Ergoregion in Magnetized Kerr Black Hole Spacetime

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Abstract

Black holes are heavy and compact objects that have strong gravitational fields. A Lorentz invariant quantity, spacetime interval defines the geometry and the exterior of the black hole. This allows us to formulate the boundaries of the black hole. Ergoregion is a region specifically present in a rotating black hole where no object can stay at stand still but has to rotate in the direction of the rotation of the black hole. This region is of great concern for extracting energy from the black hole. In the black hole spacetime, when a magnetic field is embedded in the background, the geometry of the ergoregion changes and the extraction of energy becomes more efficient as the energy can be electromagnetically extracted apart from the rotational counterpart. In this report, we will see how a Kerr black hole changes for $a \leq 1$ and $a > 1$, the ergoregion of magnetized Reissner-Nordstrom black hole and what happens when a Kerr black hole is placed in a magnetized spacetime and the ergoregion in this spacetime. Note that all equations, unless otherwise stated, are in natural units ($c = 1$, $G = 1$).

1 Introduction

Black Holes are the most dense and compact objects present in the universe. Black holes are the end stages of heavier stars. With the advent of Einstein's General theory of Relativity, the existence of black hole was theorised as a mathematical solution to the famous Einstein's field equation. These have been the topic of interest in the astrophysics society and research on the same has been going for decades. Ever since then many mathematical models have been formulated to describe the geometry and the spacetime of various types of black hole.
A black hole is characterized by only its Mass (M), angular momentum (J) and charge (Q) and its state will not be defined by the state before its formation. This is the **No Hair Theorem**. Based on the characterization parameters, black hole spacetime can be divided into 4 fundamental types: **Schwarzschild black hole** which is characterised by only the mass (M), **Reissner Nordstrom black hole** which is characterised by the mass (M) and charge (Q), **Kerr black hole** which is characterised by the mass (M) and angular momentum (J). **Kerr-Newman black hole** which is characterised by the mass (M), angular momentum (J) and the charge (Q).
In relativity, spacetime is defined as a manifold of events endowed with a metric and a metric defines the geometry of the spacetime.
In a black hole, there exists a boundary beyond which no matter can be observed by a faraway observer. This boundary is called the **Event Horizon**. **Ergosphere/Ergoregion** also called

as stationary surface limit, is another region present outside a rotating black hole's outer event horizon. Outside of this region, an object can stay static or at rest and when it slips into this region, the object can either fall into the black hole or an object can stay in orbit moving in the direction of the rotation of the black hole.
When the flat spacetime background is filled with an uniform magnetic field, the resulting background is called as **Melvin universe** or **Melvin Metric** [1]. With further studies into the energetics of black hole and electrodynamics, recent researches have been about the spacetime geometry of black holes when immersed in an external magnetic field. Henceforth, this type of spacetime will be called **magnetized spacetime**. With this addition of magnetic field, the ergoregion of the black hole changes drastically beyond comprehension.
In general relativity, any heavy rotating object can distort the spacetime and any particle in the vicinity of the object can get reoriented. Hence, the object drags any particle and the spacetime itself in the direction of its rotation. This effect is known as Frame Dragging. Similarly, when an uncharged rotating black hole is placed in a magnetic field, the black hole drags the magnetized background as well, thereby current density is produced.
In the following sections, we will examine the ergoregions of various black hole spacetimes starting with Kerr black hole then magnetized Reissner Nordstrom black hole and lastly magnetized Kerr black hole.

2 Spacetime

In simple terms, spacetime consists of 3 dimensions of space and one dimension of time that is mutually perpendicular to the 3 space axes. The mathematical generalisation of a spacetime is given by,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \tag{2}$$

where $\mu, \nu = 0, 1, 2, 3 \leftrightarrow t, x, y, z \leftrightarrow t, r, \theta, \phi$ and $x^\mu = (x^0, x^1, x^2, x^3) \leftrightarrow (t, x, y, z) \leftrightarrow (t, r, \theta, \phi)$
An equation of this form is either called the spacetime interval or line element and the coefficients of such an equation gives the metric of a given spacetime. From any given spacetime, the boundary of event horizon can be deduced by putting $1/g_{rr} = 0$ and the ergoregion by putting $g_{tt} = 0$. The simplest form of spacetime is called the **Minkowski spacetime** which defines a flat spacetime and in cartesian coordinates it is given by [2]

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{3}$$

or,

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \tag{4}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

The above metric is called the Minkowski metric which defines a flat spacetime.
Karl Schwarzschild solved the field equations to determine the spacetime geometry of a non-rotating point particle and describes the gravitational field around the body. It is defined in spherical coordinates and is given by,

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{6}$$

$$g_{\mu\nu} = \begin{pmatrix} -1 - \frac{2M}{r} & 0 & 0 & 0 \\ 0 & (1 - \frac{2M}{r})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta \end{pmatrix} \tag{7}$$

This spacetime can be extended to give the geometry of the exterior vacuum region outside any localised spherically symmetric source. Hence such a spacetime can give the geometry of any non-rotating spherical stars and planets. Any black hole defined by this spacetime geometry is said to be **Schwarzschild Black hole** which is characterised by only its mass (M).

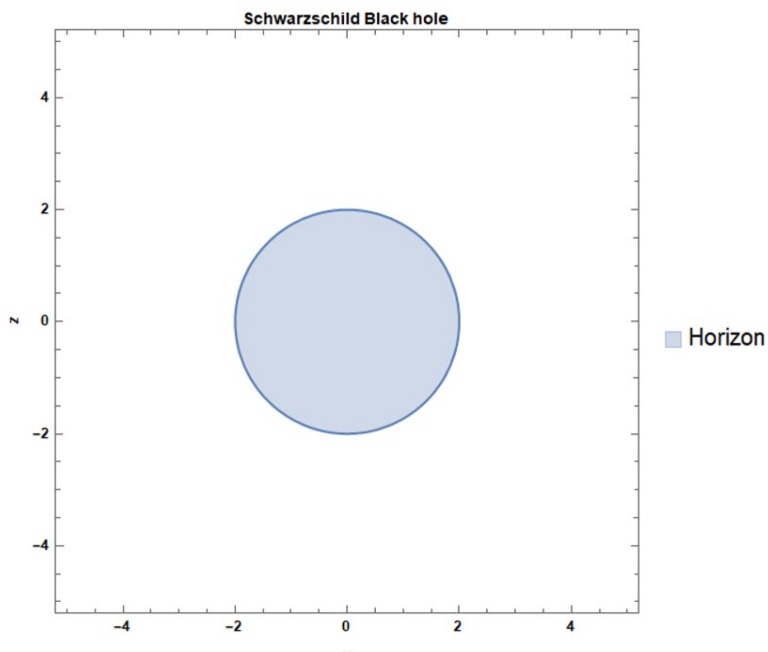

Figure 1: Cross section of the event horizon of a Schwarzschild Black hole of radius 2 wherein $G = 1$, $M = 1$

The boundary of no return or the event horizon is found out to be,

$$\frac{1}{g_{rr}} = 0$$

$$r_H = 2M \tag{8}$$

This is also known as the Schwarzschild radius. In case of this type of black hole, the ergoradius is the same as the Schwarzschild radius but the ergoregion is physically absent.

2.1 Kerr Spacetime

As discussed in the previous section about the Schwarzschild spacetime which describes the geometry of the exterior region around a non-rotating spherically symmetric object, a New Zealand mathematician, Roy Kerr discovered the Kerr spacetime/geometry which is an exact solution to the Einstein's field equation that describes the geometry of the exterior spacetime and the gravitational field around an uncharged rotating black hole. This spacetime is an extension to the Schwarzschild spacetime and is characterized by the mass (M) and the angular momentum (J). This type of black hole is not spherically symmetric but axially symmetric. Any fast and heavy rotating body can be described by the Kerr spacetime.

$$ds^2 = -(1-\frac{2Mr}{\rho^2})dt^2+\frac{\rho^2}{\Delta}dr^2+\rho^2 d\theta^2+(r^2+a^2+\frac{2Mra^2sin^2\theta}{\rho^2})sin^2\theta d\phi^2-\frac{4Marsin^2\theta}{\rho^2}dtd\phi \tag{9}$$

$$g_{\mu\nu} = \begin{pmatrix} -1-\frac{2Mr}{\rho^2} & 0 & 0 & -\frac{2Marsin^2\theta}{\rho^2} \\ 0 & \frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\frac{2Marsin^2\theta}{\rho^2} & 0 & 0 & (r^2+a^2+\frac{2Mra^2sin^2\theta}{\rho^2})sin^2\theta \end{pmatrix} \tag{10}$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 - 2Mr + a^2$$

$$a = J/M$$

The above line element defines the Kerr geometry in Boyer-Lindquist spherical coordinates. The parameter **a** is called the angular momentum parameter which is defined as the angular momentum per unit mass (J/M). The time t in the equation is the time observed by a far away stationary observer. In the sections to follow, we will look into the different coordinate systems employed to describe a Kerr black hole.
The Kerr spacetime is generally defined in two different corrdinates according to the need namely [3] ,

1. **Kerr Schild Cartesian Coordinates**
   - This is the cartesian form of representing the Kerr spacetime and is given by,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \tag{11}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2Mr^3}{r^4+a^2z^2}l_\mu l_\nu \tag{12}$$

where,

$$l_\mu = (1, \frac{rx+ay}{r^2+a^2}, \frac{ry-ax}{r^2+a^2}, \frac{z}{r}) \tag{13}$$

and r is implicitly defined as

$$\frac{x^2+y^2}{r^2+a^2} + \frac{z^2}{r^2} = 1 \tag{14}$$

2. **Boyer Lindquist Spherical Coordinates**

- This is the spherical representation of the Kerr spacetime and is given by,

$$ds^2 = -(1-\frac{2Mr}{\rho^2})dt^2+\frac{\rho^2}{\Delta}dr^2+\rho^2 d\theta^2+(r^2+a^2+\frac{2Mra^2sin^2\theta}{\rho^2})sin^2\theta d\phi^2-\frac{4Marsin^2\theta}{\rho^2}dtd\phi$$

$$g_{\mu\nu} = \begin{pmatrix} -1-\frac{2Mr}{\rho^2} & 0 & 0 & -\frac{2Marsin^2\theta}{\rho^2} \\ 0 & \frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\frac{2Marsin^2\theta}{\rho^2} & 0 & 0 & (r^2+a^2+\frac{2Mra^2sin^2\theta}{\rho^2})sin^2\theta \end{pmatrix}$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 - 2Mr + a^2$$

$$a = J/M$$

When $1/g_{rr}$ is equated to 0, we get a quadratic equation which yields two solutions i.e two radius of event horizon, one is larger than the other. The larger is called the **Outer Event Horizon** and smaller is called the **Inner Event Horizon** or **Cauchy Horizon**. The inner region is deduced as a result of mathematical conjecture.
The Outer Event Horizon is,

$$r_+ = M + \sqrt{M^2-a^2} \tag{15}$$

The Inner Event Horizon is,

$$r_- = M - \sqrt{M^2-a^2} \tag{16}$$

Similarly, the radius of ergoregion can be derived by equating g_{tt} to 0 which yields a quadratic equation resulting into two radius solutions. The larger radii is called the Outer Ergosphere and the smaller on is called the Inner Ergosphere. Usually the inner ergosphere is smaller than the Cauchy horizon.
The Outer Ergosphere is,

$$r_E^+ = M + \sqrt{M^2-a^2\cos^2\theta} \tag{17}$$

The Inner Ergosphere is,

$$r_E^- = M - \sqrt{M^2-a^2\cos^2\theta} \tag{18}$$

Hence, comparing the radii of ergosphere and event horizons, we can infer that,

$$r_E^+ \geq r_+ \geq r_- \geq r_E^- \tag{19}$$

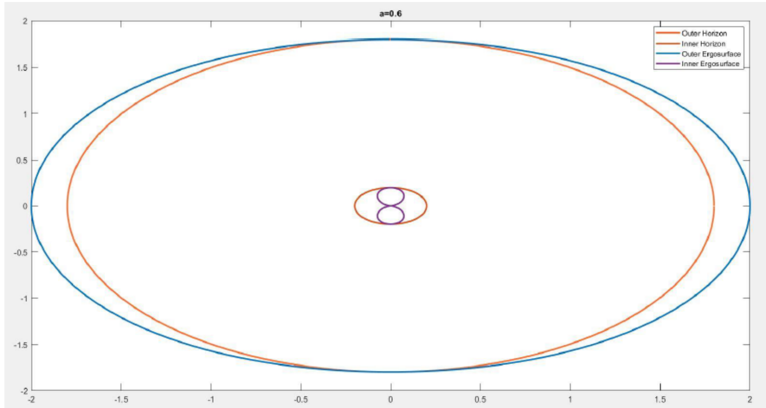

Figure 2: The boundaries of a Kerr black hole with $M = 1$ and $a = 0.6$ in BL coordinates.

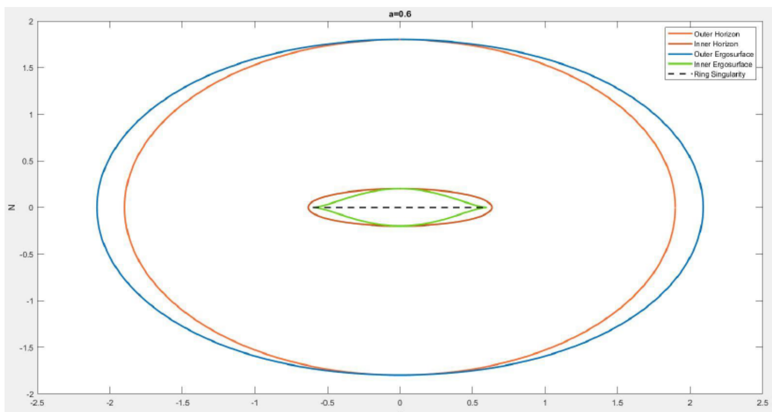

Figure 3: The boundaries of a Kerr black hole with $M = 1$ and $a = 0.6$ in KS coordinates.

Using these two coordinates, we can define the location of the horizon in Cartesian form as,

$$\frac{x^2}{2Mr_\pm} + \frac{y^2}{2Mr_\pm} + \frac{z^2}{r_\pm^2} = 1 \tag{20}$$

The above equation is the equation of an ellipsoid.
The ergosphere representation in Cartesian form is quite complicated, hence the ergosphere is parameterically represented and is given by two equations,

$$\sqrt{x_E^2 + y_E^2} = \sqrt{r_E^{\pm 2} + a^2} \sin\theta \tag{21}$$

$$z_E = r_E^\pm \cos\theta \tag{22}$$

Generally, in a Kerr black hole for $a < J/M^2$, the singularity is enclosed by the event horizon but when $a > 1$, the event horizon no longer exists and we get a Kerr naked singularity which is visible to external observers in theory. But the Cosmic Censorship Hypothesis prohibits this which states that there can be no naked singularity in existence, hence for every Kerr black hole $a < 1$. However, researchers and observers have been trying to find Superspinar ($a > 1$) Kerr black holes by investigating the x-ray spectrum of the accretion disk and if the spin parameter is greater or lesser than 1. If we consider a superspinar kerr black hole, there exists no event horizon but the ergosphere is still present but only at certain angles. The real value of radius of the ergosphere can be deduced for this range of θ. [4]

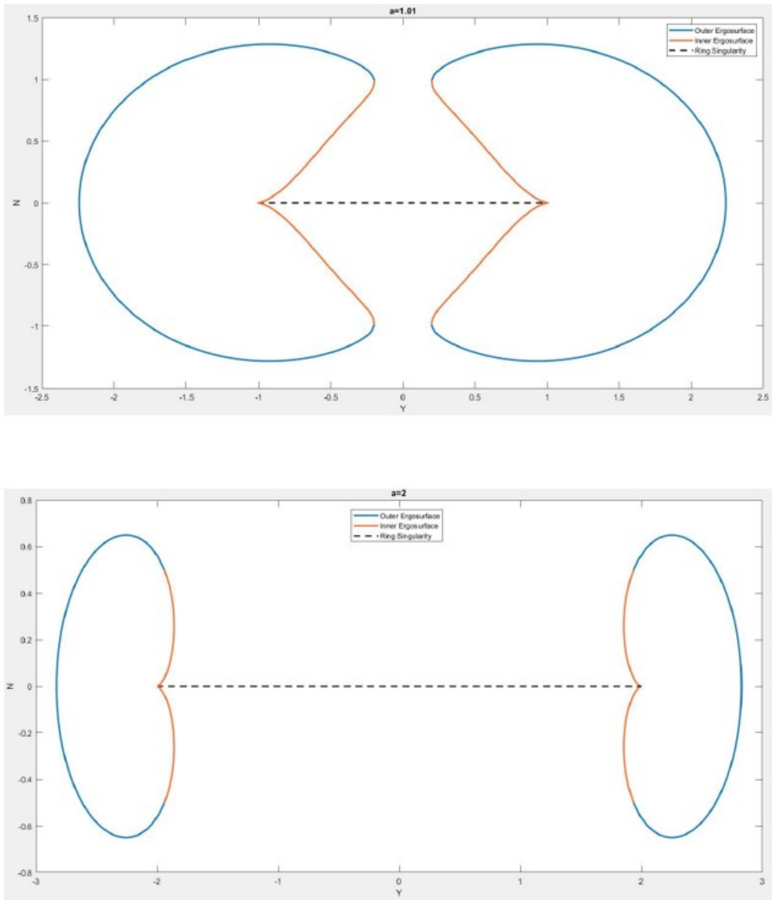

Figure 4: The boundaries of a Superspinar Kerr black hole with $M = 1$, $a = 1.01$ (Top) and $a = 2$ (Bottom) in KS coordinates.

$$r_E^{\pm} = M \pm \sqrt{M^2 - a^2 \cos^2 \theta} \tag{23}$$

From the above equation, for all $1/a < cos\theta$, the ergoradius is imaginary and the egoregion does not exist for angles $0 <= \theta < cos^{-1}(1/a)$. However, for angular regions $arccos(1/a) > \theta > \pi/2$, the ergoradii is real and the ergoregion is present in this range. The boundaries of the Kerr black hole were plotted for both types of coordinates for various values of the spin parameter (a).

The angle of no ergoregion in Boyer Lindquist coordinates is given as,

$$-cos^{-1}(1/a) < \theta_{ne} < cos^{-1}(1/a) \tag{24}$$

The opening angle for an observer to access the region near the ring singularity without crossing the ergoregion is given as $\theta_{op} = 2\theta_{ne}$. For $a = 1.01, \theta_{op} = 49.24°$ and for $a = 2, \theta_{op} = 120°$ [See Figure 5]. This tells us that for even a small increase in the spin parameter from 1, the ergoregion

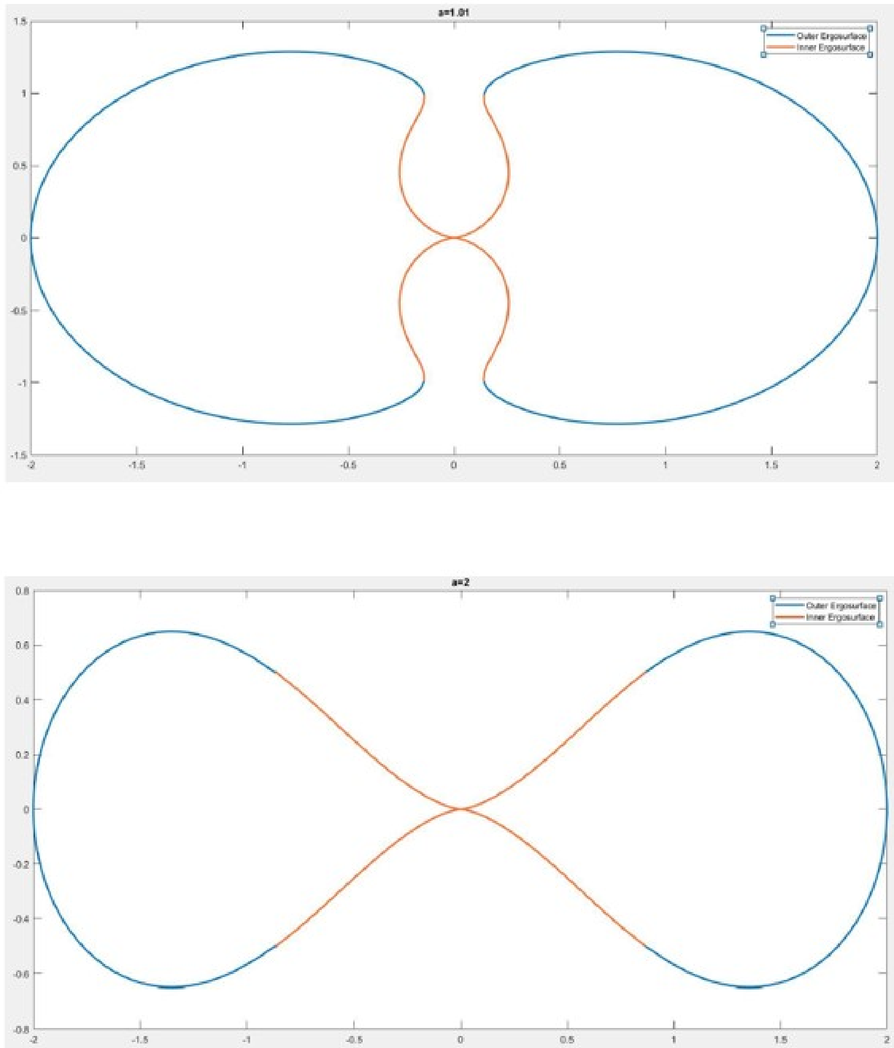

Figure 5: The boundaries of a Superspinar Kerr black hole with $M = 1$, $a = 1.01$ (Top) with no ergoregion for the region of angle $\theta_{op} = 49.24°$ and $a = 2$ (Bottom) with no ergoregion for the region of angle $\theta_{op} = 120°$ in BL coordinates.

changes drastically and at the poles, a conical region of no ergoregion is formed which allows observers to be in the vicinity of the ring singularity.

3 Magnetized spacetime

This section deals with a black hole spacetime appended in an ambient magnetic field B. Firstly, we will look into how the magnetic field affects the exterior of a Reissner-Nordstrom (charged) black hole. Then, we will examine the same for a Kerr black hole emmersed in a magnetic field.

For generating the magnetized black hole solutions, a magnetizing transformation procedure, SU(2,1) transformation is used to include the magnetic field over the regular line element of a black hole. For any black hole in a magnetized spacetime, the metric becomes asymptotic to the Melvin metric near the poles at infinity i.e the metric of the black hole at infinity will resemble that of the Melvin metric or magnetized flat spacetime. Now we will look into magnetized Reissner-Nordstrom spacetime.

3.1 Magnetized Reissner-Nordstrom spacetime

The line element of a magnetized RN black hole is represented in the form,

$$ds^2 = H(-fdt^2 + f^{-1}dr^2 + r^2 d\theta^2) + H^{-1}r^2\sin^2\theta(d\phi - \omega dt)^2 \tag{25}$$

$$g_{\mu\nu} = \begin{pmatrix} -fH + \frac{\omega^2 r^2 \sin^2\theta}{H} & 0 & 0 & -H^{-1}\omega r^2 \sin^2\theta \\ 0 & Hf^{-1} & 0 & 0 \\ 0 & 0 & Hr^2 & 0 \\ -H^{-1}\omega r^2\sin^2\theta & 0 & 0 & -H^{-1}r^2\sin^2\theta \end{pmatrix} \tag{26}$$

where

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$
$$H = 1 + \frac{1}{2}B^2(r^2\sin^2\theta + 3Q^2\cos^2\theta) + \frac{1}{16}B^4(r^2\sin^2\theta + Q^2\cos^2\theta)^2$$
$$\omega = -\frac{2QB}{r} + \frac{1}{2}QB^3 r(1 + f\cos^2\theta) + 2MQB^3$$

From the above, we can infer that there is a $d\phi dt$ dependency which means that there is a coupling between time and motion in the plane of rotation. The magnetized Reissner-Nordstrom black hole is axially symmetric unlike a non-magnetized Reissner-Nordstrom black hole which is spherically symmetric. The event horizon remains the same as that of a normal Reissner-Nordstrom black hole (RNBH) [See Figure 6].

$$r_H = M + \sqrt{M^2 - Q^2} \tag{27}$$

Unlike a RNBH, the magnetized RNBH will have an ergoregion analogous to that of a Kerr black hole at the equatorial plane. The ergoregion can be determined where g_{tt} becomes positive.

$$g_{tt} = -fH + \frac{\omega^2 r^2 \sin^2\theta}{H} \tag{28}$$

The first term of equation (28) vanishes on the horizon (r_+) whereas the second term contributes positively for all $sin\theta \neq 0$, therefore g_{tt} will be positive near to the exterior of the horizon. Also, at large r near polar axes with $sin\theta$ becoming small, g_{tt} will be positive.

Figure 6 suggests that when a magnetic field is embedded on a RNBH spacetime, an ergoregion is formed. From figure 7 it can be noted that the ergoregion is absent if the charge is 0 ($Q = 0$), in this case the metric reduces to Schwarzchild-Melvin metric.
When g_{tt} is numerically evaluated, it is evident that the ergoregion extends to infinity near the poles and asymptotically approaches the z axis as z tends to infinity. At the poles, there are regions where g_{tt} is negative which remotely depicts that of vaccuum region. [5]

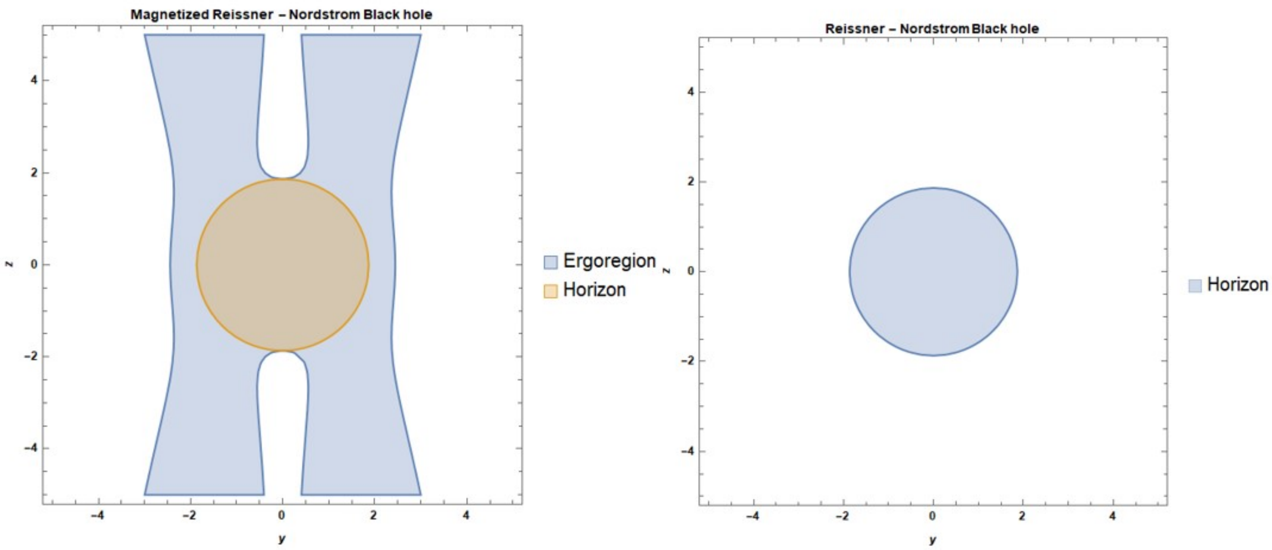

Figure 6: The cross section of the ergoregion and event horizon of a Magnetized Reissner-Nordstrom black hole (left) with $M = 1$, $B = 1$ and $Q = \frac{1}{2}$ and of a Reissner-Nordstrom black hole (Right) with $M = 1$, $B = 0$ and $Q = \frac{1}{2}$. Here the radius of event horizon remains the same.

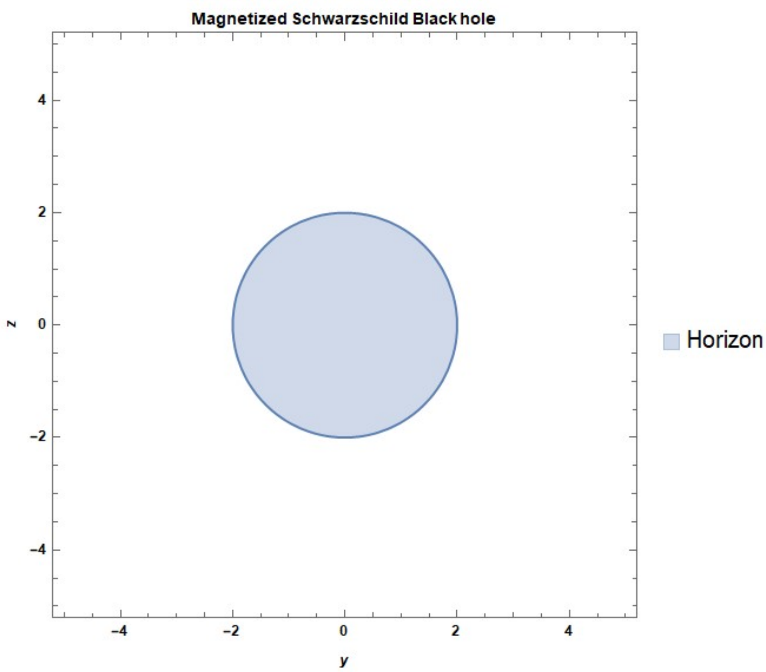

Figure 7: The cross section of the ergoregion and event horizon of a Schwarzschild-Melvin black hole with $M = 1$, $B = 1$ and Q=0.

3.2 Magnetized Kerr Spacetime

This is an extension to the previous Magnetized Reissner-Nordstrom spacetime wherein the charge is set to 0 and the black hole is made to rotate. Before we further our discussion on this, a term called **Frame Dragging** must be familiarized. Frame dragging is observed on the spacetime due to a non-static/rotating object. This is otherwise known as Lense-Thirring effect which states that the rotation of massively heavy object can influence the spacetime to distort and the particles near it to co-rotate with the object, making the orbit of the particle to change orientation. This effect is predominantly evident in the ergosphere wherein it is impossible to stay at rest and due to frame dragging, any particle inside the ergosphere will experience this effect and co-rotates with the massive object.[6]

When this study is extended to a magnetized spacetime, a rotating body with a large mass can also drag the magnetic field in the direction of its rotation. Due to this, electromagnetic storms are produced and current density is induced which result in the black hole to charge up, theoretically charges up to the Wald charge. The energy from these storms can be extracted via the electromagnetic Penrose process which is much more efficient than the mechanical Penrose process. This allows a particle to radiate out with extreme energies.

The line element of a magnetized Kerr metric is given as, [7]

$$ds^2 = (-\frac{\Delta}{A}dt^2 + \Delta^{-1}dr^2 + d\theta^2)\Sigma|\Lambda|^2 + \frac{A\sin^2\theta}{\Sigma|\Lambda|^2}(|\Lambda_0|^2 d\phi - \omega dt)^2 \tag{29}$$

$$g_{\mu\nu} = \begin{pmatrix} -\frac{\Delta\Sigma|\Lambda|^2}{A} & 0 & 0 & -\frac{\omega|\Lambda_0|^2 A\sin^2\theta}{\Sigma|\Lambda|^2} \\ 0 & \frac{\Sigma|\Lambda|^2}{\Delta} & 0 & 0 \\ 0 & 0 & \Sigma|\Lambda|^2 & 0 \\ -\frac{\omega|\Lambda_0|^2 A\sin^2\theta}{\Sigma|\Lambda|^2} & 0 & 0 & \frac{|\Lambda_0|^4 A\sin^2\theta}{\Sigma|\Lambda|^2} \end{pmatrix} \tag{30}$$

where

$$\Delta = r^2 - 2Mr + a^2$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

$$A = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta$$

$$\Lambda \equiv \Lambda(r,\theta) = 1 + \frac{B^2\sin^2\theta}{4}(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}) - i.\frac{aB^2M\cos\theta}{2}(3 + \frac{a^2\sin^4\theta}{\Sigma} - \cos^2\theta)$$

where,

$$i = \sqrt{-1}$$

$$\omega = \frac{\alpha - \beta\Delta}{r^2 + a^2} + \frac{3}{4}aM^2B^4$$

where,

$$\alpha = a(1 - a^2M^2B^4)$$

$$\beta = \frac{a\Sigma}{A} + \frac{aMB^4}{16}(-8r\cos^2\theta(3 - \cos^2\theta) - 6r\sin^4\theta + \frac{2a^2\sin^6\theta}{A}(2Ma^2 + r(r^2 + a^2)) \\ + \frac{4Ma^2\cos^2\theta}{A}[(r^2 + a^2)(3 - \cos^2\theta)^2 - 4a^2\sin^2\theta])$$

$$a = \frac{J}{M}$$

and,

$$|\Lambda_0|^2 \equiv |\Lambda(r,0)|^2 = 1 + a^2M^2B^4 \tag{31}$$

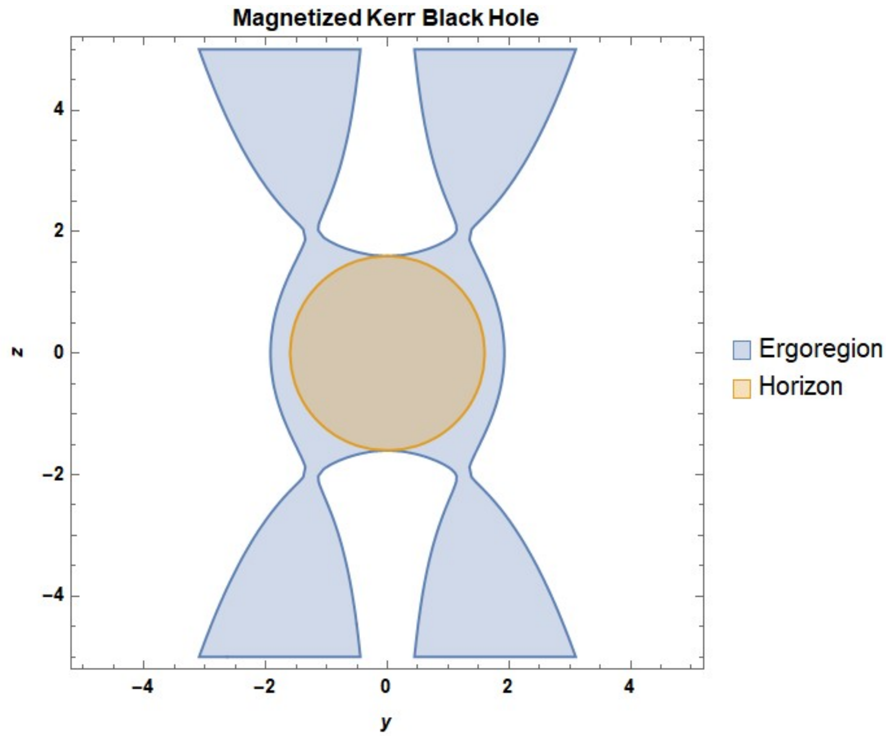

Figure 8: The cross section of the ergoregion and event horizon of a Magnetized Kerr black hole with $M = 1$, $B = 1$ and $a = 0.8$.

Again, like the Kerr black hole, the magnetized Kerr black hole is axially symmetric and not spherically symmetric. However, the event horizon remains the same as that of a Kerr black hole.

$$r_H^{\pm} = M \pm \sqrt{M^2 - a^2} \tag{32}$$

The magnetized Kerr black hole has an ergoregion where g_{tt} becomes positive.

$$g_{tt} = -\frac{\Delta\Sigma}{A}|\Lambda|^2 + \frac{\omega^2 A \sin^2\theta}{\Sigma|\Lambda|^2} \tag{33}$$

The ergoregion is numerically evaluated. Like the magnetized RNBH, the first term vanishes at the horizon while the second term contributes positively when $sin\theta \neq 0$, therefore near to the exterior of the horizon, g_{tt} will be positive. The ergoregion extends to infinity near the poles and asymptotically approaches the Melvin metric.

4 Results

For a Kerr black hole, we see that for $a < 1$ the black hole has 4 boundaries namely the outer ergosphere, inner ergosphere, outer event horizon and inner event horizon and a ring singularity which enclosed inside the event horizon. Though, the cosmic censorship hypothesis forbids the nature to have a naked singularity, we observed that naked singularities can be observed for spin parameter $a > 1$ for a specific angular region $\theta_{op} = 2cos^{-1}(1/a)$. This allows any observer

to observe the naked singularity without crossing the ergoregion. This type of black hole is called Superspinar Kerr black hole and has no event horizons making it effective to observe the singularity.
In the magnetized spacetime, the Reissner-Nordstrom black hole is develops an ergoregion unlike a normal charged black hole. The magnetized Reissner-Nordstrom becomes axially symmetric from spherically symmetric with the influence of the magnetic field. The ergoregion asymptotically approaches the z axis as z tends to infinity. An ergoregion is formed for any zero value of charge and magnetic field. When the charge is set as 0, the ergoregion disappears and reduces to Schwarzschid-Melvin metric.
When a Kerr black hole is placed in the Melvin magnetic universe, the black hole drags the background including the magnetic field in the direction of its rotation. This produces current density and induces the black hole to charge up till the Wald charge ($Q = 2aMB$) [8]. As the spin parameter a increases, the current density increases with much stronger electromagnetic storms. The ergoregion deforms slowly from the normal Kerr black hole as the spin parameter increases. For faster rotation, the ergoregion looks similar to that of the magnetized Reissner-Nordstrom black hole. The ergoregion asymptotically approaches the z-axis as z tends to infinity and at infinity the metric reduces to Melvin Metric.

5 Conclusions

Ergoregions are of important concerns in dealing with the energetics of a black hole. One such process called the Penrose process which explains the physics of extracting the mechanical/rotational energy of rotating black hole. This process is of great significance since this gives us a model of how particles can be radiated out and this process is solely possible only if the particle is in the ergosphere and can be decayed into two particles wherein one falls into the black hole and the other gains energy and radiates out of the ergosphere. Hence, the study of ergoregions have become necessary.
Observations says that there are strong axial magnetic field at the centre of our galaxy and at that centre is a rotating black hole. The presence of the magnetic field on the background of the black hole gives rise to the magnetized black hole spacetime. Since, particles and objects revolve around the black hole. The study of ergoregions and energetics becomes more viable. Extension to the penrose process is called the electromagnetic Penrose process which observed in magnetized Kerr black hole. By this process, the energy extracted can go way beyond that of the rotational energy and particles will radiate out with tremendous amount of energies.
This led to the discussion of black holes in magnetized spacetime and how the ergoregions would change with the influence of a magnetic field. Currently, my work is focused on dealing with the motion of a particle and efficiency of Penrose process in a magnetized Kerr spacetime.

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