Probability and Statistics

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- Our dice will have only 4 faces.
- As if we have 4 pieces of paper numbered $1, 2, 3, 4$. Throwing a die is same as drawing a numbered paper at random here.

Rules for our game

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- Let's begin playing to understand what's going on.

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  Let us draw a point at that location.

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- So we need to go to

$$(0.8 \times 0.25 + 0.1\ ,\ 0.8 \times 0.4 + 0.04) = (0.3, 0.36)$$

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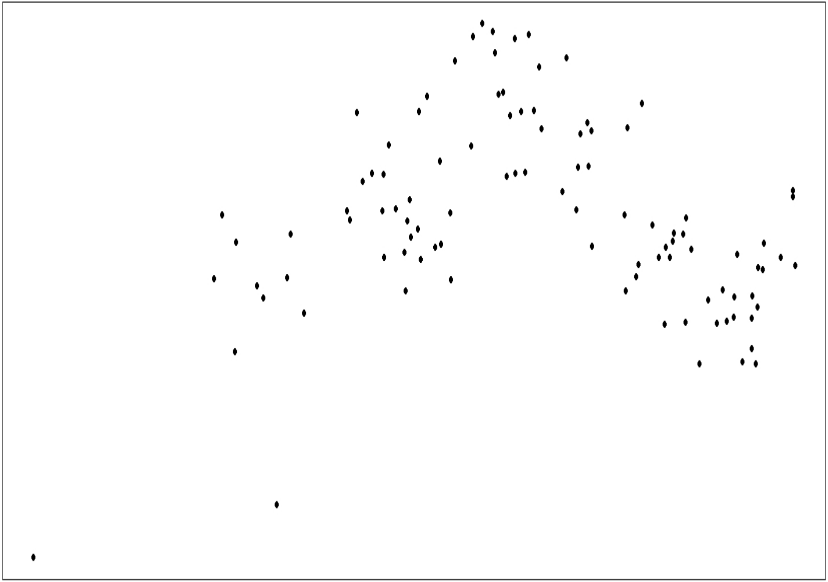
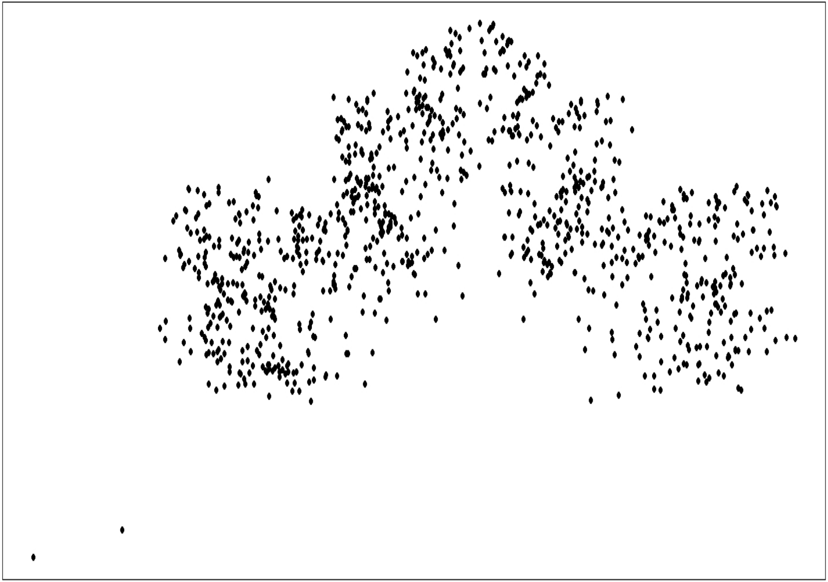
- If we keep on playing, the no of dots will keep on increasing.
- Now if we keep on playing this game, then what do we get to see? Or what can we expect to see?
- Want to see some magic?

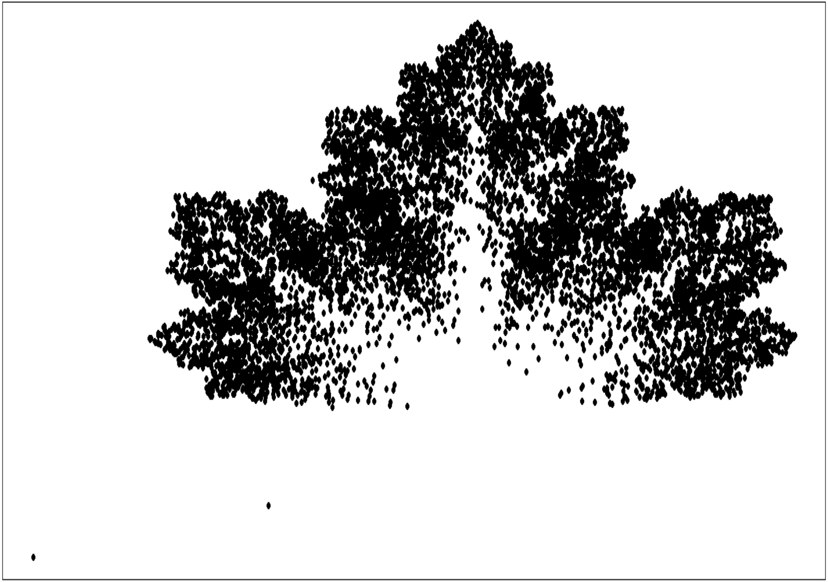
No of times we played = 100



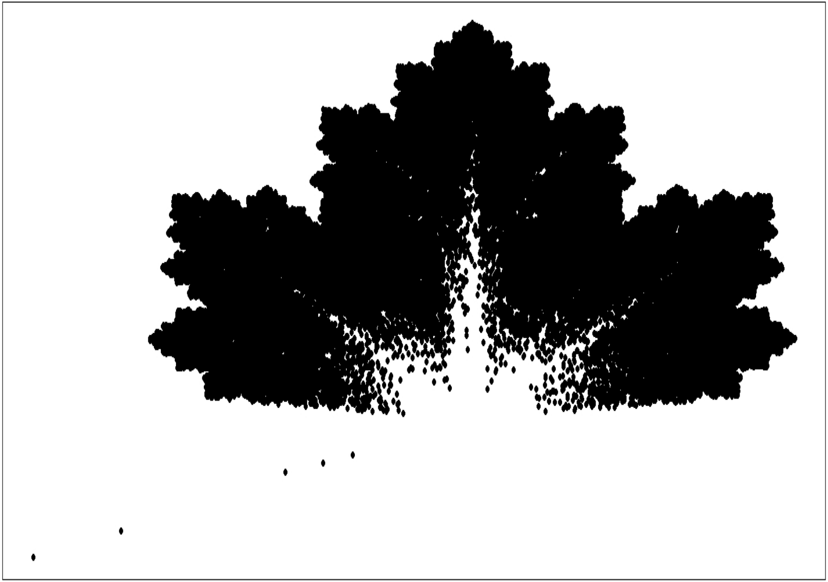
No of times we played = 1000



No of times we played = 10000



No of times we played = 1e+05



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- We didn't know what face would come up beforehand, but somehow we ended up with a picture as if it was drawn by an artist.
- If a every person in a group of friends played this game on their own, we would get a picture of a leaf everytime.
- Although note that the sequence of die face values obtained by each friend would obviously differ.

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- So, in our example, throwing of the die was random, but the output was something deterministic. This phenomenon is colloqially termed as statistical regularity.
- It's as if we kept on plotting points randomly, but ended up with a deterministic output.

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- Think of leaves on a tree or even of all our thumb impression!

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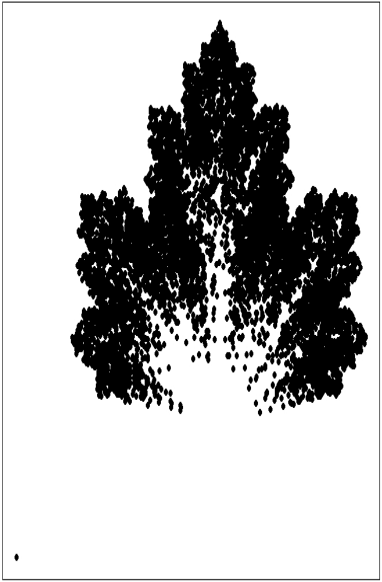

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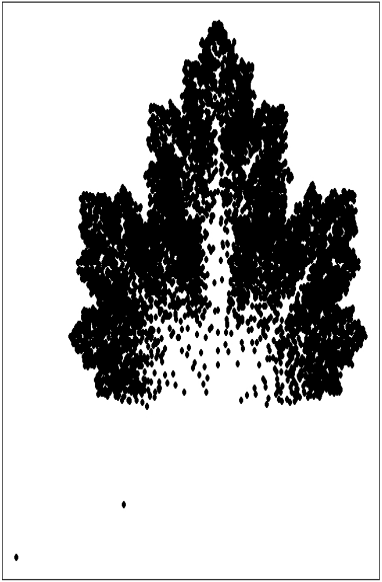

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- The ideas of probability and statistics will dictate the success of the life insurance company.

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- In our Mathematical Ludo example, the way we constructed our path (where to go corresponding to each die face value) is where the mystery lies.
- If we had taken them in the following manner:

Die face	Where to go..?
1	$(0\ ,\ 0.16y)$
2	$(0.85x - 0.04y\ ,\ -0.04x + 0.85y + 1.6)$
3	$(0.2x - 0.26y\ ,\ 0.23x + 0.22y + 1.6)$
4	$(-0.15x + 0.28y\ ,\ 0.26x + 0.24y + 0.44)$

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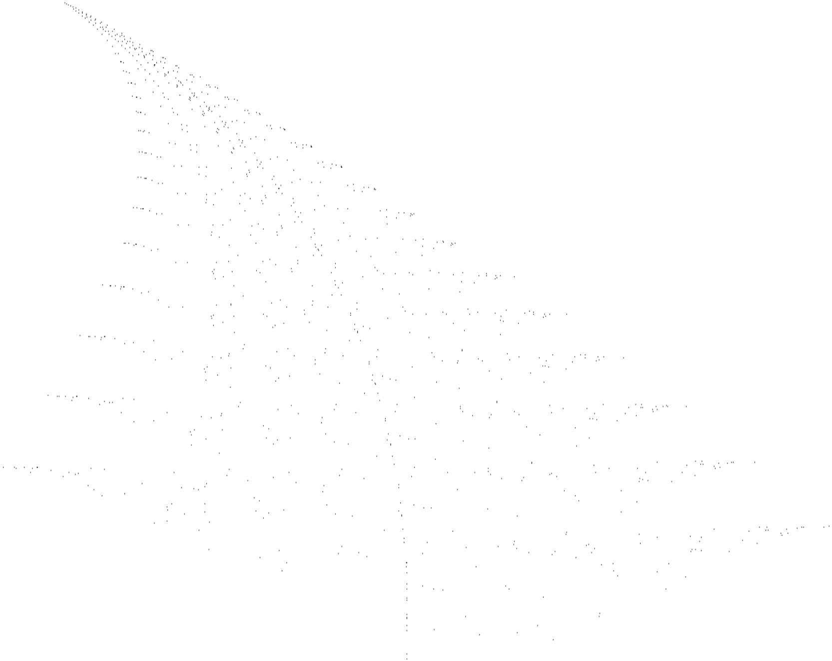

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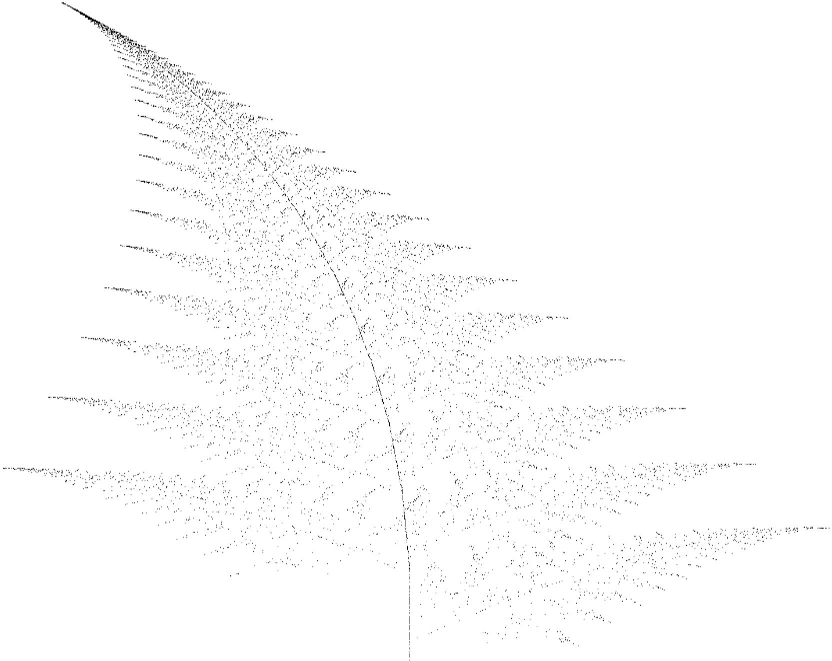
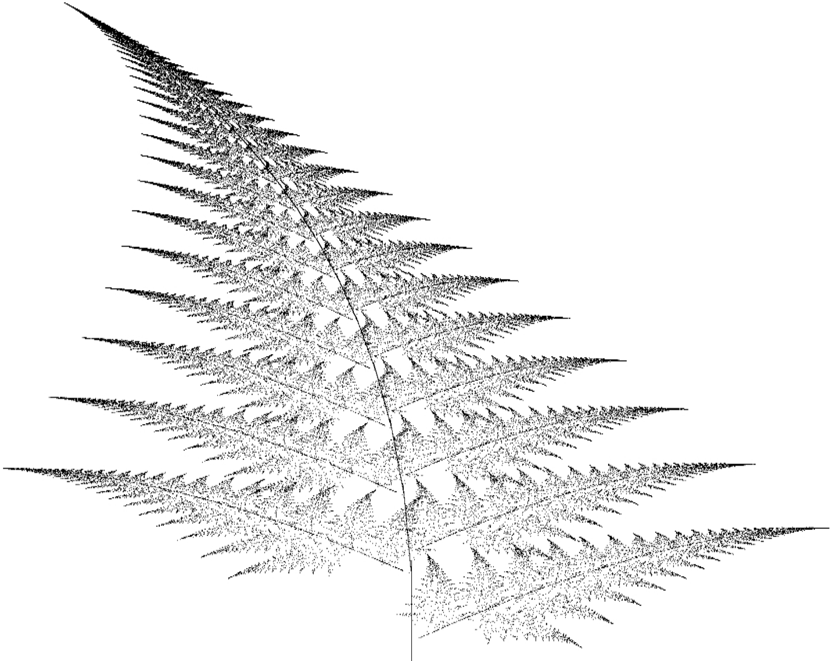


No of times we played = 100000



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- Such random practices is termed **random experiment**.
- Think in terms of a "coin toss".
- Be careful of the distinction between a random experiment and a trial.
- Since the experiment is random, there is no way to conclusively comment on the result of the experiment. What we can do is think of all the possible results that might occur on performing the expriment.

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- So the sample space is a set whose every element is a possible **outcome** of a random experiment.
- Another related concept is that of an **event**. An **event** is essentially a subset of the sample space. Let's take an example.

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- Now the sample space has 36 possible outcomes. For which of those outcomes, will we win the prize money?
- The answer is for all those (i,j) such that $i+j \geq 10$, i.e, for $\{(4,6),(5,5),(5,6),(6,4),(6,5),(6,6)\}$. Now this set is a subset of our sample space. So this can thought of as an example of an event.

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- Hint: Think of infinite sample space scenarios.

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Let a random experiment have sample space S. Then by event space of the random experiment we mean a collection $\mathcal{F}$ of subsets of S satisfying the following conditions.

- S$\in \mathcal{F}$; the entire sample space is itself an event.
- $\forall A \in \mathcal{F},\ A^c \in \mathcal{F}$; Complement of an event is also an event.
- $\forall A_1, A_2, ... \in \mathcal{F},\ A_1 \cup A_2 \cup ... \in \mathcal{F}$

These conditions also imply that $\mathcal{F}$ is closed under countable intersection. The elements of $\mathcal{F}$ are called events.

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- For a finite sample space S, the most common choice for event space is the power set $\mathcal{P}(S)$.
- We have learnt quite a few terminolgies. Let us revert back to the idea of statistical regularity.

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- So we have a table as follows:

Toss.No (n)	1	2	3	4	5
Outcome	head	tail	head	head	tail
Frequency	1	1	2	3	3

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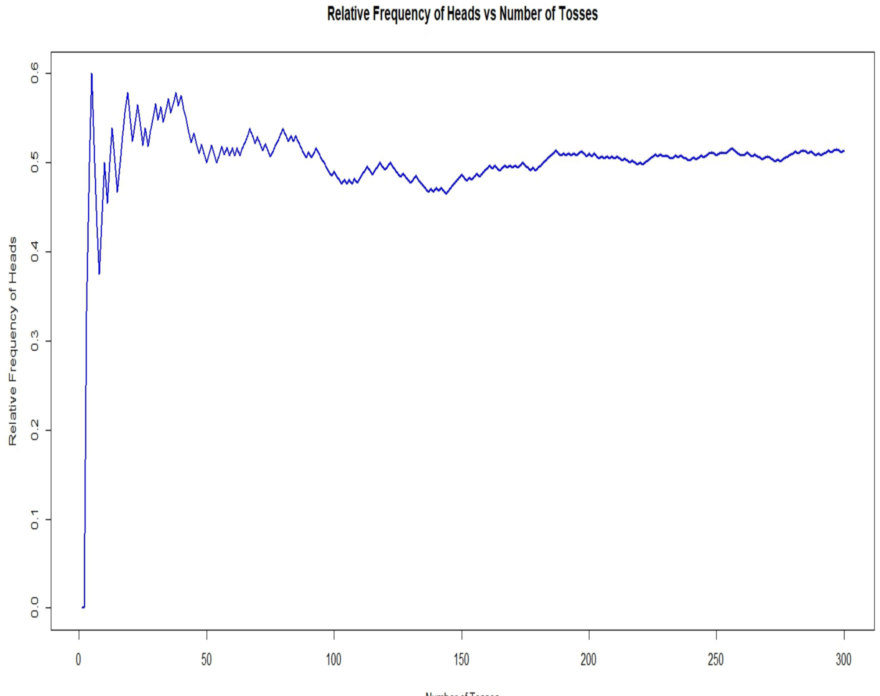
- In this example, we had conducted the experiment only 5 times. What if we had repeated this experiment multiple times?
- Now suppose we draw a graph, where on the x-axis we have the number of tosses and on the y-axis we have the relative frequency



Relative Frequency of Heads vs Number of Tosses
Relative Frequency of Heads
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
50
100
150
200
250
300

- The thing to note here is how the graph changes. See how the graph fluctuates at the beginning and then tends to stabilize at the end.

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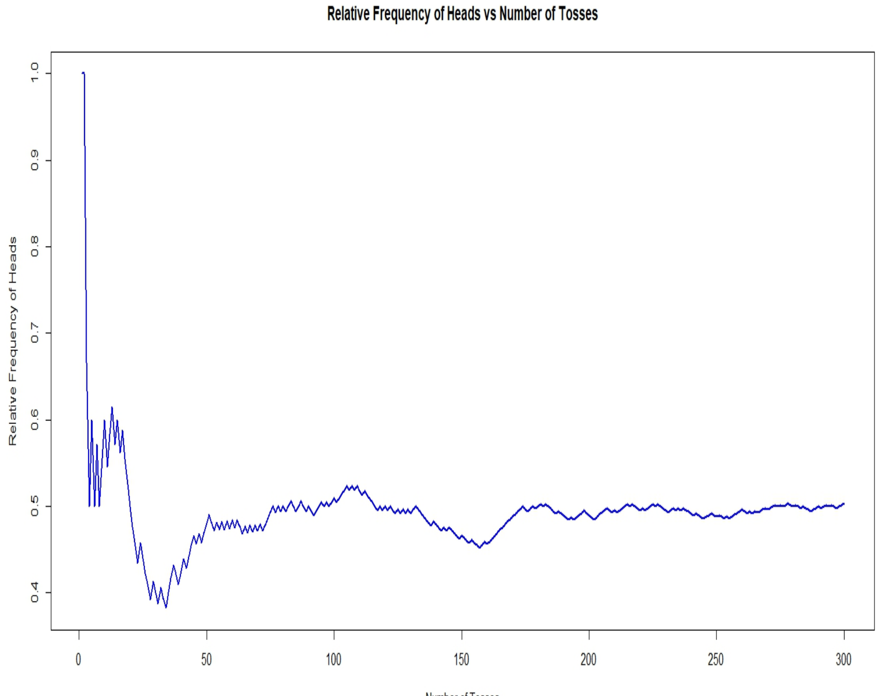
- It seems as if the random nature of the graph vanishes as no of tosses increases.
- Now suppose, if you give that coin to your friend and ask him to toss it for another 300 times (I don't know if he will remain your friend after this though!!)
- Also ask him to draw the same graph as you did previously. We see that the graph does differ. Although the horizontal nature of the graph as the no of tosses increases is evident here as well.


Relative Frequency of Heads vs Number of Tosses
Relative Frequency of Heads
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0
50
100
150
200
250
300

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- The trials need to be independent.

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- Thus we may think of randomness as “irregularity”.
- However in certain situations, outcomes of random experiments show a regularity that is similar to deterministic behaviour. This is what is called **statistical regularity**.
- If the same random experiment is repeated independently for a large number of times then certain aggregate behaviours of the outcomes often converge to a deterministic number.

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- Here as well, by aggregate we mean a value obtained from all the outcomes obtained from all the trials performed.
- Like in case of repeatedly tossing a coin, the relative frequency of obtaining head on the basis of all the tosses done.

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- Then it is seen that as $n \to \infty$ the random variable X_n approaches some fixed number. This behaviour is an example of statistical regularity. The limiting fixed number is called the probability of the event.

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- So, who came first, the egg or the chicken??
- We will explore a more rigorous definition of probability later, where this sort of a confusion does'nt arise. It is known as the axiomatic definition of probability.
- The idea exploring the idea of probaility using our understanding of relative frequency is still useful and perhaps the most easily understandable.

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- If we are unattentive on the road, then there is a higher chance of getting run over by a car.
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- We should never byepass our natural intuition.

Example

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- So we just cannot attach any probability to any event. There has to be some ground rules.
- Russian mathematician, Kolmogorov had made a list of these rules. These rules are known as axioms of probability.
- Using these axioms, we can establish a formal definition of probability.

Axiomatic definition of Probability

Let S be a nonempty set (called the sample space). Let $\mathcal{F}$ be the set of all events (subsets of S). Then a function $P : \mathcal{F} \to \mathbb{R}$ is called a probability if it satisfies the following conditions:

1. $\forall A \in \mathcal{F} \quad P(A) \geq 0$
2. $P(S) = 1$
3. For any countable collection of pairwise disjoint events $A_1, A_2, ... \in \mathcal{F}$

$$P\left(\cup_n A_n\right) = \sum_n P\left(A_n\right)$$

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Let A and B be two events such that $B \subseteq A$, then $P(B) \leq P(A)$.

Proof.

Let $C = A \backslash B$. Then $B \cap C = \phi$ and $B \cup C = A$.
So,

$$P(A) = P(B \cup C) = P(B) + P(C) \quad \text{(by axiom 3, } \because B \cap C = \phi\text{) (by ax}$$

$$\geq P(B) \quad (\because P(C) \geq 0, \text{ by axiom 1})$$

$\square$

A few results

Theorem

For any event A, we have $P(A^c) = 1 - P(A)$.

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Theorem

For any event A, we have $P(A^c) = 1 - P(A)$.

Proof.

We know that $A \cup A^c = S$, the whole sample space.
Also $A \cap A^c = \phi$.
By axiom 2, we know $P(S) = 1$.
By axiom 3, we have $P(S) = P(A) + P(A^c)$.
Combining, $P(A) + P(A^c) = 1$.
So, $P(A^c) = 1 - P(A)$, as required. $\square$

Theorem

For any two events A *and* B *belonging to event space* S,

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THANK YOU!