UNIT - IV Integral calculus

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* multiple of Integral: Double Integral and Triple Integral.

Double Integral of form $I = \iint f(x,y)\,dx\,dy$

Triple Integral of form $I = \iiint f(x,y,z)\,dx\,dy\,dz$.

To evaluate following procedure problem

1) let x_1, x_2 & y_1, y_2 be constants the Integral (I) Integrate in any order.

$$I = \int_0^1 \int_2^4 f(x,y)\,dx\,dy$$

(2) let x_1, x_2 be constants and $y = f_1(x)$ & $y_2 = f_2(x)$ then Integrate w.r.t y (then x)

ex:- $I = \int_0^1 \int_x^{\sqrt{x}} f(x,y)\,dy\,dx$

(3) let y_1, y_2 be constants and $x_1 = f_1(y)$ & $x_2 = f_2(y)$ the 1st Integrate w.r.t x & w.r.t y

$$I = \int_0^{x} \int_y^{y^2} f(x,y)\,dx\,dy$$

* Evaluation Double Integral

1) $\int_1^2 \int_0^3 xy^2\,dx\,dy$

(a)

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Sol $I = \int_{y=1}^{2}\int_{x=0}^{3} x y^2 \, dx\, dy.$

$y = 1 \quad x = 0$

$I = \int_{y=1}^{2} y^2 \cdot \frac{x^2}{2}\Big|_0^3 dy$

$I = \int_{y=1}^{2} y^2 \left(\frac{9}{2}\right) dy.$

$I = \frac{9}{2} \int_{y=1}^{2} y^2 dy.$ ~~Ferr~~ to y

$I = \frac{9}{2}\left(\frac{y^3}{3}\right)_1^2$

$I = \frac{9}{2}\left[\frac{8}{3} - \frac{1}{3}\right)$

$= \frac{\cancel{9}^{3}}{2}\left(\frac{7}{\cancel{3}}\right)$

$\boxed{I = \frac{21}{2}}$

(2) $\int_0^1 \int_0^2 x y \, dy\, dx$

sol $I = \int_{x=0}^{1}\int_{y=0}^{2} x y \, dy\, dx$

$I = \int_{x=0}^{1} x \frac{y^2}{2}\Big|_0^2 dx$

$I = \int_{x=0}^{1} \frac{\cancel{4}^{2}}{\cancel{2}_{1}}\Big)_0^2$

$I = 2\int_{x=0}^{1} x$

$I = 2 \left(\frac{x^2}{2}\right)_0^1$

$I = \cancel{2} \times \frac{1}{\cancel{2}}$ $\boxed{I = 1}$

1) $\int_0^1\int_0^2 (x^2+y^2)\,dy\,dx$

$$I=\int_{x=0}^{1}\int_{y=0}^{2}(x^2+y^2)\,dy\,dx$$

$$I=\int_{x=0}^{1} x^2 y+\frac{y^3}{3}\Big|_0^2\,dx$$

$$I=\int_{x=0}^{1} x^2+\left(\frac{8}{3}\right)\Big|_0\,dx$$

$$I=\frac{2x^3}{3}+\frac{8}{3}x\Big|_0^1$$

$$I=\frac{2}{3}+\frac{8}{3}=\frac{10}{3}$$

$$\boxed{I=10/3}$$

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2) $\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$

$$I=\int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} xy\,dy\,dx$$

$$I=\int_{x=0}^{1} x\frac{y^2}{2}\Big|_x^{\sqrt{x}}\,dx$$

$$I=\frac{1}{2}\int_{x=0}^{1} x\left((\sqrt{x})^2-x^2\right)dx$$

$$I=\frac{1}{2}\int_{x=0}^{1}(x^2-x^3)\,dx$$

$$I=\frac{1}{2}\left(\frac{x^3}{3}-\frac{x^4}{4}\right)_0^1$$

$$I=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{4}\right)$$

$$I=\frac{1}{2}\left[\frac{4-3}{12}\right]$$

$$\boxed{I=\frac{1}{24}}$$

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$I = \frac{9}{2}$

(5) $\int_1^4 \int_0^{\sqrt{4-x}} xy \, dy dx$

Sol

$I = \int_1^4 \int_0^{\sqrt{4-x}} xy \, dy dx$

$I = \int_1^4 x \cdot \left. \frac{y^2}{2} \right|_0^{\sqrt{4-x}} dx$

$I = \int_1^4 x \cdot \left(\sqrt{4-x}\right)^2$

$I = \frac{1}{2} \int_1^4 4x - x^2 = \frac{1}{2}\left(\frac{4x^2}{2} - \frac{x^3}{3}\right)_1^4$

$I = \frac{1}{2}\, 4\left(\frac{16}{2} - \frac{64}{3}\right) - \left(\frac{4}{2} - \frac{1}{3}\right)$

$I = \frac{1}{2}\left(32 - \frac{64}{3}\right) - \left(\frac{4}{2} - \frac{1}{3}\right)$

$= \frac{1}{2}\left(\frac{96-64}{3}\right) - \left(\frac{12-2}{6}\right)$

$= \frac{1}{2}\left(\frac{32}{3} - \frac{10}{6}\right)$ lcm

$32 \times 2 = 64$

$= \frac{1}{2}\left(\frac{64-10}{6}\right)$

$= \frac{1}{2}\left(\frac{64-10}{6}\right)$

$\frac{1}{2}\left(\frac{54}{6}\right)$

$= \frac{54}{12} = \frac{9}{2}$

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Evaluate $\iint xy\,dx\,dy$. where R is region bounded by x-axis ordinate. $x=2a$ and the curve $x^2=4ay$.

Sol since $x^2=4ay$ is parabola curve symmetrical y-axis To find the point integration with this curve and $x=2a$

Hence $x^2=4ay$.

$$(2a)^2=4ay$$

$$4a^2=4ay$$

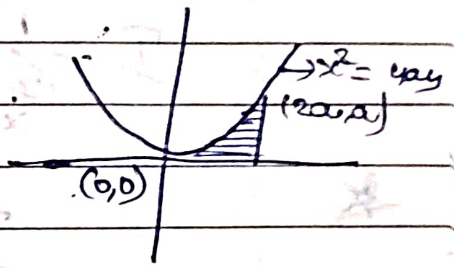
$$\cancel{4a^2}=\cancel{4a}y \Rightarrow y=a$$

$$x^2=4ay$$

$$\frac{x^2}{4a}=y$$

$$I=\iint_R xy\,dx\,dy=\int_{x=0}^{2a}\int_0^{x^2/4a} xy\,dy\,dx$$

$$I=\int_{x=0}^{2a} x\left[\frac{y^2}{2}\right]_0^{\frac{x^2}{4a}} dx$$



$$I=\int_{x=0}^{2a} x\left(\frac{x^2}{4a}\right)^2 dx$$

$$I=\int_{x=0}^{2a} x\cdot\frac{x^4}{32a^2}\,dx$$

$$I=\frac{1}{32a^2}\int_{x=0}^{2a} x^5\,dx$$

$$=\frac{1}{32a^2}\left[\frac{x^6}{6}\right]_0^{2a}$$

$$=\frac{1}{32a^2}\cdot\frac{(2a)^6}{6}$$

$$=\frac{1}{32a^2}\cdot\frac{64a^6}{6}=\frac{a^4}{3}$$

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Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circle.

$x = x^2+y^2=a^2$

$$I = \iint xy\,dx\,dy = \int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} xy\,dy\,dx$$

$$I = \int_{x=0}^{a} x\cdot\left[\frac{y^2}{2}\right]_0^{\sqrt{a^2-x^2}} dx$$

$$I = \int_{x=0}^{a} x\left(\frac{\left(\sqrt{a^2-x^2}\right)^2}{2}\right)$$

$$I = \int_{x=0}^{a} x\left(\frac{a^2-x^2}{2}\right) = \int_{x=0}^{a}\frac{xa^2-x^3}{2}$$

$$I = \frac{1}{2}\int_{x=0}^{a} xa^2-x^3 = \frac{1}{2}\left(\frac{x^2}{2}a^2-\frac{x^4}{4}\right)_0^a$$

$$I = \frac{1}{2}\left(\frac{a^2}{2}a^2-\frac{a^4}{4}\right)$$

$$I = \frac{1}{2}\left(\frac{2a^4}{4}-\frac{a^4}{8}\right)$$

$$I = \frac{1}{2}\left(\frac{a^4}{4}\right) = \left(\frac{a^4}{8}\right)$$

$$\boxed{I = \frac{a^4}{8}}$$

Evaluate the following Triple Integrals

(1) $\int_0^1 \int_0^2 \int_1^2 x^2yz\,dx\,dy\,dz$

Sol. $I = \int_{z=0}^{1} \int_{y=0}^{2} \int_{x=1}^{2} x^2yz\,dx\,dy\,dz$

$I = \int_{z=0}^{1} \int_{y=0}^{2} \left.\frac{x^3}{3}\right|_1^2 yz\,dy\,dz$

$= \int_{z=0}^{1} \int_{y=0}^{2} \frac{7}{3}\,yz\,dy\,dz$

$I = \frac{7}{3} \int_{z=0}^{1} \left.\frac{y^2}{2}\right|_0^2 z\,dz$

$I = \frac{14}{3} \int_{z=0}^{1} z\,dz$

$I = \frac{14}{3} \left.\frac{z^2}{2}\right|_0^1$

$I = \frac{\cancel{14}^{\,7}}{3} \times \frac{1}{\cancel{2}} = \boxed{\frac{7}{3}}$ Ans

(2) $\int_0^4 \int_0^5 \int_0^2 xyz\,dz\,dy\,dx$

Sol. $I = \int_{z=0}^{4} \int_{y=0}^{5} \int_{x=0}^{2} xyz\,dz\,dy\,dx$

$I = \int_{z=0}^{4} \int_{y=0}^{5} xy \cdot 2$

$\Rightarrow 2 \int_{x=0}^{4} \left.\frac{y^2}{2}\right|_0^5 \Rightarrow 2 \int_{x=0}^{4} x \cdot \frac{25}{2}$

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$$\Rightarrow \frac{50}{2}\int_{x=0}^{4} x$$

$$\Rightarrow \frac{50}{2}\left[\frac{x^2}{2}\right]_0^4$$

$$\Rightarrow \frac{50}{2}\left[\frac{\cancel{16}\,8}{\cancel{2}}\right] \Rightarrow \Rightarrow \frac{\cancel{400}\,200}{\cancel{2}} = 200$$

$$\boxed{I = 200}$$

* Evaluate the following triple Integral

$$\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dz\,dy\,dx$$

Sol:- $$I = \int_{y=0}^{1}\int_{x=y^2}^{1}\int_{z=0}^{1-x} x\,dz\,dy\,dx$$

$$I = \int_{y=0}^{1}\int_{x=y^2}^{1} x(z)_0^{1-x}\,dx\,dy.$$

$$I = \int_{y=0}^{1}\int_{x=y^2}^{1} x(1-x)\,dx\,dy.$$

$$I = \int_{y=0}^{1}\int_{x=y^2}^{1} (x-x^2)\,dx\,dy.$$

$$I = \int_{y=0}^{1}\left(\frac{x^2}{2}-\frac{x^3}{3}\right)_{y^2}^{1} dy$$

$$I = \int_{y=0}^{1}\left(\frac{1}{2}-\frac{y^4}{2}\right)-\left(\frac{1}{3}-\frac{y^6}{3}\right)dy$$

$$I = \int_{y=0}^{1}\left(\frac{1}{2}-\frac{1}{3}-\frac{y^4}{2}+\frac{y^6}{3}\right)dy$$

$$I = \int_0^1\left(\frac{1}{6}-\frac{y^4}{2}+\frac{y^6}{3}\right)$$

$$I = \left(\frac{1}{6}y - \frac{y^5}{10} + \frac{y^7}{21}\right)_0^1$$

$$I = \frac{1}{6} - \frac{1}{10} + \frac{1}{21} = \frac{4}{35}$$

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Evaluate the following double integrals by changing the order of integration

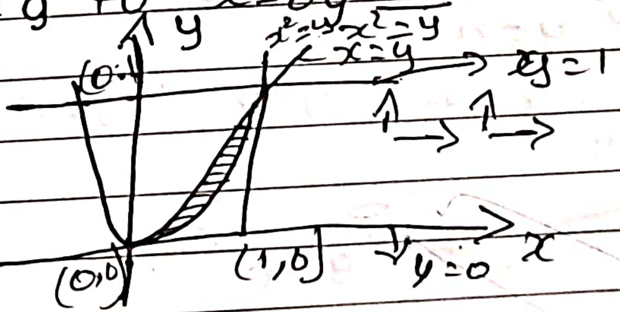
$$\int_0^1 \int_y^{\sqrt{y}} xy \, dx\, dy.$$

$$I = \int_{y=0}^{1} \int_{x=y}^{\sqrt{y}} xy \, dx\, dy$$

$\Rightarrow y = 0$ to $y = 1$

$x = y$ to $y = 1$

$x = y$ to $x = \sqrt{y}$ OR $x^2 = y$



To find the point of intersection.

consider $x = y$ and $x = \sqrt{y}$

$y = y^2 \Rightarrow y^2 - y = 0$

$y(y-1) = 0$

$\Rightarrow y = 0 \quad y = 1$

$x = y$

Put $y = 0$. $x = 0$

$y = 1$ $x = 1$

the pt of intersection $(0,0)$ & $(1,1)$

Now; we chang the order of Integration

$$I = \int_{x=0}^{1}\int_{y=x^2}^{x} xy\,dy\,dx$$

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$$I = \int_{x=0}^{1} x \left[\frac{y^2}{2}\right]_{x^2}^{x} dx$$

$$I = \frac{1}{2}\int_{x=0}^{1} x\left(x^2 - x^4\right) dx$$

$$I = \frac{1}{2}\int_{x=0}^{1} \left(x^3 - x^5\right) dx$$

$$I = \frac{1}{2}\left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^1$$

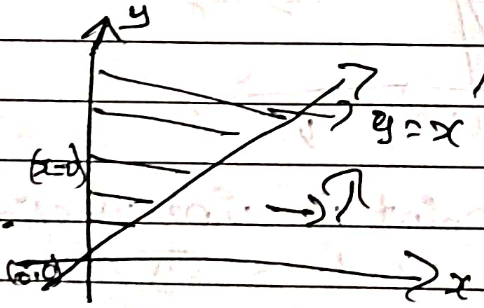
$$I = \frac{1}{2}\left[\frac{1}{4} - \frac{1}{6}\right]_0$$

$$I = \frac{1}{2}\left[\frac{6-4}{24}\right] = \frac{1}{2}\cdot\frac{2}{24} = \frac{1}{24}$$

★ $$\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx.$$

Sol: $$I = \int_{x=0}^{\infty}\int_{y=x}^{\infty} \frac{e^{-y}}{y}\,dy\,dx$$

$x=0 \cdot x=\infty$; $y=x$ to $y=\infty$.



To find pt of Intersection.

consider $y = x$

$y=0 \quad x=0$

$y=\infty \quad x=\infty$.

∴ there pt of intersection $(0,0)$ & (∞,∞)

now we change of order of Integrates

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$$I = \int_{y=0}^{\infty}\int_{x=0}^{y} \frac{e^{-y}}{y}\, dx\, dy$$

$$I = \int_{y=0}^{\infty} \frac{e^{-y}}{y}\, x \int_{0}^{y} dy$$

$$I = \int_{y=0}^{\infty} \frac{e^{-y}}{\cancel{y}}\, \cancel{y}\, dx$$

$$I = \int_{y=0}^{\infty} e^{-y} dy$$

$$I = -e^{-y}\Big|_{0}^{\infty}$$

$$I = -(e^{-\infty} - e^{-0})$$

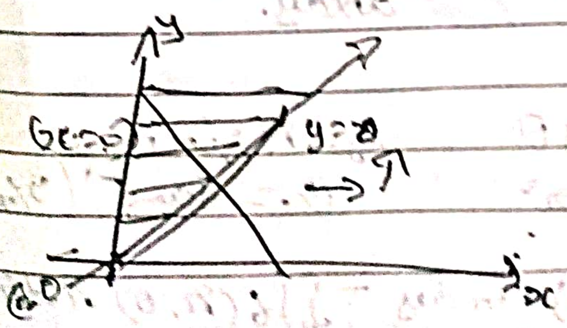
$$I = 1$$

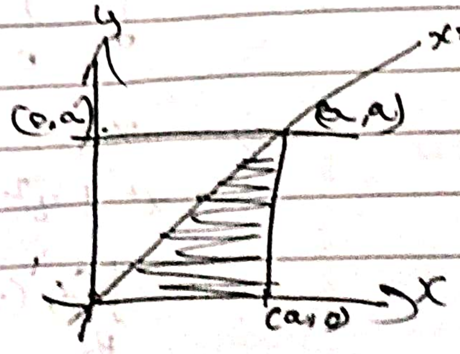
$$\int_{0}^{a}\int_{y}^{a} \frac{x}{x^2+y^2}\, dx\, dy.$$

$$\int_{y=0}^{a}\int_{x=y}^{a} \frac{x}{x^2+y^2}\, dx\, dy.$$

$x = 0$, $y = a$
$x = y$, $x = a$

~~$x=a$, $y=a$~~, $y=0$, $y=a$





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To find the pt of Intersection consider.

$y = x$.

$y = 0$ $\quad x = 0 \quad (0, 0)$

$y = a$ $\quad x = a \quad (a, 0)$

Point of Intersection $(0, 0)$ & (a, a).

Now we change the order of Integration

$$I = \int_{x=0}^{a} \int_{y=0}^{x} \frac{x}{x^2+y^2}\, dy\, dx$$

$$I = \int_{x=0}^{a} \tan^{-1}(y/x) \Big|_0^x dx$$

$$I = \int_{x=0}^{a} \tan^{-1}(1)\, dx.$$

$$I = \int_{x=0}^{a} \frac{\pi}{4}\, dx$$

$$= \frac{\pi}{4}\, x \Big|_0^a$$

$$\boxed{I = \frac{\pi a}{4}}$$

☆ Evaluate the following double integral by changing into polar Co-ordinate.

$$\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\, dx\, dy.$$

$\Rightarrow$ $x = r\cos\theta$ $\quad y = r\sin\theta$

$x^2 + y^2 = r^2$ $\quad \theta = \tan^{-1}(y/x)$

$$\iint_R f(x,y)\, dx\, dy = \iint_{R'} f(r, \theta)\, r\, dr\, d\theta$$

$dx\, dy = r\, dr\, d\theta$

$$I = \int_{y=0}^{\infty}\int_{x=0}^{\infty} e^{-(x^2+y^2)}\,dx\,dy.$$

$y=0$ to $y=\infty$
$x=0$ $\quad x=\infty$

$x^2+y^2=r^2$

$\boxed{r=0}$ $\quad \boxed{r=\infty}$

$\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(0)=0$

$\theta = \tan^{-1}(\infty)=\pi/2$

$$I = \int_{y=0}^{\infty}\int_{x=0}^{\infty} e^{-(x^2+y^2)}\,dx\,dy = \int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} e^{-r^2}\,dr\,d\theta$$

$$I = \int_{\theta=0}^{\pi/2} d\theta \int_{r=0}^{\infty} e^{-r^2} r\,dr = \int_{\theta=0}^{\pi/2} d\theta \cdot \int_{t=0}^{\infty} e^{-t}\frac{dt}{2} = [\theta]_0^{\pi/2}\frac{1}{2}$$

$$(-e^{-t})_0^{\infty}$$

put $r^2=t$

$2r\frac{dr}{dt}=1$

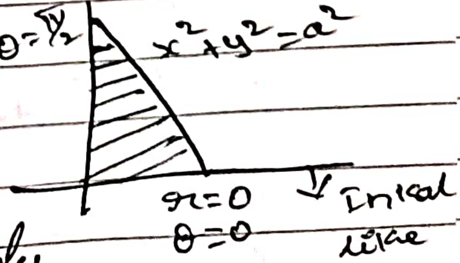
$2r\,dr=dt$

$r\,dr=\frac{dt}{2}$

$I = \frac{\pi}{2}\times\frac{1}{2}(-e^{-\infty}+e^{-0})$

$I = \frac{\pi}{2}\times\frac{1}{2} = \frac{\pi}{4}$

★ $$\int_0^a\int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dx\,dy$$



Sol: $$I = \int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dx\,dy$$

But WKT $x^2+y^2=r^2$ $\quad \theta=\tan^{-1}(y/x)$

$x=0$ to $x=a$
$y=\sqrt{a^2-x^2}$

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$x^2+y^2=a^2$

$a^2=r^2$

$\boxed{r=a}$ $r=0$

$$I=\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dx\,dy=\int_{\theta=0}^{\pi/2}\int_{r=0}^{a} r^2\sin^2\theta\sqrt{r^2}\; dr\,d\theta$$

$$I=\int_{\theta=0}^{\pi/2}\sin^2\theta\,d\theta\cdot\int_{r=0}^{a} r^2 r^2\,dr$$

Standard formula $\int_{\theta=0}^{\pi/2}\sin^n\theta\,d\theta=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{1}{2}\cdot\frac{\pi}{2}$

$$I=\frac{1}{2}\cdot\frac{\pi}{2}\times\left.\frac{r^5}{5}\right|_0^a$$

$$I=\frac{\pi}{4}\times\frac{a^5}{5}=\frac{\pi a^5}{20}$$

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★ Application of Double and Triple Integral. [formula]

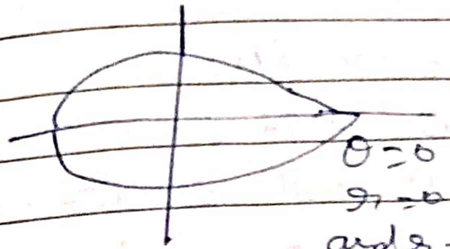
$\iint_R dx\,dy$ $\iiint_V dx\,dy\,dz$

1) $\iint_R dx\,dy$ = Area of region in the form cartesion.

$\iint_R r\,dr\,d\theta$ = " " " " Polar

$\iiint_V dx\,dy\,dz$ = Volume of the solid in the form cartesian

* Find area Enclosed by the cardioid $r = a(1+\cos\theta)$ · Betn $\theta = 0$ and $\theta = \pi$



$$\int_{\theta=0}^{\pi}\int_{r=0}^{a(1+\cos\theta)} r\,dr\,d\theta$$

$r = a(1+\cos\theta)$, Betn $\theta = 0$ and $\theta = \pi$

$$A = \int_{\theta=0}^{\pi}\int_{r=0}^{a(1+\cos\theta)} r\,dr\,d\theta$$

$$A = \int_{\theta=0}^{\pi} \left[\frac{r^2}{2}\right]_0^{a(1+\cos\theta)} d\theta$$

$$A = \int_{\theta=0}^{\pi} \frac{a^2(1+\cos\theta)^2}{2}$$

$(a+b)^2 = a^2+b^2+2ab$ formula applied

$$A = \frac{a^2}{2}\int_{\theta=0}^{\pi} (1+\cos^2\theta + 2\cos\theta)\,d\theta$$

formula

$$A = \frac{a^2}{2}\int_{\theta=0}^{\pi} \left(1+\frac{1+\cos 2\theta}{2} + 2\cos\theta\right) d\theta$$

$$A = \frac{a^2}{2}\int_{\theta=0}^{\pi}\left(1+\frac{1}{2}+\frac{\cos 2\theta}{2}+2\cos\theta\right)d\theta$$

$$A = \frac{a^2}{2}\int_{\theta=0}^{\pi}\left(\frac{3}{2}+\frac{\cos 2\theta}{2}+2\cos\theta\right)d\theta$$

$$A = \frac{a^2}{2}\left[\frac{3}{2}\theta\Big|_0^{\pi} + \frac{1}{2}\frac{\sin 2\theta}{2}\Big|_0^{\pi} + 2\sin\theta\Big|_0^{\pi}\right]$$

$$A = \frac{3a^2\pi}{4}$$

$$\boxed{A = \frac{3a^2\pi}{4} \text{ sq unit}}$$

★ Find the volume Bounded by the Surface $z = a^2 - x^2$ and the planes $x=0, x=a$, $z=0$ and $y=0$, $z=0$ and $y=b$

sol: Given $y=0$ and $y=b$

$x=0$ and $x=?$

$z=0$ and $z=a^2-x^2$

If $z=0$: $z = a^2 - x^2$

$0 = a^2 - x^2$

$a^2 = x^2$

$x = a$

WKT $\iiint dx\,dy\,dz$

$$V = \int_{y=0}^{b} \int_{x=0}^{a} \int_{z=0}^{a^2-x^2} dz\,dx\,dy.$$

$$V = \int_{y=0}^{b} \int_{x=0}^{a} z \Big|_0^{a^2-x^2} dx\,dy.$$

$$V = \int_{y=0}^{b} \int_{x=0}^{a} \left(a^2 x - \frac{x^3}{3}\right)\Big|_0^a dy.$$

$$V = \int_{y=0}^{b} \left(a^3 - \frac{a^3}{3}\right) dy$$

$$V = \frac{2a^3}{3} \int_{y=0}^{b} dy$$

$$V = \frac{2a^3}{3} (y)_0^b$$

$$\boxed{V = \frac{2a^3 b}{3}}$$

Beta Gamma Function

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Gamma function An Integral of the form called Gama Funⁿ & is denoted by

$$\int_0^{\infty} e^{-x} x^{n-1} dx \text{ is}$$

$$\Gamma n = \int e^{-x} x^{n-1} dx \text{ where } n > 0$$

Beta function:- An Integral of the form $\int_0^1 x^{n-1} (1-x)^{n-1} dx$ is call Beta Funⁿ it is denoted by

$$\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$$

where $m \& n > 0$.

properties

1) $\Gamma n+1 = n \Gamma n$

2) $\Gamma n+1 = n!$ if n is Positive Integer

3) $\Gamma \frac{1}{2} = \sqrt{\pi}$

4) $\beta(m,n) = \beta(n,m)$ β is symmetric

5) $\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$ Relⁿ Betⁿ Beta & Gamma function

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Problems

* using Beta Gamma function Evaluate

$$\int_0^{\pi/2} \sin^6\theta\, d\theta$$

Sol

$$\int_0^{\pi/2} (\sin^6\theta \cos^0\theta)\, d\theta$$

$$\frac{1}{2} B\left(\frac{6+1}{2}, \frac{0+1}{2}\right)$$

$$\frac{1}{2} B\left(\frac{7}{2}, \frac{1}{2}\right)$$

$$\frac{1}{2} \frac{\Gamma\frac{7}{2} \cdot \Gamma\frac{1}{2}}{\Gamma\frac{7}{2}+\frac{1}{2}} \Rightarrow \Rightarrow \Rightarrow \Rightarrow$$

$$\frac{1}{2} \cdot \frac{\Gamma\frac{5}{2}+1 \cdot \sqrt{\pi}}{\Gamma 4} =$$

$$\frac{7+1}{2} = \frac{8}{2}$$

$$\frac{1}{2} \frac{\frac{5}{2}\Gamma\frac{5}{2}\, \Gamma\frac{1}{2}}{6}$$

$$\Gamma 4 = (4-1)!$$

$$3! = 6$$

$$\frac{1}{2} \frac{\frac{5}{2}\Gamma\frac{3}{2}+1\, \sqrt{\pi}}{6}$$

$$\frac{\frac{1}{2} \times \frac{5}{2} \times \frac{3}{2}\Gamma\frac{1}{2}+1 \cdot \sqrt{\pi}}{6}$$

$$\frac{\frac{1}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \sqrt{\pi}}{6}$$

$$\int_0^{\pi/2} \sin^6\theta\, d\theta = \frac{15}{16 \times 6}$$

$$\int_0^{\pi/2} \sin^6\theta\, d\theta = \frac{\cancel{15}\,\pi}{\cancel{96}} = \frac{5\pi}{32}$$

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$\int_0^{\pi/2} \cos^4\theta \, d\theta$

$\int_0^{\pi/2} \sin^4\theta \cos^3\theta \, d\theta$

$\int_0^{\pi/2} \sqrt{\tan\theta} \, d\theta$

$(\tan\theta)^{1/2}$

$\left(\frac{\sin\theta}{\cos\theta}\right)^{1/2}$

$\sin^{1/2}\theta \cdot \cos^{-1/2}\theta$

$\int_0^{\pi/2} \cos^7\theta \, d\theta \Rightarrow \int_0^{\pi/2} \sin^0\theta \cdot \cos^7\theta \, d\theta$

$\Rightarrow \frac{1}{2} \beta\left(\frac{0+1}{2}, \frac{7+1}{2}\right)$

$\Rightarrow \frac{1}{2} \beta\left(\frac{1}{2}, 4\right) \Rightarrow \frac{1}{2} \frac{\sqrt{\frac{1}{2}}\sqrt{4}}{\sqrt{\frac{1}{2}+4}}$ $\qquad \sqrt{4} = (4-1)!$
$3!$

$\Rightarrow \frac{1}{2} \frac{\sqrt{\pi} \cdot 3!}{\sqrt{9/2}}$ $\qquad \sqrt{n} = (n-1)!$
$\sqrt{4} = (4-1)!$
$= 3!$

$\Rightarrow \frac{1}{\cancel{2}}\left(\frac{\sqrt{\pi} \cdot \cancel{6}3}{\sqrt{\frac{7}{2}+1}}\right) \Rightarrow \frac{\sqrt{\pi} \cdot 3}{\frac{7}{2} \sqrt{\frac{5}{2}+1}} \Rightarrow \frac{\sqrt{\pi} \cdot 3}{\frac{7}{2} \times \frac{5}{2} \sqrt{\frac{3}{2}+1}}$

$= \frac{3\sqrt{\pi}}{\frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \sqrt{\frac{1}{2}+1}}$ $\qquad = \frac{3\sqrt{\pi}}{\frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \sqrt{\frac{1}{2}}}$

$\Rightarrow \frac{3 \cdot \cancel{\sqrt{\pi}}}{\frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \cancel{\sqrt{\pi}}} = \frac{3}{\frac{105}{16}} = \frac{48}{105} = \frac{16}{35}$

$\boxed{I = \frac{16}{35}}$

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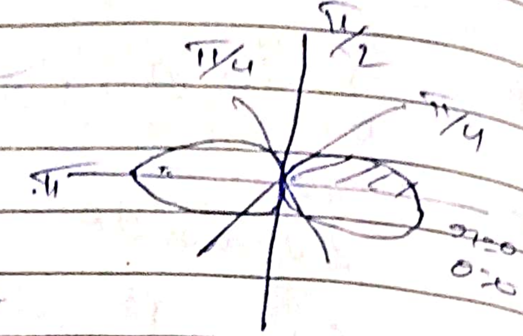
* find the total area of the lemniscate
$r^2 = a^2 \cos 2\theta$ · $\theta = 0$ to $\pi/4$

$A = \iint_R r\,dr\,d\theta$

$A = \int_{\theta=0}^{\pi/4} \int_{r=0}^{a\sqrt{\cos 2\theta}} r\,dr\,d\theta$



$A = \int_{\theta=0}^{\pi/4} \frac{r^2}{2}\Big|_0^{a\sqrt{\cos 2\theta}} d\theta$

$A = \int_{\theta=0}^{\pi/4} \frac{a^2(\sqrt{\cos 2\theta})^2}{2} d\theta.$

$A \frac{a^2}{2} \int_{\theta=0}^{\pi/4} \cos 2\theta\, d\theta.$

$A = \frac{a^2}{2} \frac{\sin 2\theta}{2}\Big|_0^{\pi/4}$

$A = \frac{a^2}{4}(\sin 2\pi/4 - \sin 0)$

$A = \frac{a^2}{4} \times 1$

$\boxed{A = \frac{a^2}{4}}$

* using Beta Gamma function show that
$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$

Sol Given $I_1 = \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \int_0^{\pi/2} \sin^{1/2}\theta\, d\theta$

$I_2 = \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} = \int_0^{\pi/2} \sin^{-1/2}\theta\, d\theta.$

$I_1 \times I_2 = \int_0^{\pi/2} \sin^{1/2}\theta \cos^0\theta\, d\theta \times \int_0^{\pi/2} \sin^{-1/2}$

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$I_1 \times I_2 = \frac{1}{2} B\left(\frac{\frac{1}{2}+1}{2}, \frac{0+1}{2}\right) \frac{1}{2} B\left(\frac{-\frac{1}{2}+1}{2}, \frac{0+1}{2}\right)$

$\cancel{I_1 \times I_2 = \frac{1}{2} B\left(\frac{\frac{1}{2}+1}{2}\right)}$

$I_1 \times I_2 = \frac{1}{4} B\left(\frac{3}{4}, \frac{1}{2}\right) B\left(\frac{1}{4}, \frac{1}{2}\right)$

$I_1 \times I_2 = \frac{1}{4} \frac{\Gamma\frac{3}{4}\,\Gamma\frac{1}{2}}{\Gamma\left(\frac{3}{4}+\frac{1}{2}\right)} \cdot \frac{\Gamma\frac{1}{4}\,\Gamma\frac{1}{2}}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}$

$\frac{1}{4} \frac{\Gamma\frac{3}{4}\sqrt{\pi}}{\Gamma\frac{3}{4}} \cdot \frac{\Gamma\frac{1}{4}\sqrt{\pi}}{\Gamma\frac{3}{4}}$

$\frac{1}{4} \frac{\pi \cdot \Gamma\frac{1}{4}}{\Gamma\left(\frac{1}{4}+1\right)}$

$\frac{1}{4} \frac{\pi \cdot \Gamma\frac{1}{4}}{\frac{1}{4} \cdot \Gamma\frac{1}{4}}$

$\boxed{I_1 \times I_2 = \pi}$

* using Beta Gamma function $\int_0^1 x^7(1-x^4)^3 dx$ and using the one results

$I = \frac{1}{n} B\left(\frac{m+1}{n}, p+1\right)$

Given $I = \int_0^1 x^7(1-x^4)^3 dx$

$m=7 \quad n=4 \quad p=3$

$I = \frac{1}{4} B\left(\frac{8}{4}, 4\right)$

$I = \frac{1}{4} B(2, 4)$

$I = \frac{1}{4} \frac{\Gamma 2\,\Gamma 4}{\Gamma(2+4)}$

$= \frac{1}{4} \frac{1 \times 6}{120}$

$I = \frac{1}{4} \frac{1 \times 6}{120}$ (120 → 40 → 20)

$\boxed{I = \frac{1}{80}}$

$I = \frac{1}{4} \frac{1!\ 3!}{\ldots}$

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