GATE PREVIOUS YEAR QUESTIONS ON FUNCTIONS

Q.1 (A) How many binary relations are there on a set A with n elements?

(B) How many one-to-one functions are there from a set A with n elements onto itself → n!

[GATE 1987]

Q.2 Let R denote the set of real numbers. Let $f: R\times R \to R\times R$ be a bijective function defined by $f(x,y)=(x+y,x-y)$. The inverse function of f is given by

(A) $f^{-1}(x,y)=\left(\frac{1}{x+y},\frac{1}{x-y}\right)$

(B) $f^{-1}(x,y)=(x-y,x+y)$

(C) $f^{-1}(x,y)=\left(\frac{x+y}{2},\frac{x-y}{2}\right)$

(D) $f^{-1}(x,y)=(2(x-y),2(x+y))$

[GATE 1996]

Q.3 Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there are exactly 97 functions from X to Y. From this, one can conclude that

(A) $|X|=1,\quad |Y|=97$

(B) $|X|=97,\quad |Y|=1$

(C) $|X|=97,\quad |Y|=97$

(D) None of the above

[GATE 1996]

Q.4 The number of functions from an m element set to an n element set is

(A) $m+n$

(B) m^n

(C) n^m

(D) $m\times n$

[GATE 1998]

Q.5 Let $f: A\to B$ be a function, and let E and F be subsets of A. Consider the following statements about images.

$E = \{1, 2\}$, $F = \{3, 4\}$

$S1: f(E\cup F)=f(E)\cup f(F)$

$S2: f(E\cap F)=f(E)\cap f(F)$

Which of the following is true about S1 and S2?

(A) Only S1 is correct

(B) Only S2 is correct

(C) Both S1 and S2 are correct

(D) None of S1 and S2 are correct

[GATE 2001]

Q.6 Let $f: B\to C$ and $g: A\to B$ be two functions and let $h = fog$. Given that h is an onto function, which one of the following is TRUE?

(A) f and g should both be onto functions

(B) f should be onto but g need not be onto

(C) g should be onto but f need not be onto

(D) both f and g need not be onto

[GATE 2005]

Q.7 Let 'f' be a function from a set A to a set B, 'g' be a function from B to C, and 'h' be a function from A to C, such that $h(a)=g(f(a))$ for all $a\in A$. Which of the following statements is always true for all such functions f and g?

GATE Since 2004

(A) g is onto $\Rightarrow$ h is onto

(B) h is onto $\Rightarrow$ f is onto

(C) h is onto $\Rightarrow$ g is onto

(D) h is onto $\Rightarrow$ f and g are onto

[GATE 2005]

Q.8 Let X, Y, Z be sets of sizes x, y and z respectively. Let $W = X \times Y$ and E be the set of all subsets of W. The number of functions from Z to E is

(A) z
(B) $z \times 2^{xy}$
(C) 2^z
(D) 2^{xyz}

[GATE 2006]

Q.9 How many onto (or surjective) functions are there from an n-element $(n \geq 2)$ set to a 2-element set?

(A) 2^n
(B) $2^n - 1$
(C) $2^n - 2$
(D) $2(2^n - 2)$

[GATE 2012]

Q.10 Consider the set of all functions $f : \{0,1,\ldots,2014\} \to \{0,1,\ldots,2014\}$ such that $f(f(i)) = i$, for all $0 \leq i \leq 2014$. Consider the following statements:

P: For each such function, it must be the case that for every $i, f(i) = i$.

Q: For each such function, it must be the case that for some $i, f(i) = i$,

R: Each such function must be onto.

Which one of the following in CORRECT?

(A) P, Q and R are true

(B) Only Q and R are true

(C) Only P and Q are true

(D) Only R is true

[GATE 2014]

Q.11 Let S denote the set of all functions $f : \{0,1\}^4 \to \{0,1\}$. Denote by N the number of functions from S to the set $\{0,1\}$. The value of $\log_2 \log_2 N$ is $\underline{2^4}$. = 16 Ans

[GATE 2014]

Q.12 Let X and Y be finite sets and f be a function .Which one of the following statements TRUE?

(A) For any subsets A and B of X, $|f(A \cup B)| = |f(A)| + |f(B)|$

(B) For any subset A and B of X, $f(A \cap B) = f(A) \cap f(B)$

(C) For any subset A and B of X, $|f(A \cap B) = \min\{|f(A)|, |f(B)|\}|$

(D) For any subsets S and T of Y, $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

[GATE 2014]

Q.13 The number of onto functions (surjective functions) from set $X = \{1,2,3,4,\}$ to set $Y = \{a,b,c\}$ is ——.

[GATE 2015]

$= {}^3C_0(3)^4 - {}^3C_1(3-1)^4 + {}^3C_2(3-2)^4 - {}^3C_3(3-3)^4$

$= 81 - 48 + 3 - 0 = 36$ Ans

Q.14 A function $f : N^+ \to N^+$, defined on the set of positive integers N^+, satisfies the following properties :

$f(n) = f(n/2)$ if n is even

$f(n) = f(n+5)$ if n is odd

Let $R = \{i \mid \exists j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is ——————.

[GATE 2016]

$f(1) = x$

$f(2) = f(2/2) = f(1) = x$

$f(3) = f(3+5) = f(8) = f(4) = f(2) = f(1) = x$

$f(4) = f(2) = f(1) = x$

$f(5) = f(5+5) = f(5)$

$\Rightarrow f(1) = f(2) = f(3) = f(4) = f(6) = f(7) = f(8) - f(9)$

$- - = x$

$\Rightarrow f(5) = f(10) = f(15) = - - - = y$

It will have only 2 value.

$$\boxed{x \;\&\; y}$$

G A T E Since 2004