UNIT :- 5

22/12/23

Differential equations

ordinary differential equations

If $y = f(x)$ is a function where y is dependent variable, x is independent variable. an equation which involves dependent variable and at least one dependent dirivation of y with respect to

The order of differential equation is the order of high dirivative present in differential equation. the degree of the differential equation is the integral part of high order derivation.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = 0$$

Solution of first order and First degree differential equation.

A first order and First degree equation will be of the form. $\frac{dy}{dx} = f(x,y)$

this eq classified into mainly 4 type as follow

1) variable sependable equation.
2) Homogenous eq^n.
3) liniar eq^n.
4) exiact eq^n.

* Exact differential equation.

an eq^n which is of the form

$M(x,y)dx + N(x,y)dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

is said to be exact differential eq^n if solution is given by

$$\int M(x,y)\,dx + \int N(y)\,dy = c.$$

A Solve the following diff eqn

① $(2x+y+1)dx + (x+2y+1)dy = 0$

Sol The given eqn is of the form $Mdx + Ndy = 0$.

$M = 2x+y+1$; $N = x+2y+1$

$\frac{\partial M}{\partial y} = 0+1+0 = 1$ $\qquad$ $\frac{\partial N}{\partial x} = 1+0+0 = 1$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Thus the given eqn exact D.E

Sol: $\int M dx + \int N(y) dy = c.$

$\int (2x+y+1)dx + \int 2y\,dy = c$

$\cancel{2}\frac{x^2}{\cancel{2}} + yx + x + \cancel{2}\frac{y^2}{\cancel{2}} = c$

$\boxed{x^2 + xy + x + y^2 = c}$

② $[y(1+1/x) + \cos y]dx + \{x + \log x - x\sin y\}dy = 0$

Sol: the given eqn is of the form. $Mdx + Ndy = 0$.

$M = y(1+1/x) + \cos y$. $N = x + \log x - x\sin y$

$\frac{\partial M}{\partial y} = 1 + 1/x - \sin y$ $\qquad$ $\frac{\partial N}{\partial x} = 1 + 1/x - \sin y.$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

thus the given eqn exact D.E

Sol: $\int M dx + \int N(y)\,dy = c$

x (siny) wr

$\int (y(1+\frac{1}{x}) + \cos y) dx + \int 0 \, dy = c$

$y \int (1+\frac{1}{x}) dx + \cos y \int dx = c$

$y \{x + \log x\} + \cos y \cdot x = c.$

$xy + y \log x + x \cos y = c.$

(3) $(\cos x \tan y + \cos(x+y)) dx + \{\sin x \sec^2 y + \cos(x+y)\} dy = 0.$

sol $M dx + N dy = 0.$

$M = \cos x \tan y + \cos(x+y)$, $N = \sin x \sec^2 y + \cos(x+y)$

$\frac{\partial M}{\partial y} = \cos x \cdot \sec^2 y + (-\sin(x+y))(0+1)$

$= \cos x \sec^2 y - \sin(x+y)$

$\cancel{\frac{\partial M}{\partial y} = \cos x + 0 - \sin(x+y) \cdot 1}$

$\frac{\partial N}{\partial x} = \cos x \sec^2 y - \sin(x+y)(1+0)$

$= \cos x \sec^2 y - \sin(x+y).$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

sol. $\int M dx + \int N(y) dy = c.$

$\int (\cos x \tan y + \cos(x+y)) dx + \int 0 \, dy = c$

$\tan y \int \cos x \, dx + \int \cos(x+y) \, dy = c$

$\sin x \tan y + \sin(x+y) = c.$

(4) $\frac{dy}{dx} + \frac{x+3y-4}{3x+9y-2} = 0.$

sol. $\frac{\partial y}{\partial x} = -(x+3y-4) dx = -(3x+9y-2) dy$

$\underbrace{(x+3y-4)}_{M} dx + \underbrace{(3x+9y-2)}_{N} dy = 0$

$\frac{\partial M}{\partial y} = 0+3-0$

$= 3$

$\frac{\partial N}{\partial x} = 3+0-0$

$= 3$

$\int m\,dx + \int N(y)\,dy = c$

$\int (x+3y-4)\,dx + \int 9y\,dy = c$

$\frac{x^2}{2} + 3y(x) - 4x + \frac{9y^2}{2} = c$

$\frac{x^2}{2} + 3xy - 4x + \frac{9y^2}{2} = c.$

(5) $y(x^2+y^2+a^2)\,dy + x(x^2+y^2-a^2)\,dx = 0.$

sol: $m\,dx + N\,dy = 0.$

~~$M = x(x^2+y^2+a^2)$ $N = y(x^2+y^2+a^2)$.~~

$m = \underbrace{x(x^2+y^2-a^2)}_{M}\,dx = 0, N = \underbrace{y(x^2+y^2+a^2)}_{N}.$

$m = (x^2+y^2-a^2)\,dx \cdot N = y(x^2+y^2+a^2)$

$m = x^3 + xy^2 - xa^2 \cdot = x^2y + y^3 + a^2y$

$\frac{\partial m}{\partial y} = 2xy \cdot \frac{\partial N}{\partial x} = 2xy$

$\therefore \frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$

sol: $\int m\,dx + \int N(y)\,dy = c.$

$\int (x^3+xy^2-a^2x)\,dx + \{y^3 ay^2\}\,dy = c$

$\frac{x^4}{4} + \frac{x^2 \cdot y^2}{2} - \frac{a^2x^2}{2} + \frac{y^4}{4} + \frac{ay^2}{2}a^2 = c$

(6) ~~$(x^2-4xy-2y^2)\,dx + (y^2-4xy-2x$~~

~~$M = x^2-4xy$~~

⑥ $(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0.$

sol $Mdx+Ndy=0.$

$M=x^2-4xy-2y^2$. $N=y^2-4xy-2x^2$

$\frac{\partial M}{\partial y}=0-4x-4y$ $\frac{\partial N}{\partial y}=0-4y-4x$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial y}.$

sol:-

$\int Mdx+\int N(y)dy=c$

$\int(x^2-4xy-2y^2)dx+\int y^2dy=c$

$\frac{x^3}{3}-4y\left(\frac{x^2}{2}\right)-2y^2x+\frac{y^3}{3}=c.$

$\boxed{\text{Reducible to Exact}}$

Suppose the given diff eqn is of the form $Mdx+N(x,y)dy=0$

$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$ ~~then we take the diff~~

$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}$ check wether it is nearer to M or N

WKT If it is nearer to M, we comput $\frac{1}{M}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=g(y)$ and find the Integrating factor If = I.F. $=e^{-\int g(y)dy}$

multiply this Integrating fraction to the given eqn and reduces to exact diff

If $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}$ is Nearer to N. then

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comput $\frac{1}{N}\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$

and find the Integrating factor $IF = e^{\int f(y)dy}$

multiply this Integrating factor to the given eq and reduce to exact differential eq

solve the

(1) $y(2x-y+1)dx + x(3x-4y+3)dy = 0$

Sol: The given eq is the form.

$Mdx + Ndy = 0$

$M = 2xy - y^2 + y$. $N = 3x^2 - 4xy + 3x$

$\frac{\partial m}{\partial y} = 2x - 2y + 1$ $\quad \frac{\partial N}{\partial x} = 6x - 4y + 3$

$\therefore \frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$

$\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x} = (2x-2y+1) - (6x-4y+3)$

$= 2x - 2y + 1 - 6x + 4y - 3$

$= -4x + 2y - 2$

$= -2(2x - y + 1)$ $\quad \therefore$ I.f. is nearest to M

$\frac{1}{m}\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y(\cancel{2x-y+1})}\left[-2(\cancel{2x-y+1})\right]$

$= \frac{-2}{y} = g(y)$

$\cancel{\int g(y)}\, dy.$

$\therefore IF = e^{\int g(y) dy}$

$= e^{-\int\left(\frac{-2}{y}\right)dy}$

$= e^{2\int \frac{1}{y} dy} = e^{2\log y} = e^{\log y^2}$

$\boxed{IF = y^2}$

$y(2x-y+1)\,dx + x(3x-4y+3)\,dy = 0$

multiplying I.F y^2, to eq^n 1 we get

$y^3(2x-y+1)\,dx + xy^2(3x-4y+3)dy = 0$

$M = y^3(2x-y+1)$ $\quad N = xy^2(3x-4y+3)$

$= 2xy^3 - y^4 + y^3$ $\quad = 3x^2y^2 - 4xy^3 + 3xy^2$

$\frac{\partial M}{\partial y} = 6xy^2 - 4y^3 + 3y^2$; $\frac{\partial N}{\partial x} = 6xy^2 - 4y^3 + 3y^2$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{x}$

sol^n. $\int M dx + \int N(y) dy = c$

$\int(2xy^3 - y^4 + y^3)\,dx + \int 0\,dy = c$

$\cancel{2}y^3 \frac{x^2}{\cancel{2}} - xy^4 + xy^3 = c.$

$x^2y^3 - xy^4 + xy^3 = c$

(2) $y(x+y)\,dx + (x+2y-1)\,dy = 0.$

sol $Mdx + Ndy = 0$

$M = y \cdot x + y^2$ $\quad N = x + 2y - 1$

$\frac{\partial M}{\partial y} = x + 2y$ $\quad \frac{\partial N}{\partial x} = 1 + 0$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$

~~Mdx + Ndy.~~

sol ~~find~~

Considers $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = x + 2y - 1$

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y}\right) = \frac{1}{x+2y-1}(x+2y-1)$

$IF = e^{\int F(x)dx}$ $\quad = 1 = f(x)$

$= e^{\int 1 \cdot dx} = e^x$

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$y(x+y)dx + (x+2y-1)\,dy = 0$

$\cancel{e^x(x+y)dx + e^x}$

$\cancel{y(e^x +}\ (ye^x x + y^2 e^x)dx + (xe^x + 2e^x y - e^x)dy = 0.$

$\frac{\partial m}{\partial y} = xe^x + 2ye^x \quad \frac{\partial N}{\partial x} = xe^x + e^x + 2e^x y - e^x$

$\therefore \frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$

Exact D.E

Sol. $\int m\,dx + \int N(y)\,dy = c$

$\int (xe^x y + e^x y^2)dx + 0 = c$

$y\int \underset{U\ V}{xe^x}\,dx + y^2 \int e^x dx = c$

$y[xe^x - 1\cdot e^x] + y^2 e^x = c$

$xe^x y - e^x y + e^x y^2 = c.$

$$\boxed{\begin{array}{l}\text{Bernoulli's} \\ \int U\,V dx = UV_{(1)} - U'V_{(2)} + U''V_{(3)} - U'''V_{(4)} + \dots\end{array}}$$

(3) $(8xy - 9y^2)dx + 2(x^2 - 3xy)dy = 0.$

Sol. $m = (8xy - 9y^2) \quad N = 2(x^2 - 3xy).$

$\frac{\partial m}{\partial y} = 8x - 9(2y) \qquad \frac{\partial N}{\partial x} = 2(2x^2) - 6y$

$= 8x - 18y \qquad 6x^2 \quad 6xy.$

$N = 2(x^2 - 3xy)$

$2x^2 - 6xy$

$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x} \qquad 4x - 6y.$

$\therefore$ the given eq is not exact.

$$\frac{\partial m}{\partial y}-\frac{\partial N}{\partial x}=8x-18y-(4x-6y)$$
$$=8x-18y-4x+6y.$$
$$4x-12y$$
$$4(x-3y)$$
$$\frac{1}{N}\left(\frac{\partial m}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{4}{2x(x-3y)}(x-3y)$$
$$=\frac{2}{x}=f(x).$$
$$I_f=e^{\int f(x)dx}$$
$$=e^{\int 2/x\,dx}$$
$$=e^{2\int 1/x\,dx}$$
$$=e^{2\log x}=e^{\log x^2}=x^2$$
multiply x^2 in eq^n - ①.

$$(8xy-9y^2)dx+2(x^2-3xy)dy=0$$
$$x^2(8xy-9y^2)dx+x^2(2(x^2-3xy)dy]=0\rightarrow ②$$
$$(8x^3y-9x^2y^2)dx+x^2(2x^2-6xy)dy)=0.$$
$$(8x^3y-9x^2y^2)dx+(2x^4-6x^3y)dy)=0$$
$$m=8x^3y-9x^2y^2 \quad N=2x^4-6x^3y$$
$$\frac{\partial m}{\partial y}=8x^3-18x^2y \quad \frac{\partial N}{\partial y}=8x^3-18x^2y.$$
$$\frac{\partial m}{\partial y}=\frac{\partial N}{\partial x}$$
sol
$$\int(8x^3y-9x^2y^2)dx+0=c$$
$$\left(8^2y\cdot\frac{x^4}{4}-9^3\cdot y^2\frac{x^3}{3}\right)=c$$
$$\underline{2x^4y-3x^3y^2=c}$$

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(4) $(x^2+y^2+x)\,dx + xy\,dy = 0$.

$m = x^2+y^2+x \qquad N = xy$.

$0+2y+0 \qquad \frac{\partial N}{\partial x} = y$.

$\frac{\partial m}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = y$.

$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$

∴ the given eqn. is not exist.

$\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x} = 2y - y$.

$= y$.

This is near to "N"

$\frac{1}{N}\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{xy}(y)$

$= \frac{1}{x} = f(x)$.

I.F $e^{\int F(x)dx}$

$= e^{\int 1/x\, dx}$

$= e^{\log x}$

$= x$

multiply x in eq 1 -

$x\{(x^2+y^2+x)\,dx + xy\,dy\} = 0$

$x(x^2+y^2+x)\,dx + x(xy)\,dy = 0$

$(x^3+xy^2+x^2)\,dx + (x^2y)\,dy = 0$.

$\frac{\partial m}{\partial y} = x^3 + xy^2 + x^2$, $\frac{\partial N}{\partial x} = x^2 y$.

$\frac{\partial m}{\partial y} = 0 + x\,2y + 0$. $\frac{\partial N}{\partial x} = 2x\,y$.

$= 2xy$. $= 2xy$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Sol I.F

$\int (x^3 + xy^2 + x^2)\,dx + 0 = c$.

$\boxed{\frac{x^4}{4} + y^2 \frac{x^2}{2} + \frac{x^3}{3} = c}$

5. $y(2xy+1)\,dx - x\,dy = 0$.

$m = y(2xy+1)$ ~~$N = -x\,dy$~~ $N = -x$

$\frac{\partial N}{\partial x} = -1$

$= 2xy^2 + y$ $\cdot N = -1$

$\frac{\partial M}{\partial y} = 4xy + 1$

$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ the given eqn is not exact

~~$\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x} = 2xy^2 + y + 1$.~~

$\frac{\partial m}{\partial y} = 4xy + 1$ $\frac{\partial N}{\partial x} = -1$

$\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right) = 4xy + 1 + 1$

$4xy + 2$

$2(2xy+1)$.

$$\frac{1}{m}\left(\frac{\partial m}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{1}{y(2xy+1)}[2(xy+1)].$$

$$\int^{5/4} \qquad e^{\log y}=y.$$

$$(2xy^3+y^2)\,dx - xy\,dy = 0$$

$m = 2xy^3+y^2$ $\qquad$ $N = -xy$

$\frac{\partial m}{\partial y} = 6xy^2+2y$ $\qquad$ $\frac{\partial N}{\partial x} = -y$

?

Orthogonal Trajectories

vimp If two family of cours or such that every member of ~~family~~ one family every member of other family at right angle then they are said to be orthogonal trajectories of each other

Cartiesion curve :- an eq of the curve $f(x,y,c)=0$ whereever c is fixed constant represent the curve

diffentation w r t x' & eleminating constant c replacing $\frac{dy}{dx}$ by $\frac{dx}{dy}$ & solve the eq to get the orthogonal Trajectory (OT)

(2) Polar curves :-
$f(r,\theta,c)=0$ wher c is fixed constant represent the curve, diffrenting w r t θ and eleminiad constant & c.
Replace $\frac{dr}{d\theta}$ by $r^2\frac{d\theta}{dr}$ & solve the eq to get the required O.T.

② self orthogonal family:-
If the differential eqⁿ of the given family given remains unaltered of for replacing $\frac{dy}{dx} \to -\frac{dx}{dy}$ the family of curves is said to be self orthogonal

Problems

① find the O.T. of the family $y = x + ce^{-x}$

Sol $y = x + ce^{-x} \to ①$

Diff w.r.t $\to x$

$\frac{dy}{dx} = 1 + ce^{-x}(-1)$

$\frac{dy}{dx} = 1 - ce^{-x}$

$ce^{-x} = 1 - \frac{dy}{dx}$

$① \Rightarrow y = x + 1 - \frac{dy}{dx}$

$\frac{dy}{dx} + y - x - 1 = 0 \to ②$

replace $\frac{dy}{dx} \to \frac{-dx}{dy}$

∴ eq ② becoms.

$-\frac{dx}{dy} + y - x - 1 = 0$

$\frac{dx}{dy} - y + x + 1 = 0$

$\frac{dx}{dy} + x = y - 1$

there is linear differntal eqⁿ. in x.
which is of the form where $P = 1$; $Q = y - 1$

$\frac{dx}{dy} + Px = Q$

$P = 1 \quad Q = y - 1$

$IF = e^{\int P dy} = e^{\int 1 dy} = e^{y}$

Sol:-

$x(IF) = \int Q(IF)\,dy + c_1$

$xe^{y} = \int (y-1)e^{y}\,dy + c_1$

$= \int (\underset{u}{y}\underset{v}{e^{y}} - e^{y})\,dy + c_1$

$xe^{y} = ye^{y} - 1 \cdot e^{y} - e^{y} + c_1$

$xe^{y} = ye^{y} - 2e^{y} + c_1$

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$\frac{\cdot}{\cdot}e^y, \quad x = y - 2 + c_1/e^y.$

$y = -x - 2 + c_1 e^{-y}$

$y = -x + c_1 e^{-y} - 2$

$\boxed{y = -x + k e^{-y} + 2}$ is the required O.T where $c_1 = k$

(2) find the O.T of family of curves $y^2 = cx^3$.

Sol $y^2 = cx^3 \rightarrow (1)$

Diff w.r.t $\rightarrow x$.

$2y \frac{dy}{dx} = c(3x^2)$

$c = \frac{2y \, dy/dx}{3x^2}$

(1) $\Rightarrow y^{\cancel{2}} = \left(\frac{2\cancel{y} \, dy/dx}{3\cancel{x^2}}\right) \cancel{x^3}$

$3y = 2x \frac{dy}{dx}$

$2x \frac{dy}{dx} = 3y \rightarrow (2)$

This is diff eqn of the given family of curves

Replace $\frac{dy}{dx} \rightarrow \frac{-dx}{dy}$

$\therefore (2) \Rightarrow 2x\left(\frac{-dx}{dy}\right) = 3y$

$-2x\,dx = 3y\,dy$

This is in the form of variable separable.

$2x\,dx + 3y\,dy = 0$

Int

$\int 2x\,dx + \int 3y\,dy = c_1$

$2\frac{x^2}{2} + 3\frac{y^2}{2} = c_1$

$*$ by 2 $\quad$ by 2

$2x^2 + 3y^2 = 2c_1$

$\boxed{2x^2 + 3y^2 = k}$ where $k = 2c_1$ is required O.T

Imp (3) show that family of parabola, $y^2 = 4a(x+a)$ is self orthogonal.

Sol: $y^2 = 4a(x+a) \rightarrow (1)$.

$y^2 = 4ax + 4a^2$.

Diff wrt 'x':

$2y \frac{dy}{dx} = 4a + 0$.

$2y \frac{dy}{dx} = 4a$.

$a = \frac{2y}{4} \cdot \frac{dy}{dx}$

$a = y/2 \cdot \frac{dy}{dx}$.

$\therefore (1) \Rightarrow y^2 = 4\left(\frac{y}{2} \frac{dy}{dx}\right)\left(x + \frac{y}{2} \frac{dy}{dx}\right)$

$y^2 = 2y\, y_1 \left(x + \frac{y}{2} y_1\right)$

$\boxed{y = 2xy_1 + yy_1^2} \rightarrow (2)$

this is diff. eq^n of the given family.

replace $\frac{dy}{dx} \rightarrow \frac{-dx}{dy}$ or $y_1 \rightarrow -1/y_1$

$\therefore (2) \Rightarrow y = 2x\left(-1/y_1\right) + y\left(-1/y_1\right)^2$

$y = -\frac{2x}{y_1} + y/y_1^2$

$y = \frac{-2xy_1 + y}{y^2}$.

$yy_1^2 = -2xy_1 + y$.

$\boxed{y = 2xy_1 + yy_1^2} \rightarrow 3$.

this is diff eq^n of the required O.T

comparing eq^n (2) & (3). the diff. eq^n are same thus the family of parabola $y^2 4a(x+a)$ or self orthogonal.

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O.T

* find the orthogonal trajectory of the family of cardioide $r = a(1+\cos\theta)$

sol: $r = a(1+\cos\theta) \rightarrow (1)$

$\log r = \log a + \log(1+\cos\theta)$

Diff w.r.t θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin\theta}{1+\cos\theta} \rightarrow (2)$

Replace $\frac{dr}{d\theta} \Rightarrow -r^2\frac{d\theta}{dr}$

$\frac{1}{r}\left(-r^2\frac{d\theta}{dr}\right) = \frac{-\sin\theta}{1+\cos\theta}$

$r\frac{d\theta}{dr} = \frac{\sin\theta}{1+\cos\theta}$

$= \frac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}$

$r\frac{d\theta}{dr} = \tan\theta/2 \rightarrow (3)$

This is diff eq of required O.T.

$\frac{d\theta}{\tan\theta/2} = \frac{dr}{r}$

$\frac{dr}{r} = \cot\theta/2\, d\theta$

Int

$\int\frac{dr}{r} = \int\cot\theta/2\, d\theta + \log c$

$\log r = \frac{\log\sin\theta/2}{1/2} + \log c$

$= 2\log\sin\theta/2 + \log c$

$\log r = \log\sin^2\theta/2 + \log c$

$\log r = \log(c\sin^2\theta/2)$

$r = c\sin^2\theta/2$

$2\sin^2\theta/2 = 1-\cos\theta$

$r = c\left(\frac{1-\cos\theta}{2}\right)$

$r = \frac{c}{2}(1-\cos\theta)$

$r = b(1-\cos\theta)$

where $b = c/2$

* obtain the O.T of the family of curves
$r^n = a^n \cos n\theta \rightarrow (1)$.

Sol:- $\log r^n = \log(a^n \cos n\theta)$

$n\log r = \log a^n + \log \cos n\theta$

$\text{—''—} = n\log a + \text{—''—}$

Diff w.r.t θ

$n\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}(-\sin n\theta \cdot n)$.

$n\frac{1}{r}\frac{dr}{d\theta} = -n\tan n\theta$

$\frac{1}{r}\frac{dr}{d\theta} = -\tan n\theta \rightarrow (2)$

Replace $\frac{dr}{d\theta} \rightarrow r^2\frac{d\theta}{dr}$

$(2) \Rightarrow$

$\frac{1}{r}\left(-r^2\frac{d\theta}{dr}\right) = -\tan\theta$

$r\frac{d\theta}{dr} = \tan n\theta \rightarrow (2)$

$\frac{d\theta}{\tan n\theta} = \frac{dr}{r}$

$\frac{dr}{r} = \cot n\theta \, d\theta$

$\int\frac{dr}{r} = \int\cot n\theta \, d\theta + \log c$

$\log r = \frac{\log \sin n\theta + n\log c}{n}$

$n\log r = \log \sin n\theta + \log c^n$

$\cancel{\log} r^n = \cancel{\log}(\sin n\theta \cdot c^n)$

$\boxed{r^n = c^n \sin n\theta}$

Newtons law of cooling

the law states that change in temperature of the body is proposnal to the diff b/w temp. of the body and that of surrounding medium.

let t_1 be the intial tempearuter of the body and t_2 be the constant tempearture of the medium

T be the temp of the body at any time t ∴ by newton's law of cooling we have an expression of temp. function given by $T = t_2 + (t_1 - t_2) e^{-kt}$
where t_1 intial temp. t_2 surrounding temp. t - time and k proportanility constant.

① A body in air at 25°c cools from 100°c to 75°c in 1 mint. find the temp of body at the end of 3 mints

sol given data

$t_2 = 25°c$

$t_1 = 100°c$

$T = 75°c$

$t = 1 min$

by Newtons law of cooling we have: $T = t_2 + (t_1 t_2$

$$T = t_2 + (t_1 - t_2) e^{-kt}$$

$$75 = 25 + (100 - 25) e^{-k(1)}$$

$$50 = 75e^{-k}.$$

$$e^{-k} = \frac{50}{75} = 2/3$$

$$e^{-k} = 2/3$$

$$\frac{1}{e^{k}} = 2/3$$

$e^{k} = 3/2$

$\log e^{k} = \log(3/2)$

$k \log_e e = \log_e(3/2)$

$k(1) = 0.405$

$\boxed{k = 0.405}$

$T = ?\ ,\ t = 3$

$T = t_2 + (t_1 - t_2)e^{-kt}$

$T = 25 + (100 - 25)e^{(-0.405)3}$

$T = 25 + 75(0.296)$

$\boxed{T = 47.25^{\circ}C}$

(2) If the temp of the A is $30^{\circ}c$ and a metel ball cools from $100^{\circ}c$ to $70^{\circ}c$ in 15min, find how log will it take for the metal ball to reach a temperature $40^{\circ}c$.

Ans. $t_2 = 30^{\circ}c$

$t_1 = 100^{\circ}c$

$T = 70^{\circ}c$

$t = 15\,min$

$T = t_2 + (t_1 - t_2)e^{-kt}$

$70 = 30 + (100 - 30)e^{-15k}$

$70 - 30 = 70e^{-15k}$

$40 = 70e^{-15k}$

$e^{-15k} = 40/70$

$e^{-15k} = 4/7$

$1/e^{15k} = 4/7$

$e^{15k} = 7/4$

$\log e^{15k} = \log(7/4)$

$15k \log_e e = \log_e (7/4)$.

$15k = 0.5596$.

$\therefore k = \frac{0.5596}{15}$

$\boxed{k = 0.0373}$

$T = 40^\circ c, t = ?$

$T = t_2 + (t_1 - t_2) e^{-kt}$

$40 = 30 + (100 - 30) e^{-(0.0373)t}$.

$40 - 30 = 70 e^{-(0.0373)t}$.

$10 = 70 e^{-(0.0373)t}$.

$e^{-(0.0373)t} = \frac{10}{70}$.

$\frac{1}{(0.0373)t} = 1/7$

$e^{(0.0373)t} = 7$.

$\log_e (0.0373)t = \log 7$.

$(0.0373) t \log_e e = 1.9459$

$(0.0373) t = 1.9459$

$t = \frac{1.9459}{0.0373}$

$t = 52.1699$.

$t \approx 52$ min

$\therefore$ It take 52 min for the metal body to reach tem p.rs $40^\circ c$.

* a bottal at room temp 72°F is kept In a refrigerator where the temp is 44°F after 30 minutes, water cooled to 61°F

(1) what is the temp. of the mineral water In another 30 minutes.

(2) How long will it take cool to 50°F.

Ans Given data.

$t_1 = 72°F$

$t_2 = 44°F$

$T = 61°F$

$t = 30min$

$T = t_2 + (t_1 - t_2)e^{-kt}$

$61 = 44 + (72-44)e^{-30k}$

$61 - 44 = 28e^{-30k}$

$17 = 28e^{-30k}$

$e^{-30k} = \frac{17}{28}$

$\frac{1}{e^{30k}} = \frac{17}{28}$

$e^{30k} = \frac{28}{17}$

$\log e^{30k} = \log(28/17)$

$30k \log e = \log(1.64)$

$30k = 0.4989$

$k = \frac{0.4989}{30}$

$\boxed{k = 0.0166}$

(i) $T = ?$, $t = 60$ min.

$T = t_2 + (t_1 - t_2)e^{-kt}$

$T = 44 + (72-44)e^{-(0.0166)60}$

$= 44 + 28(0.3693)$

$\boxed{T = 54.34}$

$T = 54.34°F$

$$T = t_2 + (t_1 - t_2)e^{-Kt}$$

$$50 = 44 + (72 - 44)e^{-(0.0166)t}$$

$$50 - 44 = 28e^{-(0.0166)t}$$

$$6 = 28e^{-(0.0166)t}$$

$$e^{-(0.0166)t} = \frac{6}{28}$$

$$\frac{1}{e^{(0.0166)t}} = \frac{6}{28}$$

$$e^{(0.0166)t} = \frac{28}{6}$$

$$\log_e e^{(0.0166)t} = \log_e(28/6)$$

$$(0.0166)t \log_e e = 1.5404$$

$$(0.0166) = \frac{1.5404}{}$$

$$t = \frac{1.5404}{0.0166}$$

$$t = 92.75$$

$$\boxed{t = 93\,m/s.}$$

Flow of Electricity

Electricity can we understood as the substance which flows through conductors such as wire the rate or flow of electricity is called the current.
Electric circuit is compsed by three elements such as, Inductance (L) resistance (R), capacitance (C) and active element being the voltage source.
the diff equation is given : $L\frac{di}{dt} + Ri = E$

(1) Series circuit with resistance (R) Inductance (L) and electro motive force (E) is governd by the Diff^n.

$L\frac{di}{dt} + Ri = E$, where L and R are constants and initially the current I is 0 find the current at any time t

Sol.
$$L\frac{di}{dt} + Ri = E$$
$$\div L, \quad \frac{di}{dt} + \frac{R}{L} i = \frac{E}{L} \Rightarrow (1).$$

$$\frac{dy}{dx} + Py = Q$$
$$P = R/L, \quad Q = E/L.$$

$$IF = e^{\int P dt} = e^{\int R/L\, dt}$$
$$= e^{R/L \int dt} = e^{R/L\, t}$$

$$sol^n = i(IF) = \int Q(IF)\,dt + c$$
$$i e^{R/L\, t} = \int \frac{E}{L} (e^{\frac{Rt}{L}}) dt + c.$$
$$= \frac{E}{L} \left[\frac{e^{R/L\, t}}{R/L}\right] + c$$
$$= \frac{E}{\cancel{L}} \cdot \frac{\cancel{L}}{R} e^{R/L} + c$$

$$\boxed{i e^{R/L\, t} = \frac{E}{R} e^{R/L\, t} + C} \quad (2)$$

to find the constant C.

$i=0,\ t=0$

(2) $\Rightarrow$ $0 = \frac{E}{R} e^{0} + C$

$$\frac{E}{R} + C = 0$$

$$\boxed{C = -E/R}$$

(2) $\Rightarrow$ $i e^{Rt/L} = \frac{E}{R} e^{R/L\, t} - E/R$

$\div\ e^{R/L\, t}$

$$i = \frac{E}{R}\left(1 - e^{-R/L\, t}\right)$$

$$i(t) = \frac{E}{R}\left(1 - e^{\frac{-R}{L} t}\right).$$

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