

**Q.23** If  $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$  then which of the following is a factor of  $\Delta$ .

- (A)  $a+b$  (B)  $a-b$  (C)  $abc$  (D)  $a+b+c$

**Q.24** Let A, B, C, D be  $n \times n$  matrices, each with non-zero determinant, If  $ABCD = I$ , then  $B^{-1}$  is

- (A)  $D^{-1}C^{-1}A^{-1}$  (B)  $CDA$   
(C)  $ADC$  (D) Does not necessarily exist

**Q.25** Consider the matrices  $X_{(4 \times 3)}$ ,  $Y_{(4 \times 3)}$  and  $P_{(2 \times 3)}$ . The order of  $[P(X^T Y)^{-1} P^T]^T$  will be

- (A)  $(2 \times 2)$  (B)  $(3 \times 3)$  (C)  $(4 \times 3)$  (D)  $(3 \times 4)$

**Q.26** Let  $x$  and  $y$  be two vectors in a 3 dimensional space and  $\langle x, y \rangle$  denote their dot product. Then the determinant

$$\det \begin{bmatrix} \langle x, x \rangle & \langle x, y \rangle \\ \langle y, x \rangle & \langle y, y \rangle \end{bmatrix}$$

- (A) Is zero when  $x$  and  $y$  are linearly independent.  
(B) Is positive when  $x$  and  $y$  are linearly independent.  
(C) Is non-zero for all non-zero  $x$  and  $y$ .  
(D) Is zero only when either  $x$  or  $y$  is zero.

**Q.27** The value  $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$  is

- (A)  $a^2 b^2 c^2$  (B)  $-a^2 b^2 c^2$  (C)  $4a^2 b^2 c^2$  (D)  $-4a^2 b^2 c^2$

**Q.28** If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

then the determinant of A is equal to \_\_\_\_\_.

**Q.29** Let  $M^4 = I$ , (where  $I$  denotes the identity matrix) and  $M \neq I$ ,  $M^2 \neq I$  and  $M^3 \neq I$ . Then, for any natural number  $k$ ,  $M^{-1}$  equals,

- (A)  $M^{4k+1}$  (B)  $M^{4k+2}$  (C)  $M^{4k+3}$  (D)  $M^{4k}$

**Statement For Linked Answer Q.30 & Q.31**

Cayley-Hamilton Theorem states that a square matrix satisfies its own characteristic equation. Consider a matrix

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

**Q.30**  $A$  satisfies the relation

(A)  $A + 3I + 2A^{-1} = 0$

(B)  $A^2 + 2A + 2I = 0$

(C)  $(A + I)(A + 2I) = 0$

(D)  $\exp(A) = 0$

**Q.31**  $A^9$  equals

(A)  $511A + 510I$

(B)  $309A + 104I$

(C)  $154A + 155I$

(D)  $\exp(9A)$

**Q.32** The matrix  $A = \begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$  has three distinct Eigen values and one of its Eigen vectors is  $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ . Which

one of the following can be another Eigen vector of  $A$ ?

(A)  $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

(B)  $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

**Q.33** The constant term of the characteristic polynomial of the matrix

$\begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix}$  is \_\_\_\_\_.

**Q.34** Let  $N$  be a  $3 \times 3$  matrix with real number entries. The matrix is such that  $N^2 = 0$ . The Eigen values of  $N$  are

(A)  $0, 0, 0$

(B)  $0, 0, 1$

(C)  $0, 1, 1$

(D)  $1, 1, 1$

**Q.35** Let  $M$  be a real  $4 \times 4$  matrix. Consider the following statements :

S1 :  $M$  has 4 linearly independent eigenvectors.

S2 :  $M$  has 4 distinct eigenvalues.

S3 :  $M$  is non-singular (invertible) matrix.

Which one among the following is TRUE?

(A) S1 implies S2

(B) S1 implies S3

(C) S2 implies S1

(D) S3 implies S2

**Q.36** The eigen values and the corresponding eigen vectors of a  $2 \times 2$  matrix are given by

Eigen value	Eigen vector
$\lambda_1 = 8$	$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2 = 4$	$v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

(A)  $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$

(B)  $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(C)  $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$

(D)  $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

**Q.37** The system of equations

$$x + y + z = 6$$

$$x + 4y + 6z = 20$$

$$x + 4y + \lambda z = \mu$$

has NO solution for values of  $\lambda$  and  $\mu$  given by

(A)  $\lambda = 6, \mu = 20$

(B)  $\lambda = 6, \mu \neq 20$

(C)  $\lambda \neq 6, \mu = 20$

(D)  $\lambda \neq 6, \mu \neq 20$

**Q.38** The value of  $q$  for which the following set of linear algebraic equations

$$2x + 3y = 0$$

$$6x + qy = 0$$

can have non-trivial solution is

**Q.39** The matrix has  $M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$  eigen values  $-3, -3, 5$ . An eigen vector corresponding to the eigenvalue 5 is  $[1 \quad 2 \quad -1]^T$ . One of the eigen vectors of the matrix  $M^3$  is

(A)  $[1 \quad 8 \quad -1]^T$

(B)  $[1 \quad 2 \quad -1]^T$

(C)  $[1 \quad \sqrt[3]{2} \quad -1]^T$

(D)  $[1 \quad 1 \quad -1]^T$

23→

B

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

, factor of  $\Delta$

Sol<sup>n</sup>

$$R_3 \rightarrow R_3 - R_1, \quad R_2 \rightarrow R_2 - R_1$$

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

(A)  $a+b$

~~(B)  $a-b$~~

(C)  $abc$

(D)  $a+b+c$

24  $\Rightarrow$

$$[ABCD]^{-1} = [I]^{-1}$$

B

$$D^T C^{-1} B^T A^{-1} = I$$

$$\cancel{D} D^T \cancel{C}^{-1} B^T \cancel{A}^{-1} A = D I A$$

$$\cancel{C} C^T B^T = C D A$$

$$B^T = C D A \quad \underline{\text{Ans}}$$

25  $\Rightarrow$

$$X \rightarrow 4 \times 3, Y \rightarrow 4 \times 3, P \rightarrow 2 \times 3$$

A

$$[P(X^T X)^{-1} P^T]^T$$

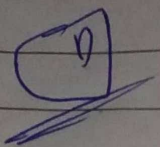
$$\begin{matrix} \swarrow & \searrow \\ (2 \times 3) & 3 \times (2) \end{matrix}$$

$$(2 \times 2)^T$$

$$= [2 \times 2]^{-1} \quad \underline{\text{Ans}}$$



26 ⇒



$$\begin{vmatrix} (x_1^2 + x_2^2 + x_3^2) & (x_1 y_1 + x_2 y_2 + x_3 y_3) \\ (x_1 y_1 + x_2 y_2 + x_3 y_3) & (y_1^2 + y_2^2 + y_3^2) \end{vmatrix}$$

∂f  $x = 0$

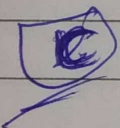
$$\begin{vmatrix} 0 & 0 \\ 0 & y_1^2 + y_2^2 + y_3^2 \end{vmatrix} = 0$$

∂f  $y = 0$

$$\begin{vmatrix} x_1^2 + x_2^2 + x_3^2 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

⇒ Is zero only when either  $x$  or  $y$  is zero.

27 ⇒

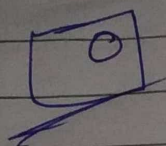


$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 4a^2 b^2 c^2$$

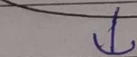
Ans

28 →



$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

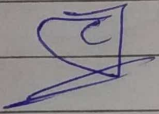
$$\rightarrow -2R_1 = R_2$$



So,  $|A| = 0$

29 →

$$M^4 = I, \quad \underline{M \neq I, M^2 \neq I, M^3 \neq I}$$



$$M^4 = I$$

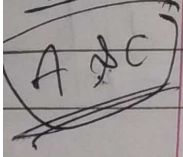
$$M^3 \cdot M = M^{-1} \cdot M$$

$$\underline{M^{-1} = M^3} \quad \text{Ans}$$

$$\rightarrow M^{-1} = M^3 \cdot (M^4)^k$$

$$\boxed{M^{-1} = M^{4k+3}}$$

30 →



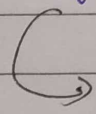
$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

$$(-3-\lambda)(0-\lambda) - (-2) = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$A^2 + 3A + 2I = 0 \Rightarrow \underline{(A+2I)(A+I) = 0}$$

mult.  $A^{-1}$



$$\boxed{A + 3I + 2A^{-1} = 0}$$

$$\rightarrow \textcircled{A}$$

$$\downarrow$$

$$\textcircled{C}$$

31)  $\Rightarrow$  $A^9 \Rightarrow$  $\lambda^9$  $\longrightarrow$ 

$\lambda = -2, -1$

$$\downarrow$$
$$(-1)^9 = (-1)$$

$$\textcircled{A} \Rightarrow s_{11} A + s_{10} I = s_{11}(-1) + s_{10}$$

$$= (-1)$$

$$\uparrow$$
$$(-1)$$

32)  $\Rightarrow$ 

$$|A - \lambda I| = \begin{vmatrix} 3/2 - \lambda & 0 & 1/2 \\ 0 & -1 - \lambda & 0 \\ 1/2 & 0 & 3/2 - \lambda \end{vmatrix}$$

$$|A - \lambda| = \left( \frac{3}{2} - \lambda \right) \left[ \left( \frac{3}{2} - \lambda \right) (-1 - \lambda) \right] + \frac{1}{2} \left[ (1 + \lambda) \frac{1}{2} \right]$$
$$= (1 + \lambda) \left[ - \left( \frac{3}{2} - \lambda \right)^2 + \left( \frac{1}{2} \right)^2 \right]$$

$$= (1 + \lambda) \left[ \left( \frac{1}{2} + \frac{3}{2} - \lambda \right) \left( \frac{1}{2} - \frac{3}{2} + \lambda \right) \right]$$

$$= (1 + \lambda) (2 - \lambda) (\lambda - 1)$$

$$\lambda = -1, 1, 2$$

Eigen Vector

$$\textcircled{C} \quad \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$



33 →

0

$$\begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix}$$

$\Rightarrow$

$$2R_1 = R_2$$

$$|A| = 0$$

34 →

$N_{3 \times 3}$

$$N^2 = 0$$

Eigen value

No change

A

So

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Ans

35 →

S1:  $M$  has 4 linear independent eigen vectors.

S2:  $M$  has 4 distinct eigen values.

S3:  $M$  is non-singular matrix.

True :-

$\Rightarrow$  S2 implies S1

36)

$$\lambda_1 = 8$$
$$\lambda_2 = 4$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

A

sol

$$\lambda_1 + \lambda_2 = 12$$

$$\lambda_1 \lambda_2 = 32$$

A

$$\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$



$$|A| = 32$$

$$6 + 6 = 12$$

37)

B

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & u \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 0 & 0 & \lambda - 6 & u - 20 \end{array} \right]$$

No sol<sup>n</sup> for

$$\lambda = 6, u \neq 20$$

38

$$\begin{aligned} 2x + 3y &= 0 \\ 6x + 4y &= 0 \end{aligned}$$

9

$$\begin{vmatrix} 2 & 3 \\ 6 & 4 \end{vmatrix} = 0$$

$$2y - 18 = 0$$

$$y = 9$$

39

$$M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{bmatrix}$$

Eigen Value = -3, -3, 5

B

at 5

Eigen Vector

$$[1 \ 2 \ -1]^T$$

not change

So

$$\textcircled{B} [1, 2, -1]^T$$