- EM Algorithm (Numerical)

- Assume that we have 2 coins namely C_1 & C_2.
- Assume that the bias of C_1 is θ_1 (probability of getting head with C_1).
- Assume that the bias of C_2 is θ_2 (probability of getting head with C_2).
- We want to find the value of θ_1 & θ_2 by performing a number of trials.

First Experiment:

- We choose 5 times one of the coin.
- We toss the chosen coin 10 times.

(B) H T T T H H T H T H → $H \to \frac{5}{10}$, $T = \frac{5}{10}$.

(A) H H H H T H H H H T H
$H = \frac{9}{10}$, $T = \frac{1}{10}$.

(A) H T H H H H T H T H H
$H = \frac{8}{10}$, $T = \frac{2}{10}$.

(B) H T H T T T H H T T
$H = \frac{4}{10}$, $T = \frac{6}{10}$.

(A) T H H H T H H H H T H.
$H = \frac{7}{10}$, $T = \frac{3}{10}$.

θ_1 = number of head using C_1 / total no. of flips using C_1.

θ_2 = number of heads using C_2 / total numbers of flips using C_2.

(B)

Coin A	Coin B.
	5H, 5T.
9H, 1T	
8H, 2T	
	4H, 6T
7H, 3T	
24H, 6T	9H, 11T.

$\theta_1 = \frac{24}{24+6} = 0.8$ | $\theta_2 = \frac{9}{9+1} = 0.45$.

More Challenging Problem.

21 Sunday

No A & B are given.

H T T T H H T H T H.
H H H H T H H H H H
H T H H H H H T H H
H T H T T T H H T T.
T H H H T H H H T H.

• We do not know the identities of the coins used for each set of tosses (we treat them as hidden variables).

? H T T T H H T H T H (5H, 5T).
? H H H H T H H H H H (9H, 1T)
? H T H H H H H T H H (8H, 2T)
? H T H T T T H H T T (4H, 6T)
? T H H H T H H H T H (7H, 3T).

Head Probability: $\theta_A = 0.60$, $\theta_B = 0.50$ { At Random Consideration }.

JUNE '20

Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30												

JULY '20

Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31									

$\theta_A = 0.6$

$\theta_B = 0.5$

First Round → 5 H, 5 T.

$$P(k) = \theta^k (1-\theta)^{n-k}.$$

Likelihood of A = $P_A(h)^h (1 - P_A(h))^{10-h}$

$= 0.65 (1 - 0.6)^{10-5}$

$= 0.0007962624.$

JUNE '20

Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30												

$= \cancel{0.000}$
$= 0.0017728025$

Likelihood of B = $P_b(h)^h \cdot (1 - P_b(h))^{10-h}$
$= .5^5 (1-0.5)^{10-5}$
$= 0.0009765625$

Normalize by using A/A+B
$= 0.00079624 / (0.00079624 + 0.0009765625)$
$= 0.45$

Normalize by using B/A+B.
$= (0.0009765625 / 0.00079624 + 0.0009765625)$
$= 0.55$

Likely No of H & T. for first Toss.
For A = H = 0.45 * 5 = 2.2, T = 0.45 * 5 = 2.2.
For B = H = 0.55 * 5 = 2.8, T = 0.55 * 5 = 2.8.

JULY '20

Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31									

25 June
Thursday

2020
Week 26 Day (177-188)

Coin A	Coin B	A	B
= 2.2H, 2.2T	2.8H, 2.8T	0.45	0.55
7.2H, 0.8T	1.8H, 0.2T	0.80	0.20
5.9H, 1.5T	2.1H, 0.5T	0.73	0.27
1.4H, 2.1T	2.6H, 3.9T	0.35	0.65
4.5H, 1.9T	2.5H, 1.1T	0.65	0.35
$\sum$ = 21.3H, 8.6T	11.7H, 8.4T		

$\rightarrow \frac{21.3}{21.3+8.6} = 0.71$

$\rightarrow \frac{11.7}{11.7+8.4} = 0.58$

Maximize

JUNE '20

Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30												

June 26
Friday

2020
Week 26 Day (178-188)

$O_A = 0.71$

$O_B = 0.58$

$O_A = 0.80$

$O_B = 0.52$

JULY '20

Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31									