unit. 2

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* linear Algebra:

matrix :- A set of mn elements written in an array of m rows n colum. embeded in [ ] { } is called a matrix of order m x n.

ex:- $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

If m = n, the matrix is called square matrix
The matrix of order 1xn 1/n is called row matrix.
The matrix m/n mxn is called a colum matrix

Elementary:- Transformation (operation associated) with the matrix).

* Inter change of any two rows
* multiplication any row by non zero constant
* addition to any row constant multiple of any other row.

Echlon form :- A non zero matrix A is said to be row echlon for If the following condition exists.

* all the ~~non~~ zero rows be below the non 0 rows
* The first non 0 entry In any non 0 row is one.
* In order to reduce the given matrix to a row for echolon form., we must prefer to have the leading entry (First entry in the that row is non zero nue A priority

* In this case entry is 0 we can Inter change suitable row

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* Taken non zero entry use r^1 to make all the entry's In the first colum to zero by elemtary row transformation.
* used r^2 to make all entrys In the second colum to 0 and countinue the process and until we get a row echlon form

Imp
* Rank of matrix :- The ranks of a matrix In its echlon form is equal to the number of non zero rows is denoted by. $\rho(A)$.

.

* consistency :- linear. of a system of linear equation A system of eq In witch all the unon quantity appear In First degree alone is called system of lenear ~~system~~ of eq.
~~A~~ set of m linear eq in n unon is as follows

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$

$$a_{21} + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_m.$$

where ~~as~~ a_{ij}s. and v_is or constant.
If b_1, b_2, b_3 etc b_m or all zero, the system is said to be Homogenuy.
the set of values x_1 and x_2 etc witch satisfied all the eq.s symultensly is called soultions of system of equations.
A system of linear eq. is said to be consistance If it poses the solution otherwise the system is in consistance
The system of eq. represented by matrix eq. ~~ax=b.~~ $Ax = b$ is consistance if so $\rho(A) = \rho(AB)$ were $[A:B]$ is Augmented matrix.

$\rho \to$ rang

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Suppose ~~Rooo~~ $\rho(A) = \rho(A:B) = r$ Then the condition for vari type of soution or as follow

<u>unique solution</u>
$$\rho(A) = \rho(A:B) = r = n \text{ (unknown)}$$
(number of unnon)

Infinet solution :- $\rho(A) = \rho(A:B) = r < n$.

If ~~finaly~~ $\rho(A) \neq \rho(A:B)$ then the system is inconsistance

Working procedures:- we first form the augmented matrix (A, B) and we can clearly Identyfy the portion of the co-efficient matrix A in it

~~pave~~ reduce matrix the $(A:B)$ to on ~~ete~~ echelon form by elementry row transformation

* This will enable us to immediately the ranks of A and $(A:B)$ wider the regulent the system of equation

the echlon form of $(A:B)$ is converted back to the eq. forms and solution till emerg easly.

① Test for consistency and Solve.

$\Rightarrow$ Augmented matrix

$x+y+z=6$
$x-y+2z=5$
$3x+y+z=8.$

$$[A:B] \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & -1 & 2 & : & 5 \\ 3 & 1 & 1 & : & 8 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 3R_1$

$$[A-B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 1 & : & -1 \\ 0 & -2 & -2 & : & -10 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : 6 \\ 0 & -2 & 1 & : -1 \\ 0 & 0 & -3 & : -9 \end{bmatrix}$$

$\therefore n =$ unknown.

$1+(x,y,z)$.

$\rho(A) = \rho(A:B) = 3 \quad r = 3 = n$.

Both $[A]$ & $(A:B)$ matrices have all there rows non-zero. Now $AX = B$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ -9 \end{bmatrix}$$

$x + y + z = 6$ — ①

$-2y + z = -1$ — ②

$-3z = 9$ — ③

$z = \frac{-9}{-3} = \boxed{z = 3}$

in eq ② $-2y + 3 = -1$.

$-2y = -1 - 3$. $\Rightarrow -2y = -4$.

$\boxed{y = 2}$

In eq ①

$x + 2 + 3 = 6$.

$x = 6 - 5$.

$\boxed{x = 1}$

Thus $x = 1, y = 2, z = 3$ is required so.

② $5x + 3y + 7z = 4$.

$3x + 26y + 2z = 9$.

$7x + 2y + 10z = 5$.

$\Rightarrow$ Augmented matrix.

$$[A:B] = \begin{bmatrix} 5 & 3 & 7 & : 4 \\ 3 & 26 & 2 & : 9 \\ 7 & 2 & 10 & : 5 \end{bmatrix}$$

$R_2 \to 5R_2 - 3R_1 \; ; R_3 \to 5R_3 - 7R_1$

$$[A:B] \sim \begin{bmatrix} 5 & 3 & 7 & : 4 \\ 0 & 121 & -11 & : 33 \\ 0 & -11 & 1 & : -3 \end{bmatrix}$$

$R_2 \to R_2/11$

$$[A:B] \begin{bmatrix} 5 & 3 & 7 & : 4 \\ 0 & 11 & -1 & : 3 \\ 0 & -11 & 1 & : -3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$[A:B] - \begin{bmatrix} 5 & 3 & 7 & : 4 \\ 0 & 11 & -1 & : 3 \\ 0 & -11 & 1 & : -3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$[A:B] - \begin{bmatrix} 5 & 3 & 7 & : 4 \\ 0 & 11 & -1 & : 3 \\ 0 & 0 & 0 & : 0 \end{bmatrix}$$

$$[A:B] - \begin{bmatrix} 5 & 3 & 7 & = 4 \\ 0 & 11 & -1 & = 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$\rho(A) = 2$, $\rho(A = B) = 2$ system is consistent

$\rho(A) = \rho(A = B) = 2 = r$.

But, number of unknowns i,e $n = 3$.

since $\rho(A) = \rho(A:B) = 2 < n$.

$= 2 < 3$.

we have infinite solution. $r = 2 < n$.

$(n - r) = 3 - 2 = 1$.

Hence, one of variable can take arbitrary value let $\boxed{z = k}$

$AX = B$

$$\begin{bmatrix} 5 & 3 & 7 \\ 0 & 11 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}.$$

$5x + 3y + 7z = 4$ — ①

$11y - z = 3$ — ②

in eq ② $11y - k = 3$

$11y = k + 3$

In eq ① $5x + 3\left(\frac{k+3}{11}\right) + 7k = 4$.

$\frac{5x + 3k + 9}{11} + 7k = 4$.

$5x = 4 - 7k - \left(\frac{3k + 9}{11}\right)$

$5x = \frac{44 - 77k - 3k - 9}{11}$

$5x = \frac{35 - 80k}{11}$

$\boxed{x = \frac{7 - 16k}{11}}$

thus, $x = \frac{7-16k}{11}$ $y = \frac{k+3}{11}$, $z=k$.
represent infinite sol
since k is arbitrary.

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(3) $x - 4y + 7z = 14$
$3x + 8y - 2z = 13.$
$7x - 8y + 26z = 5.$

$\Rightarrow$ Augmented matrix

$$(A:B) = \begin{bmatrix} 1 & -4 & 7 & 14 \\ 3 & 8 & -2 & -13 \\ 7 & -8 & 26 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 7R_1$

$$[A:B] \sim \begin{bmatrix} 1 & -4 & 7 & ; & 14 \\ 0 & 20 & -23 & ; & -29 \\ 0 & 20 & -23 & ; & -93 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$(A:B) \sim \begin{bmatrix} 1 & -4 & 7 & ; & 14 \\ 0 & 20 & -23 & ; & -29 \\ 0 & 0 & 0 & ; & -64 \end{bmatrix}$$

$\rho(A) = 2$ $\rho(A:B) = 3.$, $n=3.$
this sys. ; inconsistent.
$\rho(A) \neq \rho(A:B).$

(4) $x + 2y + 3z = 14$
$4x + 5y + 7z = 35$
$3x + 3y + 4z = 21$

$\Rightarrow$ Augmented matrix

$$(A:B) = \begin{bmatrix} 1 & 2 & 3 & ; & 14 \\ 4 & 5 & 7 & ; & 35 \\ 3 & 3 & 4 & ; & 21 \end{bmatrix}$$

$5-8 = -3$
$7-12 = -5$
$35-56 = -21$
$3-6 = -3$
$4-9 = -3$
$4-9 = -5$
$21-6-21$

$R_2 \to R_2 - 4R_1$, $R_3 \to R_3 - 3R_1$

$$(A:B) \sim \begin{bmatrix} 1 & 2 & 3 & ; & 14 \\ 0 & -3 & -5 & ; & -21 \\ 0 & -3 & -5 & ; & -21 \end{bmatrix}$$

$R_3 \to R_3 - R_2.$

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$$(A:B) \sim \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 0 & -3 & -5 & : & 21 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}.$$

$\rho(A) = 2$, $\rho(A:B) = 2$, $n = 3$.

This system is consistent.

$\rho(A) = \rho(A \cdot B) = 2 < n = 3$

$r = 2 < n$

we have Infinite solution.

$(n - r) = 3 - 2 = 1$

let $\boxed{z = k}$

$AX = B$,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ -21 \\ 0 \end{pmatrix}.$$

$x + 2y + 3z = 14 \rightarrow 1$

$-3y - 5z = -21 \rightarrow 2$

in eq 2 $\quad -3y - 5k = -21$

$-3y = -21 + 5k$.

$y = \dfrac{\cancel{-}21 + 5k}{\cancel{-}3}$

in eq 1 $\quad x + 2\left(7 - \dfrac{5k}{3}\right) + 3k = 14$

$x + 14 - \dfrac{10k}{3} + 3k = 14$

$x = 14 - \left(14 - \dfrac{10k}{3}\right) - 3k$.

$\cancel{x = \dfrac{42 - 42 + 10k - 9k}{3}}$

$= \dfrac{42 - 9k - 14 + 10k}{3}$

$= 28 + k$

$x = \dfrac{42 - 42 + 10k - 9k}{3}$

$\boxed{x = \dfrac{k}{3}}$

Thus ; $x = \dfrac{k}{3}$, $y = \dfrac{7 - 5k}{3}$, $z = k$.

represent infinite sol^n.
since k is arbitrary.

* find the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

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Ans $R_1 \to R_2$

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \longleftrightarrow R_3 - R_1$

$R_4 \longrightarrow R_4 - R_1$.

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & 3 & -3 \end{bmatrix}$$

* solution of system of linear equation.

* Gauss elemination method.

consider A system of linear equations

$$\left.\begin{array}{l} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{array}\right\} \longrightarrow (1)$$

The system of linear eq is eavalent to matrix.

$aX = B$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

is called co-effecent matrix.

consider the augmentered matrix.

$$[AB] = \left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

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for gauss elemination method the coeffi-cent method. A is a reduy upper triangul-ar matrix.

$$[A:B] \sim \left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}' & a_{23}' & b_2'' \\ 0 & 0 & a_{33}'' & b_3'' \end{array}\right] \to 2$$

from equatin (2) the linear eq is equalent ~~of linear~~ or equal to the system

$AX = B.$

$a_{11}x + a_{12}y + a_{13}z = b_1$

$a_{22}'y + a_{23}'z = b_2$

$a_{33}''z = b_3''$

by solving above eq we get the solution x, y and z

Gauss Jordon method

by elementry row transformation the co-efficentry matrix A in 2 is reduy to diagnal matrix.

$$[A:B] \sim \left[\begin{array}{ccc:c} a_{11}'' & 0 & 0 & b_1'' \\ 0 & a_{22}'' & 0 & b_2'' \\ 0 & 0 & a_{33}'' & b_3'' \end{array}\right] \to (3)$$

from three the system of eq given by

$AX = B$

$$\begin{bmatrix} a_{11}'' & 0 & 0 \\ 0 & a_{22}'' & 0 \\ 0 & 0 & a_{33}'' \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1'' \\ b_2'' \\ b_3'' \end{bmatrix} = \begin{array}{l} a_{11}''x = b_1'' \\ a_{22}''y = b_2'' \\ a_{33}''y = b_3'' \end{array}$$

* ~~by solving eq~~ ~~x+y+z=9~~

solve the system of eq by Gauss eleminat and Gauss Jordan method

$x+y+z=9$
$x-2y+3z=8$
$2x+y-z=3.$

sol:- $[A:B]$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 8 \end{array}\right]$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$

$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$

$R_3 \to 3R_3 - R_2$

$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & -11 & -44 \end{array}\right]$

$R_3 \to R_3/(-11)$

$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$R_1 \to R_1 - R_3$
$R_2 \to R_2 - 2R_3$

$\sim \left[\begin{array}{ccc:c} 1 & 1 & 0 & 5 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$R_1 \to 3R_1 + R_2$

$\sim \left[\begin{array}{ccc:c} 3 & 0 & 0 & 6 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$AX = B$

$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -9 \\ 4 \end{bmatrix}$

$3x = 6 \Rightarrow x = 2$
$-3y = -9 \Rightarrow y = 3$
$z = 4$

consider $AX = B$.

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 9 \\ -1 \\ 4 \end{bmatrix}$

$x + y + z = 9 \to (1)$
$-3y + 2z = -1 \to (2)$
$\boxed{z = 4}$

$\therefore (2) \Rightarrow -3y + 2z = -1$
$-3y + 8 = -1$
$-3y = -1 - 8$
$-3y = -9$

$\boxed{y = 3}$

(1) $\Rightarrow x+y+z=9$

$x+3+4=9$

$x=9-7$

$\boxed{x=2}$

$\therefore x=2,\ y=3,\ z=4$

H.W

Find the rank of the matrix

$$A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_1 \rightarrow R_2$

$$A=\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - 6R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 2 & 17 \end{bmatrix}$$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

$R_4 \rightarrow R_4 - R_3$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3.$

Since no of non zero rows is 3 $\rho(A)=3$

Ans
$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_1 \to R_2$

$$A = \begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$

$$A \sim \begin{pmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{pmatrix}$$

$R_3 \to 5R_3 - 4R_2.$
$R_4 \to 5R_4 - 9R_2.$

$$A \sim \begin{pmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{pmatrix}$$

$R_4 \to R_4 - R_3$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3.$

since, no of non zero rows is
$\rho(A) = 3.$

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Ay
$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_1 \rightarrow R_2$

$$A = \begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$
$R_4 \rightarrow R_4 - 6R_1$

$$A \sim \begin{pmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{pmatrix}$$

$R_3 \rightarrow 5R_3 - 4R_2$
$R_4 \rightarrow 5R_4 - 9R_2$

$$A \sim \begin{pmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{pmatrix}$$

$R_4 \rightarrow R_4 - R_3$

$$A \sim \begin{pmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\rho(A) = 3$

since no of non zero rows $\rho(A) = 3$.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_1 \to R_2$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 \to 5R_3 - 4R_2$
$R_4 \to 5R_4 - 9R_2$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3$

since no of non zero rows is 3

$\rho(A) = 3$

Ans

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_1 \to R_2$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 \to 5R_3 - 4R_2$

$R_4 \to 5R_4 - 9R_2$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3$

Since No of non zero rows is 3

$\rho(A) = 3$

completed H.W

* Solve the system of linear eq. by gauss elimination method & gauss Jordon method.

Ans.

$x+y+z=9$
$2x+y-z=0$
$2x+5y+7z=52$

Ans

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array}\right]$$

$R_3 \to R_3 + 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$AX = B$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -18 \\ -20 \end{pmatrix}$$

$x+y+z=9 \to (1)$
$-y-3z=-18 \to (2)$
$-4z=-20$
$\boxed{z=5}$

$(2) \Rightarrow -y-3z=-18$
$-y=-18+3z$
$=-18+15$
$-y=-3$

$(1) \Rightarrow x+y+z=9$
$x+3+5=9$
$x=9-8$
$x=1$

Jordon method

$R_1 \to 4R_1 + R_3$
$R_2 \to 4R_2 - 3R_3$

$$\sim \left[\begin{array}{ccc:c} 4 & 4 & 0 & 16 \\ 0 & -4 & 0 & -12 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$$\sim \left[\begin{array}{ccc:c} -4 & 0 & 0 & 4 \\ 0 & -4 & 0 & -12 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$AX=B$

$$\begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -12 \\ -20 \end{bmatrix}$$

$4x=4 \Rightarrow x=1$
$-4y=-12 \Rightarrow y=3$
$-4z=20 \Rightarrow z=5$

$x=1,\ y=3,\ z=5.$

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$$x_1 - 2x_2 + 3x_3 = 2$$
$$3x_1 - x_2 + 4x_3 = 4$$
$$2x_1 + x_2 - 2x_3 = 5$$

Ans $(A:B) \sim \begin{bmatrix} 1 & -2 & 3 & : & 2 \\ 3 & -1 & 4 & : & 4 \\ 2 & 1 & -2 & : & 5 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -2 & 3 & : & 2 \\ 0 & 5 & -5 & : & -2 \\ 0 & 5 & -8 & : & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & -2 & 3 & : & 2 \\ 0 & 5 & -5 & : & -2 \\ 0 & 0 & -3 & : & 3 \end{bmatrix}$$

$AX = B$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 5 & -5 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}.$$

(*) $x_1 - 2x_2 + 3x_3 = 2$

$x_1 \cancel{-5x_2} + 5x_3 = -2$

$-\cancel{3}x_3 = \cancel{3}.$

$\boxed{x_3 = -1}$

* $5x_2 - 5x_3 = -2$

$5x_2 = -2 + 5x_3$

$= -2 - 5.$

$5x_2 = -7$

$x_2 = -7/5 = -1.4$

$x_1 = x_1 - 2x_2 + 3x_3 = 2.$

$x_1 = 2 + 2x_2 - 3x_3$

$= 2 + 2(-1.4) - 3(-1)$

$\boxed{\Rightarrow x_1 = 2.2}$

Jordan method

* $R_1 \to R_1 + R_3$

$R_2 \to 3R_2 + 5R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & -2 & 0 & 5 \\ 0 & 15 & 0 & -2 \\ 0 & 0 & -3 & 3 \end{array}\right]$$

$R_1 \to 15R_1 + 2R_2$

$$\left[\begin{array}{ccc:c} 15 & 0 & 0 & 33 \\ 0 & 15 & 0 & -21 \\ 0 & 0 & -3 & 3 \end{array}\right]$$

$AX = B.$

$$\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 33 \\ -21 \\ 3 \end{bmatrix}$$

$15x_1 = 33 \Rightarrow x_1 = 33/15 = 2.2$

$15x_2 = -21 \Rightarrow x_2 = -21/15 = -1.4$

$-3x_3 = 3 \Rightarrow x_3 = 3/-3 = -1$

* $2x+3y-z=5$

$4x+4y-3z=3$

$2x-3y+2z=2$

$$(A:B)\quad \left[\begin{array}{ccc:c} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ 2 & -3 & 2 & 2 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - R_1$

$$\sim \left[\begin{array}{ccc:c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & -6 & 3 & -3 \end{array}\right]$$

$R_3 \rightarrow R_3/(-3)$

$$\sim \left[\begin{array}{ccc:c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 2 & -1 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_2$

$$\sim \left[\begin{array}{ccc:c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & -2 & -6 \end{array}\right]$$

$Ax = B$.

$$\begin{pmatrix} 2 & 3 & -1 \\ 0 & -2 & -1 \\ 0 & 0 & -2 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ -6 \end{bmatrix}.$$

$2x+3y-z=5 \rightarrow (1)$

$-2y-z=-7 \rightarrow 2$

$-2z=-6 \Rightarrow \boxed{z=3}$

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~~$y = 11 - 2y - z = -7$~~
~~$= -2 - 7$~~
~~$= -9$~~
~~$\boxed{y = -9}$~~

$\cancel{-2y - 3 = -7}$
$\cancel{= -9 = -7}$
$\cancel{= -7 + 9}$

$-2y = -4 \Rightarrow y = \frac{-4}{-2}$

~~$x = 2x + 3y - z = 5$~~
~~$= 2 + 3 - 5$~~
~~$= 5 - 5$~~
~~$= 0$~~
~~$\boxed{x = 0}$~~

$2x + 3y - z = 5$
$2x + 3(2) - 3 = 5$
$2x + 3 = 5 \Rightarrow 2x = 5$
$\Rightarrow 2x = 2 \ \boxed{x = 1}$

~~$z = 3, \ y = -9, \ x = 0$~~

$R_1 \to 2R_1 - R_3$
$R_2 \to 2R_2 - R_3$

$$\sim \left[\begin{array}{ccc:c} 4 & 6 & 0 & 16 \\ 0 & -4 & 0 & -8 \\ 0 & 0 & -2 & -6 \end{array}\right]$$

$R_1 \to 4R_1 + 6R_2$

$$\left[\begin{array}{ccc:c} 16 & 0 & 0 & 16 \\ 0 & -4 & 0 & -8 \\ 0 & 0 & -2 & -6 \end{array}\right]$$

$x + 2y + z = 3$
$2x + 3y + 3z = 10$
$3x - y + 2z = 13.$

$-1 - 7(1)$
$-1 - 7$
$= -8.$

Ans

$$(A:B) \sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 2 & 3 & 3 & : & 10 \\ 3 & -1 & 2 & : & 13 \end{bmatrix}$$

$R_2 = R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$2 - 2\times1 = 2 = 0.$
$3 - 2\times1 = 2 = 1$
$3 - 2\times1 = 2 = 1$
$10 - 2\times3 = 10 - 6$
$= 4$

$$(A:B) \sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & 1\times1 & 1 & : & 4 \\ 0 & -7 & -1 & : & 7 \end{bmatrix}$$

$3 - 2\times1 = 3 = 0$
$-1 - 3\times2 = 6 = -7$
$2 \times 3\times1 = 3 = -1$
$13 - 3\times3 = 6 = 7$

$R_3 \to R_3 + 7R_2$

$$\begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & 1 & 1 & : & 4 \\ 0 & 0 & 14 & : & 63 \end{bmatrix}$$

$7 + 4$
$= 11$

0
$-7\times1 + 7\times1 = 0$
$7\times1 + 1\times7 = 14.$
-8
$7\times7 + 7\times1 = 14$
$49 + 14 = 63.$

$AX = B.$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 63 \end{bmatrix}$$

$1x + 2y + 1z = 3$
$1y + 1z = 4$
$+ 14z = 63$
$= \frac{63}{14} = \boxed{4.5}$

$z = 4.5$

$1x + 2y + 1z = 3$
$1x + 2(4) + 1\cancel{4.5}$
$1x + 2 = 3$
$1x = 2 - 3$
$1x = -1 = 0$
$\boxed{x = 0}$

$1y + 1z = 4$
$= 1 + 4 - 1$
$= 1 + 3.$
$\boxed{y = 4}$

$1y + \cancel{4.5z} = 4.$
$2.\cancel{5y} = 4$
$y = \boxed{\cancel{0.72}}$
$\boxed{\cancel{y = 0.72}}$

$x + 2y + 3z = 3$
$y + z = 4$
$14 - 62 = 67$
$z = 1/1$

$\cancel{3x+4y}$

$3x+4y+5z=18$

9) $\cancel{2x+y+8z=13}$

$5x-2y+7z=20$

$$(A:B): \begin{bmatrix} 3 & 4 & 5 & : & 18 \\ 2 & 1 & 8 & : & 13 \\ 5 & -2 & 7 & : & 20 \end{bmatrix}$$

$R_2 \to 3R_2 - 2R_1$

$R_3 \to 3R_3 - 5R_1$

$$\begin{bmatrix} 3 & 4 & 5 & : & 18 \\ 0 & -11 & 14 & : & 3 \\ 0 & -26 & -4 & : & -30 \end{bmatrix}$$

$R_3 \to R_3 + 9R_1$

$$\begin{bmatrix} 3 & 4 & 5 & : & 18 \\ 0 & -11 & 14 & : & 3 \\ 0 & 0 & 41 & : & 132 \end{bmatrix}$$

$AX = B.$

$$\begin{bmatrix} 3 & 4 & 5 \\ 0 & -11 & 14 \\ 0 & 0 & 41 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 3 \\ 132 \end{bmatrix}$$

$\cancel{3x+4y+5z=18} \to ①$

$-11y+14z=3 \to ②$

$41z=132.$

$z = \frac{132}{41} = 3.21$

$-11y+4z=3$

$\cancel{-11y+14z=3}$

$-11y = 41.94$

$y = \frac{41.94}{-11}$

$\boxed{y = -3.8}$

$3x - 15.2 + 16.05 = 18$

$3x = 18 - 0.85.$

$3x = 17.15$

$\boxed{x = 5.71}$

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(1) $x+2y+z=3$
$2x+3y+3z=10$
$3x-y+2z=13$

$A=\begin{bmatrix}1&2&1\\2&3&3\\3&-1&2\end{bmatrix}$ $B=\begin{bmatrix}3\\10\\13\end{bmatrix}$ $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$(A:B)\begin{bmatrix}1&2&1&:3\\2&3&3&:10\\3&-1&2&:13\end{bmatrix}$

$R_2\to R_2-2R_1$
$R_3\to R_3-3R_1$

$(A:B)=\begin{bmatrix}1&2&1&:3\\0&-1&1&:4\\0&-7&-1&:4\end{bmatrix}$

$R_3\to R_3-7R_2$

$\begin{bmatrix}1&2&1&:3\\0&-1&1&:4\\0&0&-8&:-24\end{bmatrix}$

$AX=B$

$\begin{bmatrix}1&2&1\\0&-1&1\\0&0&-8\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\4\\-24\end{bmatrix}$

$x+2y+z=3\to(1)$
$-y+z=4\to(2)$
$-8z=-24-$

$-8z=-24$

$z=\frac{-24}{-8}$

$\boxed{z=3}$

$-y+z=4$
$-y=4-z$
$-y=4-3$
$\boxed{y=-1}$

$x+2y+z=3$
$x-2+3=3$
$\boxed{x=2}$

$\boxed{x=2},\ \boxed{y=-1}\ \boxed{z=3}$

$R_1\to 8R_1+R_3$
$R_2\to 8R_2-R_3$

$-8\ \ -16\ \ 0:0$
$0\ \ -8\ \ +16:48$
$0\ \ 0\ \ -8\ :-24$

$R_1\to 2R_1+16.$

$\begin{bmatrix}-8&0&0&:16\\0&-8&0&:48\\0&0&-8&:24\end{bmatrix}$

$-8x=16$
$-8y=48$
$-8x=16$
$x=\frac{-16}{18}$
$\boxed{x=-2}$

$y-8y=48$
$\boxed{y=-6}$

$-8z=24$
$\boxed{z=3}$

Gauss - Seidel method.

Gauss - sidel method is a Iterative process (method) for solving a system of linear eq with or diagnoly domint.

consider → The system of linear eq's with three unknowns

eq:- $a_{11}x + a_{12}y + a_{13}z = b_1 \rightarrow (1)$

$a_{21}x + a_{22}y + a_{23}z = b_2 \rightarrow (2)$

$a_{31}x + a_{32}y + a_{33}z = b_3 \rightarrow (3)$

This system of eq is said to be diagnoly domint if

$|a_{11}| > |a_{12}| + |a_{13}|$

$|a_{22}| > |a_{21}| + |a_{23}|$

$|a_{33}| > |a_{31}| + |a_{32}|$

The given system of eq can be re written as.

(1) ⇒ $a_{11}x + a_{12}y + a_{13}z = b$.

$a_{11}x = b_1 - a_{12}y - a_{13}z$

$x = \frac{1}{a_{11}}[b_1 - a_{12}y - a_{13}z]$

(2) ⇒ $a_{21}x + a_{22}y + a_{23}z = b_2$

$a_{22}y = b_2 - a_{21}x - a_{23}z$.

$y = \frac{1}{a_{22}}[b_2 - a_{21}x - a_{23}z]$.

(3) ⇒ $a_{31} + a_{32}y + a_{33} = b_3$

$a_{33}z = b_3 - a_{31}x - a_{32}y$

$z = \frac{1}{a_{33}}(b_3 - a_{31}x - a_{32}y)$

* consider the trial solution $x^{(0)}=0$ $y^{(0)}=0$ $z^{(0)}=0$

* solve the system of linear equation by gauss sidel method

(1) $10x + y + z = 12 \rightarrow (1)$

$x + y + 10z = 12 \rightarrow (3)$

$x + 10y + z = 12 \rightarrow (2)$

Ans This method is solved if $10 > 1+1$, $10 > 1+1$, $10 > 1+1$

The given eq can be re-written as. x, y & z

(1) ⇒ $10x + y + z = 12$

$10x = 12 - y - z$

(1) $x = \frac{1}{10}(12 - y - z)$

② $y = \frac{1}{10}[12 - x - z]$

③ $z = \frac{1}{10}[12 - x - y]$.

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let us consider trial solution as, $x^{(0)} = 0,\ y^{(0)} = 0,\ z^{(0)} = 0$

I iteration

$x^{(1)} = \frac{1}{10}(12 - 0 - 0) = 1.2$.

$y^{(1)} = \frac{1}{10}(12 - 1.2 - 0) = 1.08$

$z^{(1)} = \frac{1}{10}[12 - 1.2 - 1.08] = 0.972$.

II iteration.

$x^{(2)} = \frac{1}{10}(12 - 1.08 - 0.972] = 0.9948$.

$y^{(2)} = \frac{1}{10}(12 - 0.9948 - 0.972] = 1.0033$

$z^{(2)}\ \frac{1}{10}[12 - 0.9948 - 1.0033] = 1.0001$

III iteration.

$x^{(3)} = \frac{1}{10}(12 - 1.0033 - 1.0001] = 0.9996$

$y^{(3)} = \frac{1}{10}(12 - 0.9996 - 1.0001) = 1.0000$

$z^{(3)} = \frac{1}{10}(12 - 0.9996 - 1.0000] = 1.0000$.

IV

$x^{(4)} = \frac{1}{10}(12 - 1.0000 - 1.0000] = 1$

$y^{(4)} = \frac{1}{10}(12 - 1 - 1.0000] = 1$

$z^{(4)} = \frac{1}{10}[12 - 1 - 1] = 1$

$\therefore\ x \approx 1 \quad y \approx 1 \quad z \approx 1$.

* $3x + 20y - z = -18 \rightarrow (2)$
$2x - 3y + 20z = 25 \rightarrow (3)$
$20x + y - 2z = 17 \rightarrow (1)$

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A: $20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$

This solved it a $|20| > |1| + |-2| = 3$
$|20| > |3| + |-1| = 4$
$|20| > |2| + |-3| = 5$

The given eq can be re written as.
~~let us consider trial solution as.~~

$(1) \Rightarrow x = \frac{1}{20}(17 - y + 2z)$
$(2) \Rightarrow y = \frac{1}{20}(-18 - 3x + z)$
$(3) \Rightarrow z = \frac{1}{20}(25 - 2x + 3y)$

let us consider as trial solution as $x^{(0)} = 0, y^{(0)} = 0, z^{(0)} = 0$

I iteral.

$x^{(1)} \frac{1}{20}[17 - 0 + 0] = 0.85$

$y^{(1)} = \frac{1}{20}(-18 - 3(0.85) + 0] = -1.0275$

$z^{(1)} = \frac{1}{20}(25 - 2(0.85) + 3(-1.02 75)] = 1.0108$

II iteral.

$x^{(2)} \frac{1}{20}[17 - (-1.0275) \, 2(1.0108)] = 1.0024$

$y^{(2)} \frac{1}{20}(-18 - 3(1.0024) + 1.0108] = -0.9998.$

$z^{(2)} \frac{1}{20}(25 - 2(1.0024) + 3(0.9998)) = 0.9997$

III iteral.

$x^{(3)} \frac{1}{20}[17 - (-0.9998) + 2(0.9997] = 0.9999.$

$y^{(3)} \frac{1}{20}[-18 - 3(0.999) + 0.9997] =$

$z^{(3)} = \frac{1}{20}(25-2(0.9999)+3(-1)).$

$\Rightarrow 1.0000.$

IV Iteration

$x^{(4)} = \frac{1}{20}(17+1+2(1.0000)) = 1$

$y^{(4)} = \frac{1}{20}(-18-3(1)+(1)) = -1$

$z^{(4)} = \frac{1}{20}(25-2(1)+3(-1)) = 1$

$x \approx 1, \; y = -1, \; z = 1$

* $x+3y+10z = 24, \; 28x+4y-z = 32, \; 2x+17y+4z = 35.$

carry out three iteration.

Ans

$28x+4y-z = 32$

$2x+17y-4z = 35$

$x+3y+10z = 24$

$|28| > |4|+|-1| = 5.$

$|17| > |2|+|4| = 6.$

$|10| > |1|+|3| = 4.$

(1) $\Rightarrow x = \frac{1}{28}(32-4y+z)$

(2) $\Rightarrow y = \frac{1}{17}(35-2x-4z).$

(3) $\Rightarrow z = \frac{1}{10}(24-x-3y)$

$x^{(0)} = 0, \; y^{(0)} = 0, \; z^{(0)} = 0$

I iteration

$x^{(1)} = \frac{1}{28}(32-0-0) = 1.1428.$

$y^{(1)} = \frac{1}{17}[35-2(1.1428)-0] = 1.9243$

$z^{(1)} = \frac{1}{10}(24-1.1428-3(1.9243)) = 1.7084.$

II iteration:-

$x^{(2)} = \frac{1}{28}(32-4(1.9243)+1.7084) = 0.9289$

$y^{(2)} = \frac{1}{17} (35 - 2(0.9289) - 4(1.7084)] = 1.5475$

$z^{(2)} = \frac{1}{10} [24 - 0.9289 - 3(1.5475)] = 1.8428$

III iteration

$x^{(3)} = \frac{1}{28} (32 - 4(1.5475) + (1.8428)) = 0.9876$

$y^{(3)} = \frac{1}{17} (35 - 2(0.9876) - 4(1.8428)) = 1.5090$

$z^{(3)} = \frac{1}{10} [24 - 0.9876 - 3(1.5090)] = 1.8485$

$\boxed{x \simeq 0.9876}$. $\boxed{y = 1.5090}$. $\boxed{z \approx 1.8485}$.

(*) $27x + 6y - z = 85$.
$6x + 15y + 2z = 72$
$x + y + 54z = 110$.

Ans by taking $x = 2, y = 3, z = 2$ as initial approximation.

$27x + 6y - z = 85 \rightarrow (1)$
$6x + 15y + 2z = 72 \rightarrow (2)$
$x + y + 54z = 110 \rightarrow (3)$

$|27| > |6| + |-1| = 7$
$|15| > |6| + 2 = 8$
$|54| > |1| + |1| = 2$

(1) $x \Rightarrow \frac{1}{27} (85 - 6y + z) =$

$y \Rightarrow \frac{1}{15} (72 + 6x + 2z)$

$z \Rightarrow \frac{1}{54} (110 + x + y)$.

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$x^{(0)} = 2 \quad y^{(0)} = 3 \quad z^{(0)} = 2.$

I iteration.

$$x^{(1)} = \frac{1}{27}[85 - \overset{6(3)}{\cancel{6(4)}}-2) = 2.\cancel{93}55555$$

$$y^{(1)} = \frac{1}{15}[72 - \overset{6(2.5555)}{\cancel{6(2)}}+2\} = 3.51111$$

$$z^{(1)} = \frac{1}{54}[110 - 2.5555 - 3.51111\} = 1.9246.$$

II iteration.

$$x^{(2)} = \frac{1}{27}[85 - 6(3.51111) + 1.9246] = 2.4391$$

$$y^{(2)} = \frac{1}{15}[72 - 6(2.4391) + 2(1.9246)] = 3.5677$$

$$z^{(3)} = \frac{1}{54}[110 - 2.4391 - 3.5677] = 1.9258.$$

~~III iteration~~

$$x^{(3)} = \frac{1}{27}[85 - 6(3.5678) + 1.9258)] = \cancel{3.5725}\ 2.4266$$

$$y^{3} = \frac{1}{15}(72 - 6(2.6774) - 2(1.9258)) = 3.5725.$$

$$z^{(3)} = \frac{1}{54}(110 - 2.6774 - 3.4722)$$
$$= 1.92259$$

IV Iteration.

$$x^{(4)} = \frac{1}{27}(85 - 6(3.4722) - 2(1.9231))$$

$$y^{(4)} = \frac{1}{15}[72 - 6(2.4266) - (2 \times 1.9259).$$
$$= 3.5730$$

$$z^{(4)} = \frac{1}{54}(110 - 2.4255 - 3.5430]$$
$$= 1.9259$$

$x = 2.4255, \quad y = 3.5730, \quad z = 1.9259.$

Eigen values and Eigen vectors.

given a squar matrix a If there exist or scalor land x (real or complex) and non zero colum matrix x such that $AX = \lambda X$ then λ is called iden value of A & X is called correspond eifere vector of A corresponding eifere hygen value λ.

Rayleigh's power.

This method is used to find numericaly largest iden value and corresponding Ident vector of the given squar matrix A.

let us consider the initial eigen vector as.

$$X^{(0)} = \begin{bmatrix}1\\0\\0\end{bmatrix} \text{ or } \begin{bmatrix}0\\1\\0\end{bmatrix} \text{ or } \begin{bmatrix}0\\0\\1\end{bmatrix} \text{ (or) } \begin{bmatrix}1\\1\\1\end{bmatrix}.$$

* find the largest Eign value and corresponding eigen vector of the matrix. By Power.

$$A = \begin{bmatrix}2 & 0 & 1\\0 & 2 & 0\\1 & 0 & 2\end{bmatrix}.$$

consider the intial eigen vector :

$$X^{(0)} = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

Now

$$A X^{(0)} = \begin{bmatrix}2 & 0 & 1\\0 & 2 & 0\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{matrix}2+0+0\\0+0+0\\1+0+0\end{matrix}\begin{bmatrix}2\\0\\1\end{bmatrix}$$

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$$= 2\begin{pmatrix} 1 \\ 0 \\ 0.5 \end{pmatrix} = \lambda . x^{(1)}$$

$A x^{(1)} =$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2+0+0.5 \\ 0+0 \quad 0 \\ 1+0+1 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix}$$

$$= \lambda_2 x^{(2)}.$$

$A x^{(2)}$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} \quad \begin{matrix} 2+0+0.8 \\ 0+0+0 \\ 1+0+1.6 \end{matrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8\begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \lambda_3 x^{(3)}$$

$A x^{(3)}$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \begin{pmatrix} 2+0+0.93 \\ 0+0+0 \\ 1+0+1.88 \end{pmatrix} = \begin{bmatrix} 2.93 \\ 0 \\ 2.86 \end{bmatrix} = 2.93\begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} =$$

$$\lambda_4 x^{(4)}$$

$A x^{(4)}$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \begin{pmatrix} 2+0+0.98 \\ 0+0+0 \\ 1+0+1.96 \end{pmatrix} = \begin{bmatrix} 2.98 \\ 0 \\ 2.96 \end{bmatrix} \cancel{\lambda_5 x^{(5)}}$$

$$= 2.99\begin{pmatrix} 1 \\ 0 \\ 0.99 \end{pmatrix} = \lambda_5 x^{(5)}$$

$A x^{(5)}$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0.99 \end{bmatrix} = \begin{pmatrix} 2.99 \\ 0+0+0 \\ 1+0+1.98 \end{pmatrix} = \begin{bmatrix} 2.99 \\ 0 \\ 2.98 \end{bmatrix} = 2.99\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \lambda_6 x^{(6)}$$

$A x^{(6)}$

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0+1 \\ 0+0+0 \\ 1+0+2 \end{bmatrix} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} = 3\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \lambda_7 x^{(7)}$$

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where $x^{(6)} = x^{(7)}$ these largest Eigen value is 3' dic. vector is $\begin{bmatrix}1\\0\\1\end{bmatrix}$.

(x) ~~Find the largest~~ :

imp (1) Find all the eigen value & the corresponding Eigen vector of the matrix

$$A = \begin{bmatrix}-1 & 3\\-2 & 4\end{bmatrix} \qquad I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix}-1 & 3\\-2 & 4\end{bmatrix} - \lambda\begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix}$$

$$= \begin{bmatrix}-1-\lambda & 3\\-2 & 4-\lambda\end{bmatrix}$$

consider the characteristic eq $A - \lambda I = 0$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix}-1-\lambda & 3\\-2 & 4-\lambda\end{bmatrix} = 0$$

$$(-1-\lambda)(4-\lambda) - (-6) = 0$$

$$-4 \cancel{-4\lambda} + \lambda + \lambda^2 + 6 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda-2) - 1(\lambda-2) = 0$$

$$(\lambda-2)\cdot(\lambda-1) = 0$$

$$\lambda - 2 = 0 \qquad \lambda - 1 = 0$$

$$\boxed{\lambda = 2} \quad \boxed{\lambda = 1}.$$

$\therefore \lambda = 2$ and $\lambda = 1$ is case - 1

(i) $\lambda = 2$

$(A - \lambda I)\, X = 0$

$$\begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-3x + 3y = 0 \Rightarrow -x + y = 0 \rightarrow (1)$

$-2x + 2y = 0 \Rightarrow -x + y = 0 \rightarrow (2)$

(1) $\Rightarrow$

$-x + y = 0$

$\cancel{x = y} + x = +y$

$\boxed{x = y}$

$\frac{x}{1} = \frac{y}{1}$ $\qquad X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x = 1, y = 1$

case 2 (ii) $\lambda\ 1$

$[A - \lambda I]\, X = 0$

$$\begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2x + 3y = 0 \rightarrow 3$

$-2x + 3y = 0 \rightarrow 4$

(3) $\Rightarrow -2x + 3y = 0$

$+2x = +3y$

$2x = 3y$

$\frac{x}{3} = \frac{y}{2}$

$\therefore x = 3 \quad y = 2 \qquad X_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

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* Find Eigen value & the corresponding Eigen vector to the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

Ans

$$A - \lambda I = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

consider the characterystic equation

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$10 - 2\lambda - 5\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - 6\lambda - \lambda + 6 = 0$$

$$\lambda(\lambda-6) - 1(\lambda-6) = 0$$

$$\lambda = 6, \lambda = 1$$

case (i) $\lambda = 6$

$$[A - \lambda I]x = 0$$

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x + 4y = 0 \Rightarrow x - 4y = 0 \rightarrow (1)$

$x - 4y = 0$

$x = 4y$

$\frac{x}{4} = \frac{y}{1}$

$\therefore x = 4 \quad y = 1 \qquad \therefore x_1 \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

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(ii) $\lambda = 1$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 4 & 4 \\ 1 & 4 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$4x + 4y = 0 \Rightarrow x + y = 0 \rightarrow 3$

$x + y = 0 \rightarrow (A)$

(B) $x + y = 0$

$x = -y$

$\frac{x}{-1} = \frac{y}{1}$

$x = -1, \; y = 1$ $\quad x_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(7) $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

$$A - \lambda I = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix}$$

consider the characteristic equation $|A - \lambda I| = 0$

$$\begin{bmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix} = 0$$

$(4-\lambda)(3-\lambda) - 2 = 0$

$12 - 3\lambda - 4\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 7\lambda + 10 = 0$

$\lambda^2 - 5\lambda - 2\lambda + 10 = 0$

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$(\lambda - 5) - 2(\lambda - 5) = 0$

$(\lambda - 5)(\lambda - 2) = 0$

$\lambda = 5, \lambda = 2$

case(i) $\lambda = 5$.

$[A - \lambda I] X = 0$

$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x + y = 0 \Rightarrow x - y = 0 \rightarrow (1)$

$2x - 2y = 0 \Rightarrow x - y = 0 \rightarrow (2)$

$(1) \rightarrow x - y = 0$

$x = y$

$\frac{x}{1} = \frac{y}{1}$

$x = 1, y = 1$

$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(ii) $\lambda = 2$

$(A - \lambda I) X = 0$

$\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x + y = 0 \rightarrow (3)$

~~(iii) $\lambda = 5$.~~

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Ans. $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} = 0$

$\lambda^3 - (\Sigma d)\lambda^2 + (\Sigma md)\lambda - |A| = 0 \rightarrow (1)$

$\Sigma d = 8 + 7 + 3 = 18.$

$\Sigma md = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$

$= (21 - 16) + (24 - 4) + (56 - 36)$

$\Sigma md = 45$

$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$

$= 8(21 - 16) - (-6)(-18 + 8) + 2(24 - 14)$

$8(5) + 6(-10) + 2(10).$

$|A| = 0.$

$= \lambda^3 - (\Sigma d)\lambda^2 + (\Sigma md)\lambda - |A| = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\lambda(\lambda^2 - 18\lambda + 45) = 0.$

$\lambda = 0, \lambda^2 - 18\lambda + 45 = 0$

$\lambda^2 - 15\lambda - 3\lambda + 45 = 0$

$\lambda(\lambda - 15) - 3(\lambda - 15) = 0$

$(\lambda - 15)(\lambda - 3) = 0$

$\lambda - 15 = 0, \quad \lambda - 3 = 0.$

$\therefore \lambda = 0, \lambda = 15, \lambda = 3$

continue in last page

(4) $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \Rightarrow |A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)(-1-\lambda) - 3 = 0$$

$$\Rightarrow -1 - \lambda + \lambda + \lambda^2 - 3 = 0$$

$$\Rightarrow \lambda^2 - 4 = 0$$

$$\Rightarrow \lambda = 2$$

$$(A - \lambda I)(x) = 0.$$

$$\begin{pmatrix} -1 & 1 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$\Rightarrow -x + y = 0$ $\qquad \Rightarrow -x + y = 0$

$3x - 3y = 0$ $\qquad \Rightarrow x - y = 0.$

$$\Rightarrow +x = +y$$

$$\Rightarrow x = y$$

$$\Rightarrow \frac{x}{1} = \frac{y}{1} \Rightarrow x = 1, y = 1, x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

(5) $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \Rightarrow |A - \lambda I| = 0$

$$\Rightarrow \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} = 0.$$

$$\Rightarrow \begin{pmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{pmatrix} = 0$$

$\Rightarrow (1-\lambda)(3-\lambda)-8=0$

$\Rightarrow 3-\lambda-3\lambda+\lambda^2-8=0$

$\Rightarrow \lambda^2-4\lambda-5=0$

$\Rightarrow \lambda^2-5\lambda+\lambda-5=0$

$\Rightarrow \lambda(\lambda-5)\ 1(\lambda-5)=0$

$\Rightarrow \lambda+1=0 \quad \lambda-5=0$

$\Rightarrow \lambda=-1 \quad \lambda=5.$

(i) $\lambda=-1$

$(A-\lambda I)x=0$

$$\begin{pmatrix} 1+1 & 4 \\ 2 & 3+1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0.$$

$$=\begin{pmatrix} 2 & 4 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0$$

$= 2x+4y=0$

$2x+4y=0$

$= x+2y=0$

$x=-2y$

$\frac{x}{-2}=\frac{y}{1} \quad \Rightarrow x=-2,\ y=1$

$x_1=\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ //

(ii) $\lambda=5.$

$$\begin{vmatrix} -4 & 4 \\ 2 & -2 \end{vmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0 \Rightarrow -4x+4y=0$$

$2x-2y=0$

?

$\Rightarrow -x+y=0$

$x-y=0$

$\Rightarrow x=y \quad \Rightarrow x=1 \quad y=1$

$\Rightarrow x=y \quad \Rightarrow x=y \quad x_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ //

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$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$\lambda = 0, 3, 15$

i) $\lambda = 0$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$8x - 6y + 2z = 0 \rightarrow (1)$

$-6x + 7y - 4z = 0 \rightarrow (2)$

$2x - 4y + 3z = 0 \rightarrow (3)$

(1) and (2)

Rcm

$$\frac{x}{\begin{vmatrix} -6 & 2 \\ 7 & -4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 8 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}}$$

$$\frac{x}{10} = \frac{-y}{-20} = \frac{z}{20}$$

$x_1 = (x, y, z)' = (10, 20, 30)' = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$

(ii) $\lambda = 3$.

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x - 6y + 2z = 0 \rightarrow (1)$

$-6x + 4y - 4z = 0 \rightarrow (2)$

$2x - 4y + 0z = 0 \rightarrow (3)$

(1) & (2)

Rcm.

$$\frac{x}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 5 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$$

$$\frac{x}{16} = \frac{+y}{8} = \frac{z}{-16}$$

$$x_2 = (x, y, z)' = (16, 8, -16)' = \begin{pmatrix} 16 \\ 8 \\ -16 \end{pmatrix}$$

(iii) $\lambda = 15$.

$(A - \lambda I)x = 0$.

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ -y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x - 6y + 2z = 0 \rightarrow (1)$

$-6x - 8y - 4z = 0 \rightarrow (2)$

$2x - 4y - 12z = 0 \rightarrow (3)$

1 & 2

Rcm

$$\frac{x}{\begin{vmatrix} -6 & 2 \\ -8 & -4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -7 & 2 \\ -6 & 4 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -7 & -6 \\ -6 & -8 \end{vmatrix}}$$

$$\frac{x}{|40|} = \frac{-y}{40} = \frac{z}{20}.$$

$$x_3 = \begin{pmatrix} 40 \\ -40 \\ 20 \end{pmatrix}$$

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Imp

$$A = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$

$\lambda^3 - (\Sigma d)\lambda^2 + (\Sigma m)\lambda - |A| = 0 \rightarrow ①$

$\Sigma d = 1 + 5 + 1 = 7$

$\Sigma md = \begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & \cancel{5} \\ 1 & 5 \end{vmatrix}$

$= (5-1) + (1-9) + (5-1)$

$= 0$

$|A| = 1(5-1) - 1(1-3) + 3(1-15)$

$= -36$

$① \Rightarrow$

$\lambda^3 - 7\lambda^2 + 0\lambda - (-36) = 0$

$\lambda^3 - 7\lambda^2 + 36 = 0 \rightarrow ②$.

by inspection method λ.

$\boxed{\lambda = -2}$ $-8 - 28 + 36 = 0$

$\boxed{0 = 0}$

By synthetic division.

-2	1	-7	0	36
	0	-2	18	-36
	1	-9	18	0

$1 \cdot \lambda^2 - 9\lambda + 18 = 0$

$\lambda^2 - 9\lambda + 18 = 0$

$\lambda^2 - 6\lambda - 3\lambda + 18 = 0$

~~$\lambda(\lambda^2 - 9\lambda) + 18 = 0$~~

~~$\lambda^2 - 9\lambda + 18 = 0$~~

$\lambda^2 - 6$

$\lambda(\lambda - 6) - 3(\lambda - 6) = 0$

$(\lambda - 6)(\lambda - 3) = 0$

$\lambda - 6 = 0 ; \lambda - 3 = 0$

λ.

$\therefore \lambda = \cancel{2}, \cancel{3}\ 3 - 6$ are the egian value

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$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

$$(A - \lambda I) = \begin{pmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{pmatrix}$$

(i) $\lambda = -2$ $\quad (A - \lambda I) X = 0$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x + y + 3z = 0 \rightarrow (1)$

$x + 7y + z = 0 \rightarrow (2)$

$3x + y + 3z = 0 \rightarrow (3)$

Rcm for eq^{ns} (1) & (2)

$$\frac{x}{\begin{vmatrix} 1 & 3 \\ 7 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 3 & 1 \\ 1 & 7 \end{vmatrix}}$$

$$\frac{x}{-20} = \frac{-y}{0} = \frac{z}{20}$$

$$x_1 = (x, y, z) = (-20, 0, 20) = \begin{pmatrix} -20 \\ 0 \\ 20 \end{pmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(ii) $\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x + y + 3z = 0 \rightarrow (1)$

$x + 2y + z = 0 \rightarrow (2)$

$3x + y - 2z = 0 \rightarrow (3)$

(1) & (2)

$$\frac{x}{\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}} \quad \frac{-y}{\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix}}$$

$x_2 = (x, y, z) = (-5, 5, -5) = \begin{pmatrix} -5 \\ 5 \\ -5 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

(iii) $\lambda = 6$

$(A - \lambda I)X = 0$

$$\begin{vmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{vmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-5x + y + 3z = 0 \rightarrow (1)$

$x - y + z = 0 \rightarrow (2)$

$3x + y - 5z = 0 \rightarrow (3)$

$$\frac{x}{\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -5 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -5 & 1 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x}{4} = \frac{-y}{-8} = \frac{z}{4}$$

$$x_3 = \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix}$$

(2) Assignment Question

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{pmatrix} = 0$$

$\lambda^3 - (\Sigma d)\lambda^2 + (\Sigma md)\lambda - |A| = 0$

$\Sigma d = 6$

$$\Sigma md = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$$

$= 4 + 5 + 2$

$= 11$

$3 - (-2)$

$|A| = 1(6-2) - 0 - 1(2-4)$

$= 4 + 2$

$= 6.$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0 \rightarrow (2)$

$\boxed{\lambda = 2}$ $\quad 8 - 24 + 22 - 6 = 0$

$0 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 11 & -6 \\ & 0 & 2 & -8 & 6 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda-3) - 1(\lambda-3) = 0$

$\lambda - 1 = 0 \qquad \lambda - 3 = 0$

$\boxed{\lambda = 1} \qquad \boxed{\lambda = 3}$

$\lambda = 2, 1, 3.$

(i) $\lambda = 2 \quad = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$

$-x + 0y - z = 0 \quad -①$

$x + 0y + z = 0$

$2x + 2y + z = 0 \quad -②$

$$\frac{x}{\begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -1 & -1 \\ 1 & -1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix}}$$

$$\frac{x}{0} = \frac{-y}{0} = \frac{z}{0}.$$

$$x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(ii) $\lambda = 1 \quad \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$

$0x + 0y - z = 0$

$x + y + z = 0$

$2x + 2y + 2z = 0$

$$\frac{x}{\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}}$$

$$\Rightarrow \frac{x}{1} = \frac{-y}{1} = \frac{z}{0}$$

$$x_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

(iii) $\lambda = 3$ $\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$

$-2x + 0y - z = 0.$

$x - y + z = 0.$

$2x + 2y + 0z = 0.$

$$\frac{x}{\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x}{-1} = \frac{+y}{+1} = \frac{z}{2}$$

$$x_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} .//$$

---

* $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

$\lambda^3 - (\Sigma d)\lambda^2 + (\Sigma md)\lambda - |A| = 0$

$\Sigma d = 8 - 3 + 1$

$\Sigma d = 6$

$\Sigma md = \begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} + \begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix}$

$\Sigma md = (-3 - 8) + (8 + 6) + (-24 + 32)$

$= -11 + 14 + 8$

$\Sigma md = 4$

$|A| = 8(-8 - 8) + 8(4 + 6) - 2(-16 + 3)$

$= -88 + 80 - 14$

$\lambda = 1$

by synthetiimethod.

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$$\begin{array}{c|cccc} & 1 & -6 & 11 & -6 \\ 1 & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array} = 0.$$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda^2 - 5\lambda - \lambda + 6 = 0$

$\lambda(\lambda - 5) - 1(\lambda - 6) = 0$

$\boxed{\lambda - 1 = 0}$ $\boxed{\lambda - 3} = 0$

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

$\lambda = 1$

$[A - \lambda I] \times 0.$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$7x - 8y - 2z = 0 \rightarrow ①$

$4x - 4y - 2z = 0 \rightarrow ②$

$3x - 4 + 0z = 0$

$$A - \lambda \begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2. \\ 3 & -4 & 1-\lambda \end{bmatrix}$$

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\frac{x}{8} = \frac{+y}{+6} = \frac{z}{4}$$

$x = (x, y, z) = 8, 6, 4$

$$\begin{bmatrix} 8 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$

$\lambda = 2.$

$(A - \lambda I)x = 0.$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$6x - 8y - 2z = 0$
$4x - 5y - 2z = 0$
$3x - 4y - z = 0.$

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -5 & -2 \end{vmatrix}} \quad \frac{y}{\begin{vmatrix} 6 & -2 \\ 4 & -2 \end{vmatrix}} \quad \frac{z}{\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix}}$$

$$\frac{x}{6} = \frac{+y}{+4} = \frac{z}{2}.$$

$$x_2 = (6\ 4\ 2) = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}.$$

(ii) $\lambda = 3$

$(A - \lambda I)x = 0$ $\begin{pmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$5x - 8y - 2z = 0 \to (1)$
$4x - 6y - 2z = 0 \to (2)$
$3x - 4y - 2z$

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -6 & -2 \end{vmatrix}} \quad \frac{-y}{\begin{vmatrix} 5 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 5 & -8 \\ 4 & -6 \end{vmatrix}}$$

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$$\frac{x}{4} = \frac{+y}{+2} = \frac{z}{2}.$$

(x_3) $(4, 2, 2)$ $\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$

Diaganalization

If A is any squar matrix of order n having n linearly Indipendent eigan vectors, then ther exist n^{th} order matrix P. such that $P^{-1}AP$ is the diagnal matrix that is $P^{-1}AP = D$

Note:- to find $A^n = PD^nP^{-1}$

1) reduce the matrix to diagnol form.

$A = \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix}$

$|A - \lambda I| = 0.$

$\begin{vmatrix} -\lambda & 1 \\ 3 & -\lambda-1 \end{vmatrix} = 0$

$(1-\lambda)(-1-\lambda) - 3 = 0$

$-1 + \cancel{\lambda} - \cancel{\lambda} + \lambda^2 - 3 = 0$

$\lambda^2 - 4 = 0$

$\lambda^2 = 4 \Rightarrow \lambda = \pm\sqrt{4} = \pm 2$

$\boxed{\lambda = 2, -2}$

$\therefore \lambda = 2, -2$ or the eigan values.

i) when $\lambda = 2$

$\lambda = 2$

$(A - \lambda I)x = 0.$

$\begin{bmatrix} -1 & 1 \\ 3 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

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$-x+y=0 \Rightarrow$

$3x-3y=0 \Rightarrow x-y=0 \rightarrow (2)$.

$(1) \Rightarrow x-y=0$

$x=y$

$$\frac{x}{1}=\frac{y}{1}$$

$x_1=(1,1)=\begin{bmatrix}1\\1\end{bmatrix}$. ~~$x_1=(1,1)$ is eigen vec~~

(ii) $\lambda=-2$

$(A-\lambda I)x=0$.

$$\begin{bmatrix}3&1\\3&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

$3x+y=0 \rightarrow (1)$

$3x+y=0 \rightarrow (2)$.

$(1)\ 3x+y=0$

$3x=-y$.

$$\frac{x}{-1}=\frac{y}{3}$$

$\therefore x_2\ (-1,3)=\begin{pmatrix}-1\\3\end{pmatrix}$.

$P=(x_1, x_2)$

$P=\begin{pmatrix}1&-1\\1&3\end{pmatrix}$ $\quad P^{-1}=\frac{\operatorname{adj} P}{|P|}$

$\operatorname{adj} P=\begin{vmatrix}3&1\\-1&1\end{vmatrix}$

$|P|=3+1=4$

$\therefore P^{-1}=\frac{1}{4}\begin{pmatrix}3&1\\-1&1\end{pmatrix}$.

$P^{-1}AP=\frac{1}{4}\begin{pmatrix}3&1\\-1&1\end{pmatrix}\begin{bmatrix}1&1\\3&-1\end{bmatrix}\begin{bmatrix}1&-1\\1&3\end{bmatrix}$.

$=\frac{1}{4}\begin{bmatrix}3&1\\-1&1\end{bmatrix}\begin{bmatrix}1&1\\3&-1\end{bmatrix}\begin{bmatrix}1&-1\\1&3\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}3&1\\-1&1\end{bmatrix}\begin{bmatrix}2&2\\2&-6\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}8&0\\0&-8\end{bmatrix}$

$=\begin{bmatrix}2&0\\0&-2\end{bmatrix}$

$=D$

~~Assignment~~

$(*)\ \begin{bmatrix}-1&2\\2&-1\end{bmatrix}$

Assessment Quest

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$$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

(a) $|A - \lambda I| = 0.$ $\quad A = \begin{bmatrix} -1-\lambda & 2 \\ 2 & -1-\lambda \end{bmatrix}$

$\Rightarrow (-1-\lambda)(-1-\lambda) - 4 = 0$

$\Rightarrow 1 + \lambda + \lambda + \lambda^2 - 4 = 0.$

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0.$

$\Rightarrow \lambda^2 + 3\lambda - \lambda - 3 = 0$

$\Rightarrow \lambda(\lambda+3) - 1(\lambda+3) = 0$

$\Rightarrow \lambda + 3 = 0 \qquad \lambda - 1 = 0.$

$\lambda = -3 \quad (\text{or}) \quad \lambda = 1$

(i) $\lambda = -3$ $\quad \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$2x + 2y = 0 \quad \Rightarrow x + y = 0$

$2x + 2y = 0 \quad \Rightarrow x = -y$

$\dfrac{x}{1} = \dfrac{y}{1}$

$x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(ii) $\lambda = 1$

$\Rightarrow \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\Rightarrow -2x + 2y = 0 \qquad -x + y = 0$

$-2x + 2y = 0 \qquad -x = -y$

$\dfrac{x}{1} = \dfrac{y}{1}$

$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

$P = (x_1, x_2) = \qquad P = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$

$P^{-1} = \dfrac{\text{adj} P}{|P|}$

$\Rightarrow \text{adj} P = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

$$|P| = -1 - 1 = -2$$

$$P^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

$$P^{-1}AP = \frac{1}{-2}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1+2 & -1+2 \\ -2-1 & 2-1 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 3+3 & 1-1 \\ -3+3 & -1-1 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 6 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} = D$$

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7) Reduce the matrix to the diagonal form

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

A: $|A - \lambda I| = 0$ $\qquad A = \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix}$

$\Rightarrow (1-\lambda)(3-\lambda) - 8 = 0.$

$\lambda^2 - 4\lambda - 5 = 0$

$\lambda^2 - 5\lambda + \lambda - 5 = 0.$

$\lambda(\lambda - 5) + 1(\lambda - 5) = 0.$

$(\lambda - 5) \cdot (\lambda + 1) = 0$

$\lambda - 5 = 0 \qquad \lambda + 1 = 0$

$\lambda = 5 \quad , \quad \lambda = -1$

(i) $\lambda = 5$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$-4x + 4y = 0 \Rightarrow x - y = 0$

$2x - 2y = 0 \Rightarrow x - y = 0.$

Constant $x - y = 0$

$x = y$

$\frac{x}{1} = \frac{y}{1} \qquad x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

(ii) $\lambda = -1$

$(A - \lambda I)x = 0.$

$$\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$2x + 4y = 0 \qquad x + 2y = 0$

$2x + 4y = 0. \qquad x + 2y = 0$

Consider $x + 2y = 0$

$x = -2y$

$\frac{x}{-2} = \frac{y}{1} \qquad x_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$

$P = (x_1, x_2)$

$P = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}.$

$P^{-1} = \frac{\text{adj} P}{|P|}$

$\text{adj} P = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

$|P| = 1 - (-2) = 3$

$\therefore P^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}.$

$P^{-1}AP = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 5 & -1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 15 & 0 \\ 0 & -3 \end{bmatrix}.$

$= \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} = D$

Also find A^3

$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$A^n = PD^nP^{-1}$

$A^3 = PD^3P^{-1}$

$= \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 125 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

$A^3 = \frac{1}{3}\begin{bmatrix} 125 & 2 \\ 125 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 125-2 & 250+2 \\ 125+1 & 250-1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 123 & 252 \\ 126 & 249 \end{bmatrix}$

$= \begin{bmatrix} 41 & 84 \\ 42 & 83 \end{bmatrix}$

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