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Assignment - 6 : Parameter ~~Ass~~ Estimation

Q1: (Max likelihood estimation of mean $= \theta_1$ & variance $= \theta_2$ in normal pop.)

Sol: PDF : $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

& $\mu = \theta_1$, $\sigma^2 = \theta_2$

$$\therefore f(x_i) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Likelihood function : $L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i)$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

$$= \prod_{i=1}^{n} (\theta_2)^{-1/2} \cdot \prod_{i=1}^{n} (2\pi)^{-1/2} \cdot \prod_{i=1}^{n} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

$$= (\theta_2)^{-n/2} \cdot (2\pi)^{-n/2} e^{-\prod_{i=1}^{n} \frac{(x_i-\theta_1)^2}{2\theta_2}}$$

$$= (\theta_2)^{-n/2} (2\pi)^{-n/2} e^{\left(-\frac{1}{2\theta_2} \prod_{i=1}^{n} (x_i-\theta_1)^2\right)}$$

Take log on both sides

$$Z = \ln(L(\theta_1, \theta_2)) = -\frac{n}{2} \ln(\theta_2) - \frac{n}{2} \ln(2\pi) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i-\theta_1)^2 \quad \text{--- (2)}$$

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Diff. b.s. w.r.t. θ_1 (on ①)

$$\frac{\partial z}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

Now, $\frac{\partial z}{\partial \theta_1} = 0$

$$\Rightarrow \frac{1}{\theta_2} \left(\sum_{i=1}^{n} (x_i - \theta_1) \right) = 0$$

$$= \sum_{i=1}^{n} x_i - n\theta_1 = 0$$

$$= n\theta_1 = \sum_{i=1}^{n} x_i$$

$$= \theta_1 = \sum_{i=1}^{n} \frac{x_i}{n} = \bar{x}$$

$$\boxed{\theta_{1_{NLE}} = \bar{x}_n}$$

hence $\theta_1 = \bar{x}$ is therefore sample mean

Now, diff. b.s. w.r.t θ_2 (on ①)

$$\frac{\partial z}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Now, $\frac{\partial z}{\partial \theta_2} = 0$

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$$\Rightarrow \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\therefore \theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

As, $\theta_1 = \bar{x}$, so

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Sol 2: PDF of binomial dist. $f(x) = {}^{m}C_{x}\, p^{x} (1-p)^{m-x}$

Here, $n = m$ & $p = \theta$

$$f(x) = {}^{m}C_{x}\, \theta^{x} (1-\theta)^{m-x}$$

$$\Rightarrow f(x_i) = {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

<u>likelihood func:</u> $L(m, \theta) = \prod_{i=1}^{n} f(x_i)$

$$L(m, \theta) = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

$$L(m, \theta) = \prod_{i=1}^{n} {}^{m}C_{x_i} \prod_{i=1}^{n} \theta^{x_i} \prod_{i=1}^{n} (1-\theta)^{m-x_i}$$

$$\cancel{L(m, \theta) = \prod_{i=1}^{n} {}^{m}C_{x_i} \prod_{i=1}^{n} \theta^{x_i} \prod_{i=1}^{n} (1-\theta)^{m-x_i}}$$

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$$L(m,\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{mn - \sum_{i=1}^{n} x_i}$$

Taking log b.s -

$$\ln(L(m,\theta)) = \ln\left[\prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{\prod_{i=1}^{n} x_i} (1-\theta)^{mn - \sum_{i=1}^{n} x_i}\right]$$

$$\ln(L(m,\theta) = \ln\left(\prod_{i=1}^{n} {}^{m}C_{x_i}\right) + \sum_{i=1}^{n} x_i \ln\theta +$$

$$\underbrace{\left(mn - \sum_{i=1}^{n} x_i\right)\left(\ln(1-\theta)\right)}_{=Z} \quad \text{--- (1)}$$

Diff. (1) w.r.t. θ

$$\frac{\partial z}{\partial \theta} = \frac{1}{\theta}\sum_{i=1}^{n} x_i + \left(\frac{\sum_{i=1}^{n} x_i - mn}{1-\theta}\right)$$

Now, $\frac{\partial z}{\partial \theta} = 0$

$$\Rightarrow \frac{1}{\theta}\sum_{i=1}^{n} x_i + \left(\frac{\sum_{i=1}^{n} x_i - mn}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i - mn}{\theta - 1} = \left(\frac{\sum_{i=1}^{n} x_i}{\theta}\right)$$

$$\cancel{-}\frac{mn}{\sum_{i=1}^{n} x_i} = \cancel{-}\frac{1}{\theta}$$

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$$\theta = \frac{\overline{x_n}}{m}$$

$$\boxed{\theta_{MLE} \; \mathcal{E}(0,1) = \frac{\overline{x_n}}{m}}$$