Unit:-5 Vector calculus

1) $\nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$

2) $\nabla^2 = \left(\frac{\partial^2}{\partial x^2}\hat{i} + \frac{\partial^2}{\partial y^2}\hat{j} + \frac{\partial^2}{\partial z^2}\hat{k}\right)$

3) Gradient $= \nabla\phi$ where ϕ is scalar function

4) Divergence $\text{div}\,\vec{A} = \nabla\cdot\vec{A}$

5) curl $\vec{A} = \nabla\times\vec{A}$

6) Laplaction of $\phi = \nabla^2\phi = \nabla(\nabla\phi)$.

① Find the directional derivative of $\phi = xy + yz + zx$ at $(1, 2, 3)$ in direction of $3\hat{i} + 4\hat{j} + 5\hat{k}$.

sol: Direction derivative $(D.D) = \nabla\phi\cdot\hat{n}$

$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(xy + yz + zx)$

$\nabla\phi = \left(\frac{\partial}{\partial x}(xy + yz + zx)\hat{i} + \frac{\partial}{\partial y}(xy + yz + zx)\hat{j} + \frac{\partial}{\partial z}(xy + yz + zx)\hat{k}\right)$

$\nabla\phi = \left((\overset{2+3}{y+z})\hat{i} + (\overset{1}{x}+\overset{3}{z})\hat{j} + (\overset{2}{y}+x)\hat{k}\right)$

$\nabla\phi_{\text{at}(1,2,3)} = (5\hat{i} + 4\hat{j} + 3\hat{k})$

$\hat{n} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{\sqrt{9+16+25}}$ $\quad \left| \frac{\nabla\phi}{|\nabla\phi|} \right|$

$\frac{\nabla\phi}{|\nabla\phi|}$

$\hat{n} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{\sqrt{50}}$

$\therefore D.D = \nabla\phi\,\hat{n}$

$= 5\hat{i} + 4\hat{j} + 3\hat{k} = \dfrac{3\hat{i} + 4\hat{j} + 5\hat{k}}{\sqrt{50}}$

$D\hat{n} = \dfrac{15 + 16 + 15}{\sqrt{50}} = \dfrac{46}{\sqrt{50}}$

$D.D =$ $\sqrt{49}$ 7

* ~~find the D.D of~~ find the D.D of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction of $2\hat{i} - 3\hat{j} + 6\hat{k}$

Sol:

$DD = \Delta\phi\hat{n} = \nabla\phi = \left(\dfrac{\partial}{\partial x}\hat{i} + \dfrac{\partial}{\partial y}\hat{j} + \dfrac{\partial}{\partial z}\hat{k}\right)$
~~$(4xz^3 - 3x^2 - yz^2)$~~

$= \dfrac{\partial}{\partial x}(4xz^3 - 3x^2y^2z)\hat{i} + \dfrac{\partial}{\partial y}(4xz^3 - 3x^2y^2z)\hat{j} + \dfrac{\partial}{\partial z}(4xz^3 - 3x^2y^2z)\hat{k}$

$= (4z^3 - 6xy^2z)\hat{i} + (-6x^2yz)\hat{j} + (12xz^2 - 3x^2y^2)\hat{k}$

$= (32 - 24)\hat{i} + (48)\hat{j} + (96 - 12)\hat{k}$

$\nabla\phi = 8\hat{i} + 48\hat{j} + 84\hat{k}$

~~$\hat{n} = \dfrac{8\hat{i} + 48\hat{j} + 84\hat{k}}{\sqrt{64 + 2304 + 7056}} = \dfrac{8\hat{i} + 48\hat{j} + 84\hat{k}}{\sqrt{9424}}$~~

$\dfrac{\nabla\phi}{|\nabla\phi|}$ $\hat{n} = \dfrac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{4 + 9 + 36}} = \dfrac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$ $= \sqrt{49}$

$\therefore DD = \nabla\phi\cdot\hat{n} = (8\hat{i} + 48\hat{j} + 84\hat{k})\cdot\dfrac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$

$= \dfrac{16 - 144 + 504}{7}$

$DD = \dfrac{376}{7}$ $= \boxed{DD = \dfrac{376}{7}}$

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★ find the unit vector normal to the surface $xy^3z^2 = 4$ at $(-1,-1,2)$

Ans The unit vector $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$

The scalar fun $\phi = c$

$\therefore \phi = xy^3z^2$ is in form of $\phi = c$

$\boxed{c=4}$

$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(xy^3z^2)$

$\nabla\phi = \left(\frac{\partial}{\partial x}(xy^3z^2)\hat{i} + \frac{\partial}{\partial y}(xy^3z^2)\hat{j} + \frac{\partial}{\partial z}(xy^3z^2)\hat{k}\right)$

$\nabla\phi = (y^3z^2\hat{i} + 3xy^2z^2\hat{j} + \frac{\partial}{\partial z}(xy^3z^2)\hat{k})$

$\nabla\phi = (y^3z^2\hat{i} + 3xy^2z^2\hat{j} + 2xy^3z\hat{k})$

$\nabla\phi$ at $(-1,-1,2)$

$\nabla\phi = -4\hat{i} - 12\hat{j} + 4\hat{k}$
~~at (-1,-1,2)~~
at $(-1,-1,2)$

$|\nabla\phi| = \sqrt{16+144+16} = \sqrt{176}$

$\hat{n} = \frac{+4\hat{i} - 12\hat{j} + 4\hat{k}}{\sqrt{176}}$

H.W $x^2y + y^2z + z^2x = 5$ at $(1,-1,2)$

→ $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$

$\nabla\phi = \frac{\partial}{\partial x}(x^2y + y^2z + z^2x)\hat{i} + \frac{\partial}{\partial y}(x^2y + y^2z + z^2x)\hat{j} + \frac{\partial}{\partial z}(x^2y + y^2z + z^2x)\hat{k}$

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$\Rightarrow (2xy + z^2)\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 + 2zx)\vec{k}$

$\Rightarrow (-2 + 4)\vec{i} + (1 - 4)\vec{j} + (1 + 4)\vec{k}$

$\Rightarrow 2\vec{i} - 3\vec{j} + 5\vec{k}$

$|\nabla\phi| = \sqrt{4 + 9 + 25} = \sqrt{38}$

$$\hat{n} = \frac{2\vec{i} - 3\vec{j} + 5\vec{k}}{\sqrt{38}}$$

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* find Angal ~~Anlsod~~ b/w Surface $xyz^2 = 1$ and $x^2 + y^2 + z^2 = 3$ at the point $(1, 1, 1)$

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

$\phi_1 = c_1 \quad \phi_2 = c_2$

$\phi_1 = xyz^2 \quad c_1 = 1 \quad \phi_2 = x^2 + y^2 + z^2 \quad c_2 = 3$

$\nabla\phi_1 = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi_1$

$\nabla\phi_1 = \left(\frac{\partial\phi_1}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)$

$\nabla\phi_1 = \left(\frac{\partial}{\partial x}(xyz^2)\hat{i} + \frac{\partial}{\partial y}(xyz^2)\hat{j} + \frac{\partial}{\partial z}(xyz^2)\hat{k}\right)$

$\nabla\phi = (yz^2)\hat{i} + (xz^2)\hat{j} + (2zxy)\hat{k}$.

$\nabla\phi_{(1,1,1)} = (\hat{i} + \hat{j} + 2\hat{k})$

$$|\nabla\phi_1| = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{1 + 1 + 4}}$$

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$$|\nabla|\phi_1 = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$$

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$$\nabla\phi_2 = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} \frac{\partial}{\partial z}\hat{k}\right)\phi_2$$

$$= \cancel{\left(\frac{\partial\phi_2}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi}$$

$$= \left(\frac{\partial\phi_2}{\partial x}\hat{i} + \frac{\partial\phi_2}{\partial y}\hat{j} + \frac{\partial\phi_2}{\partial z}\hat{k}\right)$$

$$\nabla\phi_2 = \left(\frac{\partial}{\partial x}(x^2+y^2+z^2)\hat{i} + \frac{\partial}{\partial y}(x^2+y^2+z^2) \frac{\partial}{\partial z}(x^2+y^2+z^2)\hat{k}\right)$$

$$= (2x)\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\nabla\phi_2 \text{ at } (1,1,1) = \frac{2\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{4+4+4}} = \frac{2\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{12}}$$

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

$$\cos\theta = \frac{\hat{i}+\hat{j}+2\hat{k} \cdot 2\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{6}\ \sqrt{12}}$$

$$\cos\theta = \frac{2+2+4}{\sqrt{6}\,\sqrt{4\times3}} = \frac{8}{2\sqrt{6}\sqrt{3}} = \frac{4}{\sqrt{6}\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{6}\sqrt{3}}\right)$$

θ

★ find angle b/w normal surface $xy + yz + zx + 1 = 0$ at $(1,1,-1)$ and $(1,-1,1)$

Sol

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

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$\frac{\partial}{\partial x}(xy+yz+2x+1)$ $\frac{\partial}{\partial y}(xy+yz+2x+1)$

$\frac{\partial}{\partial z}(xy+yz+2x+1)$

$\nabla \phi_1 = (y+z)\hat{i} + (x+z)\hat{j} + (y+x)$

At $(1,1,-1) \Rightarrow 0\hat{i} + 0\hat{j} + 2\hat{k}$.

$\sqrt{0+0+4}$.

$|\nabla\phi| = \vec{A} = \sqrt{4} = 2$

$\nabla\phi_2 = \frac{\partial}{\partial x}(xy+yz+2x+1)\hat{i} + \frac{\partial}{\partial y}(xy+yz+2x+1)\hat{j} + \frac{\partial}{\partial z}(xy+yz+2x+1)\hat{k}$.

$= (y+z)\hat{i} + (x+z)\hat{j} + (y+x)\hat{k}$.

$0\hat{i} + 2\hat{j} + 0\hat{k}$.

$= 2\hat{j}$.

$|\nabla\phi_2| = |\vec{B}| = \sqrt{4} = 2$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{2\hat{k}\cdot 2\hat{j}}{\sqrt{4}\sqrt{4}} = \frac{0}{4} = 0$

$\frac{2\hat{k}\cdot 2\hat{j}}{4} =$ $\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\cos\theta = 0$

$\theta = \cos^{-1}(0) = \pi/2$ (Q1) 1.

Q. If $\vec{F} = x^2y\hat{i} + 2xy^2z\hat{j} - y^2z^3\hat{k}$, find
i) $\text{div}\,\vec{F}$ ii) $\text{curl}\,\vec{F}$

Sol. $\vec{F} = x^2y\hat{i} + 2xy^2z\hat{j} - y^2z^3\hat{k}$

$\Rightarrow \text{div}\,\vec{F} = \nabla\cdot\vec{F}$

$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(x^2y\hat{i} + 2xy^2z\hat{j} - y^2z^3\hat{k})$

$= \left(\frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(2xy^2z) + \frac{\partial}{\partial z}(-y^2z^3)\right)$

$\boxed{\text{div}\,\vec{F} \Rightarrow (2xy + 4xyz + 3y^2z^2)}$

(ii) $\text{curl} = \nabla\times\vec{F}$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 2xy^2z & -y^2z^3 \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(-y^2z^3) - \frac{\partial}{\partial z}(2xy^2z)\right) - j\left(\frac{\partial}{\partial x}(-y^2z^3) - \frac{\partial}{\partial z}(x^2y)\right) + k\left(\frac{\partial}{\partial x}(2xy^2z) - \frac{\partial}{\partial y}(x^2y)\right)$$

$= i(-2yz^3) - (2xy^2) - j(0) - 0)$
$+ k(2y^2z - x^2)$

$(i(-2yz^3 - 2xy^2) + k(2y^2z - x^2)$ //

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Q) If $\vec{F} = xy^2\hat{i} + 2x^2yz\hat{j} - 3yz^2k$, find i) curl $\vec{F}$

ii) div (curl $\vec{F}$).

Sol.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & 2x^2yz & -3yz^2 \end{vmatrix}$$

$$\left(i \frac{\partial}{\partial y}(-3yz^2) - \frac{\partial}{\partial z}(2x^2yz) - j\left(\frac{\partial}{\partial x}(-3yz^2 - \frac{\partial}{\partial z}(xy^2)\right) \right.$$

$$\left. k\left(\frac{\partial}{\partial x}(2x^2yz) - \frac{\partial}{\partial y}(xy^2)\right)\right)$$

$$\left(i(-3z^2) - (2x^2y) + k(4xyz - 2xy) \right)$$

$$\text{div}(\text{curl}\,\vec{F}) = \nabla \cdot \text{curl}\,F$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(i(-3z^2) - (2x^2y) + k(4xyz - 2xy)\right)$$

$$= \frac{\partial}{\partial x}(-3z^2) - (2x^2y) + \frac{\partial}{\partial z}k(4xyz - 2xy))$$

$$= 0 + 0$$

H.W practice

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* Find div $\vec{F}$ and curl $\vec{F}$ if $\vec{F}$. ~~$=\nabla(2x^3y^2z^4)$~~

Sol To find $\vec{F} = \nabla(2x^3y^2z^4)$

$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(2x^3y^2z^4)$

$= \left(\frac{\partial}{\partial x}(2x^3y^2z^4)\hat{i} + \frac{\partial}{\partial y}(2x^3y^2z^4)\hat{j} + \frac{\partial}{\partial z}(2x^3y^2z^4)\hat{k}\right)$

$\vec{F} = 6x^2y^2z^4\hat{i} + 4x^3yz^4\hat{j} + 8x^3y^2\hat{k}$

$\text{div}\,\vec{F} = \nabla\cdot\vec{F}$

$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(6x^2y^2z^4\hat{i} + 4x^3yz^4\hat{j} + 8x^3y^2z^3\hat{k})$

$= \frac{\partial}{\partial x}(6x^2y^2z^4) + \frac{\partial}{\partial y}(4x^3yz^4) + \frac{\partial}{\partial z}(8x^3y^2z^3)$

~~$\nabla\cdot F = \frac{\partial}{\partial x}(6x^2y^2z^4) + \frac{\partial}{\partial y}(4x^3yz^4) + \frac{\partial}{\partial z}(8x^3y^2z^3)$~~

$\text{div}\,\vec{F} = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$

$\text{curl}\,\vec{F} = \nabla\times\vec{F}$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6x^2y^2z^4 & 4x^3yz^4 & 8x^3y^2z^3 \end{vmatrix}$$

$= i\left(\frac{\partial}{\partial y}(8x^3y^2z^3) - \frac{\partial}{\partial z}(4x^3yz^4)\right) - j\left(\frac{\partial}{\partial x}(8x^3y^2z^3) - \frac{\partial}{\partial z}(6x^2y^2z^4)\right) + k\left(\frac{\partial}{\partial x}(4x^3yz^4)\right)$

$= i(16x^3yz^3 - 16x^3yz^3) - j(24x^2y^2z^3 - 24x^2y^2z^3) + k(12x^2yz^4 - 12x^2yz^4)$

curl $= 0$

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$\vec{F} = \frac{\partial}{\partial x}(xy^3z^2)\hat{i} + \frac{\partial}{\partial y}(xy^3z^2)\hat{j} + \frac{\partial}{\partial z}(xy^3z^2)\hat{k}$

$\frac{\partial}{\partial x}(xy^3z^2)\hat{i} + \frac{\partial}{\partial y}(xy^3z^2) + \frac{\partial}{\partial z}(xy^3z^2)k$

$\hat{i}(xy^3z^2) + \hat{j}(xy^3z^2) + \hat{k}(xy^3z^2)$

$\text{div}\,\vec{F}\ \frac{\partial}{\partial x}(y^3z^2)\hat{i} + \frac{\partial}{\partial y}(x\cdot z^2)\hat{j} + \frac{\partial}{\partial z}(xy^3)$

$(y^3z^2)\hat{i} + (3xy^2z^2)\hat{j} + (2xy^3z)\hat{k}$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^3z^2 & 3xy^2z^2 & 2xy^3z \end{vmatrix}$$

$\cancel{= \frac{\partial}{\partial y}(xy^3)\ \frac{\partial}{\partial z}(y^3z^2)(3xy^2z^2) + \frac{\partial}{\partial x}(xy^3)}$

$\cancel{- \frac{\partial}{\partial z}(y^3z^2) + \frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial y}(y^3z^2)}$

$= \cancel{3y^2x} -$

$i\left(\frac{\partial}{\partial y}\, 2xy^3z\right) - \frac{\partial}{\partial z}(3xy^2z^2) - j\left(\frac{\partial}{\partial x}\, 2xy^3z\right.$

$\left. - \frac{\partial}{\partial z}(y^3z^2)\right) + \left(\frac{\partial}{\partial x}\, 3xy^2z^2 - \frac{\partial}{\partial y}\, y^3z^2\right)$

$= \left(\cancel{6y^2xz} - \cancel{6xyz}\right) - \left(\cancel{2y^3z} - \cancel{2y^3z}\right)$

$+ \left(\cancel{3y^2z^2} - \cancel{3y^2z^2}\right)$

$= \underline{0}$

Find div F and curl F where F = grad $(x^3y + y^3z$
$+ z^3x - x^2y^2z^2)$

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Sol $\frac{\partial}{\partial x}(x^3y + y^3z + z^3x - x^2y^2z^2) + \frac{\partial}{\partial y}(x^3y + y^3z + z^3x - x^2y^2z^2) + \frac{\partial}{\partial z}(x^3y + y^3z + (z^3x - x^2y^2z^2$

$(3x^2y + z^3 - 2xy^2z^2)i -$

$(x^3 y^2 z + 3y^2z + 2yx^2z^2)\hat{j} +$

$(y^3 + 3z^2x + 2x^2y^2z) \cdot \hat{k} +$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3x^2y + z^3 - 2xy^2z^2) & (x^3 3y^2 z + 3y^2z + 2yx^2z^2) & (y^3 + 3z^2x + 2x^2y^2z) \end{vmatrix}$$

$\left(\frac{\partial}{\partial y}(y^3 + 3z^2x + 2x^2y^2z) - \frac{\partial}{\partial z}(x^3 3y^2z + 3y^2z + 2yx^2z^2\right.$

$-\left(\frac{\partial}{\partial z}(3x^2y + z^3 - 2xy^2z^2) - \frac{\partial}{\partial x}(y^3 + 3z^2x + 2x^2y^2z)\right)$

$+ \frac{\partial}{\partial x}(x^3 3y^2z + 3y^2z + 2yx^2z^2) - \frac{\partial}{\partial y}(3x^2y + z^3 - 2xy$

$3y^2 + 4y\,x^2 z +$

Prove that $\vec{F} = (-x^2+yz)\hat{i} + (4y - z^2x)\hat{j} + (2xz - 4z)\hat{k}$ is solenoidal. A vector field $\vec{F}$ is said to be solenoidal if $\operatorname{div}\vec{F} = 0$

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Sol $\operatorname{div}\vec{F} = \nabla\cdot\vec{F}$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(-x^2+yz)\hat{i} + (4y-z^2x)\hat{j} + (2xz-4z)\hat{k}$$

$$= \frac{\partial}{\partial x}(-x^2+yz) + \frac{\partial}{\partial y}(4y-z^2x) + \frac{\partial}{\partial z}(2xz-4z)$$

$$= (\cancel{-2x} + \cancel{4} + \cancel{2x} - 4)$$

$$\boxed{\operatorname{div}\vec{F} = 0}$$

$\therefore \vec{F}$ is solenoidal

Irrotational

A field $\vec{F}$ is said to be irrotational if curl. When $\vec{F}$ is irrotational then there always exist scalar functional φ such that $\nabla\phi = \vec{F}$

A Show that the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^3z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find the scalar potential φ such that $\nabla\phi = \vec{F}$

Given $\vec{F} = 2xyz^3\hat{i} + x^3z^3\hat{j} + 3x^2yz^2\hat{k}$

$\operatorname{curl}\vec{F} = \nabla\times\vec{F}$

$$\operatorname{curl}\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz^3 & x^2z^3 & 3x^2yz^2 \end{vmatrix}$$

$$\operatorname{curl}\vec{F} = \left(\frac{\partial}{\partial y}(3x^2yz^2) - \frac{\partial}{\partial z}(x^2z^3)\right)\hat{i} - \hat{j}\left(\frac{\partial}{\partial x}(3x^2yz^2) - \frac{\partial}{\partial z}(2xyz^3)\right) + \hat{k}\left(\frac{\partial}{\partial y}(x^2z^3) - \frac{\partial}{\partial y}(2xyz^3)\right)$$

$$\operatorname{curl}\vec{F} = (3x^2z^2 - 3x^2z^2)\hat{i} - \hat{j}(6xyz^2 - 6xyz^2) + \hat{k}(2xz^3 - 2xz^3)$$

To find φ

$\nabla\phi = \vec{F}$

$$\left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right) = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$$

$\Rightarrow \frac{\partial \phi}{\partial x} = 2xyz^3 \Rightarrow \int \frac{\partial \phi}{\partial x}\, \partial x = \int (2xyz^3)\, \partial x$

$\phi = 2\frac{x^2}{2} yz^3 + f(y,z) \Rightarrow \phi = x^2yz^3 + f(y,z)$

$\frac{\partial \phi}{\partial y} = x^2z^3 \Rightarrow \int \frac{\partial \phi}{\partial y}\, \partial y = \int (x^2z^3)\, \partial y$

$\phi = x^2yz^3 + g(x,z) \rightarrow ②$

$\frac{\partial \phi}{\partial z} = 3x^2yz^2 \Rightarrow \int \frac{\partial \phi}{\partial z}\, \partial z = \int (x^2yz^2)\, \partial z$

$\phi = \cancel{3}x^2y\frac{z^3}{\cancel{3}} + h(x,y) \Rightarrow \phi = x^2yz^3 + h(x,y) \rightarrow ③.$

comparing Eqn ① ② & ③.

$f(y,z) = 0$

$g(x,z) = 0$

$h(x,y) = 0$

$\phi =$ common terms $+ f(x,z) + g(x,z) + h(x,y)$

$\phi = x^2yz^3 + 0 + 0 + 0$

$\phi = x^2yz^3$

* Vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Show that the field irrotational. Also find solar potent

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2+xy^2) & (y^2+x^2y) & 0 \end{vmatrix}$$

$\left(\frac{\partial}{\partial y} 0 - \frac{\partial}{\partial z} y^2 + x^2y\right) - \left(\frac{\partial}{\partial x} 0 - \frac{\partial}{\partial z} (x^2 + xy^2)\right)$

$+ \left(\frac{\partial}{\partial x}(y^2 + x^2y) - \frac{\partial}{\partial y}(x^2 + xy^2)\right)$

$= k\,(2x\cancel{+}y) - (2xy)$

$= 0$

To find ~~di~~

div $\vec{F}$ of

$\left(\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}\right) = (x^2 + xy^2) + (y^2 + x^2 \ldots$

$\frac{\partial \phi}{\partial x} = (x^2 + xy^2) \Rightarrow \frac{\partial \phi}{\partial x} \int x^2 + x^2y^2\, \partial x$

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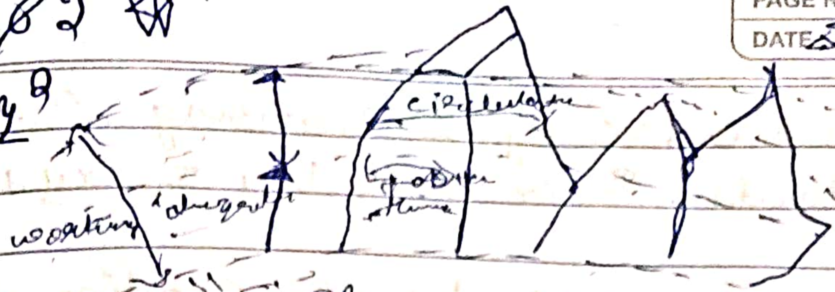
$\frac{2x^3}{6} + \frac{3x^2y^2}{6\,2}$

$\frac{x^3}{3} + \frac{x^2y^2}{2}$



$\frac{2x^3 + 3x^2y^2}{6}$

"SUPR LINEAR"

$= \cancel{2x+}$

$= \frac{x^3}{3} + \frac{x^2}{2}y^2 + f(y,z)$

$= \frac{2x^3 + 3x^2y^2}{6} + f(y,z)$

$= \frac{x^3}{3} + \frac{x^2y^2}{2} + f(y,z).$

$\frac{\partial\phi}{\partial y} = y^2 + x^2y \Rightarrow \phi = \frac{y^3}{3} + \frac{x^2y^2}{2}$

$= \frac{2y^3 + 3x^2y^2}{6} + g(x,z)$

$= \frac{y^3}{3} + \frac{x^2y^2}{2}$

$= \frac{x^3}{3} + \frac{y^3}{3} + \frac{x^2y^2}{2}$

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(1) form the PDE by Eliminating the arbitrary constants. $z=(x-a)(y-b)$, $z=xy+y\sqrt{x^2-a^2}$

(2) form the PDE by eliminating the arbitrary functions. $z=f(x^2-y^2)$, $\phi(xy+z^2, x+y+z)=0$

3) Solve $\frac{\partial^3 u}{\partial x^2 \partial y}=\cos(2x+3y)$ by direct Integration method.

(4) $4\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=3z$ where $z(0,y)=2e^{5y}$ use the method of separation of variables

(1) $z=(x-a)(y-b) \rightarrow (1)$

Differents w.r.to x partialy

$\frac{\partial z}{\partial x}=(1)(y-b) \rightarrow (2)$

Differents w.r.to y paritially.

Types of drive = crossbeltdrive

$c=6m$

Assume larger dia $d_1=400mm$ & $r_1=0.2m$

Smaller dia $d_2=300mm=0.3m$; $r_2=0.15m$

i) cross belt drive; $L_1=$ 13.18 m

ii) open belt drive, $L_2=$ 13.10 m