Joint Probability distribution

We have discussed probability distribution associated with single random variable.

In this topic, we discuss probability distribution associated with two random variable referred to as joint probability distribution.

Random variable is a function that assigns a real number to every sample point in the sample space of a random experiment.

Eg:- When a coin is tossed, the number of heads turning up is denoted by a real value 1 and for tail, the real value 0.

$S = \{H, T\}$

$x(H) = 1$

$x(T) = 0$ where X is a random variable.

Joint probability function and joint probability distribution

If X and Y are two discrete random variables, we define the joint probability function by

$P\{X = x, Y = y\} = f(x, y)$

where $f(x,y)$ satisfies the condition

a) $f(x,y) \geq 0$

b) $\sum_x \sum_y f(x,y) = 1$

The set of values of this function

$f(x_i, y_j) = J_{ij}$ for $i = 1, 2, 3, \ldots m$, $j = 1, 2, 3, \ldots n$ is called joint probability distribution of x and y.

Joint probability distribution table:

$x \backslash y$	y_1	y_2	....	y_n	$f(x_i)$
x_1	J_{11}	J_{12}		J_{1n}	$f(x_1)$
x_2	J_{21}	J_{22}	....	J_{2n}	$f(x_2)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	
x_m	J_{m1}	J_{m2}	....	J_{mn}	$f(x_m)$
$g(y_j)$	$g(y_1)$	$g(y_2)$ ~~$g(y_3)$~~		$g(y_n)$	1

Independent random variable

Two discrete random variables X and Y are said to be independent if $J_{ij} = f(x_i)\, g(y_j)$ otherwise they are dependent.

Formulas:

1. Expectation :

i. Expectation of $x = E(X) = \mu_x = \sum_i x_i f(x_i)$

ii. Expectation of $y = E(Y) = \mu_y = \sum_j y_j g(y_j)$.

iii. Expectation of $XY = E(XY) = \mu_{xy} = \sum_i \sum_j x_i y_j J_{ij}$

2. Variance of $x = V(x) = E(x^2) - [E(x)]^2 = \sigma_x^2$ (σ_x^2)

3. Co-variance of X and $Y = CoV(X,Y) = E(X,Y) - E(X)E(Y)$

4. Co-relation of X and $Y = \rho(X,Y) = \dfrac{CoV(X,Y)}{\sigma_x \sigma_y}$

Problems:-

1. The joint probability distribution table for two random variables X and Y is as follows:

$x \backslash y$	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine i. marginal distribution of x & y
ii. $Cov(X, Y)$
iii. Are X and Y independent?
iv. $\rho(X, Y)$

$x \backslash y$	-4	2	7	$f(x_i)$
1	1/8	1/4	1/8	1/2
5	1/4	1/8	1/8	1/2
$g(y_j)$	3/8	3/8	1/4	1

$X = \{x_i\} = \{1, 5\}$
$Y = \{y_j\} = \{-4, 2, 7\}$

i. Marginal distribution of x.

x_i	1	5
$f(x_i)$	1/2	1/2

Marginal distribution of y

y_j	-4	2	7
$g(y_j)$	3/8	3/8	1/4

ii. $Cov(X, Y) = E(X,Y) - E(X)E(Y)$ — ①

$E(X) = \sum_i x_i f(x_i)$
$= x_1 f(x_1) + x_2 f(x_2)$
$= 1(1/2) + 5(1/2)$
$= 3$

$E(Y) = \sum_j y_j g(y_j)$
$= y_1 g(y_1) + y_2 (g(y_2)) + y_3 g(y_3)$
$= -4(3/8) + 2(3/8) + 7(1/4)$
$= 1$

$$E(XY) = \sum_i \sum_j x_i y_j J_{ij}$$

$$= x_1 y_1 J_{11} + x_1 y_2 J_{12} + x_1 y_3 J_{13} + x_2 y_1 J_{21} + x_2 y_2 J_{22} + x_2 y_3 J_{23}$$

$$= (1)(-4)(1/8) + (1)(2)(1/4) + (7)(1)(1/8) + (5)(-4)(1/4) + (5)(2)(1/8) + (5)(7)(1/8)$$

$$= 1.5$$

$$Cov(X,Y) = E(X,Y) - E(X)E(Y)$$

$$= 1.5 - (3)(1)$$

$$= -1.5$$

iv. $$\rho(x,y) = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

$$\cancel{= -1.5}$$

$$\sigma_x^2 = [E(x^2) - [E(x)]^2 \quad \text{—} (2)$$

$$E(x^2) = \sum_i x_i^2 f(x_i)$$

$$= x_1^2 f(x_1) + x_2^2 f(x_2)$$

$$= 1^2(1/2) + 5^2(1/2)$$

$$= \frac{26}{2}$$

$$= 13$$

Eq. (2) becomes

$$\sigma_x^2 = 13 - (3)^2$$

$$= 13 - 9$$

$$= 4$$

$$\sigma_x = 2$$

$$\sigma_y^2 = E(Y)^2 - [E(Y)]^2 \quad \text{③}$$

$$E(Y^2) = \sum_j y_j^2 g(y_j)$$

$$= (-4)^2(3/8) + (2)^2(3/8) + (7)^2(1/4)$$

$$= 6 + 1.5 + 12.25$$

$$= 19.75$$

Eq. ③ becomes.

$$\sigma_y^2 = 19.75 - (1)^2$$

$$= 19.75 - 1$$

$$\sigma_y^2 = 18.75$$

$$\sigma_y = 4.33$$

$$\rho(x,y) = \frac{-1.5}{(2)(4.33)}$$

$$= -0.1732$$

iii. To check if x and Y are independent.

$$J_{ij} = f(x_i)g(y_j)$$

Consider $J_{11} = 1/8$

$$f(x_1) = 1/2$$

$$g(y_1) = 3/8$$

$$= (1/2)(3/8) = \frac{3}{16} \neq J_{11}$$

$\therefore$ The random variables X and Y are dependent.

2. Suppose X and Y are independent random variables with the following respective distribution. Find the joint probability distribution of X and Y. Also verify $Cov(X,Y) = 0$.

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

Since X and Y are independent random variables,

$\therefore J_{ij} = f(x_i)\, g(y_j)$

x \ y	-2	5	8	$f(x_i)$
1	0.21	0.35	0.14	0.7
2	0.09	0.15	0.06	0.3
$g(y_j)$	0.3	0.5	0.2	1

$J_{11} = f(x_1)\, g(y_1) = (0.7)(0.3) = 0.21$

$J_{12} = (0.7)(0.5) = 0.35$

$J_{13} = (0.7)(0.2) = 0.14$

$J_{21} = (0.3)(0.3) = 0.09$

$J_{22} = (0.3)(0.5) = 0.15$

$J_{23} = (0.3)(0.2) = 0.06.$

$Cov(X,Y) = E(XY) - E(X)E(Y)$ — ①

$E(X) = \sum_i x_i f(x_i)$

$= (1)(0.7) + (2)(0.3)$

$= 0.7 + 0.6$

$= \cancel{0.13}\ 1.3$

$E(Y) = \sum_j y_j g(y_j)$

$= (-2)(0.3) + (5)(0.5) + (8)(0.2)$

$= -0.6 + 2.5 + 1.6$

$= 3.5$

$E(XY) = \sum_i \sum_j x_i y_j J_{ij}$

$= x_1y_1J_{11} + x_1y_2J_{12} + x_1y_3J_{13} + x_2y_1J_{21} + x_2y_2J_{22} + x_2y_3J_{23}$

$= (1)(-2)(0.21) + (1)(5)(0.35) + (1)(8)(0.14) + (2)(-2)(0.09) + (2)(5)(0.15) + (2)(8)(0.06)$

$= -0.42 + 1.75 + 1.12 - 0.36 + 1.5 + 0.96$

$= 4.55$

(1) $\Rightarrow Cov(x, y) = 4.55 - (1.3)(3.5)$

3. The joint probability distribution for two variables is as shown below.

$x \backslash y$	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find i) marginal distribution of x and y.

ii) $f.(x,y) \times Cov(x,y)$

iii) Also find $P(X+Y>0)$

$x \backslash y$	-2	-1	4	5	$f(x_i)$
1	0.1	0.2	0	0.3	0.6
2	0.2	0.1	0.1	0	0.4
$g(y_j)$	0.3	0.3	0.1	0.3	1

i) Marginal distribution of x.

x_i	1	2
$f(x_i)$	0.6	0.4

Marginal distribution of y

y_j	-2	-1	4	5
$g(y_j)$	0.3	0.3	0.1	0.3

ii) $Cov(x,y) = E(XY) - E(X)E(Y)$ — (1)

$E(x) = \sum_i x_i f(x_i)$

$= (1)(0.6) + (2)(0.4)$

$= 1.4$

$E(Y) = \sum_j y_j g(y_j)$

$= (-2)(0.3) + (-1)(0.3) + (4)(0.1) + (5)(0.3)$

$= -0.6 - 0.3 + 0.4 + 1.5$

$= 1$

$$E(XY) = \sum_i \sum_j x_i y_j J_{ij}$$

$$= (1)(-2)(0.1) + (1)(-1)(0.2) + (1)(4)(0) + (1)(5)(0.3) + (2)(-2)(0.2) + (2)(-1)(0.1) + (2)(4)(0.1) + (2)(0)(5)$$

$$= -0.2 - 0.2 + 0 + 1.5 - \cancel{0.8} - 0.2 + \cancel{0.8} + 0$$

$$= 0.9$$

$$① \Rightarrow Cov(X,Y) = 0.9 - (1.4)(1)$$

$$= -0.5$$

iii) $$P(X+Y>0) = P(1,4) + P(1,5) + P(2,-1) + P(2,4) + P(2,5)$$

$$= 0 + 0.3 + 0.1 + 0.1 + 0$$

$$= 0.5$$

4. The joint probability distribution of two discrete random variables X and Y is given by $f(x,y) = k(2x+y)$ where x and y are integers such that $0 \le x \le 2$, $0 \le y \le 3$. Find the value of constant k.

ii) Find the marginal distribution of X and Y.

iii) ST the random variables X and Y are dependent.

$X = \{x_i\} = \{0, 1, 2\}$

$Y = \{y_j\} = \{0, 1, 2, 3\}$

X\Y	0	1	2	3	$f(x_i)$
0	0	k	2k	3k	6k
1	2k	2k+k	2k+2k	2k+3k	14k
2	4k	5k	6k	7k	22k
$g(y_j)$	6k	9k	12k	15k	42k

$f(x,y) = k(2x+y)$

If $x=0$, $y=0$, $f = k(0+0) = 0$

If $x=0$, $y=1$, $f = k(0+1) = k$

If $x=0$, $y=2$, $f = k(0+2) = 2k$

If $x=1$, $y=0$, $f(1,0) = 2k$

If $x=1$, $y=1$, $f(1,1) = 2k+k$

If $x=1$, $y=2$, $f(1,2) = 4k$

$42k = 1$

$k = \frac{1}{42}$

ii) Marginal distribution of X.

x_i	0	1	2
$f(x_i)$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{22}{42}$

Marginal distribution of Y.

y_j	0	1	2	3
$g(y_j)$	$\frac{6}{42}$	$\frac{9}{42}$	$\frac{12}{42}$	$\frac{15}{42}$.

ii) $J_{23} = f(x_2) \cdot g(y_3)$

$4k = 14k \cdot 12k$

$\frac{4}{42} \neq \frac{14}{42} \cdot \frac{12}{42}$

$\therefore$ X and Y are dependent.

5. A fair coin is tossed thrice. The random variables X and Y are defined as follows: X = 0 or 1 according as head or tail occurs on the first toss. Y = number of heads.

i) Determine the probability distribution of X and Y.
ii) Determine the joint probability distribution of X and Y.
iii) Co-variance of X and Y.

Let S be the sample space and association of the random variable X and Y is given by the table.

S	HHH	HHT	HTT	HTH	TTT	TTH	THH	THT
X	0	0	0	0	1	1	1	1
Y	3	2	1	2	0	1	2	1

ii) Probability distribution of X and Y

$X = \{x_i\} = \{0, 1\}$

$Y = \{y_j\} = \{0, 1, 2, 3\}$

x_i	0	1
$f(x_i)$	$\frac{1}{2}$	$\frac{1}{2}$

y_j	0	1	2	3
$g(y_j)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$P(X=0) = \frac{4}{8} = \frac{1}{2}$

$P(X=1) = \frac{1}{2}$

ii)

$X \backslash Y$	0	1	2	3	$f(x_i)$
0	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0	$\frac{1}{2}$
$g(y_j)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

$J_{ij} = f(x_i, y_j) = P(x = x_i, y = y_j)$

$J_{11} = P(x=0, y=0) = 0$ $\qquad$ $J_{21} = P(x=1, y=0) = \frac{1}{8}$

$J_{12} = P(x=0, y=1) = \frac{1}{8}$ $\qquad$ $J_{22} = P(x=1, y=1) = \frac{1}{4} = \frac{2}{8}$

$J_{13} = P(x=0, y=2) = \frac{2}{8}$ $\qquad$ $J_{23} = P(x=1, y=2) = \frac{1}{8}$

$J_{14} = P(x=0, y=3) = \frac{1}{8}$ $\qquad$ $J_{24} = P(x=1, y=3) = 0$

iii) $Cov = E(XY) - E(X)\,E(Y)$

$E(X) = \sum_i x_i f(x_i)$

$= 0 + \frac{1}{2}$

$= \frac{1}{2}$

$E(Y) = \sum_j y_j g(y_j)$

$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$

$= \frac{12}{8} = \frac{3}{2}$

$E(XY) = \sum_i \sum_j x_i y_j J_{ij}$

$= 0+0+0+0+0+\frac{1}{4}+\frac{2}{8}+0$

$= \frac{1}{2}$.

$CoV = \frac{1}{2} - \left(\frac{1}{2}\right)\left(\frac{3}{2}\right)$

$= \frac{1}{2} - \frac{3}{4}$

$= \frac{2-3}{4}$

$= \frac{-1}{4}$

6. If X and Y are independent random variables. X takes values 2, 5, 7 with probability ½, ¼, ¼ respectively, Y takes values 3, 4, 5 with probability ⅓, ⅓, ⅓. i) Find the joint probability distribution of X & Y.
ii) Find $f(X, Y)$ iii) Find the probability distribution of $Z = X+Y$.

Given,

x_i	2	5	7
$f(x_i)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

y_j	3	4	5
$g(y_j)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Since, X & Y are independent random variables,

$J_{ij} = f(x_i)\, g(y_j)$

$x \backslash y$	3	4	5	$f(x_i)$
2	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$
5	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
7	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
$g(y_j)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1

ii) $\rho(X,Y) = \dfrac{Cov(X,Y)}{\sigma_x \sigma_y}$

$Cov(X,Y) = E(XY) - E(X)E(Y)$

$E(X) = \sum_i x_i f(x_i)$

$= 1 + \frac{5}{4} + \frac{7}{4}$

$= 4$

$E(Y) = \sum_j y_j g(y_j)$

$= 1 + \frac{4}{3} + \frac{5}{3}$

$= 4$

$E(XY) = \sum_i \sum_j x_i y_j J_{ij}$

$= 1 + \frac{8}{6} + \frac{10}{6} + \frac{15}{12} + \frac{20}{12} + \frac{25}{12} + \frac{21}{12} + \frac{28}{12} + \frac{35}{12}$

$= 1 + \frac{18}{6} + \frac{144}{12}$

$= 16$

$Cov = 16 - (4)(4)$

$= 0$

$\rho(X,Y) = \dfrac{0}{\sigma_x \sigma_y} = 0.$

iii.

$z = x_i + y_j$	5	6	7	8	9	10	11	12
$P(z)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

Analysis of variance (Anova)

Analysis of variance is used to compare 3 or more population means. Anova is developed by R.A. Fischer.

There are two types of Anova

1. One way Anova - It is classified according to only one factor.

Eg:- Three machines in a factory producing bolts

A	B	C
3	4	1
4	5	2
	6	3
		4

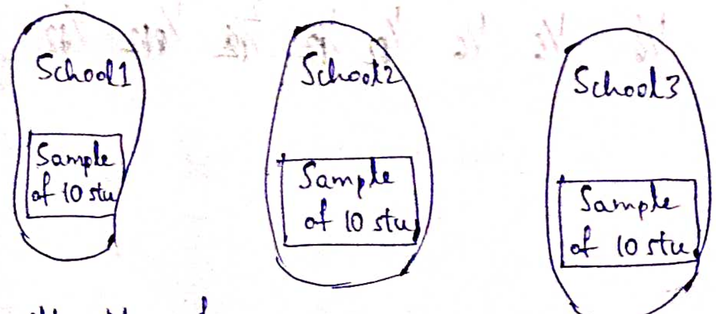
2. Two way Anova - It is classified according to two factors.

It can be applied i. B/w the columns.
ii. b/w the rows.

Eg:-

	Students			
Days	A	B	C	D
Mon	2	2	2	1
Tue	1	3	2	2
Wed	6	1	4	6

$$\text{Anova} = \frac{\text{Variability b/w the means}}{\text{Variability within the distribution.}}$$



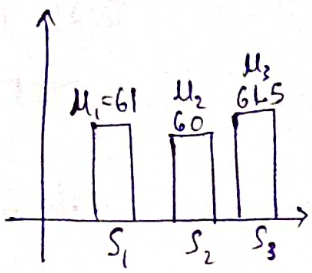
μ_1 - Mean of population $\quad \mu_2 \quad \mu_3$

$\bar{x}_1$ - Mean of sample $\quad \bar{x}_2 \quad \bar{x}_3$

$H_0: \mu_1 = \mu_2 = \mu_3$

H_a: Atleast one of the mean differ from other.

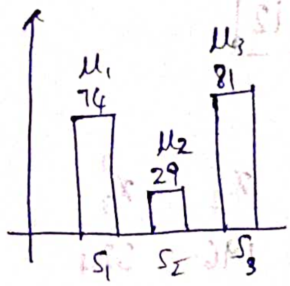
Case 1:



$\mu_1 = \mu_2 = \mu_3$

Fails to reject H_0.

Case 2:



$\mu_1 \neq \mu_2 \neq \mu_3$

Rejects H_0.

1. The following figures relate to the production in kgs of 3 varieties A, B, C of wheat grown in 12 plots.

A: 14 16 18

B: 14 13 15 22

C: 18 16 19 19 20

Make an analysis of variance using F-test to examine whether there is any significant difference in the production of varieties. Value at (2,9) degree of freedom is 4.26.

$A(x_1)$	$B(x_2)$	$C(x_3)$
14	14	18
16	13	16
18	15	19
	22	19
		20
$\Sigma x_1 = 48$	$\Sigma x_2 = 64$	$\Sigma x_3 = 92$
$n_1 = 3$	$n_2 = 4$	$n_3 = 5$

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$

$= 48 + 64 + 92$

$\boxed{T = 204}$

N - size of the population

n_1 = size of the sample A.

$N = n_1 + n_2 + n_3$

$= 3 + 4 + 5$

$\boxed{N = 12}$

x_1^2	x_2^2	x_3^2
196	196	324
256	169	256
324	225	361
	484	361
		400

$\Sigma x_1^2 = 776 \quad \Sigma x_2^2 = 1074 \quad \Sigma x_3^2 = 1702$

Corrector factor.

$= \frac{T^2}{N} = \frac{204 \times 204}{12}$

$= 3468$

SST - Total Sum of Squares.

$SST = \left(\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2\right) - \left(T^2/N\right)$

$= 776 + 1074 + 1702 - 3468$

$= 84$

SSC - Sum of Squarer due to Columns (Treatment).

$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \left(T^2/N\right)$

$= \frac{(48)^2}{3} + \frac{(64)^2}{4} + \frac{(92)^2}{5} - 3468$

$= 3484.8 - 3468$

$= 16.8$

SSE – ~~SST~~ Sum of Squares due to Errors.

$SSE = SST - SSC$

$= 84 - 16.8$

$= 67.2$

Anova table:-

Source of variation	Sum of squares	Degree of freedom	Mean's squares	F-ratio
Varieties	$SSC = 16.8$	$k-1 = 3-1 = 2$	$MSC = \frac{SSC}{k-1} = 8.4$	$F = \frac{MSC}{MSE} = \frac{8.4}{7.47} = 1.12$
Errors	$SSE = 67.2$	$N-k = 12-3 = 9$	$MSE = \frac{SSE}{N-k} = 7.47$	
	$SST = 84$	$N-1 = 12-1 = 11$		

~~F=1.12~~

Null hypothesis: There is no significant difference in the production of wheat of 3 varieties A, B, C.

Alternative hypothesis: Atleast two means are different, (there is a significant difference in the production of wheat of 3 varieties A, B, C).

$H_0: \mu_1 = \mu_2 = \mu_3$.

$H_a: \mu_1 \neq \mu_2$ or $\mu_2 \neq \mu_3$ or $\mu_1 \neq \mu_3$ or $\mu_1 \neq \mu_2 \neq \mu_3$.

F calculated is less than F critical.

i.e $1.12 < 4.26$.

Hence, H_0 is accepted.

$\mu_1 = \mu_2 = \mu_3$

2. The following accompanying data ~~resulting~~ resulted from an experiment. Compare the degree of railing of fabric copolymerized with the three different mixtures of Metoacrylic acid.

Mixture 1: 0.56. 1.12 0.90 1.02 0.94

Mixture 2: 0.72 0.69 0.87 0.78 0.91

Mixture 3: 0.62 1.08 1.07 0.99 0.93

Make a analysis of variance using F-test to examine whether there is any significance difference in the mixture of three variables $F_{0.05}$ at (2,12) degrees of freedom is 3.89.

Mixture 1 (x_1)	Mixture 2 (x_2)	Mixture 3 (x_3)
0.56	0.72	0.62
1.12	0.69	1.08
0.90	0.87	1.07
1.02	0.78	0.99
0.94	0.91	0.93

$\Sigma x_1 = 4.54$ $\Sigma x_2 = 3.97$ $\Sigma x_3 = 4.69$.

$n_1 = 5$ $n_2 = 5$ $n_3 = 5$

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$

$= 4.54 + 3.97 + 4.69$

$\boxed{T = 13.2}$

$N = n_1 + n_2 + n_3$

$\boxed{N = 15}$

Corrector factor

$= \frac{T^2}{N} = \frac{13.2 \times 13.2}{15}$

$= 11.616$

x_1^2	x_2^2	x_3^2
0.3136	0.5184	0.3844
1.2544	0.4761	1.1664
0.81	0.7569	1.1449
1.0404	0.6084	0.9801
0.8836	0.8281	0.8649

$\Sigma x_1^2 = 4.302 \quad \Sigma x_2^2 = 3.1879 \quad \Sigma x_3^2 = 4.5407$

$$SST = \left(\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2\right) - T^2/N$$

$$= 4.302 + 3.1879 + 4.5407 - 11.616$$

$$= 0.4146$$

$$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \left(T^2/N\right)$$

$$= \frac{20.6116}{5} + \frac{15.7609}{5} + \frac{21.9961}{5} - 11.616$$

$$= 11.67372 - 11.616$$

$$= 0.05772$$

$$SSE = SST - SSC$$

$$= 0.4146 - 0.05772$$

$$= 0.35688$$

Source of variation	Sum of squares	Degree of freedom	Mean's squares	F-ratio.
Varieties	$SSC = 0.05772$	$k-1=2$	$MSC = \frac{SSC}{k-1} = 0.02886$	$F = \frac{MSC}{MSE} = 0.9704$
Errors	$SSE = 0.35688$	$N-k=12$	$MSE = \frac{SSE}{N-k} = 0.02974$	
	$SST = 0.4146$	$N-1=14$		

$F_{calculated} < F_{critical}$.

$0.9704 < 3.89$.

Hence, H_0 is accepted.

3. The following data represents the performance of three detergent and three different temperatures with specially designed equipments.

Water temperature	Detergent		
	X	Y	Z
Cold water	57	55	67
Warm water	49	52	68
Hot water	54	46	58

Test whether three significant for three varieties of detergents $F_{0.05}$ for degree of freedom $(2,6) = 5.14$.

$X(x_1)$	$Y(x_2)$	$Z(x_3)$
57	55	67
49	52	68
54	46	58

$\Sigma x_1 = 160$ $\quad \Sigma x_2 = 153$ $\quad \Sigma x_3 = 193$

$n_1 = 3$ $\quad n_2 = 3$ $\quad n_3 = 3$

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$

$= 160 + 153 + 193$

$T = 506$

Correction factor

$N = 3+3+3$

$= 9$

$$\frac{T^2}{N} = \frac{506 \times 506}{9}$$

$= 28448.44$

x_1^2	x_2^2	x_3^2
3249	3025	4489
2401	2704	4624
2916	2116	3364

$\Sigma x_1^2 = 8566$, $\Sigma x_2^2 = 7845$ $\Sigma x_3^2 = 12477$

$$SST = \left(\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2\right) - T^2/N$$

$$= 8566 + 7845 + 12477 - 28448.44$$

$$= 439.56$$

$$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N}$$

$$= \frac{25600}{3} + \frac{23409}{3} + \frac{37249}{3} - 28448.44$$

$$= 28752.66 - 28448.44$$

$$= 304.22$$

$$SSE = SST - SSC$$

$$= 439.56 - 304.22$$

$$= 135.34$$

Source of variation	Sum of squares	Degree of freedom	Means squares	F-ratio
Varieties	$SSC = 304.22$	$k-1=2$	$MSC = 152.11$	$\frac{MSC}{MSE} = 6.74$
Errors	$SSE = 135.34$	$N-k=6$	$MSE = 22.56$	
	$SST = 439.56$	$N-1=8$		

$F_{calculated} > F_{critical}$

$6.74 > 5.14$

$\therefore H_0$ is rejected.