i.  $0 \in \phi$
- ☒ ii.  $\phi \in \{0\}$
- ☐ iii.  $\{0\} \subset \phi$
- ☒ iv.  $\phi \subset \{0\}$
- ☒ v.  $\{0\} \in \{0\}$
- ☐ vi.  $\{0\} \subset \{0\}$
- ☒ vii.  $\phi \subseteq \phi$
- ☒ viii.  $\{\phi\} \subseteq \{\phi\}$
- ☒ ix.  $\phi \in \phi$
- ☒ x.  $\phi \subseteq \{\phi\}$
- ☐ xi.  $\{\phi\} \subseteq \phi$
- ☒ xii.  $\phi \in \{\phi\}$
- ☒ xiii.  $\{\phi\} \in \phi$
- ☒ xiv.  $\phi \in \{\phi, \{\phi\}\}$
- ☐ xv.  $\{\phi\} \in \{\phi\}$
- ☒ xvi.  $\{\phi\} \in \{\{\phi\}\}$
- ☒ xvii.  $\{\phi\} \subset \{\phi, \{\phi\}\}$
- ☐ xviii.  $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$
- ☐ xix.  $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$
- ☒ xx.  $x \in \{x\}$
- ☒ xxi.  $\{x\} \subseteq \{x\}$
- ☐ xxii.  $\{x\} \in \{x\}$
- ☒ xxiii.  $\{x\} \in \{\{x\}\}$
- ☒ xxiv.  $\phi \subseteq \{x\}$
- ☐ xxv.  $\phi \in \{x\}$
- ☒ xxvi.  $\{a, b\} \subseteq \{a, b, c, \{a, b, c\}\}$
- ☐ xxvii.  $\{a, b\} \in \{a, b, c, \{a, b, c\}\}$
- ☒ xxviii.  $\{a, b\} \subseteq \{a, b, \{\{a, b\}\}\}$
- ☐ xxix.  $\{a, b\} \in \{a, b, \{\{a, b\}\}\}$
- ☐ xxx.  $\{a, \phi\} \subseteq \{a, \{a, \phi\}\}$
- ☒ xxxi.  $\{a, \phi\} \in \{a, \{a, \phi\}\}$

Q.11 Given  $S = \{2, a, \{3\}, 4\}$  and  $R = \{\{a\}, 3, 4, 1\}$ , indicate whether the following are true or false.

- F i)  $\{a\} \in S$   
 T ii)  $\{a\} \in R$   
 T iii)  $\{a, 4, \{3\}\} \subseteq S$   
 T iv)  $\{\{a\}, 1, 3, 4\} \subseteq R$   
 F v)  $R = S$   
 T vi)  $\{a\} \subseteq S$   
 F vii)  $\{a\} \subseteq R$   
 T viii)  $\phi \subseteq R$   
 T ix)  $\phi \subseteq \{\{a\}\} \subseteq R$   
 T x)  $\phi \subseteq S$   
 F xi)  $\phi \in R$   
 T xii)  $\phi \subseteq \{\{3\}, 4\}$

(12)

$A = \{a\}$   
 $B = \{A, \text{extra}\}$   
 $C = \{\emptyset, \text{extra}\}$   
 not in A

$A = \{a\}$   
 $B = \{\{a\}, e\}$   
 $C = \{\{\{a\}, e\}, e\}$

(13)

$A = \{a\}$   
 $B = \{\{a\}, e\}$   
 $C = \{\{a\}, \{a\}, e, e\}$

(14)

$A = \{a\}$   
 $B = \{a, \{a\}, e\}$

Q.12 Give an example of sets A, B and C such that  $A \in B$ ,  $B \in C$  and  $A \notin C$

Q.13 Give an example of sets A, B and C such that  $A \in B$ ,  $B \in C$  and  $A \in C$

Q.14 Find two sets A and B such that  $A \in B$  and  $A \subseteq B$

Q.15 Determine whether each of the following statements is true for arbitrary sets A, B, C.

- T i) If  $A \in B$  and  $B \subseteq C$ , then  $A \in C$   $\rightarrow A = \{a\}, B = \{A, e\}, C = \{A, e, e\}$   
 F ii) If  $A \in B$  and  $B \subseteq C$ , then  $A \subseteq C$   
 F iii) If  $A \subseteq B$  and  $B \in C$ , then  $A \in C$   $\rightarrow A = \{a\}, B = \{a, e\}, C = \{\{a, e\}, e\}$   
 F iv) If  $A \subseteq B$  and  $B \in C$ , then  $A \subseteq C$

Q.16 Find the possible number of elements in A if it is a subset of following sets:--

i.  $\{a, s, r\}$  and

ii.  $\{r, k, p\}$

$\Rightarrow A = \emptyset, \{a\}$   
 $|A| = 0$   
 $|A| = 1$  Ans

Q.17 What is the minimum cardinality of A if it is a Superset of following sets:--

a)  $\{r, m, a\}$  and

b)  $\{s, h, a, r\}$

$\Rightarrow A = \{r, m, a, s, h\}$   
 $|A| \geq 5$  Ans

Q.18 What is/are the possible value(or values) of the cardinality of A if the given statements are true?

i. A is a superset of  $\{1, 3\}$  and  $\{4, 3, 3\}$

$\rightarrow A = \{1, 3, 4, \dots\}$   $|A| \geq 3$

ii. A is a subset of  $\{4, 7, 5, 1, 3\}$

$\rightarrow A_1 = \{1, 3, 4, 5, 7\}, A_2 = \emptyset \Rightarrow |A_1| = 5, |A_2| = 0$

Q.19 What should be possible value(or values) of x, if  $\{3, 7, x\}$  and  $\{3, 9, x\}$  are subsets of  $\{3, 7, 9, 13\}$ ?

$\rightarrow x = 13$  Ans

Q.20  $A = \{1, 2, x\}$ ,  $B = \{1, 2, 7\}$ , What is the value of the x if the both of given statements are true?

a) A is a subset of B and

$\Rightarrow A \subseteq B \wedge B \subseteq A \Rightarrow A = B \rightarrow x = 7$  Ans

b) B is a subset of A

Q.21 What is the number of possible subsets of a set with cardinality 5?

$\rightarrow \frac{n!}{5! * (n-5)!}$  Ans

Q.22 What is the number of possible subsets with even cardinality of  $\{a, b, c\}$ ?

$\rightarrow 2^3 = 8$  even  $\rightarrow 4$

Q.23 What is the number of possible subsets with odd cardinality of  $\{a, b, c\}$ ?

$\rightarrow 4$  odd

Q.24 What is the number of subsets possible with cardinality as a multiple of 4 of set S? if

i.  $n(S) = 16 \rightarrow {}^{16}C_0 + {}^{16}C_4 + {}^{16}C_8 + {}^{16}C_{12} + {}^{16}C_{16} = 16512$  Ans

ii.  $n(S) = 19 \rightarrow {}^{19}C_0 + {}^{19}C_4 + {}^{19}C_8 + {}^{19}C_{12} + {}^{19}C_{16} = 130816$  Ans



Q.25 Find the power set of following sets

- i.  $\{r\} \rightarrow \{\emptyset, \{r\}\}$
- ii.  $\{\{a\}\} \rightarrow \{\emptyset, \{\{a\}\}\}$
- iii.  $\{r, a\} \rightarrow \{\emptyset, \{r\}, \{a\}, \{r, a\}\}$
- iv.  $\{r, \{a\}\} \rightarrow \{\emptyset, \{r\}, \{\{a\}\}, \{r, \{a\}\}\}$
- v.  $\{1, \emptyset\} \rightarrow \{\emptyset, \{1\}, \{\emptyset\}, \{1, \emptyset\}\}$
- vi.  $\{X, Y, Z\} \rightarrow \{\emptyset, \{X\}, \{Y\}, \{Z\}, \{X, Y\}, \{Y, Z\}, \{X, Z\}, \{X, Y, Z\}\}$
- vii.  $\emptyset \rightarrow \{\emptyset\}$
- viii.  $\{\emptyset\} \rightarrow \{\emptyset, \{\emptyset\}\}$
- ix.  $\{\{\emptyset\}\} \rightarrow \{\emptyset, \{\{\emptyset\}\}\}$
- x.  $\{\emptyset, \{\emptyset\}\} \rightarrow \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

Q.26 Find the cardinality of following sets.

- i.  $P(\{a, b, \{a, b\}\}) \rightarrow 2^3 = 8$
- ii.  $P(\{\emptyset, a, \{a\}, \{\{a\}\}\}) \rightarrow 2^4 = 16$
- iii.  $P(P(\emptyset)) \rightarrow 2^{2^0} = 2$
- iv.  $P(\{\{\emptyset\}\}) \rightarrow 2^{2^1} = 2$

Q.27 Let  $A = \{\emptyset\}$ . Let  $B = P(P(A))$ .

- T i) Is  $\emptyset \in B$ ?  $\emptyset \subseteq B$ ?
- T ii) Is  $\{\emptyset\} \in B$ ?  $\{\emptyset\} \subseteq B$ ?
- T iii) Is  $\{\{\emptyset\}\} \in B$ ?  $\{\{\emptyset\}\} \subseteq B$ ?

$$\begin{aligned} A &= \{\emptyset\} \\ P(A) &= \{\emptyset, \{\emptyset\}\} \\ P(P(A)) &= \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\} = B \end{aligned}$$

Q.28 Let  $A = \{\emptyset, \{\emptyset\}\}$ . Determine whether each of the following statements is true or false.

- T i)  $\emptyset \in P(A)$
- T ii)  $\emptyset \subseteq P(A)$
- T iii)  $\{\emptyset\} \subseteq P(A)$
- T iv)  $\{\emptyset\} \subseteq A$
- T v)  $\{\emptyset\} \in P(A)$
- T vi)  $\{\emptyset\} \in A$
- T vii)  $\{\{\emptyset\}\} \subseteq P(A)$
- T viii)  $\{\{\emptyset\}\} \subseteq A$
- T ix)  $\{\{\emptyset\}\} \in P(A)$
- F x)  $\{\{\emptyset\}\} \in A$

$$\begin{aligned} A &= \{\emptyset, \{\emptyset\}\} \\ P(A) &= \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\} \end{aligned}$$

Q.29 Let  $A = \{a, \{a\}\}$ . Determine whether each of the following statements is true or false.

- T i)  $\emptyset \in P(A)$
- T ii)  $\emptyset \subseteq P(A)$
- T iii)  $\{a\} \in P(A)$
- F iv)  $\{a\} \subseteq P(A)$
- T v)  $\{\{a\}\} \in P(A)$
- T vi)  $\{\{a\}\} \subseteq P(A)$
- T vii)  $\{a, \{a\}\} \in P(A)$
- F viii)  $\{a, \{a\}\} \subseteq P(A)$
- F ix)  $\{\{\{a\}\}\} \in P(A)$
- T x)  $\{\{\{a\}\}\} \subseteq P(A)$

$$\begin{aligned} A &= \{a, \{a\}\} \\ P(A) &= \{\emptyset, \{a\}, \{\{a\}\}, \{a, \{a\}\}\} \end{aligned}$$

T Q.30 Can you conclude that  $|A| = |B|$ , if  $|P(A)| = |P(B)|$ ?

T Q.31 Can you conclude that  $A = B$ , if  $P(A) = P(B)$ ?

- Q.32 Can you conclude that  $A \subseteq B$ , if  $P(A) \subseteq P(B)$ ?
- Q.33 Can you conclude that  $A \subset B$ , if  $P(A) \subset P(B)$ ?
- Q.34 Determine whether each of these sets is the power set of a set, where  $a$  and  $b$  are distinct elements.

i.  $\phi$

ii.  $\{\phi, \{a\}\}$

iii.  $\{\phi, \{a\}, \{\phi, a\}\}$

iv.  $\{\phi, \{a\}, \{b\}, \{a, b\}\}$

v.  $\{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, b, c, d\}\}$

vi.  $\{\phi, \{a\}, \{b\}, \{c\}\}$

- Q.35 In which of the following cases set  $A$  is a proper subset of Set  $B$ ?

i.  $A \subseteq B$  and  $A \neq B$

ii.  $A \subseteq B$  and  $B \subseteq A$

iii.  $A \subseteq B$  and  $A = B$

iv. None of these

### GATE and other engineering exams previous questions

- Q.1 The number of elements in the power set  $P(S)$  of the set  $S = \{\{\emptyset\}, 1, \{2, 3\}\}$  is:

(A) 2

(B) 4

(C) 8

(D) None of the above

[1995 : 1 mark]

- Q.2 Let  $P(S)$  denotes the power set of set  $S$ . Which of the following is always true?

(A)  $P(P(S)) = P(S)$

(B)  $P(S) \cap P(P(S)) = \{\phi\}$

(C)  $P(S) \cap S = P(S)$

(D)  $S \notin P(S)$

$$\begin{cases} S = \{\emptyset, 1\} \\ P(S) = \{\emptyset, \{\emptyset, 1\}\} \\ P(P(S)) = \{\emptyset, \{\emptyset, \{\emptyset, 1\}\}, \{\{\emptyset, 1\}, \{\emptyset, \{\emptyset, 1\}\}\}\} \end{cases}$$

[2000 : 2 marks]

- Q.3 Let  $A$  be a set with  $n$  elements. Let  $C$  be a collection of distinct subsets of  $A$  such that for any two subsets  $S_1$  and  $S_2$  in  $C$ , either  $S_1 \subset S_2$  or  $S_2 \subset S_1$ . What is the maximum cardinality of  $C$ ?

(A)  $n$

(B)  $n+1$

(C)  $2^{n-1} + 1$

(D)  $n!$

[IT 2005 : 2 marks]

- Q.4 The cardinality of the power set of  $\{0, 1, 2, \dots, 10\}$  is

$$2^{11} = 2048$$

[2015 set-2 : 1 mark]

- Q.5 For a set  $A$ , the power set of  $A$  is denoted by  $2^A$ . If  $A = \{5, \{6\}, \{7\}\}$ , which of the following options are TRUE?

i.  $\phi \in 2^A$

ii.  $\phi \subseteq 2^A$

iii.  $\{5, \{6\}\} \in 2^A$

iv.  $\{5, \{6\}\} \subseteq 2^A$

(A) I and III only

(B) II and III only

(C) I, II and III only

(D) I, II and IV only

[2015 set-1 : 1 mark]

Q.6 Let  $X$  be a set of size  $n$ . How many pairs of sets  $(A, B)$  are there that satisfy the condition  $A \subseteq B \subseteq X$ ?

(A)  $2^{n+1}$

(B)  $2^{2n}$

☒ (C)  $3^n$

(D)  $2^{n+1}$

(E)  $3^{n+1}$

[TIFR 2010]

Q.7 How many pairs of sets  $(A, B)$  are there that satisfy the condition  $A, B \subseteq \{1, 2, \dots, 5\}$ ,  $A \cap B = \{\}$ ?

(A) 125

(B) 127

(C) 130

☒ (D) 243

(E) 257

[TIFR 2012]

Q.8 Let  $X$  be a set with  $n$  elements. How many subsets of  $X$  have odd cardinality?

(A)  $n$

(B)  $2^n$

☒ (C)  $2^{n/2}$

(D)  $2^{n-1}$

(E) Can not be determined without knowing whether  $n$  is odd or even

[TIFR 2019]

Q.9 The number of elements in the power set of the set  $\{\{A, B\}, C\}$  is

(A) 7

(B) 8

(C) 3

☒ (D) 4

[ISRO 2013]

Q.10 The number of elements in the power set of  $\{\{1, 2\}, \{2, 1, 1\}, \{2, 1, 1, 2\}\}$  is

(A) 3

☒ (B) 8

(C) 4

☒ (D) 2

[ISRO 2017]

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