Section 2

Q.1 Determine the cardinalities of the sets:

(a) $A = \{n^7 \mid \text{n is a positive integer}\}$ $\longrightarrow \{1^7, 2^7, 3^7, 4^7, \ldots\}$

(b) $B = \{n^{109} \mid \text{n is a positive integer}\}$ $\longrightarrow \{1^{109}, 2^{109}, 3^{109}, 4^{109}, \ldots\}$

(c) $A \cup B \longrightarrow \{1, 2^7, 2^{109}, 3^7, 3^{109}, \ldots\}$

(d) $A \cap B \longrightarrow \{1 \ldots\}$

Q.2 Determine the following sets :

(a) $\phi \cup \{\phi\} \longrightarrow \{\phi\}$ (b) $\phi \cap \{\phi\} \longrightarrow \phi$

$\{A \oplus B = A \cup B - A \cap B$

$\{a, \phi, \{\phi\}\} \longleftarrow$ (c) $\{\phi\} \cup \{a, \phi, \{\phi\}\}$ (d) $\{\phi\} \cap \{a, \phi, \{\phi\}\} \longrightarrow \{\phi\}$

$\{a, \phi, \{\phi\}\} \longleftarrow$ (e) $\phi \oplus \{a, \phi, \{\phi\}\}$ (f) $\{\phi\} \oplus \{a, \phi, \{\phi\}\} \longrightarrow \{a, \{\phi\}\}$

Q.3 Let A = {1, 2, 3, 4, 5} and B ={0, 3, 6} Find

$\{1, 2, 3, 0, 4, 5, 6\} \longleftarrow$ a) $A \cup B$ b) $A \cap B \longrightarrow \{3\}$

$\{1, 2, 4, 5\} \longleftarrow$ c) $A - B$ d) $B - A \longrightarrow \{0, 6\}$

Q.4 Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. Find

$B \longleftarrow$ a) $A \cup B$ b) $A \cap B \longrightarrow A$

$\{\}$ $\longleftarrow$ c) $A - B$ d) $B - A \longrightarrow \{f, g, h\}$

$A = \{a\}$
$P(A) = \{\phi, \{a\}\}$
$B = \{b\}$
$P(B) = \{\phi, \{b\}\}$
$A \cup B = \{a, b\}$
$A \cap B = \{\}$
$P(A \cup B) = \{\phi, \{a\}, \{b\}, \{a,b\}\}$
$P(A \cap B) = \{\phi\}$

Q.5 Determine whether each of the following statements is true or false.

F (a) $A \cup p(A) = p(A)$ F (b) $A \cap p(A) = A$

T (c) $\{A\} \cup p(A) = p(A)$ F (d) $\{A\} \cap p(A) = A$ → $\{A\} \neq A$

T (e) $A - p(A) = A$ F (f) $p(A) - \{A\} = p(A)$

T (g) $p(A \cap B) = p(A) \cap p(B)$ F (h) $p(A \cup B) = p(A) \cup p(B)$

Q.6 Let $A_i = \{1, 2, 3, ..., i\}$ for $i = 1, 2, 3, ...$ find

(a) $\bigcup_{i=1}^{n} A_i = \boxed{A_n}$ (b) $\bigcap_{i=1}^{n} A_i = \boxed{A_1}$

Q.7 Let $A_i = \{..., -2, -1, 0, 1, ..., i\}$ find

(a) $\bigcup_{i=1}^{n} A_i = \boxed{A_n}$ (b) $\bigcap_{i=1}^{n} A_i = \boxed{A_1}$

Q.8 Let A_i be the set of all nonempty bit strings (that is, bit strings of length at least one) of length not exceeding i. Find → $\{0, 1, 00, 01, 10, 11, \ldots\}$

(a) $\bigcup_{i=1}^{n} A_i = \boxed{A_n}$ (b) $\bigcap_{i=1}^{n} A_i = \boxed{A_1}$

Q.9 Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ for if for every positive integer i,

(a) $A_i = \{i, i+1, i+2, ...\}$

(b) $A_i = \{0, i\}$

(c) $A_i = (0, i)$ that is, the set of real numbers x with $0 < x < i$.

(d) $A_i = (i, \infty)$ that is, the set of real numbers x with $x > i$.

	$\bigcup_{i=1}^{\infty} A_i$	$\bigcap_{i=1}^{\infty} A_i$
(a)	$\{1, 2, \ldots\}$	ϕ
(b)	$\{0, 1, \ldots\}$	$\{0\}$
(c)	$(0, \infty)$	$(0, 1)$
(d)	$(1, \infty)$	ϕ

Q.10 What can you say about the relationship between sets P and Q if

(a) $P \cap Q = P$? $\Rightarrow P \subseteq Q$ (b) $P \cup Q = P$? $\Rightarrow Q \subseteq P$

(c) $P \oplus Q = P$? $\Rightarrow Q = \phi$ (d) $P \cap Q = P \cup Q \Rightarrow P = Q$

Q.11 From which of the following you can conclude B=C?

(a) $A \cup B = A \cup C$,

(b) $A \cap B = A \cap C$,

(c) $A \oplus B = A \oplus C$, ✓

(d) A - B = B - C

Q.12 Let A = {0, 2, 4, 6, 8, 10}, B = {0, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}. Find

a) $A \cap B \cap C \rightarrow \{4, 6\}$

b) $A \cup B \cup C \rightarrow \{0, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c) $(A \cup B) \cap C \rightarrow \{4, 5, 6\}$

d) $(A \cap B) \cup C \rightarrow \{4, 5, 6, 7, 8, 9, 10\}$

Q.13 For A = {a,b,{a,c,},Ø}. Determine the following sets :

(a) A-{a} = {φ, b, {a,c}} (b) A-Ø → A

(c) A – {Ø} → {a, b, {a,c}} (d) A –{a,b} → {φ, {a, c}}

(e) A- {a,c} $\longrightarrow$ {Φ, a, {a, c}}

(f) A-{{a,b}} $\longrightarrow$ {Φ, a, b, {a, c}} = A

(g) A- {{a,c}} $\rightarrow$ {Φ, a, b}

(h) {a} –A $\longrightarrow$ Φ

(i) Ø – A $\longrightarrow$ Φ

(j) {Ø} - A $\longrightarrow$ Φ

(k) {a,c} –A $\longrightarrow$ {c}

(l) {{a,c}} –A $\rightarrow$ ~~{Φ}~~ Φ

(m) {a}-{A} $\rightarrow$ {a}

Q.14 let A, B, C be subsets of U. Which of the following must be satisfied for B=C?

(1) $A \cup B = A \cup C$

(2) $A \cap B = A \cap C$

(a) Only (1)

(b) only (2)

(c) either (1) or (2)

(d) Both (1) and (2)

Q.15 What can you say about the relationship between sets A and B for the given conditions?

(a) $A \cup B = A?$ $\Rightarrow$ $B \subseteq A$

(b) $A \cap B = A?$ $\Rightarrow$ $A \subseteq B$

(c) $A - B = A?$ $\Rightarrow$ $A \cap B = \Phi$

(d) $A \cap B = B \cap A?$ $\Rightarrow$ $A = B$

(e) $A - B = B - A?$ $\Rightarrow$ $A = B$

(f) A - B = B $\Rightarrow$ $A = \Phi$, $B = \Phi$

(g) A - B = B – A $\Rightarrow$ $A = B$

Q.16 Suppose that the universal set is U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Express each of these sets with bit strings where the ith bit in the string is 1 if I is in the set and 0 otherwise.

a) {3, 4, 5} $\longrightarrow$ 0011100000

b) {1, 3, 6, 10} $\longrightarrow$ 1010010001

c) {2, 3, 4, 7, 8, 9} $\longrightarrow$ 0111001110

Q.17 Using the same universal set as in the last problem, find the set specified by each of these bit strings,

a) 11 1100 1110 $\rightarrow$ 1,2,3,4,7,8,9 b) 01 0111 1000 $\rightarrow$ 2,4,5,6,7

c) 10 0000 0001 $\rightarrow$ 1, 10

Q.18 What subsets of a finite universal set do these bit strings represent?

a) The string with all zero $\rightarrow$ Φ

b) The string with all ones $\rightarrow$ U

Q.19 Find the sets A and B if A-B = {1,5,7,8,}, B-A = {2,10}, and $A \cap B$ = {3,6,9,}.

A = {1, 3, 5, 6, 7, 8, 9}

B = {2, 3, 6, 9, 10}

Q.20 let A ={ϕ,b}, Construct the following sets :

(a) A-ϕ $\longrightarrow$ A (b) {ϕ} – A $\longrightarrow$ Φ

(c) $A \cup$ P(A) $\rightarrow$ {Φ, b, {Φ}, {b}, {Φ, b}} (d) $A \cap$ P(A) $\rightarrow$ {Φ}

P(A) = {Φ, {Φ}, {b}, {Φ, b}}