Differential equation of Higher order.

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linear differential equation of second and Higher order with constant co-efficient. Basically dependent variable and its derivatives are first degree.

A differential eq^n of form:-

$$\frac{d^n y}{dx} + \frac{a d^{n-1}}{dx^{n-1}} + \cdots + a_{n-1}\frac{dy}{dx} + a_n y = \phi(x)$$

where, $a_1, a_2 \ldots a_n$ are all constants called linear differential eq^n nth order.

using familier notation of differential operator $D = \frac{d}{dx}$, $D^2 = \frac{d^2}{dx^2} \cdots D^n = \frac{d^n}{dx^n}$

The differential eq^n can be put form

$$(D^n y + a_1 D^{n-1} y + \cdots a_{n-1} D + a_n y] = \phi$$

$$(D^n + a_1 D^{n-1} + \cdots + a_{n-1} D + a_n] y = \phi(x)$$

i.e $f(D)\,y = \phi(x)$.

~~where $f(D) y = \phi(x)$~~

where $f(D) = D^n + a_1 D^{n-1} + \cdots + a_{n-1} D +$

If $\phi(x) = 0$ The eq^n called homogenous equation

If $\phi(x) \neq 0$ The eq^n called non homogenous eq^n.

Solution of Homogenous linear Diff eq^n.

consider diff eq of form

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0. \quad -(1)$$

where a_1 and a_2 are constants.

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i.e. $(D^2y + D_1Dy + a_2y) = 0$.
$(D^2 + a_1 D + a_2)y = 0$ -- ②.
$f(D)y = 0$

Where $f(D) = D^2 + a_1 D + a_2$
first we going establish following fundamental property -
If y_1 and y_2 are two linear independent solution eq^n - ②.

The $c_1y_1 + c_2y_2$ is also sol^n eq^n ②
where c_1 and c_2 arbitrary const.
Since y_1 y_2 sol^n eq^n ② we have.

$f(D)y_1 = 0$ and $f(D)y_2 = 0$.
$f(D)[c_1y_1 + c_2y_2] = 0$
$c_1f(D)y_1 + c_2f(D)y_2 = 0$
$c_1(0) + c_2(0) = 0$
Thus $f(D) = (c_1y_1 + c_2y_2) = 0$.

implies that $c_1y_1 + c_2y_2$ is sol^n eq^n ②
Since general sol^n second order diff eq^n as to connect two arbitrary constants - c_1 & c_2 denoted by.
$y_c = c_1y_1 + c_2y_2$
where y_c is called complementary function and it sol^n homogeneous differential equations.

method of finding complementary function - consider 2nd order homogeneous Different equation

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

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using D-operator $D = \frac{d}{dx}$

$$(D^2 y + D_1 D y + a_2 y) = 0$$
$$(D^2 + a_1 D + a_2) y = 0$$
$$f(D) y = 0.$$

Taking $y = e^{mx}$

$$\frac{dy}{dx} = m e^{mx}$$
$$\frac{d^2y}{dx^2} = m^2 e^{mx} = D^2 y.$$

$$(m^2 + a_1 m + a_2) e^{mx} = 0.$$
$$(m^2 e^{mx} + a_1 m e^{mx} + a_2 e^{mx}) = 0$$
$$m^2 + a_1 m + a_2) = 0.$$

This being quadratic eqn in m will 2 roots.

1) Real distinct.
2) Real & coincident
3) complex.

1) Real and distinct → suppose the roots are m_1 and m_2. $m_1 \neq m_2$, Then $y_1 = e^{m_1 x}$ &
$y_2 = e^{m_2 x}$ are two Independent solution of

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0.$$

Hence by fundamental property

$$y = C_1 y_1 + C_2 y_2$$
$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} - (3).$$

Thus eqn (3) is general. eqn (1)

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★ If m_1, m_2, m_3 and ---- m_n are 'n' real and distinct roots of Auxiliary equation associated with n^{th} order equation. then general sol^n is given by · $y = c_1 e^{m_1 x} + c_2 e^{m_2 x} \dots c_n e^{m_n x}$

2) Real and coincident.
Suppose roots Auxiliary eq^n - are really coincident i.e $m_1 = m_2 = m$
Then · $f(D)y = 0$.

$$(D-m)^2 y = 0$$
$$(D-m)(D-m) y = 0$$
$$(D-m) y = t$$
$$(D-m) t = 0$$
$$Dt - mt = 0$$
$$\frac{dt}{dx} - mt = 0$$
$$\frac{dt}{dx} = mt$$
$$\int \frac{dt}{t} = m dx$$
$$\int \frac{dt}{t} = \int m dx + \alpha$$
$$\log t = mx + \alpha$$
$$\log_e t = e^{mx} + \alpha$$
$$t = e^{mx + \alpha}$$
$$(D-m) y = e^{mx + \alpha}$$
$$(D-m) y = e^{mx} e^{\alpha}$$
$$(D-m) y = e^{mx} K$$
$$\frac{dy}{dx} - my = e^{mx} \cdot K$$

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$\Rightarrow \frac{dy}{dx} + Py = Q_1$.

$P = -m \quad Q = e^{mx} k$.

$y e^{\int P dx} = \int Q e^{\int dx} k$.

$y e^{\int -mx} = \int Q e^{\int -mdx} dx + c$.

$y e^{-mx} = \int e^{\cancel{mx}} k e^{\cancel{-mx}} dx + c$

$y e^{-mx} = kx + c$.

$y = \frac{kx + c}{e^{-mx}}$

$y = (kx + c_1) e^{mx}$ — (2)

Then we can say that the general sol^n of eq(1) in case of the roots are real and ~~constant~~ coincident or repeated roots can be written as.

$$y = (c_1 + c_2 x) e^{mx} \quad \text{— (2)}$$

3) complex.

suppose roots of auxillary equation are complex which always occur in phase. of say that $p \pm iq$ they general sol^n of diff eq in basic from is

$$y = a e^{(p+iq)x} + b e^{(p-iq)x}$$

where a & b are arbitrary constant ~~this form~~ can simplified further.

$$y = e^{px} (a e^{+iqx} + b e^{p-iqx})$$

$$y = e^{px} [a (\cos qx + i \sin qx) + b (\cos qx$$
$$i \sin qx)].$$
$$= e^{px} \cdot [\cos qx + i \sin qx) + b (\cos qx + i \sin q$$
$$x)]$$

where $c_1 = (a+b)$ and $c_2 = i(a-b)$ are arbitary constants.

$y = e^{px}\,[c_1 \cos qx + c_2 \sin qx]$ PAGE NO: (3)

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However if roots pair is purely imaginary.

i.e $p = 0$, ∴ sol given by

$y = c_1 \cos qx + c_2 \sin qx$ (3)

1) solve the following equations.

1) $\frac{d^2y}{dx^2} - \frac{2dy}{dx} - 3y = 0$

we have $D = \frac{d}{dx}$

(1)

$D^2y - 2Dy - 3y = 0$

$(D^2 - 2D - 3)y = 0$

$(D^2 - 2D - 3) = 0$

The Auxillary eqn ∴ $D = m$

$m^2 - 2m - 3 = 0$

$m^2 - 3m + m - 3 = 0$

$m(m-3) + 1(m-3) = 0$

$(m-3) = 0 \quad (m+1) = 0$

$(m-3) = 0 \quad (m+1) = 0$

$m = 3 \quad m = -1$

$m_1 = 3$ & $m_2 = -1$

It is real and distinct

∴ $y = c_1 e^{mx} + c_2 e^{mx}$

$y = c_1 e^{3x} + c_2 e^{-x}$ //

2) $y'' - 6y' + 13y = 0$

$\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 13y = 0$

(2)

$D = \frac{d}{dx}$

$D^2y - 6Dy + 13y = 0$

$(D^2 - 6D + 13)y = 0$

$(D^2 - 6D + 13) = 0$

the Auxillary eq^n

$m^2 - 6m + 13 = 0$

$\therefore$ use :-

$$\boxed{m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$a = 1, \quad b = -6 \quad c = 13$

$$= \frac{6 \pm \sqrt{36 - 52}}{2}$$

$$= \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{\cancel{6}3 \pm \cancel{4}i\,2}{\cancel{2}}, \quad 3 \pm 2i$$

$\therefore \sqrt{-1} = i$

$\{p \pm qi\}$ complex

$\therefore y = e^{px}(c_1 \cos qx + c_2 \sin qx)$

$y = e^{3x}(c_1 \cos 2x + c_2 \sin 2x)$ //

③ $4y''' + 4y'' + y' = 0$

sol $\frac{4d^3y}{dx^3} + \frac{4d^2y}{dx^2} + \frac{dy}{dx} = 0$

$$D = \frac{d}{dx}$$

$$4D^3y + 4D^2y + Dy = 0$$

$$(4D^3 + 4D^2 + D)y = 0$$

$$4D^3 + 4D^2 + D = 0$$

$$4m^3 + 4m^2 + m = 0$$

$m(4m^2 + 4m + 1) = 0$

$m = 0 \quad (2m+1)^2 = 0$

$(2m+1)(2m+1) = 0$

$(2m+1) = 0 \quad (2m+1) = 0$

$2m = -1$ $\quad$ $2m = -1$

$m = -1/2$ $\quad$ $m = -1/2$

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$m_1 = 0 \quad m_2 = -1/2 \quad m_3 = -1/2$

$$y = c_1 e^{0x} + (c_2 + c_3 x) e^{-x/2}$$

$$y = c_1 + (c_2 + c_3 x) e^{-x/2}$$

4) $\dfrac{d^3y}{dx^3} - \dfrac{2d^2y}{dx^2} + \dfrac{4dy}{dx} - 8y = 0.$

$$D^3y - 2D^2y + 4Dy - 8y = 0$$

$$(D^3 - 2D^2 + 4D - 8)\, y = 0$$

$$D^3 - 2D^2 + 4D - 8 = 0$$

The Auxillary eqn.

$$m^3 - 2m^2 + 4m - 8 = 0.$$

$$m^2(m-2) + 4(m-2) = 0.$$

$m^2 + 4 = 0 \qquad m - 2 = 0$

$m^2 = -4 \qquad m = 2.$

$m = \sqrt{-4}$

$m = \pm 2i \qquad m = 2.$

$$y = c_1 e^{2x} + (c_2 \cos 2x + c_3 \sin 2x$$

5) $\dfrac{d^3y}{dx^3} + y = 0$

$$D^3y + y = 0$$

$$(D^3 + 1)\, y = 0$$

$$D^3 + 1 = 0$$

$$m^3 + 1 = 0$$

By inspection method. $m = -1$

$$-1 + 1 = 0$$

By Synthetic Division Method

-1	1	0	0	1
	0	-1	1	-1
	1	-1	1	0

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$m^2 - m + 1 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = -1,\ c = 1$

$\cancel{m = 0} \cdot m = \dfrac{1 \pm \sqrt{1-4}}{2}$

$= \dfrac{1 \pm \sqrt{-3}}{2}$

$m = \frac{1}{2} \pm i\sqrt{\frac{3}{2}}$

$y = c_1 e^{-x} + e^{x/2}\left(c_2 \cos \sqrt{\tfrac{3}{2}} + c_3 \sin \sqrt{\tfrac{3}{2}}\,x\right)$ //

(6) $(D^3 - 3D + 2)y = 0$

$m^3 - 3m + 2 = 0$

$m_1 = 1$

$1 - 3 + 2 = 0$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$m^2 + m - 2 = 0$

$m^2 + 2m - m - 2 = 0$

$m(m+2) - 1(m+2) = 0$

$(m-1) = 0 \quad (m+2) = 0$

$m_2 = 1 \quad m_3 = -2$

$y = (c_1 + c_2 x)e^{x} + c_3 e^{-2x}$

7.

8/1

7) $(D^3 - 6D^2 + 11D - 6)y = 0$

Ans $m^3 - 6m^2 + 11m - 6 = 0$

$m = 1$

By synthetic division

(3)

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & \cancel{1} & -5 & 6 & \boxed{0} \end{array}$$

$m^2 - 5m + 6 = 0$

$m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

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$m-3=0$ $\quad$ $m-2=0$

$m=3$ $\quad$ $m=2$

$y = C_1 e^{x} + C_2 e^{3x} + C_3 e^{2x}.$

(8) linear homogenous equations

Ans $(D^3+8)y=0$

Auxillary eq. is

$m^3+8=0$

$m=-2 \Rightarrow (-2)^3+8 \Rightarrow -8+8=0$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & 0 & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & \boxed{0} \end{array}$$

$m^2-2m+4=0.$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4-16}}{2} \Rightarrow \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm i\sqrt{4\times3}}{2} \Rightarrow \frac{\cancel{2}}{\cancel{2}} + \frac{i\cancel{2}\sqrt{3}}{\cancel{2}}$$

$= 1 \pm i\sqrt{3}$

$= p \pm iq.$

$y = C_1 e^{-2x} + e^{x}(C_2 \cos\sqrt{3}x + C_3 \sin\sqrt{3}$

(9) $(D^3-4D^2+5D-2)y=0$

$m^3-4m^2+5m-2=0$

$m=1$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 5 & -2 \\ & 0 & 1 & -3 & 2 \\ \hline & 1 & -3 & 2 & \boxed{0} \end{array}$$

$m^2-3m+2=0$

$m^2-2m-m+2=0$

$m(m-2) -1(m-2)$

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$m-2=0 \quad m-1=0$
$m=2 \quad m=1$
$y=(c_1+c_2x)e^{x}+c_3e^{2x}$

(10) $(D^4+2D^3-5D^2-6D)y=0$
$m^4+2m^3-5m^2-6m=0$ Put $= m=-1$
$\cancel{m=0}.\ m_2=-1$ $-1+2+5-6=0$
$m(m^3+2m^2-5m-6)=0$ $= 7-7$
$m_1=0$ $0=0$
$m^3+2m^2-5m-6=0$
$m_2=-1$

-1	1	2	-5	-6
	0	-1	-1	$+6$
	1	1	-6	$\boxed{0}$

$m^2+m-6=0$
$m^2-2m+3m-6=0$ 6, -2 $+3$
$m(m-2)+3(m-2)=0$
$m_3=2 \quad m_4=-3$
$y=c_1+c_2e^{-x}+c_3e^{2x}+c_4e^{-3x}$

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(11) $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$

$4m^4 - 8m^3 - 7m^2 + 11m + 6 = 0$

$$\begin{array}{r|rrrrr} -1 & 4 & -8 & -7 & 11 & 6 \\ & 0 & -4 & 12 & -5 & -6 \\ \hline & 4 & -12 & 5 & 6 & \underline{0} \end{array}$$

$4m^3 - 12m^2 + 5m + 6 = 0$

$m = 2$.

$$\begin{array}{r|rrrr} 2 & 4 & -12 & 5 & 6 \\ & 0 & 8 & -8 & +6 \\ \hline & 4 & -4 & -3 & 0 \end{array}$$

$4m^2 - 4m - 3 = 0$

$4m^2 - 6m + 2m - 3 = 0$

$2m(2m-3)\ 1(2m-3) = 0$

$2m + 1 = 0 \qquad 2m - 3 = 0$

$2m = -1 \qquad 2m = 3$

$m = \frac{-1}{2} \qquad m = 3/2$

$y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-x/2}$

$+ c_4 e^{\frac{-3x}{2}}$

$4(0)^4 - 8(0)^3 - 7(0)^2 + 11(0) + 6 = 0 \neq 0$

$4(1)^4 - 8(1$

solve non Homogeneous linear PAGE NO: Diff eqn.
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1) Type → Particular Integral form $\frac{e^{ax}}{f(D)}$

$\Rightarrow D(e^{ax}) = a\,e^{ax}$

$$\frac{e^{ax}}{a} = \frac{e^{ax}}{D}$$

$$\Rightarrow \frac{e^{ax}}{f(D)} = \frac{e^{ax}}{f(a)}$$

In exponential to D replaced by a

1) $\frac{e^{ax+b}}{f(D)} = \frac{e^{ax+b}}{f(a)}$ 2) $\frac{e^{-ax}}{f(D)} = \frac{e^{-ax}}{f(-a)}$.

3) $$\frac{K}{f(D)} = \frac{K e^{(0)x}}{f(D)} = \frac{K}{f(0)}$$

4) $$\frac{a^x}{f(D)} = \frac{x \log_e a}{f(D)} = \frac{e^{x\log a}}{f(D)} = \frac{e^{x\log a}}{f(\log a)}$$

$$= \frac{e^{x\log a}}{(\log 4)^2 + 5(\log a) + 6}$$

5) $$\frac{\cosh ax}{f(D)} = \frac{\frac{1}{2}(e^{ax} + e^{-ax})}{f(D)}$$

$$\Rightarrow \frac{1}{2}\left[\frac{e^{ax}}{f(D)} + \frac{e^{-ax}}{f(D)}\right]$$

$\cosh ax = \frac{1}{2}(e^{ax} + e^{-ax})$

$\sinh ax = \frac{1}{2}(e^{ax} - e^{-ax})$

$$\frac{1}{2}\left(\frac{e^{ax}}{f(a)} + \frac{e^{-ax}}{f(-a)}\right)$$

special case

$$\frac{e^{ax}}{f(D)} = \frac{e^{ax}}{f(a)} \Rightarrow \frac{xe^{ax}}{f'(D)}$$

$f(a) = 0$

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Solve Non-Homogeneous L.D.E

1) ~~Type I~~ : PI of the form $\frac{e^{ax}}{f(D)}$

$\Rightarrow D(e^{ax}) = ae^{ax}$

$\frac{e^{ax}}{a} = \frac{e^{ax}}{D}$

$\Rightarrow \frac{e^{ax}}{f(D)} = \frac{e^{ax}}{f(a)}$

1) $\frac{e^{ax+b}}{f(D)} = \frac{e^{ax+b}}{f(D)}$

2) $\frac{e^{-ax}}{f(D)} = \frac{e^{-ax}}{f(-a)}$

3) $\frac{a^x}{f(D)} = \frac{x\log a}{f(D)} = \frac{e^{x\log a}}{f(D)} \quad \frac{e^{x\log a}}{f(\log a)}$.

4) $\frac{K}{f(D)} = \frac{Ke^{(0)\cdot x}}{f(D)} = \frac{K}{f(0)}$

5) $\frac{\cosh ax}{f(D)} = \frac{\frac{1}{2}[e^{ax}+e^{-ax})}{f(D)} = \frac{1}{2}\left[\frac{e^{ax}}{f(D)} + \frac{e^{-ax}}{f(D)}\right)$

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problems

$(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$

sol:- $\boxed{GS = CF + PI}$

CF = complementary function.
PI = particular Integral

To find CF

$(D^3 - 6D^2 + 11D - 6)y = 0.$

$D^3 - 6D^2 + 11D - 6 = 0.$

The Auxiliary eq^n is

$m^3 - 6m^2 + 11m - 6 = 0$

put $m=1$ $1 - 6 + 11 - 6 = \cancel{12} - \cancel{12} = 0$

By inspection method $m_1 = 1$

By synthetic Division method.

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$\Rightarrow m^2 - 5m + 6 = 0$

$\Rightarrow m^2 - 2m - 3m + 6 = 0$

$m(m-2) - 3(m-2) = 0.$

$(m-2)(m-3) = 0$

$m - 2 = 0$ $m - 3 = 0$

$\boxed{m_2 = 2}$ $\boxed{m_3 = 3}$

$-5m$

$-2 \quad -3$

$CF = y = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x}.$

To find P.I

$\dfrac{e^{-2x} + e^{-3x}}{f(D)} \Rightarrow \dfrac{e^{-2x} + e^{-3x}}{D^3 - 6D^2 + 11D - 6}$

$= \dfrac{e^{-2x}}{D^3 - 6D^2 + 11D - 6} + \dfrac{e^{-3x}}{D^3 - 6D^2 + 11D - 6}$

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$$\Rightarrow \frac{e^{-2x}}{-8-24-22-6} + \frac{e^{-3x}}{-27-54-33}$$

$$\Rightarrow \frac{e^{-2x}}{+60} + \frac{e^{-3x}}{-120}$$

$$G.S = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x} - \frac{e^{-2x}}{60} - \frac{e^{-3x}}{120}$$

(2) $$\frac{d^2y}{dx^2} - \frac{6\,dy}{dx} + 9y = 3^x + 5$$

Sol: $D = \frac{d}{dx}$

$$(D^2 y - 6Dy + 9y) = 3^x + 5$$

$$(D^2 - 6D + 9)y = 3^x + 5$$

$$GS = CF + PI$$

To find CF

$$(D^2 - 6D + 9)y = 0$$

$$D^2 - 6D + 9 = 0$$

The A eqn is.

$$m^2 - 6m + 9 = 0$$

$$m^2 - 3m - 3m + 9 = 0$$

$$m(m-3) - 3(m-3) = 0$$

$$(m-3)(m-3) = 0$$

$$\boxed{m_1 = 3}\ \boxed{m_2 = 3}$$

$$CF = y = (c_1 + c_2 x)e^{3x}$$

To find PI

$$PI = \frac{3^x + 5}{D^2 - 6D + 9}$$

$$= \frac{3^x}{D^2 - 6D + 9} + \frac{5}{D^2 - 6D + 9}$$

$$= \frac{e^{x\log 3}}{D^2 - 6D + 9} + \frac{5e^{ax}}{D^2 - 6D + 9}$$

$$PI \left[\frac{e^{x\log 3}}{(\log 3)^2 - 6(\log 3) + 9} + \frac{5}{9} \right]$$

$a^x = 3^x$.

$\log(3)$.

$$GS = CF + PI$$

$$GS = (c_1 + c_2 x)e^{3x} + \frac{3^x}{(\log 3^x) - 6(\log 3) + 9} + 5/9$$

Type II

(1) P.I of the form $\frac{\sin ax}{f(D)}$ (OR) $\frac{\cos ax}{f(D)}$

$D(\sin ax) = a\cos ax$

$D^2(\sin ax) = -a^2 \sin ax$.

$$\frac{\sin ax}{-a^2} = \frac{\sin ax}{D^2}$$

$$\Rightarrow \frac{\sin ax}{f(D^2)} = \frac{\sin ax}{f(-a^2)}$$

$$\Rightarrow \frac{\cos ax}{f(D^2)} = \frac{\cos ax}{f(-a^2)}$$

$$\frac{d^2y}{dx^2} + \frac{7dy}{dx} + 6y = 2\sin 3x$$

$D = \frac{d}{dx}$

$(D^2 + 7Dy + 6y) = 2\sin 3x$.

$(D^2 + 7D + 6)y = 2\sin 3y$.

C.S = CF + PI

To find CF

$(D^2 + 7D + 6)y = 0$

$D^2 + 7D + 6 = 0$

The A eqn is

$m^2 + 7m + 6 = 0$

$m^2 - 6m - 1m + 6 = 0$.

$m(m-6) - 1(m-6) = 0$

$(m+6)(m-1) = 0$

$m_1 = 1 \quad m_2 = -6$

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$CF = y = c_1 e^{-x} + c_2 e^{-6x}$

To find PI $\frac{2\sin 3x}{D^2+7D+6}$

$= \frac{2\sin 3x}{-9+7D+6}$

$= \frac{2\sin 3x}{7D-3}$

$= \frac{2\sin 3x}{(7D-3)} \times \frac{(7D+3)}{(7D+7)}$

$= \frac{2\sin 3x\,(7D+3)}{49D^2-9}$

$\frac{2(7D+3)\sin 3x}{49(-9)-9}$

$= \frac{\cancel{2}(7D+3)\sin 3x}{-\cancel{450}_{225}}$

$= \frac{(7D+3)\sin 3x}{-225}$

$PI = \frac{(21\cos 3x + \sin 3x)}{-225}$

$= \frac{\cancel{3}(7\cos 3x + \sin 3x)}{-\cancel{225}_{75}} \Rightarrow \frac{7\cos 3x + \sin 3x}{-75}$

$G.S = c_1 e^{-x} + c_2 e^{-6x} + \frac{7\cos 3x + \sin 3x}{-75}$

$G.S = c_1 e^{-x} + c_2 e^{-6x} + \frac{7\cos 3x + \sin 3x}{-75}$

$G.S = c_1 e^{-x} + c_2 e^{-6x} - \left(\frac{7\cos 3x + \sin 3x}{75}\right)$

Problems

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1) $y'' - 4y' + 4y = e^{2x} + \cos 2x + 4$

Sol

$G.S = CF + P.I$

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx}$

$CF = (C_1 + C_2 x)e^{2x}$

$PI = \frac{e^{2x} + \cos 2x + 4}{(D^2 + 4D + 4)}$

$= \frac{e^{2x}}{D^2 - 4D4} + \frac{\cos 2x}{D^2 - 4D + 4} + \frac{4}{D^2 - 4D + 4}$

$= P_1 = \frac{e^{2x}}{D^2 - 4D + 4}$

$= \frac{e^{2x}}{4 - 8 + 4}$

$\boxed{= \frac{e^{2x}}{0}}$

$x \frac{e^{2x}}{f'(D)}$

$x \cdot \frac{e^{2x}}{2D - 4}$

$\frac{x e^{2x}}{4 - 4} = \frac{x e^{3x}}{0}$

$\frac{x^2 e^{2x}}{f''(D)}$

$\frac{x^2 e^{2x}}{2} = P_1$

$P_2 = \frac{\cos 2x}{D^2 - 4D + 4}$

$= \frac{\cos 2x}{-4 - 4D + 4}$

$= \frac{\cos 2x}{-4D}$

$D^{-1} = \int$

$P_2 = -\frac{1}{4} \int \cos 2x$

$= -\frac{1}{4} \frac{\sin 2x}{2}$ → formula

$P_2 = -\frac{1}{8} \sin 2x$

$P_3 = \frac{4}{D^2 - 4D + 4}$

$= \frac{4e^{2x}}{4}$

$P_3 = \frac{4}{4} = 1$

$PI = P_1 + P_2 + P_3$

$= \frac{x^2 e^{2x}}{2} - \frac{\sin 2x}{8} + 1$

$GS = CF + PI$

$= (C_1 + C_2 x)e^{2x} + \frac{x^2 e^{2x}}{2} - \frac{\sin 2x}{8}$

(2) $\frac{d^2y}{dx^2} + \frac{4dy}{dx} - 12y = e^{2x} \cdot 3\sin 2x$

Sol

$(D^2 + 4D - 12) = 0$

$m^2 + 4m - 12 = 0$

$m^2 + 6m - 2m - 12 = 0$

$m(m+6) - 2(m+6) = 0$

$m - 2 = 0 \qquad m + 6 = 0$

$m = 2 \qquad m = -6$

$CF = y = c_1 e^{2x} + c_2 e^{-6x}$

$PI = \frac{e^{2x} - 3\sin 2x}{D^2 + 4D - 12}$

$= \frac{e^{2x}}{D^2 + 4D - 12} - \frac{3\sin 2x}{D^2 + 4D - 12}$

$= \frac{e^{2x}}{4 + 8 - 12} - \frac{3\sin 2x}{-4 + 4D - 12}$

$= \frac{e^{2x}}{6} - \frac{3\sin 2x}{4D - 16}$

$= \frac{xe^{2x}}{2D+4} - \frac{3\sin 2x}{4(D-4)} \times \frac{D+4}{D+4}$

$= \frac{xe^{2x}}{8} - \frac{3(D+4)\sin 2x}{4D^2 - 16}$

$= \frac{xe^{2x}}{8} - \frac{3(2\cos 2x + 4\sin 2x)}{4(-20)}$

$= \frac{xe^{2x}}{8} - \frac{3(\cos 2x + 2\sin 2x)}{-40}$

$= \frac{xe^{2x}}{8} + \frac{3(\cos 2x + 2\sin 2x)}{40}$

$GS = c_1 e^{2x} + c_2 e^{-6x} + \frac{xe^{2x}}{8} + 3\frac{(\cos 2x + 2\sin 2}{40}$ //

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(3) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = e^{3t}\cosh 2t \, 2^t$

$GS = CF + PI$

Sol: $(D^2y - 4dy + 13y) = 0$

$(D^2 - 4D + 13)y = 0$

$\Rightarrow (D^2 - 4D + 13) = 0$

$\Rightarrow (m^2 - 4m + 13) = 0$

The A eqⁿ is $m^2 - 4m + 13 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -4$
$c = 13$

$$m = \frac{-4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{-4 \pm \sqrt{36}}{2} = \frac{-\cancel{4}^{2} \pm \cancel{6}^{3}}{\cancel{2}_{1}}$$

$m = 2 \pm 3i$

$CF = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

To find $PI := \frac{e^{3t} \cosh 2t + 2^t}{D^2 - 4D + 13}$

$$PI = \frac{e^{3t} \frac{1}{2}(e^{2t} + e^{2t}) + 2^t}{D^2 - 4D + 13}$$

$\cos 2^t = (e^{2t} + e$

$$= \frac{\frac{1}{2}(e^{5t} + e^{t}) + 2^t}{D^2 - 4D + 13}$$

$$= \frac{1}{2}\left[\frac{e^{5t}}{D^2 - 4D + 13} + \frac{e^{t}}{D^2 - 4D + 13}\right] - \left[\frac{2^t}{D^2 + 4D + 13}\right]$$

$$PI = \frac{1}{2}\left[\frac{e^{5t}}{25 - 20 + 13} + \frac{e^{t}}{1 - 4 + 13}\right] + \left[\frac{e^{t\log 2}}{D^2 - 4D + 13}\right]$$

$$PI = -\frac{1}{2}\left[\frac{e^{5t}}{18} + \frac{e^{t}}{10}\right] + \left[\frac{e^{(\log 2)t}}{(\log 2)^2 - 4(\log) + 13}\right]$$

$G.S = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{1}{2}\left(\frac{e^{st}}{18} + \frac{e^t}{} + \frac{e^{\log 2}}{(\log 2)^2 - 4(\log 2) + 5} + \right.$

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Type III

In terms of x write the f(x) in decending powers of x f(D). In acceding power b using normal division method. that division get completed with any reminder that ~~of~~ cosh out ~~to~~ ~~aptend~~ opteanded in thes process the cothant is called perticular Integral.

① $(D^2+3D+2)y = 1+4x+2x^2$

$GS = CF + PI$

To find CF $(D^2+3D+2)y=0$

$D^2+3D+2=0$

The A & n is $m^2+3m+2=0$

$m^2+2m+1m+2=0$

$m(m+2)+1(m+2)=0$

$(m+1)\quad(m+2)=0$

$m+1=0 \qquad m+2=0$

$CF = c_1e^{-x} + c_2e^{-2x}$

To find $P.I = \frac{\phi(x)}{f(D)}$

$P.I = \frac{1+4x+2x^2}{D^2+3D+2}$

$$
\begin{array}{r|l}
2+3D+D^2 & 2x^2+4x+1 \;(\; x^2-x+1 \\
 & 2x^2+6x+2 \\
\hline
 & -2x-1 \\
 & \cancel{-2x}\;\cancel{-3} \\
 & +\quad\;\; + \\
\hline
 & 2 \\
 & 2 \\
\hline
 & 0
\end{array}
$$

The Reminder is zero. The quotient is called PI

$+x^2-x+1$

$PI = x^2-x+1 \quad Gs = c_1e^{-x} + c_2e^{-2x}$

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2) $(D^2+3D+2)y = 12x^2$

D^2+3D+2

m^2+3m+2

$m=-1 \cdot m=-2$

$CF = C_1e^{-x} + C_2e^{-2x}$

$$PI = \frac{12x^2}{D^2+3D+2}$$

$2+3D+D^2) 12x^2 (6x^2$
$12x^2$
0

$(D^2y+3Dy+2y) = 1+3x+x^2$

Sol : $GS = CF + PI$

To find CF $(D^2+3D+2)y=0$

$D^2+3D+2=0$

The eqn.

$m^2+3m+12=0$

$m^2+2m+1m+2=0$

$m(m+2)+1(m+2)=0$

$m+2=0 \quad m+1=0$

$m_1=-2 \quad m_2=-1$

$CF = C_1e^{-2x} + C_2e^{-x}$

$$\text{To find } PI = \frac{1+3x+x^2}{D^2+3D+2}$$

$2+3D+D^2) \cancel{x^2}+3x+1 (\frac{x^2}{2}$
$\cancel{x^2}+3x+1$
0

The reminder is zero the quotient is well. $\therefore PI = \frac{x^2}{2}$

$$GS = c_1e^{-2x} + c_2e^{-x}\frac{x^2}{2}$$

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$$\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x + 2^{-x}$$

sol:- $G_S = CF + PI$

To find CF $\quad \frac{d^2y}{dx^2} + 4y = 0$

$$(D^2y + 4y) = 0$$

$$(D^2+4)y = 0$$

the A eqn

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm\sqrt{-4}$$

$$m = \pm 2i$$

$$CF = (c_1 \cos 2x + c_2 \sin 2x).$$

To find PI $= \dfrac{x^2 + \cos 2x + 2^{-x}}{D^2+4}$

$$4+D^2 \overline{)\; x^2 \;}\left(\frac{x^2}{4} - \frac{1}{8}\right.$$

$$\frac{x^2 + \frac{1}{2}}{-\frac{1}{2}}$$

$$-\frac{1}{2}$$

$$\mp\frac{1}{2}$$

$$0$$

The quotient will be $P_1 = \frac{x^2}{4} - \frac{1}{8}$

$P_2 = \dfrac{\cos 2x}{D^2+4}$

$\dfrac{\cos 2x}{-4+4}$

$= \dfrac{x\cos 2x}{f'(D)}$

$= \dfrac{x\cos 2x}{2D}$

$= \frac{x}{2}\int \cos 2x$

$P_2 = \dfrac{x\sin 2x}{4}$

~~$P_3 = \dfrac{\cos 2x}{D^2+4}$~~

~~$= \dfrac{\cos 2x}{-4+4}$~~

~~$= \dfrac{x\cos 2x}{f'(D)}$~~

~~$= x\cos 2x$~~

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$$P_3 = \frac{D^{-x}}{D^2+4}$$

$$= \frac{e^{-\log 2x}}{D^2+4}$$

$$= \frac{e^{-\log 2x}}{(-\log 2)^2+4}$$

$$P_3 = \frac{2^{-x}}{(\log 2)^2+4}.$$

$$PI = P_1 + P_2 + P_3$$

$$= \frac{x^2}{4} - \frac{1}{3} + \frac{x\sin 2x}{4} + \frac{2^{-x}}{(\log 2)^2+4}$$

$$GS = (C_1\cos 2x + C_2\sin 2x) + \left(\frac{x^2}{4} - \frac{1}{3}\right) + \frac{x\sin 2x}{4} + \frac{2^{-x}}{(\log 2)+4}$$

~~Type IV~~ PI of the form $\frac{e^{ax}}{f(D)}$

where v is fun of 'x'

$$PI = e^{ax}\frac{1}{f(D)}v = \frac{e^{ax}}{(D+a)}v$$

ex! $$\frac{e^{2x}\sin 3x}{D^2-1} = e^{2x}\frac{1}{(D+2)^2-1}\sin 3x$$

$$= \frac{e^{-2x}}{D^2+4+4D-1}$$

$$= \frac{e^{2x}}{D^2+4D+3}\sin 3x$$

$$= \frac{e^{2x}\sin 3x}{-9+4n+3}$$

$$= \frac{e^{2x}\sin 3x}{4D-6}$$

$$= \frac{e^{2x}\sin 3x}{4D-6} \times \frac{(4D+6)}{(4D+6)}$$

$$= \frac{e^{2x}(4D+6)\sin 3x}{16D^2-36}$$

$$= \frac{e^{2x}(4n+6)\sin 3x}{16D^2-36}$$

$$= \frac{e^{2x}(4n+6)\sin 3x}{16(-n)-36}$$

$$= \frac{e^{2x}(4D+6)\sin 3x}{-180}$$

$$= \frac{e^{2x}(12\cos 3x+6\sin 3x)}{-180}$$

$$= \frac{e^{2x}\,6(2\cos 3x+\sin 3x)}{-\cancel{180}\ 30}$$

$$P.I = \frac{-e^{2x}(2\cos 3x+\sin 3x)}{30}$$

* $(D^2+2D+5)y = e^{-x}\sin 2x$

$GS = CF + PI$

Sol To find CF $(D^2+2D+5)y=0$

$D^2+2D+5=0$

The A eq^n

$m+2m+5=0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

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$a=1 \quad b=2 \quad c=5$

$$m = \frac{-2 \pm \sqrt{4-20}}{2}$$

$$m = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2}$$

$$m = -1 \pm 2i$$

$$CF = e^{-x}(c_1 \cos 2x + c_2 \sin 2x)$$

$$PI = \frac{e^{-x} \sin 2x}{D^2+2D+5}$$

$$PI = \frac{e^{-x} \sin 2x}{D^2+1-2D+2D-2+5}$$

$$= \frac{e^{-x} \sin 2x}{D^2+4}$$

$$= e^{-x} \frac{\sin 2x}{-4+4}$$

$$PI = e^{-x} \frac{x \sin 2x}{f'(D)}$$

$$= e^{-x} \frac{x \sin 2x}{2D}$$

$$= e^{-x} \frac{x}{2} \int \sin 2x$$

$$PI = -e^{-x} \frac{x \cos 2x}{4}$$

$$GS = e^{-x}(c_1 \cos 2x + c_2 \sin 2x) - e^{-x} \frac{x \cos 2x}{4}$$

H.W

(2) $(D^2+5D+6)y = e^{-2x} \cos 2x.$

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Solve the following eqn by the method of Variation parameter.

(1) $ay'' + by' + cy = x$

$(aD^2y + bDy + cy) = x$

$(aD^2 + bD + c)y = x$

$GS = CF + PI$

To find CF $(aD^2 + bD + c)y = 0$

$aD^2 + bD + c = 0$

The eqn $am^2 + bm + c = 0$

$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$\boxed{CF = c_1 y_1 + c_2 y_2}$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$w = y_1 y_2' - y_2 y_1'$

$PI = y_2 \int \frac{y_1 x}{w} - y \int \frac{y_2 x}{w}.$

$y'' + a^2 y = \sec ax.$

$GS = CF + PI$

To find CF $y'' + a^2 y = 0$

$(D^2y + a^2 y) = 0$

$(D^2 + a^2)y = 0$

$D^2 + a^2 = 0$

The ~~Ax~~ eqn is

$m^2 + a^2 = 0$

$m^2 = -a^2$

$\sqrt{m^2} = \sqrt{-a^2}$

$m = \pm ai$

$CF = c_1 \cos ax + c_2 \sin ax$

$CF = c_1 y_1 + c_2 y_2$

$y_1 = \cos ax \qquad y_2 = \sin ax$

$y_1' = -a\sin ax \qquad y_2' = a\cos ax$

$$w \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix} = a\cos^2 ax + a\sin^2 ax = a(\cos^2 ax + \sin^2 ax)$$

$w = a(1)$

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$$PI = \frac{1}{2}\int \frac{\cos ax \sec ax}{a} - y_1 \int \frac{\sin ax \sec ax}{a}$$

$$= y_2 \int \frac{\cos ax \times \frac{1}{\cos ax}}{a} - y_1 \int \frac{\frac{\sin ax}{\cos ax}}{a}$$

$$= \frac{\sin ax}{a} x - \frac{\cos ax}{a} \frac{\log(\sec ax)}{a}$$

$$\frac{x \sin ax}{a} - \frac{\cos ax \log \sec x}{a^2}$$

* $\frac{d^2y}{dx^2} + y = \tan x$

$GS = CF + PI$

To find CF $\frac{d^2y}{dx^2} + y = 0$

$(D^2y + y) = 0$

$(D^2 + 1)y = 0$

The Auxiliary eq^n.

$m^2 + 1 = 0$

$m^2 = -1$

$m = \pm\sqrt{-1}$

$m = \pm i$

$CF = c_1 \cos x + c_2 \sin x$

$CF = c_1 y_1 + c_2 y_2$

$y_1 = \cos x \qquad y_2 = \sin x$

$y_1' = -\sin x \qquad y_2' = \cos x$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x =$$

$$PI = y_2 \int \frac{y_1 x}{w} - y_1 \int \frac{y_2 x}{w}$$

$$= y_2 \int \frac{\cos x \tan x}{1} - y_1 \int \frac{\sin x \tan x}{1}$$

$$= y_2 \int \frac{\cos x \tan x}{1} - y_1 \int \frac{\sin x \tan x}{1}$$

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(3) $(D^2+5D+6)y = e^{-2x}\cos 2x$

To find CF

$D^2+5D+6=0$

Auxillary equation

$m^2+5m+6=0$

$m^2+2m+3m+6=0$

$m(m+2)+3(m+2)=0$

$m+2=0 \qquad m+3=0$

$m=-2 \qquad m=-3$

$CF = C_1e^{-2x} + C_2e^{-3x}$

To find PI

$$\frac{e^{-2x}\cos 2x}{D^2+5D+6}$$

$$= \frac{e^{-2x}\cos 2x}{(D-2)^2+5(D-2)+6}$$

$$= \frac{e^{-2x}\cos 2x}{D^2+4-2(D)+5D-10+6}$$

$$= \frac{e^{-2x}}{D^2+4-4D+5D-10+6}$$

$$= \frac{e^{-2x}\cos 2x}{-4+D}$$

$$= \frac{e^{-2x}\cos 2x}{D-4} \times \frac{D+4}{D+4}$$

$$= \frac{e^{-2x}(D+4)\cos 2x}{D^2-16}$$

$$= \frac{e^{-2x}(D+4)\cos 2x}{-4-16}$$

$$= e^{-2x}\frac{-\sin 2x+4\cos 2x}{20}$$

$$= e^{-2x}\frac{-2(\sin 2x+2\cos 2x)}{\cancel{20}\,10}$$

$$PI = e^{-2x}\frac{(\sin 2x+2\cos 2x)}{10}$$

$y_s = CF+PI$

$C_1e^{-2x}+C_2e^{-3x}+e^{-2x}\dfrac{\sin 2x+2\cos 2x}{10}$

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$y'' - 2y' + y = e^{x}\log x$

$D^2y - 2Dy + y = e^x \cdot \log x$

$CF = (D^2 - 2D + 1)\, y = 0$

$D^2 - 2D + 1 = 0$

The auxillary eq^n is

$m^2 - m - m + 1 = 0$

$\Rightarrow m(m-1) - 1(m-1) = 0$.

$m = 1$ & $m = 1$

$C.F = (c_1 + c_2 x)e^x = c_1 e^x + c_2 x \cdot e^x$

$CF = c_1 y_1 + c_2 y_2$

$y_1 = e^x$

$y_2 = x \cdot e^x$

$y_1' = e^x$

$y_2' = xe^x + e^x = (x+1)e^x$

$$w = \begin{vmatrix} e^x & x \cdot e^x \\ e^x & (x+1)e^x \end{vmatrix}$$

$w = (e^x) \cdot (x+1)e^x - x(e^x)^2$

$w = (e^x) \cdot 2/x + (e^x)^2 - x(e^x)^2$

$w = +e^{2x}$

$PI = y_2 \int \frac{y_1 x}{w} - y_2 \int \frac{y_2 x}{w}$

$= y_2 \int \frac{e^x e^x \log x}{+ e^{2x}} + y_1 \int \frac{x \cdot e^{2x} \cdot \log x}{-e^{2x}}$

$= y_2 \int \frac{e^{2x} \log x}{+e^{2x}} + y_1 \int \frac{x \cdot e^{2x} \cdot \log x}{-e^{2x}}$

$= +y_2 \int \log x - y_1 \int x \cdot \log x$

$+ y_2 x(\log x - 1) - y_1 \left(\frac{x}{2}(\log x - \frac{1}{2})\right)$

$= - x^2 \cdot e^x(\log x - 1) - e^x \frac{x^2}{2}(\log x - \frac{1}{2})$

$GS = (c_1 + c_2 x)e^x - e^x \frac{x^2}{2}(\log x - \frac{1}{2}) + x^2 \cdot e^x (\log x - 1)$ //

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$$\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = \frac{e^{3x}}{x^2}$$

$GS = CF + PI$

Sol $D^2y - 6Dy + 9y = \frac{e^{3x}}{x^2}$

The auxillary eqn is $m^2y - 6my + 9y = 0$

$m^2 - 6m + 9 = 0$.

$\rightarrow CF \rightarrow$

$m_1 = 3 \quad . \quad m_2 = 3$

$C.f = (c_1 + c_2 x)e^{3x} = c_1 e^{3x} + c_2 e^{3x}$

$y_1 = e^{3x} \qquad y_2 = x \cdot e^{3x}$

$y_1' e^{3x} - 3 \qquad y_2'' e^{3x} + 3 + e^{3x}$.

$3e^{3x} \quad = \quad e^{3x}(3+1)$

$$w = \begin{vmatrix} e^{3x} & x \cdot e^{3x} \\ 3 \cdot e^{3x} & e^{3x}(3x+1) \end{vmatrix}$$

$= e^{3x}(e^{3x}(3x+1)) - 3x(e^{3x})^2$

$= e^{3x}(3x \cdot e^{3x}) - 3x \cdot (e^{6x})$

$= \cancel{3x \cdot e^{6x}} + e^{6x} - \cancel{3xe^{6x}}$

$w = e^{6x}$

$P.I = y_2 \int \frac{y_1 x}{w} - y_1 \int \frac{y_2 x}{w}$

$= y_2 \int \frac{\cancel{e^{3x}} \cdot e^{3x}}{\cancel{e^{6x}} \, x^2} - y_1 \int \frac{x \cdot e^{3x} \cdot e^{3x}}{x^2 \cdot e^{6x}}$

$= y_2 \int \frac{1}{x^2} dx - y_1 \int \frac{1}{x} dx$.

$= y_2\left(\frac{-1}{x}\right) - y_1 \log x$

$= x \cdot e^{3x}(-1/x) - e^{3x} \log x$

$P.I = -e^{3x} - e^{3x} \cdot \log x$

$GS = CF + PI$

$= (c_1 + c_2 x)e^{3x} - e^{3x} - e^{3x} \cdot \log$

$G.S = e^{3x}[(c_1 + c_2 x) - 1 - \log x]$

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H.W

~~$\frac{d^2y}{dx^2}+y = \sec x \tan x$.~~

problems on Cauchy's and legendre's Diff eqn

$x^3\frac{d^3y}{dx} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$

put $x = e^t$

$\log x = t$

$\Rightarrow t = \log x$

Different x.

$\frac{dt}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{d'y}{dt} \times \frac{1}{x}$

$x\frac{dy}{dx} = \frac{dy}{dt}$ $D = \frac{d}{dt}$.

$xy' = Dy$.

$x^2y' = Dy$.

$x^2y'' = D(D-1)y$.

$x^3y''' = D(D-1)(D-2)y$.

$x^4y^{iv} = D(D-1)(D-2)(D-3)y$

$\Rightarrow (D(D-1)(D-2)y + D(D-1)y + Dy + y) = x$

$= (D(1)-1)(D-2) + (1)(D-1) + D + 1)\, y = x$

continue Next Page.

* $\frac{d^2y}{dx^2} + y = \sec x + \tan x$

Sol $GS = CF + PI$

to find CF : $\frac{d^2y}{dx^2} + y = 0$

$(D^2 y + y) = 0$

$(D^2 + 1) y = 0$

The Auxillation eqn :

$m^2 + 1 = 0$

$m = \sqrt{-1}$

$m = \pm i$

$CF = \sec x + \tan x$

$= c_1 y_1 + c_2 y_2$

$y_1 = \sec x + c_2 \tan x$

$y_1' = -\tan x + \sec x$

$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{pmatrix} \sec x & \tan x \\ -\tan x & \sec x \end{pmatrix} = \sec^2 x + \tan^2 x$

$y_2 = \int \frac{y_1 x}{w}$ $\quad y_1 = \int \frac{y_2 - x}{w}$

$y_2 = \frac{\sec x \tan x}{w}$ $\quad y_1 = \frac{-\tan x \sec x}{w}$

$y_2 = \int \frac{\sec x \tan x}{1}$ $\quad -y_1 = -\frac{\tan x \sec x}{1}$

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Cauchy's linear differential eq^n

(1)

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+9y=3x^2+\sin(3\log x)$$

Sol:- It is cauchy's LD eq^n

put $x=e^t$

$\log x=t$

$\Rightarrow t=\log x$

Diff wrt x

$$\frac{dt}{dx}=\frac{1}{x}$$

we write $\dfrac{dy}{dx}=\dfrac{dy}{dt}\cdot\dfrac{dt}{dx}$

$$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{1}{x}$$

$$x\frac{dy}{dx}=\frac{dy}{dt}\qquad \therefore \frac{d}{dt}=D$$

$xy'=Dy$

$x^2y''=D(D-1)y$

using these sub in given eq^n.

$(D(D-1)y+Dy+9y)=3e^{2t}+\sin(3t)$

$[(D^2-D)y+Dy+9y]=3e^{2t}+\sin 3t$

$((D^2-D)+D+9)y=3e^{3t}+\sin 3t$

$(D^2+9)y=3e^{2t}+\sin 3t.$

(GS = CF + PI.)

To find CF

$(D^2+9)y=0$

$D^2+9=0$

The A.eq^n is.

$m^2+9=0$

$m=-9$

$m=\pm 3i$

$C_1=(C_1\cos 3t+C_2\sin 3t).$

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to find P.I

$\Rightarrow \frac{3e^{2t} + \sin 3t}{(D^2+9)}$

$= \frac{3e^{2t}}{(D^2+9)} + \frac{\sin 3t}{(D^2+9)}$

$\Rightarrow \frac{3e^{2t}}{4+9} + \frac{\sin 3t}{-9+9} = \frac{3e^{2t}}{4+9} + \frac{\sin 3t}{0}$

$\Rightarrow \frac{3e^{2t}}{4+9} + t\frac{\sin 3t}{2D} \Rightarrow \frac{t}{2}\int \sin 3t$

$\Rightarrow \frac{3e^{2t}}{3} + \frac{t}{2} - \frac{\cos 3t}{3}$

$\Rightarrow \frac{3e^{4}}{13} - \frac{t \cdot \cos 3t}{3}$

$G_S = C_1 \cos 3t + C_2 \sin 3t + \frac{3e^{2t}}{13} -$

$\frac{t \cdot \cos 3t}{6}$

$= C_1 \cos 3(\log x) + C_2 \sin 3(\log x) + \frac{3 \cdot x^2}{13}$

$-\frac{\log \cos \cdot \log x}{6}$ //

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(1) $x\frac{d^2y}{dx^2} - \frac{2y}{x} = x + \frac{1}{x^2}$

$\frac{x^2\frac{d^2y}{dx^2} - 2y}{x} = x + \frac{1}{x^2}$

$x^2\frac{d^2y}{dx^2} - 2y = x^2 + 1/x$

Put $x = e^t$

$\log x = e^t$

$\log x = t$

$\frac{dt}{dx} = 1/x$

$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{1}{x}$

$x\frac{dy}{dx} = \frac{dy}{dt}$

$xy' = Dy$

$x^2y'' = D(D-1)y.$

$(D(D-1)y - 2y) = e^{2t} + 1/e^t$

$(D^2 - D - 2)y = e^{2t} + e^{-t}$

$GS = CF + PI$

$(D^2 - D - 2) = 0$

The A eqn.

$m^2 - m - 2 = 0.$

$m^2 - 2m + 1m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m+2) = 0$

$m_1 = 1 \quad m_2 = 2$

$CF = c_1e^t + c_2e^{2t}$

To find PI

$PI = \frac{e^{2t} + e^t}{D^2 - D - 2}$

$= \frac{e^{2t}}{D^2 - D - 2} + \frac{e^{-t}}{D^2 - D - 2}$

$= \frac{e^{2t}}{4 - 2 - 2} + \frac{e^{-t}}{1 + 1 - 2}$

$= \frac{t\, e^{2t}}{2D - 1} + \frac{t\, e^{-t}}{2D - 1}$

$= \frac{t\, e^{2t}}{3} + \frac{t\, e^{-t}}{-3}$

$= \frac{te^{2t}}{3} - \frac{-te^{-t}}{3}$

$= \frac{t}{3}(e^{2t} - e^{-t})$

$GS = CF + PI$

$= c_1e^{-t} + c_2e^{2t} + \frac{t}{3}(e^{2t} - e^{-t})$

$= c_1x^{-1} + c_2x^2 + \frac{\log x}{3}(x^2 - x^{-1})$

H.W

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* $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log$

* $x^2 \frac{d^2y}{dx^2} - \cancel{\frac{3x}{dy}}\, 3x\frac{dy}{dx} + 4y = (1+x)^2$

* legendre's linear Diff eqn.

(1) $(2x+1)^2 \cdot \frac{d^2y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y =$

$= 6x+5.$

Sol:- Put $(2x+1) = e^t$

$\log(2x+1) = t$

$\frac{dt}{dx} = \frac{1}{2x+1} \times 2$

$\frac{dt}{dx} = \frac{2}{2x+1}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dy}{dt} = \frac{2}{(2x+1)}$

$(2x+1)\frac{dy}{dx} = 2\frac{dy}{dt}$

$(2x+1)y' = 2Dy.$

$(2x+1)^2 y'' = 4D(D-1)y$

using these subin given D eqn

$((4D(D-1))y - 2(2Dy) - 12y = 6^3\left(\frac{e^t-1}{2}\right) + 5$

$4D^2 - 4D - 4D - 12]y = 3(e^t - 1) + 5$

$(4D^2 - 8D - 12]y = 3e^t + 2$

G.S = CF + PI

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$4(D^2-2D-3)y = 3e^t + 2$

$(D^2-2D-3)y = \frac{3e^t}{4} + \frac{2'}{4\,2}$

To find CF.

$(D^2-2D-3)y=0$

$D^2-2D-3=0.$

A.E. eq.n is.

$m^2-2m-3=0.$

$m^2-3m+1m-3=0$

$m(m-3)+1(m-3)=0$

$(m+1)(m-3)=0$

$m_1=-1 \quad m_2=3$

$CF = C_1e^{-t}+C_2e^{3t}$

$PI = \frac{\frac{3}{4}e^t + \frac{1}{2}}{D^2-2D-3}$

$= \frac{3}{4}\cdot\frac{e^t}{1-2-3} + \frac{1}{2}\times\frac{1}{3}$

$-\frac{3}{6}e^t - \frac{1}{6}$

$GS = c_1e^{-t} + c_2e^{3t} - \frac{3}{16}e^{t(x+1)} - \frac{1}{6}.$