Q.23 If $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ then which of the following is a factor of Δ.

(A) $a+b$ (B) $a-b$ (C) abc (D) $a+b+c$

Q.24 Let A, B, C, D be $n \times n$ matrices, each with non-zero determinant, If ABCD = I, then B^{-1} is

(A) $D^{-1}C^{-1}A^{-1}$ (B) CDA

(C) ADC (D) Does not necessarily exist

Q.25 Consider the matrices $X_{(4\times3)}, Y_{(4\times3)}$ and $P_{(2\times3)}$. The order of $[P(X^TY)^{-1}P^T]^T$ will be

(A) (2×2) (B) (3×3) (C) (4×3) (D) (3×4)

Q.26 Let x and y be two vectors in a 3 dimensional space and $\langle x, y \rangle$ denote their dot product. Then the determinant

$$\det\begin{bmatrix} \langle x,x \rangle & \langle x,y \rangle \\ \langle y,x \rangle & \langle y,y \rangle \end{bmatrix}$$

(A) Is zero when x and y are linearly independent.

(B) Is positive when x and y are linearly independent.

(C) Is non-zero for all non-zero x and y.

(D) Is zero only when either x or y is zero.

Q.27 The value $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$ is

(A) $a^2b^2c^2$ (B) $-a^2b^2c^2$ (C) $4a^2b^2c^2$ (D) $-4a^2b^2c^2$

Q.28 If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

then the determinant of A is equal to ________.

Q.29 Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals,

(A) M^{4k+1} (B) M^{4k+2} (C) M^{4k+3} (D) M^{4k}

Statement For Linked Answer Q.30 & Q.31

Cayley-Hamilton Theorem states that a square matrix satisfies its own characteristic equation. Consider a matrix

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

Q.30 A satisfies the relation

(A) $A+3I+2A^{-1}=0$ (B) $A^2+2A+2I=0$

(C) $(A+I)(A+2I)=0$ (D) $\exp(A)=0$

Q.31 A^9 equals

(A) $511\ A+510\ I$ (B) $309\ A+104\ I$ (C) $154\ A+155\ I$ (D) $\exp(9A)$

Q.32 The matrix $A=\begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ has three distinct Eigen values and one of its Eigen vectors is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Which one of the following can be another Eigen vector of A?

(A) $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Q.33 The constant term of the characteristic polynomial of the matrix

$\begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix}$ is ______.

Q.34 Let N be a 3×3 matrix with real number entries. The matrix is such that $N^2=0$. The Eigen values of N are

(A) 0, 0, 0 (B) 0, 0, 1 (C) 0, 1, 1 (D) 1, 1, 1

Q.35 Let M be a real 4×4 matrix. Consider the following statements :

S1 : M has 4 linearly independent eigenvectors.

S2 : M has 4 distinct eigenvalues.

S3 : M is non-singular (invertible) matrix.

Which one among the following is TRUE?

(A) S1 implies S2 (B) S1 implies S3 (C) S2 implies S1 (D) S3 implies S2

Q.36 The eigen values and the corresponding eigen vectors of a 2×2 matrix are given by

Eigen value	Eigen vector
$\lambda_1=8$	$v_1=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2=4$	$v_2=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

(A) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

Q.37 The system of equations

$$x+y+z=6$$
$$x+4y+6z=20$$
$$x+4y+\lambda z=\mu$$

has NO solution for values of λ and μ given by

(A) $\lambda=6,\ \mu=20$ (B) $\lambda=6,\ \mu\neq 20$ (C) $\lambda\neq 6,\ \mu=20$ (D) $\lambda\neq 6,\ \mu\neq 20$

Q.38 The value of q for which the following set of linear algebraic equations

$$2x+3y=0$$
$$6x+qy=0$$

can have non-trivial solution is

Q.39 The matrix has $M=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ eigen values $-3,-3,5$. An eigen vector corresponding to the eigenvalue 5 is $[1 \quad 2 \quad -1]^T$. One of the eigen vectors of the matrix M^3 is

(A) $[1 \quad 8 \quad -1]^T$ (B) $[1 \quad 2 \quad -1]^T$ (C) $\left[1 \quad \sqrt[3]{2} \quad -1\right]^T$ (D) $[1 \quad 1 \quad -1]^T$

23)

B

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$, factor of Δ

Soln $R_3 \to R_3 - R_1$, $R_2 \to R_2 - R_1$

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

(A) $a+b$ (B) $a-b$ (C) abc (D) $a+b+c$

24 ⇒ $[ABCD]^{-1} = [I]^{-1}$

B

$D^{-1}C^{-1}B^{-1}A^{-1} = I$

$\cancel{D}D^{-1}C^{-1}B^{-1}\cancel{A^{-1}A} = DIA$

$\cancel{C}C^{-1}B^{-1} = CDA$

$\boxed{B^{-1} = CDA}$ Ans

25 ⇒ $X \rightarrow 4\times3$, $Y \rightarrow 4\times3$, $P \rightarrow 2\times3$

A

$[P(X^T X)^{-1} P^T]^T$

2×3 3×2

$(2\times2)^T = \boxed{2\times2}$ Ans

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26 ⇒

D

$$\begin{vmatrix} (x_1^2+x_2^2+x_3^2) & (x_1y_1+x_2y_2+x_3y_3) \\ (y_1x_1+y_2x_2+y_3x_3) & (y_1^2+y_2^2+y_3)^2 \end{vmatrix}$$

If $x = 0$

$$\begin{vmatrix} 0 & 0 \\ 0 & y_1^2+y_2^2+y_3^2 \end{vmatrix} = 0$$

If $y = 0$

$$\begin{vmatrix} x_1^2+x_2^2+x_3^2 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

D → Is zero only when either x or y is zero.

27 ⇒

C

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

+4

$$= \boxed{4a^2b^2c^2}$$

Ans

28 ⇒

$\boxed{D}$

$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$

$\rightarrow$ $-2R_1 = R_2$

$\downarrow$

So, $\boxed{|A| = 0}$

29 ⇒ $M^4 = I$, $M \neq I$, $M^2 \neq I$, $M^3 \neq I$

C

$M^4 = I$

$M^3 \cdot \cancel{M} = M^{-1} \cdot \cancel{M}$

$M^{-1} = M^3$ Ans

$\rightarrow$ $M^{-1} = M^3 \cdot (M^4)^K$

$\boxed{M^{-1} = M^{4K+3}}$

30 ⇒

A & C

$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$

$(-3-\lambda) \times (0-\lambda) - (-2) = 0$

$\lambda^2 + 3\lambda + 2I = 0$

$A^2 + 3A + 2I = 0 \Rightarrow \boxed{(A+2I)(A+I) = 0}$

$\downarrow$

mul. A^{-1}

$\boxed{A + 3I + 2A^{-1} = 0} \rightarrow$ A

C

31 $\Rightarrow$ $A^9 \Rightarrow \lambda^9 \longrightarrow \lambda = -2, -1$

A

$(-1)^9 = -1$

~~A~~ $511 A^9 + 510 I = 511(-1) + 510$

-1 $= -1$

32 $\Rightarrow$

$$|A - \lambda I| = \begin{bmatrix} 3/2 - \lambda & 0 & 1/2 \\ 0 & -1-\lambda & 0 \\ 1/2 & 0 & 3/2 - \lambda \end{bmatrix}$$

C

$$|A - \lambda I| = \left(\frac{3}{2} - \lambda\right)\left[\left(\frac{3}{2} - \lambda\right)(-1-\lambda)\right] + \frac{1}{2}\left[(1+\lambda)\frac{1}{2}\right]$$

$$= (1+\lambda)\left[-\left(\frac{3}{2} - \lambda\right)^2 + \left(\frac{1}{2}\right)^2\right]$$

$$= (1+\lambda)\left[\left(\frac{1}{2} + \frac{3}{2} - \lambda\right)\left(\frac{1}{2} - \frac{3}{2} + \lambda\right)\right]$$

$$= (1+\lambda)(2-\lambda)(\lambda - 1)$$

$$\lambda = -1, 1, 2$$

Eigen vector

C $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

33 ⇒

$$\begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix} = \phi$$

$2R_1 = R_2$

0

$|A| = 0$

34 ⇒ $N_{3\times3}$, $N^2 = 0$, Eigen value

No change

A

So — $\boxed{0, 0, 0}$ Ans

35 ⇒ S1 : M has 4 linear independent eigen vectors.
S2 : M has 4 distinct eigen values.
S3 : M ~~φ~~ is non-singular matrix.

C

True :-

→ S_2 implies S_1

36 ⇒

$\lambda_1 = 8$ | $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = 4$ | $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(A)

Solⁿ

$\lambda_1 + \lambda_2 = 12$

$\lambda_1 \lambda_2 = 32$

(A) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ → $6 + 6 = 12$

⇓

$|A| = 32$

37 ⇒

(B)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & u \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 0 & 0 & \lambda - 6 & u - 20 \end{array}\right]$$

No solⁿ for

$\lambda = 6, \quad u \neq 20$

38 ⇒

$2x + 3y = 0$
$6x + qy = 0$

(9)

$$\begin{vmatrix} 2 & 3 \\ 6 & q \end{vmatrix} = 0$$

$2q - 18 = 0$

$\boxed{q = 9}$

39 ⇒

$$M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{bmatrix}$$, Eigen Value = -3, -3, 5

(B)

at 5 Eigen Vector [1 2 -1]ᵀ

not change

So

(B) $[1, 2, -1]^T$