standerd divation :- ~~Sd~~ = SD = $\sqrt{V}$

* ——— * ——— * ———

*1) find the value of K such that the following represent of a finite probability distribution hence find its mean & standerd divation also find $P(x \le 1)$, $P(x > 1)$, $P(-1 < x \le 2)$.

x	-3	-2	-1	0	1	2	3
$P(x)$	K	2K	3K	4K	3K	2K	K

Since X is the p.d.f we must have $P(x_i) \ge 0$,

$\Sigma P(x_i) = 1$, consider $\sum_{1}^{7} P(x_i) = 1$

$P(x_1) + P(x_2) + P(x_3) + P(x_4) + P(x_5) + P(x_6) + P(x_7) = 1$

$\cancel{16K = 1}$

$K + 2K + 3K + 4K + 3K + 2K + K = 1$

$\boxed{K = 1/16}$

mean :- $\mu = \sum_{i=1}^{7} x_i (P x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 (x_3) + x_4 P(x_4) + x_5 P(x_5) + x_6 P(x_6) + x_7 P($

$\boxed{\mu = 0}$

$E(x^2) \sum_{i=1}^{7} x_i^2 P(x_i) = 9 \times \frac{1}{16} + 4\left(2/16\right) + 1\left(3/16\right) + 0 + 1\left(3/16\right) + 4\left(2/16\right) + 9\left(1/16\right)$

$\boxed{E(x^2) = 2{,}5}$

$V = E(x^2) - \mu^2$

$= 2.5 - 0$

$V = 2.5$

$SD = \sqrt{V}$

$= \sqrt{2.5}$

$\boxed{SD = 1.58}$

$P(x \leq 1) = P(x=-3) + P(x=-2)$
$+ P(x=-1) + P(x=0) + P(x=1)$
$= 1/16 + 2/16 + 3/16 + 4/16 + 3/16.$
In fraction
$= 0.81 = 13/16$

$P(x>1) = P(x=2) + P(x=3)$

$= 2/16 + 1/16$

$\boxed{= 0.18 \;/\; 3/16}$

$P(-1 < x \leq 2) = P(x=0) + P(x=1)\, P(x=2)$
$= 4/16 + 3/16 + 2/16$

$= 9/16$

$\boxed{= 0.56}$

---

(2) A Random varible 'x' as following probablity fun as various valus of x

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

find K.

(1) find K $P(x<6)$
(2) evaluate $P(x \geq 6)$.
(3) probablity $P(3 < x \leq 6)$

Since x is PDF we must have $P(x_i) \geq 0$ & $\sum P(x_i) =$
consider $\sum_{i=1}^{8} P(x_i) = 1$

$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$
$10K^2 + 9K = 1$
$-10, 1, 9 \qquad 10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$(10K-1)(K+1) = 0$

$10K - 1 = 0$; $K + 1 = 0$

$10K = 1$ $\boxed{K = -1}$

$K = 1/10$

$\boxed{K = 0.1}$

since $P(x_i) \geq 0$ we neglet $K = -1$

x	0	1	2	3	4	5	6	7
P_x	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

$P(x<6)$, $P(x \geq 6)$ $P(3 < x \leq 6)$

$P(x<6) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$+ P(x=5)$

$= 0 + 0.1 + 0.2 + 0.2 + 0.3 + 0.01$

$= 0.81$

$P(x \geq 6) = P(x=6) + P(x=7)$

$= 0.02 + 0.17$

$= 0.19$

$P(3 < x \leq 6) : P(x=4) + P(x=5) + P(x=6)$

$= 0.3 + 0.01 + 0.02$

$= 0.33.$

(3) The probablity distribution of finate rand variable of the given by the following table

x	-2	-1	0	1	2	3
$P(x)$	0.1	K	0.2	$2K$	0.3	K

find the value of K, mean & variance

=> ~~$K + 2K + 3K = 1$~~

~~$4K = 1$~~

~~$K = \frac{1}{4}$~~

~~$0.1 + 0.2 + 3.3K = 1 \Rightarrow 0.3 + 3.3K = 1$~~

~~$3.3K = \frac{1}{0.3}$~~

~~$3.3K = 3.3$~~

~~$\boxed{K = 1}$~~

x	-2	-1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0.1

$0.6 + 4K = 1$

$4K = 1 - 0.6$

$4K = 0.4$

$\boxed{K = 0.1}$

mean $= \sum_i x_i (P(x_i)) \Rightarrow -0.2 - 0.1 + 0.2 + 0.6 + 0.3$

$\mu \Rightarrow 0.8$

$V = \sum_i (x_i)^2 P(x_i) - \mu^2$

$= 0.4 + 0.1 + 0.2 + 0.6 + 0.3 - (0.8)^2$

$= 0.16 \quad - 0.64$

$V = -0.48$ //

(ii) A coin is tossed is twoised A Random variable X Represent number of heads turning up find the discret p.d. for 'x' also. find its mean & variance.

* $S = \{HH, HT, TH, TT\}$

The association of the event of S to the Rand variable X (or) (2,1,1,0)

$P(H,H) = 1/4$ $P(HT) = 1/4$ $P(TH) = 1/4$; $P(TT) = 1/4$

$P(x=0, TT) = 1/4$

let X represent the no: of heads turning up.

$P(x=0$ no of Head$) = P(TT) = 1/4$

$P(x=1$ no, of head$) = P(HT \cup TH) = P(HT) + P(TH)$
$= 1/4 + 1/4 = 1/2$

$P(x=2) = P(HH) = 1/4.$

The discrete probablity. X is.

$X = x_i$	0	1	2
$P(x_i)$	1/4	1/2	1/4

we aboserve that $P(x_i) \geq 0$ & $\Sigma P(x_i) = 1$

mean = $\mu = \sum_{i=1}^{3} x_i P(x_i)$ $x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$

$= 0 + 1 \times 1/2 \; \cancel{2} \times 1/\cancel{4}_2$

$= 1$

variance :-

$E(x^2) = \sum_{i=1}^{3} = x_1^2 P(x_1) + x_2^2 (P(x_2) + x_3^2 (P(x_3)$

$= 0 + 1(1/2) + \cancel{4} \times 1/\cancel{4}$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

$$V = E(x^2) - \mu^2$$

$$= \frac{3}{2} - 1^2$$

$$V = \frac{1}{2}$$

(1) When coin tossed 4 times, find the probability of getting exactly one head, second one atmost 3 heads, at least 2 heads.

sol:- $n=4$ $\quad p+q=1$

$p = P(H) = \frac{1}{2} = 0.5$

$q = 1-p = 1-\frac{1}{2} = \frac{1}{2} = 0.5$

wkt probability of 'x' success out of n trial is given by

$P(x) = {}^nc_x\, p^x q^{n-x}$

$= 4c_x (0.5)^x (0.5)^{4-x}$

$= 4c_x (0.5)^{x+4-x}$

$P(x) = 4c_x (0.5)^4$

$$\boxed{P(x) = (0.0625)\, 4c_x}$$

(i) P(exactly one head $P(x=1) = (0.0625)\, 4c_1 = 0.25$

(ii) P(atmost 3 heads $P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= (0.0625)\, 4c_0 + (0.0625)\, 4c_1 + (0.0625)\, 4c_2 + (0.0625)\, 4c_3$

$= 0.9375$

(iii) P(at least 2 heads $P(x \geq 2) = 1 - P(x<2)$

$= 1 - \{P(x=0) + P(x=1)\}$

* The probability that a bomb drop from plain will strike the target is $1/5$ is 6 bombs are droped, find the probability that
(i) exactly Two will strike the tragit
(ii) at least 2 will strike the tragat

lets 't' probability is that bomb droped from a plain

$n = 6$

$p = 1/5$

$q = 1 - p = 1 - 1/5 = 4/5$

probability of x succes out of n trails is given by $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$P(x) = 6C_x (1/5)^x (4/5)^{6-x}$

(i) P(exactly 2 will strike the tragt $P(x=2)$

$= 6C_2 (1/5)^2 (4/5)^{6-2}$

$= 6C_2 (1/5)^2 (4/5)^4$

$= 0.2457$

(ii) P(at least 2 will strike the tragat)

$= P(x \geq 2) = 1 - P(x < 2)$

$= 1 - \{P(x=0) + P(x=1)\}$

$= 1 - \{6C_0 (1/5)^0 (4/5)^{6-0} + 6C_1 (1/5)^1 (4/5)^{6-1}\}$

$= 1 - \{(4/5)^6 + 6(1/5)(4/5)^5\}$

$\boxed{= 0.3446}$

* The ~~ovaro~~ overall % of failurs in certin examination is 20 if 6 canditas appear in the examination what is the probability that

(i) at least 5 pass the examination

(ii) at most 3 pass the examination.

(iii) non of them pass the examination

let p be the probability of candidates pass the examination

$q = 20\% = \frac{20}{100} = 0.2$

$P = 1 - q = 1 - 0.2 = 0.8$ $\boxed{p = 0.8}$

$\boxed{n = 6.}$

(i) $P(x) = {}^nC_x \, p^x q^{n-x}$.

$P(x) = {}^6C_x (0.8)^x (0.2)^{6-x}$

$P(x \geq 5) = P(x=5) + P(x=6)$.

$= {}^6C_5 (0.8)^5 (0.2)^{6-5} + {}^6C_6 (0.8)^6 (0.2)^{6-6}$

$= 6 \times 0.3276 \times 0.2 + 1(0.2621) \times 1$

$= \underline{0.6553}$

(ii) $P(x \leq 3) = 1 - P(x > 3)$

$= 1 - \{P(x=4) + P(x=5) + P(x=6)\}$

$= 1 - \{{}^6C_4 (0.8)^4 (0.2)^{6-4} + {}^6C_5 (0.8)^5 (0.2)^{6-5}$

$15 \times 0.4096 \times 0.04$

$+ {}^6C_6 (0.8)^6 (0.2)^{6-6}\}$

0.90106

$= 0.0989$

(ii) $P(x=0) = {}^6C_0 (0.8)^0 (0.2)^{6-0}$

$\boxed{= 0.000064}$

* If on an average 1 ship in every 10 is wrecked, find that probablity that out of 5 sheets expecteted to arriv.

(i) atleast 4 will arrive shafly

(ii) at most 2 will arriv.

(iii) none

$n=5$

$q = 1/10 = 0.1$

$P = 1-0.1 = 0.9$

(i) $P(x) = nc_x P^x q^{n-x}$

$\cancel{= 5c_x (0.9^x)}$

$= 5c_x (0.9)^x (0.1)^{5-x}$

(i) $P(x) = 5c_4 (0.9)^4 (0.1)^{5-4} + 5c_5 (0.9)^5 (0.1)^{5-5}$

$5 \times 0.6561 \times 0.1 + 1 \times 0.59049 \times 1$

$= 0.3280 +$

$\boxed{= 0.91849}$

(ii) $P(x \le 2) = 5c_0 (0.9)^0 (0.1)^{5-0} + 5c_1 (0.9)^1 (0.1)^{5-1}$

$+ 5c_2 (0.9)^2 (0.1)^{5-2}$

$= 1 \times 1 \times 0.00001 + 5 \times 0.9 \times 0.0001 +$

$10 \times 0.81 \times 0.001$

$= 0.00001 + 0.00045 + 0.0081$

$\boxed{= 0.00856}$

(iii) $^5C_0 (0.9)^0 (0.1)^{5-0}$

$P(x=0)$

$= \boxed{0.00001}$

(5) In a certain factory producing cycle tires, their is a small change of 1 in 500 tires to be diffective the tires or supply in a lot of 10 using paison distribution calculate the approximate number of lots containing i) no diffective i diffective ; 2 diffective tires respectively in a consignment of 10000 lots.

⇒ let p be the probability of diffective tyre

$\therefore p = \frac{1}{500} = 0.002$

$\boxed{n = 10}$

$\therefore m = np = (0.002) 10$

$= 0.02$

posion distribution is given by

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$$= \frac{(0.02)^x e^{-(0.02)}}{x!}$$

$$P(x) = \frac{(0.9801)(0.02)^x}{x!}$$

probability of no diffective $p(x=0)$

$= (0.9801)(0.02)^0$

$= 0.9801$

In a consignment of 10000 lots.

$= 10000 \times P(x=0)$

$= 10000 \times 0.9801$

$= 9801$

probability of 1 deffective $P(x=1)$

$$\frac{(0.9801)(0.02)^1}{1!}$$

$= 0.0196$.

In a consignment of 10000 lots

$(0.0196) \times 1000$

$= 196$

probability of 2 deffective $P(x=2)$

$$= \frac{(0.9801)(0.02)^2}{2}$$

$= 0.0001$

In a consignment of 10000 lots

$10000 \times 0(x=2)$

$= 10000 \times 0.0001$

$= 1$ 0.00019

(6) A communication channel receives independent pulses at the rate of 12 pulses per microsecond the probability of transmission error is 0.001 for each micro second compute the probabilities of

(i) no error ~~the probabilities~~ during micro second

(ii) 1 error per micro second

iii) Atleast one error per micro second

iv) 2 errors

v) Atmost 2 errors.

⇒ The probability of transmition error for each micro second $p = 0.001$

$(n=12)$

$m = np = (0.001)(12)$

$m = 0.012$

$$P(x) = \frac{m^x e^{-m}}{x!}$$

$$= \frac{(0.012)^x e^{-0.012}}{x!}$$

$$= \frac{(0.9880)(0.012)^0}{0.9880^0}$$

(i) $P(x=0) = (0.9880)(0.012)^0$

$= \cancel{0.0118}$

$= 0.9880$

(ii) $P(x=1) = \frac{(0.9880)(0.012)^1}{0.0118}$

(iii) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - 0.9880$

$= 0.012$

(iv) $P(x=2) = \frac{(0.9880)(0.012)^2}{2!}$

$= 0.00007$

(v) $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= 0.9880 + 0.0118 + 0.00007$

$= 0.9998$

(Q) If the probability that an indivial suffers a bad reaction from a certain injection in 0.001 Determine the probability that out of 2000 individuals.

(i) Exactly 3

(ii) more than 2 individuals.

(iii) more than 1 individual will suffer a bad reaction

$\Rightarrow$ let p be the Probability that an individual suffer a bad reaction $p = 0.001$

$\boxed{n = 2000}$

$m = np = (2000)(0.001)$

$\boxed{m = 2}$

$P(x) = \dfrac{m^x e^{-m}}{x!}$

$= \dfrac{2^x e^{-2}}{x!}$

$= \dfrac{(0.1353) 2^x}{x!}$

(i) $P(x=3) = \dfrac{(0.1353)(2)^3}{3!}$

$= 0.1804$

(ii) $P(x \geq 2) = 1 - P(x \leq 2)$

$1 - \{P(x=0) + P(x=1) + P(x=2)\}$

$1 - \{\dfrac{(0.1353) 2^0}{0!} + \dfrac{(0.1353)}{1!} 2^1 + \dfrac{(0.1353)}{2!} 2^2\}$

$1 - \{0.1353 + 0.2706 + 0.2706$

$1 - 0.6765$

$\boxed{= 0.3235}$

(iii) $P(x > 1) = 1 - P(x \leq 1)$

$= 1 - \{P(x=0) + P(x=1)\}$

$= 1 - \{0.1353 + 0.2705\}$

$= 1 - 0.4059$

$= 0.5941$

The probability of posion variant taking the values 3 and 4 are equal calculate the probab-ility of variant taking the value 0 and 1.

$\Rightarrow$ Given $P(x=3) = P(x=4)$

$P(x) = \dfrac{m^x e^{-m}}{x!}$

$$\frac{\cancel{m^3} \cancel{e^{-m}}}{3!} = \frac{m^4 e^{-m}}{4!}$$

$$\frac{1}{3!} = \frac{m^4 e^{-m}}{4!}$$

$$1/3! = \frac{m}{4 \times 3!}$$

$$\boxed{m=4}$$

(i) $P(x=0) = \frac{4^0 e^{-4}}{0!}$

$= 0.0183$

(ii) $P(x=1) = \frac{4^1 e^{-4}}{1!}$

$= 0.0732$

⑧ If x is the posion variant such that $P(x=2) = 9P(x=4) + 90P(x=6)$ computer distribution

$\Rightarrow P(x=2) = 9P(x=4) + 90P(x=6)$

$$\frac{m^2 e^{-m}}{2!} = \frac{9m^4 e^{-m}}{4!} + 90\frac{m^6 e^{-m}}{6!}$$

$$\frac{\cancel{m^2} \cancel{e^{-m}}}{2} = m^4 e^{-m} \left[\frac{9}{4!} + \frac{90m^2}{6!}\right]$$

$$1/2 = \frac{m^2}{2} \left[\frac{\cancel{9}\,3}{\cancel{24}\,4} + \frac{90m^2}{\cancel{360}\,4}\right]$$

$$= 1/4 \{3m^2 + m^4\}$$

$4 = 3m^2 + m^4$

$m^4 + 3m^2 - 4 = 0$

$m^4 + 4m^2 - m^2 - 4 = 0$

$m^2(m^2+4) - 1(m^2+4) = 0$

$(m^2+4)(m^2-1) = 0$

$m^2+4=0, \quad m^2-1=0$

$m^2=-4, \quad m^2=1$

$m = \pm\sqrt{-4} \quad m=\pm 1$

$m = \pm 2i \quad m=1 \quad x=1$

-4

$4 \quad -1$

* The number of Telephone lines busy at an instant of time is a binomial variant with probability 0.1 that a line is busy if 10 lines are choosen at random what is the probability that

(i) No line is busy

(ii) All lines are busy

(iii) Atleast one line is busy.

(iv) Atmost 2 line is busy

$p=0.1$
$n=10$

$\Rightarrow n=10, \quad p=0.1$

$p+q=1$

$q=1-p$

$q=1-0.1$

$q=0.9$

$$\boxed{P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}}$$

$$P(x) = {}^{10}C_{x} (0.1)^{x} (0.9)^{10-x}$$

i) $P(x=0) = {}^{10}C_{0} (0.1)^{0} (0.9)^{10}$

ii) $P(x=10) = {}^{10}C_{10} (0.1)^{10} (0.9)^{10-10}$

$= 0.0000000000.01$

iii) $P(x \geq 1) = 1 - P(x \leq 1)$

$= 1 - P(x=0)$

$= 1 - 0.3486$

$= \underline{0.6514}$

iv) $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= {}^{10}C_{0} (0.1)^{0} (0.9)^{10-0} + {}^{10}C_{1} (0.1)^{1} (0.9)^{10-1}$

$+ {}^{10}C_{2} (0.1)^{2} (0.9)^{10-2}$

$= 0.9298$ //

* An underground mine as 5 pumps installed for pumping out storm water the probability of any one of the pump failing during the storm is $\frac{1}{8}$ what is the probability that (i) at least 2 pumps will be working

(ii) All the pumps will be working during a particular storm. $x=5$

$\Rightarrow n=5, \; q=\frac{1}{8}$

WKT $p+q=1$

$p=1-q$

$p=1-\frac{1}{8}$

$$\boxed{p=\frac{7}{8}}$$

$P(x) = \cancel{}\; n_{C_x} \; p^x q^{n-x}$

$P(x) = 5_{C_x} \left(\frac{7}{8}\right)^x \cdot \left(\frac{1}{8}\right)^{5-x}$

(i) $P(x \geq 2) = 1 - P(x<2)$

$= 1 - \{P(x=1) + P(x=0)\}$

$= 1 - \{5_{C_1} \left(\frac{7}{8}\right)^1 \left(\frac{1}{8}\right)^4 + 5_{C_0} \left[\frac{7}{8}\right)^0 \left[\frac{1}{8}\right]^5\}$

$= 1 - 0.00010986$

$= 0.9989$

(ii) $P(x=5) = 5_{C_5} \left(\frac{7}{8}\right)^5 \left(\frac{1}{8}\right)^{5-5}$

$= \underline{\underline{0.5129}}$

Continuous probability distribution

If a variant can take any value in an interval It will give rise to continuation distribution

If for every x, belonging to the range of a continuous random variable (x) chearsig a real number $f(x)$ such (that) $f(x)$ satisfies the condition $f(x) \geq 0$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$f(x)$ is called continuous probability function or pd

If x lies in the interval a, b then.

$$P(a \leq x \leq b) = \int_a^b f(x)dx.$$

mean $- \mu - \int_{-\infty}^{\infty} x f(x) dx$

problems

1) find the value of c such that $f(x) =$

$$\{ \frac{x}{6} + c, \; 0 \leq x \leq 3 \}$$

0

else where is a probability density function also find $P(1 \leq x \leq 2)$

since $f(x)$ is pdf we must have $f(x) \geq 0$ and

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

consider $\int_{-\infty}^{\infty} f(x) dx = 1$

$$\int_0^3 (\frac{x}{6} + c) dx = 1$$

$$\frac{1}{6} \cdot \frac{x^2}{2} + c \Big|_2^3 = 1$$

$$\frac{1}{12} (9 - 0) + c \{3 - 0\} = 1$$

$\frac{9}{12} + 3c = 1$

$3c = 1 - 9$

$\cancel{3}c = \cancel{3}/12$

$\boxed{c = 1/2}$

by substituting c values.

$f(x) = \begin{cases} x/6 + 1/12, & 0 \le x \le 3 \\ & \text{else where} \end{cases}$

$P(1 \le x \le 2) = \int_1^2 f(x)\,dx$

$= \int_1^2 \left(\frac{x}{6} + \frac{1}{12}\right)dx$

$= \frac{1}{6}\left(\frac{x^2}{2}\right) + \frac{1}{12}(x)\Big]_{(2-1)}^{2}$

$= \frac{1}{12}(4-1) + 1/12$

$= 3/12 + 1/12$

$= 4/12$

$= 1/3$

(2) find the constant K such that

$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0 & \text{other wise} \end{cases}$

is a pdf also compute

i) $P(1 < x < 2)$ ii) $P(x \le 1)$ iii) $P(x > 1)$

iv) mean and variance

Since $f(x)$ is a pdf we must have.

$f(x) \ge 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^3 Kx^2\,dx = 1$

$K \frac{x^3}{3}\Big|_0^3 = 1$

$\frac{K}{3}(27-0)=1$

$9K=1$

$\boxed{K=1/9}$

(i) $P(1<x<2)=\int_1^2 f(x)\,dx$

$=\int_1^2 \frac{x^2}{9}\,dx$

$=\frac{1}{9}\left(\frac{x^3}{3}\right)_1^2$

$=\frac{1}{27}\cdot(8-1)$

$=7/27.$

(iii) $P(x\leq 1)$

$=\int_0^1 \frac{x^2}{9}\,dx$

$=\frac{1}{9}\left(\frac{x^3}{3}\right)_0^1$

$=\frac{1}{27}(1-0)$

$=\frac{1}{27}$

(iii) $P(x>1)$

$=\int_1^3 \frac{x^2}{9}\,dx$

$=\frac{1}{9}\left(\frac{x^3}{3}\right)_1^3$

$=\frac{1}{27}(27-1)$

$=\frac{26}{27}$

$=$

iv) Mean $\mu=\int_{-\infty}^{\infty} x f(x)\,dx$

$\int_0^{\infty} x\left(\frac{x^2}{9}\right)dx$

$\int_0^3 \frac{x^3}{9}\,dx$

$=\frac{1}{9}\left(\frac{x^4}{4}\right)_0^3$

$=\frac{1}{36}(81-0)$

$=\frac{81}{36}$

$=9/4$

variance $V=\int_{-\infty}^{\infty} x^2 f(x)\,dx-\mu^2$

$\int_0^3 x^2\cdot\frac{x^2}{9}\,dx-\left(\frac{9}{4}\right)^2$

$=\frac{1}{9}\int_0^3 x^4\,dx-\frac{81}{36}$

$=\frac{1}{9}\left(\frac{x^5}{5}\right)_0^3-\frac{81}{6}$

$=\frac{1}{45}(243-0)-\frac{81}{6}$

$=\frac{243}{45}-\frac{81}{36}$

$=\frac{27}{80}$

(*) A random variable x as following density function $p(x) = \begin{cases} kx^2, & -3 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$

evaluate k and find

(i) $P(1 \le x \le 2)$ ii) $P(x \le 2)$ iii) $P(x \ge 1)$

(i) $P(1 \le x \le 2)$

$$\int_{-3}^{3} kx^2\,dx = 1$$

$$k \cdot \left.\frac{x^3}{3}\right]_{-3}^{3} dx \Rightarrow k\left(\frac{3^3}{3} - \frac{(-3^3)}{3}\right)dx = 1$$

$$\Rightarrow k\left[\frac{27}{3} + \frac{27}{3}\right]dx = 1$$

$$\Rightarrow k\left\{\frac{\cancel{54}^{18}}{\cancel{3}_{1}}\right\}$$

$$\Rightarrow k(18) = 1 \qquad \boxed{k = 1/18}$$

i) $P(1 \le x \le 2)$

$$\int_1^2 f(x)\,dx$$

$$= \int_1^2 \tfrac{1}{18} x^2\,dx$$

$$= \tfrac{1}{18}\left(\tfrac{x^3}{3}\right)_1^2$$

$$= \tfrac{1}{18}\left(\tfrac{2^3}{3} - \tfrac{1^3}{3}\right)$$

$$= \tfrac{1}{18}\left(\tfrac{8}{3} - \tfrac{1}{3}\right)$$

$$= \tfrac{1}{18}\left(\tfrac{7}{3}\right)$$

$$\Rightarrow \frac{7}{54}.$$

(ii) $P(x \le 2)$

$$\int_0^2 f(x)\,dx$$

$$= \int_0^2 \tfrac{1}{18} x^2\,dx$$

$$= \tfrac{1}{18}\left(\tfrac{x^3}{3}\right]_0^2$$

$$= \tfrac{1}{18}\left(\tfrac{2^3}{3} - \tfrac{0^3}{3}\right)$$

$$= \tfrac{1}{18}\left(\tfrac{8}{3}\right)$$

$$\boxed{P = \frac{8}{54}}$$

iii) $P(x \ge 1)$

$$\int_1^3 f(x)\,dx$$

$$= \int_1^3 \tfrac{1}{18} x^2\,dx$$

$$= \tfrac{1}{18}\left(\tfrac{x^3}{3}\right)_1^3$$

$$= \tfrac{1}{18}\left(\tfrac{3^3}{3} - \tfrac{1^3}{3}\right)$$

$$= \tfrac{1}{18}\left(\tfrac{27}{3} - \tfrac{1}{3}\right)$$

$$\Rightarrow \tfrac{1}{18}\left[\tfrac{26}{3}\right]$$

$$= \frac{26}{54}$$

* Exponential Distribution

The continues probability destribution having pdf $f(x)$ given by.

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha > 0$ is called exponeptial Distribution.

$\mu = 1/\alpha$ iV $1/\alpha^2$

① In a certain town the duration of a shower is exponationtionlly distributed with mean equal to 5 min. what is the probability that a shower will last for.

(i) less than 10 mintes

(ii) 10 mintes ~~om~~ or ~~more~~ more

(iii) b/w 10 & 12 min.

given $\mu = 5$ but μ is $1/\alpha = 5$ $\alpha = 1/5$

$$f(x) = \begin{cases} \frac{1}{5} e^{-x/5} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$P(x < 10) = \int_0^{10} f(x)\,dx$$

$$= \int_0^{10} \frac{1}{5} e^{-x/5}\,dx$$

$$= \frac{1}{5}\left[\frac{e^{-x/5}}{(-1/5)}\right]_0^{10}$$

$$= \frac{1}{\cancel{5}}(-\cancel{5})\left[e^{-10/5} - e^{-0}\right]$$

$$= -\left[e^{-2} - 1\right]$$

$$= 0.8646$$

$$P(x \geq 10) = \int_{10}^{\infty} P(x)\,dx$$

$$\int_{10}^{\infty} \frac{1}{5} e^{-x/5}\,dx$$

$$= \frac{1}{5}\left[\frac{e^{-x/5}}{(-1/5)}\right]_{10}^{\infty}$$

$$P(x \geq 10) = \frac{1}{\cancel{5}}(-\cancel{5})\left\{e^{-\infty} - e^{-10/5}\right\}$$

$$= -\left\{0 - e^{-2}\right\}$$

$$= 0.1353$$

$$P(10 < x \leq 12) = \int_0^{12} f(x)\,dx$$

$$= \int_0^{12} \frac{1}{5} e^{-x/5}\,dx$$

$$= \frac{1}{5}\left\{\frac{e^{-x/5}}{-1/5}\right\}_{10}^{12}$$

$$= \frac{1}{\cancel{5}}(-\cancel{5})\left\{e^{-12/5} - e^{-10/5}\right\}$$

$$= -\left[e^{-12/5} - e^{-2}\right]$$

$$= 0.0446$$

(2) The length A Telephone conversation as an exponatial distribution with mean of 3 mints. find the probability A call
(i) ends in less than 3 min's
(ii) Takes b/w 3 & 5 min's

⇒ Given, $\frac{1}{\alpha} = 3$

$\frac{1}{\alpha} = 3$

$\alpha = \frac{1}{3}$

$$f(x) \begin{cases} \frac{1}{3} e^{-x/3} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(i) $P(x < 3) = \int_0^3 f(x)\,dx$

$$= \int_0^3 \frac{1}{3} e^{-x/3}\,dx$$

$$= \frac{1}{3}\left\{\frac{e^{-x/3}}{(-1/3)}\right\}_0^3$$

$$= \frac{1}{\cancel{3}}(-\cancel{3})\left\{e^{-1} - e^{-0}\right\}$$

$$= -\left[e^{-1} - 1\right]$$

$$= 0.6321$$

(ii) $P(3 < x < 5) = \int_3^5 f(x)\,dx$

$= \int_3^5 \frac{1}{3} e^{-x/3}\,dx$

$= \frac{1}{3}\left[\frac{e^{-1/3}}{(-1/3)}\right]_3^5$

$\frac{1}{\cancel{3}}(-\cancel{3})\left[e^{-5/3}\ e^{-1}\right]$

$= -\left[e^{-5/3} - e^{-1}\right]$

$\boxed{= 0.1790}$

* The kilo meter run (in 100 km) without any salt of problem in respect of an certain vehicles is a random variable have pdf $(fx) = \begin{cases} \frac{1}{40} e^{-x/40} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$

find probability that the vechaile is trouble free

(i) Atleast 25,000 kms $P(x \geq 25)$

(ii) Atmost for $25{,}000$ kms $P(x \leq 25)$

(iii) b/w 16,000 to 32,000 kms. $P(16 < x < 32)$

let x be A random variable representing km in multiples of 1000 regarding Trouble free run by ~~the~~ ~~roto~~ vhicle.

(i) $P(x \geq 25,000)$

$f(x) \begin{cases} \frac{1}{40} e^{-x/40} \cdot & x \geq 0 \\ 0 & x \leq 0. \end{cases}$

$\alpha = 1/40$

$P(x \geq 25,000) \int_{25}^{2500} f(x)\,dx$

$= \int_{25}^{\infty} \frac{1}{40} e^{-x/40} dx$

$= \frac{1}{40} \left[\frac{e^{-x/40}}{(-1/40)} \right]_{25}^{\infty}$

$= \frac{1}{40}(-40) \left[e^{-\infty} - e^{-25/40} \right]$

$= -\left[0 - e^{-25/40} \right]$

$\boxed{= 0.5352}$

(iii) $P(x = 25)$

$f(x) \begin{cases} \frac{1}{40} e^{-x/40} & x \geq 0 \\ 0 & -x \leq 0 \end{cases}$

$= \int_0^{25} \frac{1}{40} e^{-x/40} dx$

$= \frac{1}{40} \left[\frac{e^{-x/40}}{(-1/40)} \right]_0^{25}$

$= \frac{1}{40}(-40) \left[e^{-25/40} - e^{-0} \right]$

$= -(e^{-25/40} - 1)$

$= 0.4647$

(iii) $P(16 < x < 32)$

$$= \frac{1}{40}(-40)\left\{e^{-32/40}\ e^{-16/40}\right)$$

$$= -1\left\{e^{-32/40}\ e^{-16/40}\right\}$$

$$\boxed{= 0.2209.}$$

(3) The life of TV tube manufactured by a company is known to have a mean of 200 months assuming that the life as an exponential distribution find the probability that the life of tube manufactured by the company is

(i) less than 200 months.

(ii) b/w 100 & 300 months

$\mu = 200$

$\alpha = 1/200$

$$f(x) \begin{cases} 1/200\ e^{-x/200} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(i) $P(x < 200)$ $\int_0^{200} f(x)\,dx$

$$= \int_0^{200} \frac{1}{200} e^{-x/200}\,dx$$

$$= \frac{1}{200}\left\{\frac{e^{-x/200}}{(-1/200)}\right\}_0^{200}$$

$\frac{1}{\cancel{200}}(-\cancel{200})\left\{ e^{-x/200} \right.$

$\left. e^{-1} \quad e^{-0} \right\}$

$= -1 \left\{ e^{-1} \; \cancel{e^{0}} \atop -1 \right\}$

$= 0.6321$

(ii) $P(100\,x < 300) = \int_{200}^{300} f(x)\,dx$

$= \int_{200}^{300} \frac{1}{200} e^{-x/200}\,dx$

$= \frac{1}{\cancel{200}} \left[\frac{e^{-x/200}}{(-1/200)} \right]_{200}^{1}$

$= \frac{1}{200}(-200) \left\{ e^{-300/200} - e^{-100/200} \right\}$

$= -\left\{ e^{-3/2} - e^{-1/2} \right\}$

$= 0.3834$

① find the area under the normal curve in the following

(i) $z=0$ & $z=1.2$

(ii) $z=-0.68$ & $z=0$

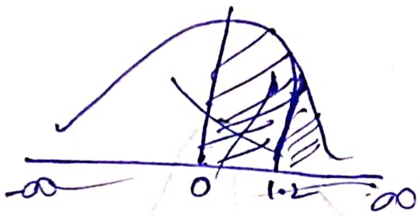
(iii) $z=-0.46$ & $z=2.21$

(iv) $z=0.81$ & $z=1.94$

v) to the left of $z=0.6$

vi) to the right of $z=-1.28$

(i) The area under the normal curve for $z=0$ & $z=1.2$ is.



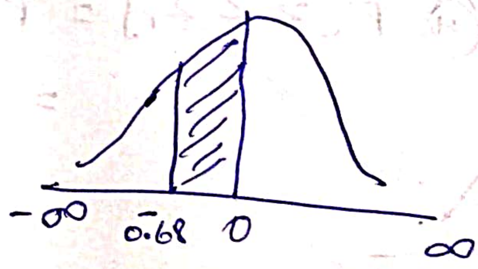
$P(0 \leq z \leq 1.2) = \phi(1.2) = 0.3849$

(ii) The area under the normal curve for $z=-0.68$ & $z=0$

$P(-0.68 \leq z \leq 0)$

$= \phi(0.68)$

$= 0.2517$



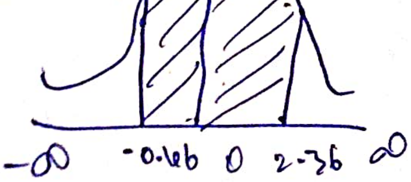
(iii) The area under the normal curve for $z=-0.46$ & $z=2.21$

$P(-0.46 \leq z \leq 2.21) = P(-0.46 \leq z \leq 0) + P(0 \leq z \leq 2.21)$

$= \phi(0.46) + \phi(2.21)$

$= 0.1772 + 0.4864$

$= 0.6636.$



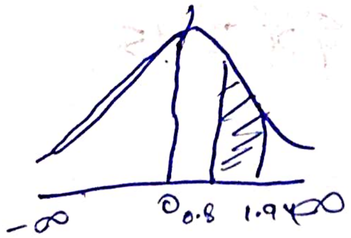
iv) the area under the normal curve for $z=-0.81$ & $z=1.94$

$= P(0.81 \leq z \leq 1.94) = P(0 \leq z \leq 1.94) - P(0 \leq z \leq 0.81)$

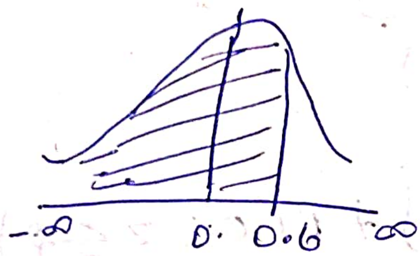
$= \phi(1.94) - \phi(0.81)$

$= 0.4738 - 0.2910$

$= 0.1823$



v) area of the curve for left of $z = 0.6$



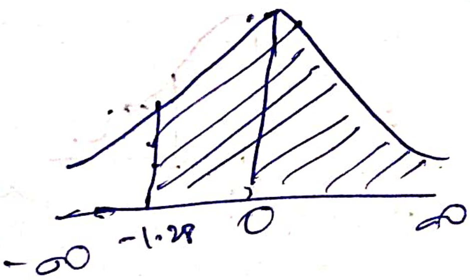
$P(-\infty \le z \le 0) - P(0 \le z \le 0.6)$

$= 0.5 + \phi(0.6)$

$= 0.5 + 0.2257$

$= 0.7257$

vi) area of the curve for right $z = -1.28$



$P(-1.28 \le z \le 0) + P(0 \le z \le \infty)$

$= \phi(1.28) + 0.5$

$= 0.3997 + 0.5$

$= 0.8997$

1 (i) If x is a normal variate with mean 30 & S.D is 5 find probability $26 \le x \le 40$ (i) & $x \ge 45$

Ans:- Given $\mu = 30$, $\sigma = 5$

we have SNV, $z = \frac{x-\mu}{\sigma}$

$$z = \frac{x-30}{5}$$

when $x = 26$, $z = \frac{26-30}{5}$ $\boxed{-0.8 = z}$

when $x = 40$, $z = \frac{40-30}{5} = \boxed{z = 2}$

$P(26 \le x \le 40) = P(-0.8 \le z \le 2)$

$P(-0.8 \le z \le 2)$

$= P(-0.8 \le z \le 0) + P(0 \le z \le 2)$

$= \phi(0.8) + \phi(2)$

$= 0.2881 + 0.4772$

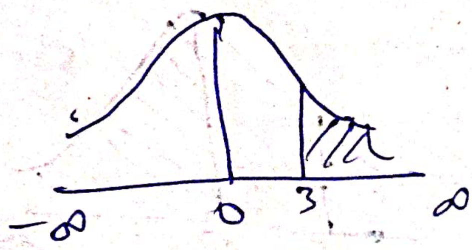
$= 0.7653$

$P(x \ge 45) = P(z \ge 3) = P(0 \le z \le \infty) - P(0 \le z \le 3)$

$= 0.5 - \phi(3)$

$= 0.5 - 0.4987$

$= 0.0013$



problems

(1) If x is normaly distributed with me 12 and standard divation for find the following: $P(x \geq 20)$ (ii) $P(x \leq 20)$

Ans Given

$\mu = 12 \quad \sigma = 4$ (sigma)

we have SNV $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-12}{4}$

when $x = 20$ $\quad z = \frac{20-12}{4}$

$= P(x \geq 20) + P(z \geq 2)$

$= P(0 \leq z \leq \infty) - P(0 \leq z \leq 2)$

$= 0.5 - \phi(2) = 0.5 - 0.4772$

$= 0.0228$.

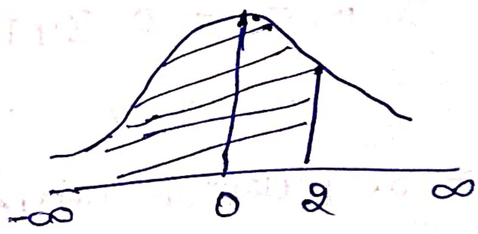
(ii) $P(x \leq 20) = P(z \leq 2)$

$= P(-\infty \leq z \leq 0) + P(0 \leq z \leq 2)$

$= 0.5 + \phi(2)$

$= 0.5 + 0.4772$

$= 0.9772$



(2) A manifacturer air main envalour from experiany that the weight of the examples is norma envalops distribution with mean 1.95g & S.D 0.05g About how many examples weighing (i) 2g or more.

(ii) 2.05g or more can be expected in a given packet of 100 envalops

1 $\xrightarrow{w.}$ $\mu = 1.95g$, $\sigma = 0.05g$

we have SNV. $z = \frac{x-\mu}{\sigma}$

$$z = \frac{x - 1.95}{0.05}$$

when $x = 2$, $z = \frac{2 - 1.95}{0.05}$ $\quad x = 2.05 = \frac{2.05 - 1.95}{0.05}$

$$z = \frac{0.05}{0.05} \qquad \boxed{z = 2}$$

$\boxed{z = 1}$

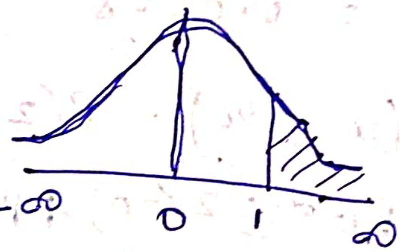
(i) $P(x \leq 2)$ $P(z \geq 1)$

$P(z \geq 1) = P(0 \leq z \leq \infty) - P(0 \leq z \leq 1)$

$= 0.5 - \phi(1)$

$= 0.5 - 0.3413$

$= 0.1587$



for a packet ~~of~~ 100 ~~at~~ ~~of~~ envovalps $100 \times P(z \geq 1) =$
100×0.1587

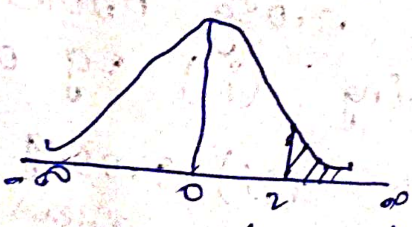
$= 15.87 = 16$ envalops

(ii) $P(x \geq 2.05) = P(z \geq 2)$

$= P(z \geq 2) = P(0 \leq z \leq \infty) - P(0 \leq z \leq 2)$

$= 0.5 - \phi(2)$

$= 0.0228$



for packet 100 envolps $100 \times P(z \geq 2)$
100×0.0228
are weig 2.05g or more $= 2.28 \simeq 2$ envalps

(3) A sample of 100 dry battery cells tested to find the length of life produce the following result with me 12 h, variants 3h. assuming the data to be normally distributed, what the % of battery cells or expected to have life

(i) more than 15h

(ii) b/w 10 & 14 h

$\mu = 12$

$V = 3 \quad \sigma = \sqrt{V} = \sqrt{3} = 1.732$

$\sigma = 1.732$

we have SNV $\cdot z = \dfrac{x - 12}{1.732}$

we $x = 15 \quad z = \dfrac{15-12}{1.732}$

$\boxed{z = 1.732}$

$x = 14 \cdot z = \dfrac{14-12}{1.732}$

$z = 1.15$

$x = 10 = \dfrac{10-12}{1.732}$

$= -1.15$

(i) $P(x \leq 15)$ $P(z \geq 1.732)$

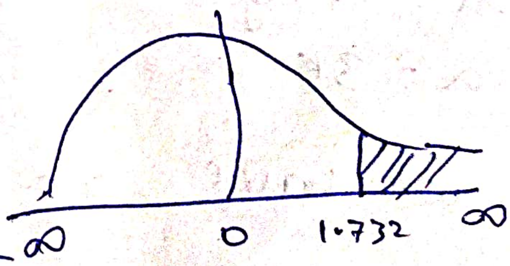
$P(z \geq 1.732) = P(0 \geq z \infty)$

$- P(0 \leq z \leq 1.732)$

$= 0.5 - \phi\, 1.732$

$= 0.5 - 0.4582$

$= \boxed{0.0418}$



The % of Battery cells or expected to have life more than 15 h. is equal to 0.0418 X100

$= 4.18\%$

1\. (ii) $P(10 \le z \le 14)$ $P(-1.15 \le z \le 1.15)$

$= P(-1.15 \le z \le \cancel{1.15}\ 0) + P(0 \le z \le 1.15)$

$= \phi(1.15) + \phi(1.15)$

$= 0.3749 + 0.3749$

$= 0.7498$

The % of the Battery cell is expected in the life more 10 & 14 0.7498×100

$= 74.98\%$

(4) The mean cold for plot of crop is 17kg and standard deviation is 3kg if distribution cold for cold per plot is normaly find, the percentage for plots giving colds

Ans (i) b/w 15.5kg & 20kg

(ii) more than 20kg

$\mu = 17$ kg

$\sigma = 3$k

we have $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-17}{3}$

$x = 15.5$ $z = \frac{15.5-17}{3}$ $= \frac{20-17}{3}$

$z = -0.5$ $= 1$

(i) $P(15.5 \le z \le 20)$ $(-0.5 \le z \le 1)$

$= P(0 \le z \le -0.5) \cdot P(0 \le z \le 1)$

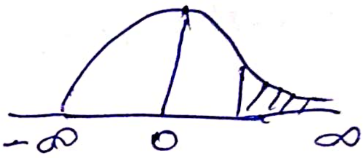
$= \phi(0.5) + \phi(1)$

$= 0.1915 + 0.\cancel{3441}3413$

$\boxed{= 0.532}$

$\boxed{0.532 \times 100 = 53.2\,\%}$

(ii) $P(x > 20) = P(z > 1)$



$= 0.5 + \phi(1)$

$= 0.5 + 0.3413$

$\boxed{= 0.1587}$

0.1587×100

$\boxed{= 15.87\,\%}$

The marks of 1K students In an exam nation a follow a normal distribution with me 70 & standard divation 5 find the number of student who's marks will be

(i) less than 65 $\qquad \mu = 1000$, $\sigma = 5$

(ii) more than 75

(iii) b/w 65 & ~~75~~ 75

we have $z = \dfrac{x - \mu}{\sigma}$

$z = \dfrac{x - \cancel{1000}\;70}{5}$ $\quad x = 65 = \dfrac{65 - \cancel{1000}\;70}{5} = \cancel{-187000}\;910.$

$$x = 75 = \frac{75-70}{5}$$

$$\boxed{z = 1}$$