UNIT - 2

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Numerical methods

First difference :-

The difference two consecutive value of $y = f(x)$ is called first difference.

consider A function $y = f(x)$ let $x_0, x_1 = x_0 + h$, $x_2 = x_1 + h$ and so on..... $x_n = x_{n-1} + h$.
Be a set of points at common Interval h let the corresponding the value of 'y' that is $y_0 = f(x_0)$ $y_1 = f(x_1) \cdot y_n = f(x_n)$.

The value of Independent variable x is called argument and corresponding values of ~~100~~ is non as entry.

* Forward difference

let $y = f(x)$ then $y_0, y_1 ---- y_n$ be the corresponding values x_0 x_1 ------ x_n. $y_0 - y_1 = \Delta y_0$.

consider A function $y = f(x)$ let $y_0, y_1, y_2 ---- y_n$. be the values of y corresponding values of

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\Delta y_2 = y_3 - y_2$$
$$\vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

$\Delta y(0), \Delta y(1), \Delta y(2)$ are called first forward differences.

The differences of the first forward differences is called the second forward differences

similarly the other higher order differences of obtain and tabulated Such Tabular arrangement is called forward difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^4 y_1 - \Delta^4 y_0 = \Delta^5 y_0$
x_1	y_1	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4 y_1$	
x_2	y_2	Δy_2	$\Delta^2 y_2$	$\Delta^3 y_2$		
x_3	y_3	Δy_3	$\Delta^2 y_3$			
x_4	y_4	Δy_4				
x_5	y_5					

$\Delta \rightarrow$ forward difference opperator.

The first entery's the table namely Δy_0 $\Delta^2 y_0$ $\Delta^3 y_0$ $\Delta^4 y_0$. $\Delta^5 y_0$ ---- are called leading forward difference.

A back word difference table.

let us consider $y = f(x)$ let $(y(0), y(1), y(2) \ldots y_n$ be the by the values of y corresponding value $x_0, x_0 x_1, \ldots\ldots x_n$.

$\therefore y_1 = y_0 = \nabla y_1$

$y_2 - y_1 = \nabla y_2$ ....

$y_n - y_{n-1} = \Delta y_n$ are called

first Backword differences III y Other higher order differences obtained & tabulated. such tabular table is called back word difference table.

Back word table.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
x_1	y_1	∇y_1			
x_2	y_2	∇y_2	$\nabla^2 y_2$		
x_3	y_3	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$	
x_4	y_4	∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_4$

The last entery's $\nabla^2 y_4$, $\nabla^2 y_4$ $\nabla^3 y_4$, and $\nabla^4 y_4$ are called the leading backword difference.

Intrapollution :- If $y(0)$, $y(1)$, $y(2)$ ... $y(n)$ be a set of value of an unknown function $y = f(x)$ corresponding to the values of $x(0)$, x_1, x_2 --- x_n the process of finding value for any given value of x b/w x_0 to x_n is called Interpolation also process of finding the value y out the given range of x is called extra pollution.

Imp

★ **Newtons forwar Interpollution differe**

The value of $y = f(x)$ $x = x_0 + RH$ that is $y = f(x_0 + \cancel{RH}$ is approximately given in
$y = y_0 + r\Delta y_0 + x\left(\frac{r-1}{2!}\right.$

To find the value of R $x = x_0 - xh$.
$x - x_0 = rn$. $\boxed{r = \frac{x - x_0}{n}}$

Newton's back word interpolution differences
The value of $y = f(x)$ at $x = x_n + rh$
~~3×2×1~~ $\Rightarrow y = f(x_n + rh)$ is approximately
~~4×3×2×1~~ given by $y = y_n + r\nabla y_n + \frac{r(r+1)}{2!}$
~~5×4×3×2×1~~ $\nabla^2 y_n + \ldots \frac{r(r+1)(r+2) \cdots (r+n-1)}{n!}$
$\nabla^n y_n$:
$x - x_n = rn$
$r = \frac{x - n}{n}$

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problems :-

The table gives the distances in nautical miles of the visible horizon ~~the table~~ for the given heights in feet above the earth surface. Find y at a distance when height is ~~200~~ 218 feet, 610 feet.

Ans: x = height: 100, 150, 200, 250, 300, 350, 400

y = distance: 10.63, 13.03, 15.04, 16.81, 18.42, 19.90, 21.27

Sol:

x	y	Ist	IInd	IIIrd	IVth	Vth
100	$10.63 = y_0$	2.4	-0.39	0.15	-0.07	0.02
150	$13.03 = y_1$	2.01	-0.24	0.08	-0.05	0.04
200	$15.84 = y_2$	1.77	-0.16	0.03	-0.01	
218 / 250	$16.81 = y_3$	1.61	-0.13	0.02		
300	$18.42 = y_4$	1.48	-0.11			
350	$19.90 = y_5$	1.37				
x_n 400	$21.27 = y_6$					

218 Vth.

0.02

0.04

$(1+r)r$ at $x = x_0 + rh$

$x - x_0 = rh$

$\frac{x - x_0}{h} = r$

Newton's forward Interpolation formula

at $x = 218$, $y = ?$ $\cancel{x}\ = \frac{x - x_0}{h} = \frac{218 - 100}{50} = 2.36$

$$y = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3! \times 2 \times 1}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0 + \frac{r(r-1)(r-2)(r-3)(r-4)}{5!}\Delta^5 y_0$$

$2! \times 1$, $3 \times 2 \times 1$, $4 \times 3 \times 2 \times 1$

$\Delta y_0 = 2.4$, $\Delta^2 y_0 = -0.39$, $\Delta^3 y_0 = 0.15$, $\Delta^4 y_0 = -0.07$

$\Delta^5 y_0 = 0.02$.

~~$y = 10.63 + 2.36(2.4) + \frac{2.36(2.36-1)}{2} \times (-0.39)$.~~

$$y = (10.63) + (2.36(2.4)) + \left(\frac{2.36(2.36-1)}{2} \times (-0.39)\right) + \frac{2.36(2.36-1)(2.36-2)}{3 \times 2 \times 1 = 6} \, 0.15 + \frac{2.36(2.36-1)(2.36-2)(2.36-3)}{4 \times 3 \times 2 \times 1 = 24} \, -0.07 + \frac{2.36(2.36-1)(2.36-2)(2.36-3)(2.36-4)}{5 \times 4 \times 3 \times 2 \times 1 = 120} \, 0.02$$

1.36, 0.36, -0.64, -1.64

$10.63 + 5.664 - 0.6258 + 0.0288 + 0.002 + 0.000$

$\cancel{\frac{(1.36)(0.36)}{6}\ (0.45) + \frac{(2.36)(1.36)(0.36)(-0.64)}{4}}$

$+ \frac{2.36\,(1.36)(0.36)(-0.64)(-1.64)(-0.07)}{120}$

$y = 10.63 + 5.66 - 0.6258 + 0.0288 \cancel{+ 0.002}$

$+ \frac{(2.36)(1.36)(0.36)(-0.64)(-1.64)(0.02)}{120}$ $\cancel{0.0002}$

$\cancel{y = 15.69}$ $= \boxed{15.69}$

Backward.

$x = 618 \quad r = \frac{618 + 400}{50} = 4.362$

Backward formul

$r = \frac{x - x_n}{n}$ at $= 610$

$r = \frac{610 - 400}{50} = 602$

$r = 4.2$

$y = y_n + r \nabla y_n + \frac{r(r+1)}{2!} \nabla^2 y_n + \frac{r(r+1)(r+2)}{3!} \nabla^3 y_n$

$+ \frac{r(r+1)(r+2)(r+3)}{4!} \nabla^4 y_n + \frac{r(r+1)(r+2)(r+3)(r+4)}{5!}$

at $x = x_n + rh$

$x = x_n = rh$. $\quad r = \frac{610 - 400}{50} = 4.2$

$\frac{x - x_n}{n} = r$

$\therefore\ y = 21.27 + (4.2)(1.37) + \frac{(4.2)(5.2)(6.2)}{} \times \frac{(6.2)(7.2)(-0.01)}{24}$

$+ \frac{(4.2)(5.2)(6.2)(7.7)(8.2)(0.04)}{120}$

$= 21.27 + 8.754 + (-1.2012) + 0.451367$

$(0.406224) + 2.664$

$= 28.83$

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(7) the area of a circle corresponding to the diameter (D) is given below.

80	85	90	95	100
5026	5674	6362	7088	7854

find the area when a diameter is 105

sol:- Difference table.

D=x	A=y	Ist	IInd	IIIrd	IVth
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

using Newton's Backword Interpolation.

$$y = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{x(x+1)(x+2)}{3!}\nabla^3 y_n$$

$$+ \frac{x(x+1)(x+2)(x+3)}{4!}\nabla^4 y_n$$

$y_n = 7854$ To find the value of $r = \frac{x - x_n}{n}$

$\Rightarrow r = \frac{105-100}{5}$ = $\nabla y_n = 766$.

$\nabla^2 y_n = 40$, $\nabla^3 y_n = 2$ $\nabla^4 y_n = 4$.

$y = 7854 + 766 + 40 + 2 + 4$.

$y = 8666$

when a Diameter is 105 the Area is 8666. This is req soln.

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(3) from the following table estimate the number of student who optaind mark's 40 & 45

marks :	30-40	40-50	50-60	60-70	70-80
No of stads :	31	42	51	35	31

x marks	y = std	I^{st}	II^{nd}	III^{rd}	IV^{th}
Below 40	31				
		42			
" 50	73		9		
		51		-25	
" 60	124		-16		37
		35		12	
" 70	159		-4		
		31			
" 80	190				

Identify Newton forward Interpolation formula

$$y = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0$$

$$+ \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0$$

$y_0 = 31$ to find r value $= \frac{x - x_0}{h}$

$$r = \frac{45-40}{10} = \frac{5}{10}\ 0.5$$

$\Delta y_0 = 42$, $\Delta^2 y_0 = 9$ $\Delta^3 y_0 = -25$, $\Delta^4 y_0 = 37$

$$y = 31 + 0.5 \times 42 + \frac{0.5(0.5-1)}{2!}\,9 + \frac{0.5(0.5-1)(0.5-2)}{3!}$$

$$-25 + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}\,37$$

$y = 31 + 21 - 1.125 - 1.5628 - 1.445$.

$y = 47.8672$

~~No of Students~~ marks Below 45 $\approx$ 48.

No of std $48 - 31 = 17$

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(ii) In the table below the value of y or conjugative terms of series of which 23.6 is the 6th term. find the first term and Tenth term of series.

x : 3 4 5 6 7 8 9

y : 4.08 8.04 14.5 23.6 36.2 ~~52.8~~ 73.9

Sol.

x	y	Ist	IInd	IIIrd	IVth		
3.	4.8						
		3.6					
4	8.4		2.5				
		6.1					
5	14.5		3	0.5			
		9.1			0		
6	23.6		3.5	0.5		-20	
		12.6			0		120
7	36.2		~~16~~ 4	~~-19.5~~ 0.5		100	
8	~~52.8~~	~~-3.4~~ 16.6			0		
9	73.9	~~41.1~~ 21.1	~~44.5~~ 4.5	~~60.5~~ 0.5			

forward

$$y = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0$$

$$+ \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0.$$

$y_0 = 4.8$. $\quad r = \frac{x - x_0}{h} = \frac{1-3}{1} = -2$

$y =$

$$= 4.8 + (-2)(3-6) + \frac{(-2)(-2-1)(2.5)}{2}$$

$$+ \frac{(-2)(-2-1)(-2-2)}{6} \times (0.5) + \frac{(-2)(-2-1)(-2-2)(-2-3)}{24 \times 0}$$

$= 4.8 - 7 - 2 + 7.5 - 5$

$= 3.1$

$r_3 = \frac{10-9}{1} = 1/1 = 1$

$y = y_n + r\nabla y_n + r\frac{(r+1)}{2!}\nabla^2 y_n$

$+ \frac{(r+1)(r+2)}{3!}\nabla^3 y_n + \frac{r(r+1)(r+2)(r-3)}{4!}$

$\nabla^4 y_n$

$= 73.9 + 1(21.1) + \frac{1(2)}{2} \times 4.5 + 0.5 + 1$

$\boxed{y = 100}$

(4) Newton's backward interposition formula

H.3 find the interpoliting polynamial for the given function by the following data

sol =

x	10	11	12	13
$f(x)$	21	23	27	33

Hence find $f(12.$

(5) find the cubic Inter politic polynomial which takes following values

x =	0	1	2	3
$f(x)$ =	1	2	1	10

Sol

x	$f(x)$	Ist	IInd	IIIrd
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

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using Newton's forward Interpolation formula

$$f(x) = f(x) + r\Delta f(x) + \frac{r(r-1)}{2!}\Delta^2 f_0(x) + \frac{r(r-1)(r-2)}{3!}\Delta^3 f_0(x)$$

To find the value of $r = \frac{x - x_0}{h} = \frac{x-0}{1} = x$

$$f(x) = 1 + x + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12)$$

$$f(x) = 1 + x - (x^2 - x) + 2(x^2 - x)(x-2)$$

$$f(x) = 1 + x - x^2 + x + 2x^3 - 4x^2 - 2x^2 + 4x$$

$$f(x) = 2x^3 - 7x^2 + 6x + 1$$

Q(4)

x	$f(x)$	I st	II nd	III rd
10	21			
		2		
11	23		2	
		4		0
12	27		2	
		6		
13	33			

using Newton's backward Interpolation formula

$$y_n = y_n + r\nabla f(x) + \frac{r(r-1)}{2!}\nabla^2 y(x) + \frac{r(r-1)(r-2)}{3!}\nabla^3 y(x)$$

To find the value of $r = \frac{x - x_n}{h} = \frac{x-13}{1} = x - 13$

$$f(x) = 33 + (x-13)6 + \frac{(x-13)(x-13+1)}{2}2$$

$$f(x) = 33 + 6x - 78 + (x-13)(x-12)$$

$$f(x) = 33 + 6x - 78 + x^2 - 12x - 13x + 156$$

$$f(x) = x^2 - 19x + 111$$

$$f(10) = 100 - 190 + 111 = 21$$

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Hence

$f(x) = x^2 - 19x + 111$

$f(12.51) = (12.51)^2 - 19(12-51) + 111$

$f(12.51) = 29.81$

at $x = 13.1$

$f(13.1) = (13.1)^2 - 19(13.1) + 111$

$f(13.1) = 33.71$

(6) given $\sin 45° = 0.7071$, $\sin 50° = 0.7660$, $\sin 55° = 0.8192$; $\sin 60° = 0.8660$

find $\sin 57°$ using appropriate interpolation for

x	y	1st			
sin45°	0.7071				
		0.0589			
sin50°	0.7660		-0.0057		
		0.0532		0.0007	
sin55°	0.8192		-0.0064		
		0.0468			
sin 60°	0.8660				

$r = \frac{x - x_n}{n} = \frac{57 - 60}{5} = -0.6$

Backword.

$f(x) = y_n + r\nabla f(x) + \frac{r(r-1)}{2!}\nabla^2 y(x)$

$+ \frac{r(r-1)(r-2)}{3!} \times \nabla^3 y^3(x)$

$\cancel{r = \frac{57-60}{5}} =$

$= 0.8660 + -0.6(0.0468) + \frac{(-0.6)(-0.6+1)}{2} \times (-0.0064)$

$+ \frac{(-0.6)(-0.6+1)(-0.6+2)}{6} \times (0.0007)$

$= 0.8860 - 0.002808 + 0.000768$
$+ 0.0000392$

$= 0.8387$

(7) A survey conducted slum located locality the following information classifed below estimat the probable number of persons in the income proof 20-25

Below 10	20					
		45				
Below 20	65		70			
		115		25		
Below 30	180		95		-215	
		210		-190		
Below 40	390		-95			
		115				
Below 50	505					

$$r = \frac{x - x_0}{h} \Rightarrow \frac{25 - 10}{10} = 1.5$$

$$= y_0 + r \Delta y_0 + \frac{r(r-1) \cdot \Delta^2 y_0}{2!} + \frac{r(r-1)(r-2)\Delta^3 y_0}{3!}$$

$$+ \frac{r(r-1)(r-2)(r-3)}{4!} \cdot \Delta^4 y_0$$

$$= 20 + 1.5 \times 45 + \frac{1.5 \times 0.5 \times 70}{2} + \frac{1.5(0.5)(0.5) \times 25}{6}$$

$$+ \frac{1.5(0.5)(1.5) \times (-215)}{24}$$

$$= 20 + 67.5 + 26.25 - 0.93 - 5.03$$

$$y(25) = 107.79$$

Number of persons taking 20 to 25 R is

$y(25) - y(20) = 107.79 - 65$

$= 42.79$

$= 43$ persons.

Interpolation formula for unequal Interval Divided Difference.

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If $f(x_0)\ f(x_1)\ f(x_2) \cdots\cdots f(x_n)$ be a set value of an unknown $y = f(x)$ corresponding to the value $x_0, x_1, x_2 \cdots\cdots x_n$ for unequal Interval ~~A set~~

$$y = f(x) = f(x_0) + (x - x_0) \cdot f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1) \cdots (x - x_n) f(x_0, x_1, \cdots x_n).$$

where $f(x_0; x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$f(x_1, x_2) = f(x_2) - f(x_1)$

It is called 1st order Divided differences,,

III α $f(x_0; x_1, x_2) = f(x_1, x_2) - f(x_0, x_1)$

$f(x_1, x_2, x_3) = \dfrac{f(x_2, x_3) - f(x_1, x_2)}{x_2 - x_1}$

The Higher order Divide order Differences arrangement intabulated such tab

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use Newton's divided difference formula given the data. $f(4)$

x	$f(x)$	I	II	III
0	$-4 = f(x_0)$			
		3		
2	$2 = f(x_1)$		3	
		12		1
3	$14 = f(x_2)$		9	
		48		
6	$158 = f(x_3)$			

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{2-(-4)}{2} = 3.$$

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{14-2}{1} = 12$$

$$f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = \frac{158-14}{3} = 48.$$

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{12-3}{3} = 9/3 = 3$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{48-12}{4} = 9$$

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{9-3}{6} = 6/6 = 1$$

$$y = f(x) = f(x_0) + (x - x_0) f(x_0 - x_1) + (x_0, x_0) + (x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3)$$

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$y = f(4) = -4 + (4-0)(3) + (4-0)(4-2)(3) + (4-0)$
$(4-2)(4-3)(1)$

$y = f(4) = -4 + 12 + 24 + 8$

$y = f(4) = 40$

* The values of x and $\log_{10} x$ to are given below.

x	300	304	305	307
$f(x)$	2.4771	2.4829	2.4843	2.4871

find $\log_{10} 310$ by newtons' divided difference

x	$f(x)$		
$300 = x_0$	2.4771		
		0.00145	
$304 = x_1$	2.4829		-0.00001
		0.0014	
$305 = x_2$	2.4843		0
		0.0014	
$307 = x_3$	2.4871		

$$(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{2.4829 - 2.4771}{304 - 300}$$

$$= 0.0014$$

$$(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{2.4843 - 2.48}{305 - 304}$$

$$= 0.0007$$

$$(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = \frac{(2.4871 - 2.48}{307 - 305}$$

$$= 0.0014$$

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{0.00145 - 0.0014}{5} = 0.00001$$

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$$f(x_1 x_2 x_3) = \frac{f(x_2, x_3) - f(x_1 x_2)}{x_3 - x_1}$$

$$= \frac{0.0014 - 0.0014}{3}$$

$$= \frac{0}{3} = 0.$$

$$f(x_0, x_1 x_2 x_3) = \frac{f(x_1 x_2 x_3) - f(x_0, x_1 x_2)}{x_3 - x_0}.$$

$$= \frac{0 - (-0.00001)}{7}$$

$$= 0.00000142.$$

$y = f(x_0) + (x - x_0)\, f(x_0 - x_1) + (x_0, x_0) +$
$f(x_0 x_1 x_2) + (x - x_0)(x - x_1)(x - x_2)$
$f(x_0 x_1 x_2 x_3).$

$2.4771 + (10)(0.0014) + (10)(x)$
$- 0.00001 + (10) \times (6)(5)(0.00000142)$

$y = 2.491$

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(8) Given
$y_{20} = 24.37, y_{22} = 49.28, y_{29} = 162.86,$
$y_{32} = 240.5$
Find y_{28} by Newton's.

⇒

x		$f(x)$	I		
20	x_0	$24.37\ f(x_0)$	12.455		
22	x_1	$49.28\ f(x_1)$		0.418	
29	x_2	$162.86\ f(x_2)$	16.217	0.968	0.045
32	x_3	$240.5\ f(x_3)$	25.9		

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = 12.455$$

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad 16.217$$

$$f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = 25.9$$

$$f(x_0, x_1, x_2) = \frac{f(x_1 x_2) - f(x_0 x_1)}{x_2 - x_0} = 0.418$$

$$f(x_1 x_2 x_3) = \frac{f(x_2 x_3) - f(x_1 x_2)}{x_3 - x_1} = 0.968$$

$$f(x_0, x_1, x_1, x_2) = \frac{f(x_1 x_2 x_3) - f(x_0 x_1 x_2)}{x_3 - x_0}$$

$$= 0.0458$$

$y = f(x) + (x - x_0) f(x_0 x_1) + (x - x_0) f$
$(x_1 x_2) f (x - x_0)(x - x_1)(x - x_2) f$
$(x_0, x_1 x_2 x_3) = 24.37$
$+ (8)(12.455) + (8)(0-418) + 8(6)(-1)$
(0.0458)

$y_{28} = 125.1556$

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Lagrange's Interpolation formula and Inverse Langrange's Interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)}{(x_3-x_0)(x_3-x_1)} \frac{(x-x_2)}{x_3-x_2} y_3$$

Inverse lagrange's Interpolation formula

$$x = f(y) = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_2-y_3)} x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} x_1$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} x_2 + \frac{(y-y_0)(y-y)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} x^3$$

① using lagrange's Interpolation formula find Interpolation polynomial that approximate to function described by following table.

x	$0\ x_0$	$1\ x_1$	$2\ x_2$	$5\ x_3$
$f(x)$	$2\ y_0$	$3\ y_1$	$12\ y_2$	$147\ y_3$

$$y = f(x) \quad \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-5)} \cancel{2} + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)} 3$$

$$+ \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)} \cancel{12} + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} \cancel{147}\ 49$$

$$y = -\frac{1}{5}\left[(x-1)(x^2-7x+10)\right] + \frac{3}{4}\left(x(x^2-7x+10)\right. - 2\left(x(x^2-6x+15)\right)\Big] + \frac{49}{20}\left(x(x^2-3x+12)\right)$$

$$y = f(x) = -\frac{1}{5}(x^3-7x^2+10x-x^2+7x-10 = x^3-8x^2+17x-10)$$

$$+\frac{3}{4}(x^3-7x^2+10x) - 2(x^3-6x^2+15x) + \frac{49}{20}(x^3-3x^2+7x)$$

$$\left\{\frac{1^{\times 4}}{5} + \frac{3}{4}^{\times 5} - 2^{\times 20} + \frac{49}{20}^{\times 1}\right\} \text{Lcm}$$

$$\overline{}$$
$$20$$

$$y = f(x) = \frac{1}{20}\left(-4x^3+15x^3-40x^3+49x^3+32x^2-105x^2 + 240x^2 - 147x^2 - 68x + 150x - 200x + 40\right].$$

$$y = f(x) = \frac{1}{20}(20x^3+20x^2-20x+40)$$

$$= \frac{20}{20}\left[x^3+x^2-x+2\right)$$

$$= x^3+x^2-x+2$$

(2) using largne's Interpolution formula find $f(9)$ from following data

x	5 x_0	7 x_1	11 x_2	13 x_3	17 x_4
$f(x)$	150 y_0	392 y_1	1452 y_2	2366 y_3	5202 y_4

$$y = \frac{(9-7)(9-11)(9-13)150}{(5-7)(5-11)(5-13)} + \frac{(9-5)(9-11)(9-13)392}{(7-5)(7-11)(7-13)}$$

$$+\frac{(9-5)(9-7)(9-13)(9-17)1452}{(11-5)(11-7)(11-13)(11-17)} + \frac{(9-5)(9-7)(9-11)(9-17)2366}{(13-5)(13-7)(13-11)(13-17)} + \frac{(9-5)(9-7)(9-11)(9-13)5202}{(17-5)(17-7)(17-11)(17-13)}$$

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$$y = \frac{2400}{-96} + \frac{(-100,352)}{48} + \frac{371712}{288}$$

$$+ \frac{302848}{(-384)} + \frac{332928}{2880}$$

$$y = -25 - 2090.667 + 1290.667$$
$$- 788.667 + 115.6$$
$$= 810A_3$$

* obtain value of T when A = 85 from following table using langarang from

=)

T	2	5	8	14
A	94.8	87.9	81.3	68.7

$x_0 = 2$ $x_1 = 5$ $x_2 = 8$ $x_3 = 14$

$y_0 = 94.8$ $y_1 = 819$ $y_2 = 813$ $y_3 = 68.7$

using Inverse langaranges formula.

$$f(y) = \frac{(y-y_1)(y-y_2)(y-y_3)x_0}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} +$$

$$\frac{(y-y_0)(y-y_0)(y-y_3)x_1}{(y_1-y_0)(y_1-y_2)(y_1-y_3} + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)}$$

$$\frac{(y-y_3)x_2}{(y_2-y_3)} + \frac{(y-y_0)(y-y_1)(y-y_2)x_3}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}$$

$$= \frac{(85-87.9)(85-81.3)(85-68.7)(2)}{(94.8-879)(94.8-8.13)(94.8-68.7)}$$

$$+ \frac{(85-94.8)(85-81.3)(85-68.7)(5)}{(87.9-94.8)(87.9-81.3)(879-68.7)} = 3.379$$

$$+\frac{(85-94.8)(85-81.3)(85-68.7)(5)}{(87.9-94.8)(87.9-81.3)(87.9-68.7)}$$

$$+\frac{(85-94.8)(85-87.9)(85-68.7)(8)}{(81.3-94.8)(81.3-87.9)(81.3-68.7)}$$

$$+\frac{(85-94.8)(85-87.9)(85-81.3)(14)}{(68.7-94.8)(68.7-87.9)(68.7-8}$$

$$= 0.143 + 3.379 + 3.301 + 0.23$$

$$= 6.3$$

Regula falsi method (or) falsi position method If $f(x)=0$ is real value continous fun^F of real variable x following foundamental property. as follows if there exist two vale a, b. f(x) as opposite signal say $f(a)<0$ & $f(b)>0$ equivalently always $f(a)\,f(b)$ then there exist atleast one real root in interval (a,b).

The regula falsi method formula gi

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}.$$

Remarks: If a, b, are close enough we can obtain approxn. root desired accuracy value quickly problems are workout by finding a & b diff is 0.1 to terminate ittirate process

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Q(1) find real root of eqn. $x^3 - 2x - 5 = 0$ using regula falsi method.

$f(x) = x^3 - 2x - 5$.

put $x = 0$

$f(0) = 0^3 - 2(0) - 5$.

$f(0) = -5 < 0$.

$x = 1 \Rightarrow f(1) = 1^3 - 2(1) - 5 \Rightarrow -6 < 0$

$x = 2 \Rightarrow f(2) = 2^3 - 2(2) - 5 \Rightarrow -1 < 0$

$x = 3 \Rightarrow f(3) \quad 3^3 - 2(3) - 5 \Rightarrow 16 > 0$.

$x = 2.1 \Rightarrow f(2.1) \quad 2.1^3 - 2(2.1) - 5 = 0.061$

3 ~~tagore~~ ~~se jasti~~ iteration ~~karnudet~~

The real root lies 2, 3.

$a = 2 \qquad f(a) = f(x) = -1$

$b = 2.1 \qquad f(b) = f(2.1) = 0.061 > 0$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_1 = \frac{2(0.061) - 2.1(-1)}{(0.061) + 1} \qquad \frac{2.222}{1.061}$$

$x_1 = 2.0942$

$f(2.0942) = (2.0942)^3 - 2(2.0942) - 5$.

$= -0.00392 < 0$

$a = 2.0942 \qquad f(a) = 0.00392$

$b = 2.1 \qquad f(b) = 0.061$

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.0942(0.061) + 2.1(0.00392)}{0.061 + 0.00392}$$

$x_2 = 2.0942$

Thus regula root 2.0942

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(9) solve $xe^x - 3 = 0$ using regula falsi method

$f(x) = xe^x - 3 = 0$

Soln put $x = 0 = xe^x - 3 = 0$

$= -3 < 0$

$x = 1 \Rightarrow xe^x - 3 = 0 \Rightarrow -0.281 < 0$

$x = 2 \quad 2e^{(2)} - 3 = 0 \Rightarrow 11.778 > 0$

$x = 1.1 \quad = (1.1).e^{(1.1)} - 3 = 0 \Rightarrow \cancel{0.00416} \Rightarrow 0.3045$

$a = 1 \quad f(a) = -0.2810$

$b = 1.1 \quad f(b) = 0.3045$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{1(0.304) + 1.1(0.281)}{0.304 + 0.281}$$

$x_1 = 1.048$

$a = 1.048 \quad f(1.048) = -0.011$

$b = 1.1 \quad f(1.1) = 0.304$

$$x_2 = \frac{(1.048)(0.304) + (1.1)(-0.011)}{0.304 + 0.011}$$

$x_2 = 1.0498$

H.W. i) using Langrange's formula obtain polynomial for follow data

$x = 1^{x_0} \quad 2^{x_1} \quad 3^{x_2} \quad 4^{x_3}$

$y = 2^{y_0} \quad 4^{y_1} \quad 8^{y_2} \quad 128^{y_3}$

Lem

$$y = \frac{(x - x_1)(x - x_2)(x - x_3)y_0}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} + \frac{(x - x_0)(x-\dots}{(x_1 - x_0)(x_1-\dots}$$

$$= \frac{(x - x_3)y_1}{(x_1 - x_3)}$$

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$$+\frac{(x-x_0)(x-x_1)(x-x_3)y_2}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}+\frac{(x-x_0)(x-x_1)(x-x_2)y_3}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$y \Rightarrow \frac{(x-2)(x-3)(x-4)\cancel{x}}{(1-2)(1-3)(1-4)}+\frac{(x-1)(x-3)(x-4)\cancel{y}2}{(2-1)(2-3)(2-4)}$$

$$+\frac{(x-1)(x-2)(x-4)\cancel{x}4}{(3-1)(3-2)(3-4)}+\frac{(x-1)(x-2)(x-3)128/64}{(4-1)(4-2)(4-3)}$$

$$\Rightarrow \frac{1}{-3}\left[(x-2)(x^2-4x-3x+12)-2(x-1)(x^2-4x-3x+12)\right.$$
$$+-4\left[(x-1)(x^2-4x-2x+8)\right]+\frac{64}{3}(x-1)$$
$$(x^2-3x-2x+6).$$

$$\Rightarrow \frac{1}{-3}(x^3-7x^2+12x-2x^2-14x+24)+2(x^3-7x^2$$
$$\cancel{+12x-x^2+7x-12)}$$
$$-4(x^3-6x^2+8x-x^2+6x-8)+\frac{61}{3}(x^3-5x^2+6x-$$
$$x^2+5x-6)$$

$$\frac{1}{3}(-x^3\cdot 6x^3-12x^3+61x^3+7x^2-42x^2+7x^2$$
$$-305x^2$$

wrong

~~★ compute real root $x\log_{10}x=1.2$ by method false position method carryout three iteration.~~

~~$\Rightarrow$ Given $x\log_{10}x=1.2$~~
~~$f(x)=x\log_{10}x=1.2.$~~
~~Regular falsi method formula.~~

$$\cancel{x=\frac{af(b)-bf(a)}{f(b)-f(a)}}$$

~~$x=0 \Rightarrow f(0)=0\times\log 0-1.2=-1.2<0$~~
~~$x=1 \Rightarrow f(1)=1\times\log 1-1.2=-1.2<0$~~
~~$x=2 \Rightarrow f(2)=2\times\log 2-1.2=-0.5979<0$~~
~~$x=3 \Rightarrow f(3)=3\times\log 3-1.2=0.2313>0$~~

* compute the real root of $x \log_{10} x = 1.2$ by the method false position method correct three iteration.

Sol: Given $x \log x = 1.2$

let $f(x) = x \log x - 1.2$

regular false method formula.

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Put $x = 0$. $f(0) = -1.2 < 0$

$x = 1$. $f(1) = -1.2 < 0$

$x = 2$ $f(2) = -1.2 < -0.5979$

$x = 3$ $f(3) = -1.2 > 0.2313$

$a = 2$ $f(a) = -0.5979$

$b = 3$ $f(b) = 0.2313$

$$x_1 = \frac{2(0.2313) + 3(0.5979)}{0.2313 + 0.5979} = \frac{2.2563}{0.8292}$$

$x_1 = 2.7210$

$f(2.7210) = 2.7210 \log_{10}(2.7210) - 1.2$

$f(2.7210) = -0.0171$

$a = 2.7210$ $f(a) = -0.0171$

$b = 3$ $f(b) = 2.2313$.

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x_2 = \frac{2.7210(0.2313) + 3(0.0175)}{0.2313 + 0.0171}$$

$x_2 = 2.7402$ //

$f(2.7402) = 2.7402 \log_{10}(2.7402) - 1.2$

$= -0.000389$

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-1.3311

$a = 2.7402$ $\qquad f(a) = 0.000389$

$b = 3$ $\qquad f(b) = 0.2313$

$$x_3 = \frac{2.7402(0.2313) + 3(0.000389)}{0.2313 + 0.000389}$$

$$x_3 = 2.7406$$

* use the regular falsi method, 4th root of 12 correct to 3 decimal $\sqrt[4]{12}$

sol let $x = \sqrt[4]{12}$

$x^4 = 12$

$f(x) = x^4 - 12 \quad -(1)$

put ~~f(x)=0~~ $x = 0 \quad f(0) = -12 < 0$

$x = 1 \quad f(1) = 1 - 12 = -11 < 0$

$x = 2 \quad f(2) = 16 - 12 = 4 > 0.$

$a = 1 \qquad f(a) = -11$

$b = 2 \qquad f(b) = 4.$

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x_1 = \frac{1(4) + 2(11)}{4 + 11} = 1.7333$$

x_2 ~~f(a)~~ -2.9740

$a = 1.7333 \qquad f(a) -2.9740$

$b = 2 \qquad f(b) = 4$

$$x_2 = \frac{1.7333(4) + 2(2.9740)}{4 + 2.9740}$$

$= 1.8470$

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$x_3 =$

$a = 1.8470$ $\quad f(a) = -0.3622$

$b = 2$ $\quad f(b) = 4$

$$x_3 = \frac{af(b) - b(f(a))}{f(b) - f(a)}$$

$$= \frac{1.8470(4) + 2(-0.3622)}{4 + 0.3622}.$$

$x_3 = 1.8597$

$a = 1.8597$ $\quad f(a) = -0.0388$

$b = 2$ $\quad f(b) = 4.$

$$\frac{1.8597(4) + 4(0.0388)}{4 + 0.0388}.$$

$x_4 = 1.8597$

$a = 1.8597$ $\quad f(a) = -0.0388.$

$b = 2$ $\quad f(b) = 4.$

$$= \frac{1.8597(4) + 4(0.0388)}{4 + 0.0388}.$$

$x_5 = 1.861$

Newton's Raphson method.

N-R method. Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

i) find real root of eq^n $xe^x - 2 = 0$ correct 3 decimal point

$xe^x - 2 = 0$

let $f(x) = xe^x - 2.$

$\therefore f'(x) = x.e^x + e^x$

$= e^x(x+1)$

using N-R method formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

put $n = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

we find x_0 $f(x) = xe^x - 2$

$x = 0 \cdot f(0) = -2 < 0$

$x = 1$ $f(1) = 1e^1 - 2 = 0.7182 > 0$

we choose, $x_0 = 1$

$$x_1 = 1 - \frac{f(1)}{f'(1)}$$

$f(x) = xe^x - 2.$ $\quad f'(x) = e^x(x+1)$

$f(1) = 0.7182$ $\quad f'(1) = e^1(1+1)$

$= 5.4365$

$$x_1 = 1 - \frac{0.7182}{5.4365}$$

$x_1 = 0.8678$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.8678 - \frac{f(0.8678)}{f'(0.8678)}$$

$$x_2 = 0.8678 - \frac{f(0.8678)}{f'(0.8678)}$$

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$f(x) = xe^{x} - 2$

$= 0.8678e^{0.8678} - 2$

$= 0.0668.$

$f'(x) = e^{x}(x+1)$

$= e^{0.8028}(0.8678+1).$

$= 4.4484$

$x_2 = 0.8678 - \frac{0.0668}{(4.4484)}$

$x_2 = 0.8527$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 0.8527 - \frac{f(0.8527)}{f'(0.8527)}$

$f(x) = xe^{x} - 2 \Rightarrow 0.8527e^{0.8527} - 2$

$= 0.00040.$

$f'(x) = e^{x}(x+1) = e^{0.8527}(0.8527+1)$

$= 4.3463.$

$\therefore x_3 = 0.8527 - \frac{0.0041}{4.3463}$

$x_3 = 0.8526.$

The reqd sol. - 0.8526

(2) find real root of $x\sin x + \cos x = 0$ using NR method carryout 4 decimal places.

$f(x) = x\sin x + \cos x$

$f'(x) = x\cos x + \sin x - \sin x$

$f'(x) = x\cos x$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_0 = \pi$

$x_1 = \pi - \frac{f(\pi)}{f'(\pi)}.$

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$f(x) = x\sin x + \cos x$
$= \pi \sin\pi + \cos\pi$
$= -1$

$f'(x) = -3-14.15.$

$x_1 = \pi - \left(\frac{-1}{-3+4.15}\right)$

$= 2.8232$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.8232 - \frac{f(2.8232)}{f'(2.8232)}$

$= 2.8232 - \frac{(-0.06591)}{(-2.6813)}$

$= 2.7986.$

$x_3 = x_2 - \frac{f(2.7986)}{f'(2.7986)}$

$= 2.7986 - \frac{(-0.00056)}{(-2.6355)}.$

$= 2.7983$

$x_4 = x_3 - \frac{f(2.7983)}{f'(2.7983)}$

$= 2.7983 - \frac{(0.0002)}{(-2.6350)}.$

$= 2.7983$ //

x^2
$2x$

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3) Find approximate value of $\sqrt{5}$ using NR.

$x = \sqrt{5}$

$x^2 = 5$ $\qquad f(x) = x^2 - 5$

$x^2 - 5 = 0$ $\qquad f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$2 - \frac{2^2}{2(2)}$

$x = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x = 0 \quad f(0) = 0^2 - 5 = -5 < 0$

$x = 1 \quad f(1) = 1 - 5 = -4 < 0$

$x = 2 \quad f(2) = 4 - 5 = -1 < 0$

$x = 3 \quad f(3) = 9 - 5 = 4 > 0$ $\qquad$ Roots lies (2, 3)

choose $x_0 = 2$

$$x_1 = 2 - \frac{f(2)}{f'(2)}$$

I

$$= 2 - \frac{(-1)}{4} = -0.25$$

$2 + 0.25$

$$\boxed{= 2.25}$$

$2 - \frac{x^2 - 5}{2x}$

$$x_2 = x_1 \frac{f(2.25)^2}{f'(2.25)^2}$$

$= 2 - \frac{4 - 5}{4}$

$$= 2.25 - \frac{0.0625}{4.5}$$

$\frac{2.25^2 - 5}{(2.25)^2}$

II

$$\boxed{= 2.236}$$

$$x_3 = x_2 - \frac{f(2.236)^2 - 4}{f'(2.236)^2}$$

-0.013888

$\frac{(0.0625)^2 - 5}{(0.0625)^2}$

$$= 2.236 - \frac{(-0.0003)}{4.472}$$

III

$$= 2.236$$

23/11/24

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* using newton's rapson method find root that lies near $x = 4.5$ $\tan x = x$ to 4 decimal places correct to four decimal places. (where is irreducible)

$x = 4.5$ $\tan = x$. (Here x is in radians)

Given $x_0 = 4.5$. $\tan x = x$

$f(x) = \tan x - x$

$f'(x) = \sec^2 x - 1 = \tan^2 x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 4.5 - \frac{f(4.5)}{f'(4.5)}$$

$f(x) = \tan x - x$.

$f(4.5) = \tan(4.5) - 4.5 = 0.1373$

$f'(4.5) = \tan^2(4.5) = 21.5048$.

$$x_1 = 4.5 - \frac{0.1373}{21.5048}$$

$x_1 = 4.5 - 0.00638$

$x_1 = 4.4936$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 4.4936 - \frac{f(4.4936)}{f'(4.4936)}$$

$f(4.4936) = \tan(4.4936) - 4.4936$

$= 0.00385$

$f'(4.4936) = \tan^2(4.4936) = 20.2270$.

$$x_2 = 4.4936 - \frac{0.00385}{20.2270}$$

$x_2 = 4.4934$.

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

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$$= 4.4934 + \frac{0.00019}{20.1889}$$

$$\boxed{x_3 = 4.4934}$$