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Find the pedal equation of the follow curves

(1) $\frac{2a}{r} = (1+\cos\theta)$

Sol

$\log\left[\frac{2a}{r}\right] = \log(1+\cos\theta)$

$\log 2a - \log r = \log(1+\cos\theta)$.

Diff w.r.t 'θ'

$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta}(-\sin\theta)$.

$+\cot\phi = +\frac{2\sin\theta/2\,\cos\theta/2}{2\cos^2\theta/2}$

$\cot = \tan\theta/2$

$= \cot(\pi/2 - \theta/2)$

$\therefore \boxed{\phi = \pi/2 - \theta/2}$

we have

$p = r\sin\phi$

$p = r\sin(\pi/2 - \theta/2)$

$p = r\cos\theta/2 \rightarrow (2)$

from (1)

$\frac{2a}{r} = 1+\cos\theta$

$2\cos^2\theta/2 = \frac{2a}{r}$

$\cos^2\theta/2 = \frac{a}{r}$

$\cos\theta/2 = \sqrt{\frac{a}{r}}$

$\therefore 2 \Rightarrow p = r\cos\theta/2$

$p = r\cdot\sqrt{a/r}$

$p^2 = r^2\left(\frac{a}{r}\right)$

$\boxed{p^2 = ar}$

(2) $r = a(1+\cos\theta) \rightarrow (1)$

$\log(r=a) = \log(1+\cos\theta)$.

$\log r - \log a = \log(1+\cos\theta)$.

diff. w.r.t.

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta)$.

$\cot\phi = \frac{-2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}$

$\cot\phi = -\tan\theta/2$
$= \cot(\pi/2+\theta/2)$

$\therefore \phi = \boxed{\pi/2+\theta/2}$

we have

$p = r\sin\phi$

$p = r\sin(\pi/2+\theta/2)$

$p = r\cos\theta/2 \rightarrow (2)$

from (1)

$r = a(1+\cos\theta)$.

$r = 2a\cos^2\theta/2$

$\cos^2\theta/2 = \frac{r}{2a}$

$\cos\theta/2 = \sqrt{\frac{r}{2a}}$

$\therefore 2 \Rightarrow p = r\cos\theta/2$

$p = r\sqrt{\frac{r}{2a}}$

$p^2 = r^2\left(\frac{r}{2a}\right)$

$p^2 = \frac{r^3}{2a}$

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(3) $r^n = a^n \cos n\theta$

$\log(r^n = a^n) = \log(\cos n\theta)$.

$\log r^n - \log a^n = \log(\cos n\theta)$.

~~diff w~~

$n \log r = n \log a + \log \cos n\theta$

$n \log r = n \log a + \log \cos n\theta$

diff w.r.t. θ.

$n \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}(-\sin n\theta\, n)$.

$\cancel{n} \cot\phi = -\cancel{n} \tan n\theta$

$\cot\phi = \cot(\pi/2 + n\theta)$

$\therefore \phi = \pi/2 + n\theta$

$p = r\sin\theta$

$= r\sin(\pi/2 + n\theta)$

$p = r^n = a^n \cos n\theta$

$\cos n\theta = \frac{r^n}{a^n}$

(2) $\Rightarrow p = r \cdot \frac{r^n}{a^n}$

$$\boxed{p = \frac{r^{n+1}}{a^n}}$$

(4) $r^m = a^m \sin m\theta$

$\log r^m = a^m (\log \sin m\theta)$

$\log r^m = \log a^m \sin m\theta$

$m \log r - m \log m \sin m\theta$

diff w.r.t θ

$m \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\sin m\theta} (\cos m\theta \cdot m)$

$\cancel{m} \cot \phi = \cancel{m} \cot m\theta$

$\cot \phi = \cot m\theta$

$\boxed{\phi = m\theta}$

$p = r \sin\theta$

$p = r \sin m\theta \rightarrow (2)$

(1) $\Rightarrow r^m = a^m \sin m\theta$

$\sin m\theta = \frac{r^m}{a^m}$

(2) $\Rightarrow p = \frac{a \cdot r^m}{a^m}$

$\boxed{p = \frac{r^{m+1}}{a^m}}$

(5) $r = a\theta \rightarrow (1)$

$\log r = \log (a\theta)$

$= \log a + \log \theta$

diff w.r.t θ.

$\frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\theta}(1)$

$\boxed{\cot \phi = \frac{1}{\theta}}$

upto here same

we have

$p = r\sin\phi$

$p^2 = r^2\sin^2\phi$

$\frac{1}{p^2} = \frac{1}{r^2\sin^2\phi}$

$= \frac{1}{r^2}\csc^2\phi$

$= \frac{1}{r^2}(1+\cot^2\phi)$

$= \frac{1}{r^2}\left(1+\left(\frac{1}{\theta}\right)^2\right) \rightarrow (2)$

from

$(1) \Rightarrow r = a\theta$

$\boxed{\theta = r/a}$

$(2) \Rightarrow \frac{1}{p^2} = \frac{1}{r^2}\left(1+\frac{1}{r/a}\right)^2$

$\frac{1}{p^2} = \frac{1}{r^2}\left[1+\frac{a}{r}\right)^2$

$= \frac{1}{r^2}\left(1+\frac{a^2}{r^2}\right)$

$\boxed{\frac{1}{p^2} = \frac{1}{r^2}+\frac{a^2}{r^4}}$

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$r = a\theta$

Diff w.r.t θ:

$\frac{dr}{d\theta} = a \cdot 1 = a$

W.K.T.

$\frac{1}{p^2} = \frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

$= \frac{1}{r^2}+\frac{1}{r^4}$

$\boxed{\frac{1}{p^2} = \frac{1}{r^2}+\frac{a^2}{r^4}}$

(6) $r^2 = a^2\sec 2\theta$

$\log r^2 = a^2(\log\sec 2\theta)$.

$2\log r = 2\log a^2 + \log\sec 2\theta$

diff w.r.t θ.

$2\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sec 2\theta}(2\sec 2\theta\tan 2\theta)$

$\cancel{2}\cot\phi = \cancel{2}\tan 2\theta$

what if we measuring as angle and arclength

$\cot\phi = \cot(\pi/2 - 2\theta)$

$$\boxed{\phi = \pi/2 - 2\theta}$$

$p = r\sin\phi$

$= r\sin(\pi/2 - 2\theta)$

$p = r\cos 2\theta \rightarrow (2)$

$(1) \Rightarrow r^2 = a^2 \sec 2\theta$

$$\frac{1}{\sec 2\theta} = \frac{a^2}{r^2}$$

$$\cos 2\theta = \frac{a^2}{r^2}$$

$$(2) \Rightarrow p = \frac{r \cdot a^2}{r^2}$$

$$\boxed{p = a^2/r}$$

Important formulas:-

$\sin[90-\theta] = \cos\theta$

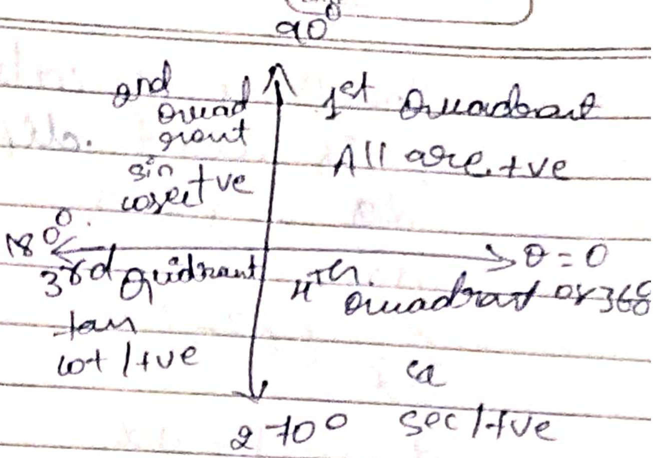
$\sin[90+\theta] = \cos\theta$

$\cos[90 - \theta] = \sin\theta$

$\cos[90+\theta] = -\sin\theta$

$\tan[90-\theta] = \cot\theta$

$\tan[90+\theta] = -\cot\theta$



$[a+b]^2 = a^2+b^2+2ab$

$[a-b]^2 = a^2-b^2+2ab$

$\sin 2\theta = 2\sin\theta.\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1-2\sin\theta$

$= 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

* where the angle $90 \pm \theta$ or $270 \pm \theta$

$\sin \rightleftarrows \cos$

$\tan \rightleftarrows \cot$

$\sec \rightleftarrows \operatorname{cosec}$

* when the angle is $180 \pm \theta$ or $360 \pm \theta$ there is no changes in the trigonometric function

$\sin[A+B] = \sin A.\cos B + \cos A.\sin B$

$\cos[A+B] = \cos A\cos B - \sin A.\sin B$

$\cos[A-B] = \cos A.\cos B + \sin A.\sin B$

$\tan[A+B] = \dfrac{\tan A + \tan B}{1-\tan A.\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A.\tan B}$

$\sin[A-B] = \sin A\cos B - \cos A\sin B$

$\tan[\pi/4+\theta] = \dfrac{\tan\pi/4 + \tan\theta}{1-\tan\pi/4\ \tan\theta}$

* Differentiation:-

$y = f(x)$ be a function.

$\dfrac{dy}{dx} = \lim_{x\to 0} \dfrac{f(x+h) - f(x)}{h}$

$y' \cdot \dfrac{d^2y}{dx^2} \quad \dfrac{d^3y}{dx^3}$

$y'' \quad y^2 \quad y^3$

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~~y~~ ~~is~~ x is called independent variable
y is called dependent variable

$\frac{d}{dx}\{a\} = 0$

$\frac{d}{dx}\{x\} = 1$

$\frac{d}{dx}(x^2) = 2x$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\frac{d}{dx}(e^{3x}) = 3e^{3x}$

$\frac{d}{dx}\{\sin ax\} = a\cos ax$

$\frac{d}{dx}\{\cos ax\} = -a\sin ax$

$\frac{d}{dx}(\tan x) = a\sec^2$

$\frac{d}{dx}\{\cot ax\} = \operatorname{cosec}^2 x$

$\frac{d}{dx}\{\sec x\} = \sec x\tan x$

$\frac{d}{dx}\{\operatorname{cosec} x\} = -\operatorname{cosec} x \times \cot x$

* Inverse :-

(1) $\frac{d}{dx}\{\sin^{-1}x\} = \frac{1}{1-x^2}$

(2) $\frac{d}{dx}\{\tan^{-1}x\} = \frac{1}{\sqrt{1+x^2}}$

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(3) $\frac{d}{dx}[\tan^{-1}x] = \frac{1}{1+x^2}$

(4) $\frac{d}{dx}\{\cot^{-1}x\} = \frac{-1}{1+x^2}$

(5) $\frac{d}{dx}(\sec^{-1}x) =$

(6) $\frac{d}{dx}\{\operatorname{cosec}^{-1}x\} =$

* product rule:-

$\frac{d}{dx}\{uv\} = u\frac{d}{dx}[v] + v\frac{d}{dx}[u]$

$= uv' + vu'$

$\frac{d}{dx}[u \pm v] = \frac{d}{dx}\{u\} \pm \frac{d}{dx}\{v\}$

* coitient rule:-

$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{d}{dx}\{u\} - u\frac{d}{dx}\{v\}}{v^2} = \frac{vu' - uv'}{v^2}$

Integration:-

(1) $\int 1 \cdot dx = x$

(2) $\int a \cdot dx = ax$

(3) $\int x\,dx = \frac{x^2}{2}$

(4) $\int x^2\,dx = \frac{x^3}{3}$

(5) $\int x^4\,dx = \frac{x^{n+1}}{n+1}$

(6) $\int e^x\,dx = e^x$

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(7) $\int e^{ax}\,dx = \frac{e^{ax}}{a}$

(8) $\int \frac{1}{x}\,dx = \log x$

(9) $\int \log x \cdot dx = x \log x - x$

(10) $\int \sin x \cdot dx = -\cos x$

(11) $\int \cos x\,dx = \sin x$

(12) $\int \sec^2 x\,dx = \tan x$

(13) $\int \operatorname{cosec}^2 x\,dx = -\cot x$

(14) $\int \tan x\,dx = \log(\sec x)$

(15) $\int \cot x\,dx = \log(\sin x)$

Questions:-

(1) $\frac{d}{dx}[x^3 \cdot \sin x]$

$\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$

$u = x^3;\ v = \sin x$

$\frac{d}{dx}[x^3 \sin x] = x^3 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^3)$

$= x^3 \cos x + \sin x\,(3x^2)$

$= x^3 \cdot \cos x + 3x^2 \sin x$

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② $\frac{d}{dx}[e^{2x}\tan x)$

Sol $e^{2x}\frac{d}{dx}\tan x + \tan x\frac{d}{dx}e^{2x}$

$e^{2x}[\sec^2 x]\cdot + \cdot\tan x[2e^{2x}]$

$e^{2x}\sec^2 x + 2e^{2x}\tan x.$

③ $\frac{d}{dx}\{\frac{1}{x}\cdot\cos x\}$

Sol $\frac{1}{x}\{\frac{d}{dx}\cos x\} + \cos x\{\frac{d}{dx}\frac{1}{x}\}$

$\frac{1}{x}\cdot(-\sin x] + \cos x\{\frac{-1}{x^2}\}$

$= \frac{-\sin x}{x} \quad \frac{-\cos x}{x^2}.$

④ $\frac{d}{dx}(e^{-3x}\sec x)$

Sol $= e^{-3x}\frac{d}{dx}(\sec x \tan x] + \sec x(-3e^{-3x})$

$= e^{-3x}\sec x\tan x - 3e^{-3x}\sec x.$

⑤ $\frac{d}{dx}[1+\cos 2x]$

$\Rightarrow \frac{d}{dx}(1). + \frac{d}{dx}(\cos 2x)$

$= 0 \cdot \sin 2x \cdot 2$

$= -2\sin 2x$

⑥ $\frac{d}{dx}[3^x - \log x]$

$\Rightarrow \frac{d}{dx}(3x) - \frac{d}{dx}(\log x).$

$= 3^x\log 3 - \frac{1}{x}$

(7) $\frac{d}{dx}\left[\frac{\operatorname{cosec} x}{x^2}\right]$ [∵ use quotient rules]

$$\frac{x^2 \frac{d}{dx}[\operatorname{cosec} x] - \operatorname{cosec} x \frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 \operatorname{cosec} \cot x - 2x \operatorname{cosec} x}{x^4}.$$

(8) $\frac{d}{dx}\left[\frac{e^{4x}}{x^3}\right] \begin{matrix} .u \\ v. \end{matrix}$

$\rightarrow \frac{vu' - uv'}{v^2}$ $\quad u = e^{4x}. \quad v = x^3$

$$= \frac{x^3 (4e^{4x}) - e^{4x}\, 3x^2}{x^6}.$$

$$= \frac{4e^{4x} \cdot x^3 - 3e^{4x} \cdot x^2}{x^6}.$$

logarithms.

(1) $\log_e (m/n) = \log_e m + \log_e n.$

(2) $\log_e (m/n) = \log_e m - \log_e n$

(3) $\log_e (m^n) = n \log_e m.$

Problems:

(1) $y = 1 + \cos x$

$\rightarrow \log y = \log(1 + \cos x)$

diff wrt to x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{1+\cos x}[0 - \sin x]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{-\sin x}{1+\cos x}$$

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(8) $y = 1 + \sin x$

$\rightarrow$ T.C.B.S, $\log y = \log(1+\sin x)$

we write $\frac{dy}{dx}$ because y is a function of f(x) y = f(x).

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+\sin x}$$

$$= \frac{\cos x}{1+\sin x}$$

(3) $y(1-\tan x) = 3$

Sol T.C.B.S $\log\{y(1+\tan x)\} = \log 3$

$$\log y + \log(1-\tan x) = \log 3$$

diff w.r.t o x

$$\frac{1}{y}\frac{dy}{dx} + \frac{1}{1-\tan x}(0-\sec^2 x) = 0$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sec^2 x}{1-\tan x}$$

Alternate method:-

$$y(1-\tan x) = 3.$$

$$y = \frac{3}{1-\tan x}$$

$$\log y = \log\left\{\frac{3}{1+\tan x}\right\}$$

$$\log y = \log 3 - \log(1-\tan x)$$

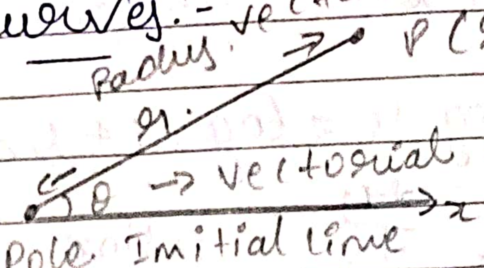
diff w.r.to: x

$$\frac{1}{y}\cdot\frac{dy}{dx} = 0 - \frac{1}{1-\tan x}\{0-\sec^2 x\}$$

$$= \frac{\sec^2 x}{1-\tan x}$$

Differential (calculus)

* polar curves :- vector.

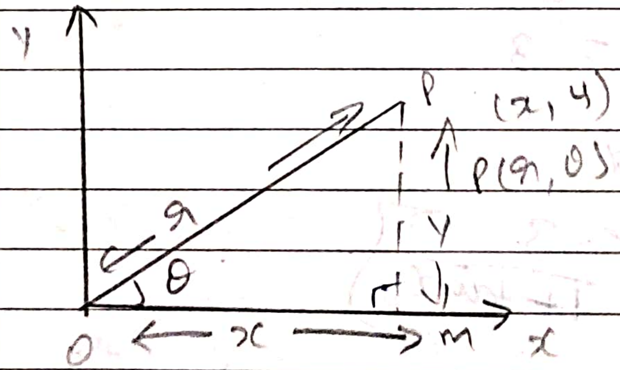


r & θ = Polar co-ordinates

$r = f(\theta)$ polar curve

* A point o with the plain is called pole
* A line ox drawn through o is called. initial line.
* let p be any point int the plane Join op that makes an angle θ with the initial line ox is called vectorial angle
* The length op = r is called radius vector
* The pair r & θ represented by P(r, θ) or P = (r, θ) are called polar co-ordinates by the point P.
* The polar curve is given by $r = f(\theta)$.

Relation between cartesion & Polar co-ordinates.



let (x, y) & (r, θ) be respectively represents cartesian and polar co-ordinates at any point P.
ox is the initial line join op which makes an angles θ with the initial line and op = r Draw pm perpendicular to the initial line so that om = x pm = y.

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* from the right angle triangle we have.

$\cos\theta = \frac{OM}{OP}$ $\sin\theta = \frac{PM}{OP}$.

$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$.

$\boxed{x = r\cos\theta} \rightarrow (1)$ $\boxed{y = r\cdot\sin\theta} \rightarrow (2)$

∴ eq^n (1) and (2) represents the relation b/w cartesian and polar coordinates.

- squaring and adding eq^n (1) & (2) we get. $x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta$.

$= r^2(\cos^2\theta + \sin^2\theta)$.

$= r^2(1)$.

$r^2 = x^2 + y^2$

$\boxed{r = \sqrt{x^2 + y^2}}$

∴ divide eq^n (2) by eq^n (1) we get

$\frac{y}{x} = \frac{r\sin\theta}{r\cdot\cos\theta}$

$\frac{y}{x} = \tan\theta$.

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left[\frac{y}{x}\right] \rightarrow (4)$

∴ eq^n (3) and (4) represents the relation b/w polar and cartesian co-ordinates.

Angle b/w Radius vector & tangent :-

Angle b/w radius vector & tangent is given by $\boxed{\tan\phi = r\frac{d\theta}{dr}}$.

$$\boxed{\cot\phi = \frac{1}{r}\frac{dr}{d\theta}}$$ x_1

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length of the perpendicular from the pole to the tangent.

$$\boxed{p = r\sin\phi} \rightarrow (1).$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^n}\left[\frac{dr}{d\theta}\right]^2$$

Angle b/w two curves :-

* the angle b/w two curves is the angle b/w their tangent and is given by $|\phi_1 - \phi_2|$

$$\tan|\phi_1 - \phi_2| = \left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1 - \tan\phi_2}\right|$$

$\tan\phi_1 \; \tan\phi_2 = -1$

[If $\tan\phi_1 \cdot \tan\phi_2 = -1$] we say that

The angle b/w the curves is $\pi/2$ (90^0) or we say that the two curves intersect each other orthogonally.

Note :- $\log mn = \log_e m + \log_e n$

$\log_e (m/n) = \log_e m - \log_e n$.

$\log_e m^n = n \log_e m$.

Problems

* find the angle b/w radius vector and the tangents for the following curves.

(i) $r = a(1-\cos\theta) \rightarrow (1)$ at $\theta = \pi/3$

→ Taking log on both sides

$\log r = \log[a(1-\cos\theta)]$

$\log r = \log a + \log(1-\cos\theta)$

Diff w.r.t θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}\cdot(0-(-\sin\theta))$$

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$$\cot\phi = \frac{\sin\theta}{1-\cos\theta}$$

$$\cot\phi = \frac{2\sin\theta/2\ \cos\theta/2}{2\ \sin\theta/2}$$

$$\cot\phi = \frac{\cos\theta/2}{\sin\theta/2}$$

$$\cot\phi = \cot\theta/2$$

$$\therefore \boxed{\phi = \theta/2}$$

At $\theta = \pi/3$ $\phi = \frac{\pi/3}{2} = \pi/6$

$\therefore$ The angle b/w the radius vector & tangent for the given curve is $\pi/6$.

(ii) $r^n = a^n \cos n\theta$

Sol Taking log on both sides

$\log r^n = \log(a^n \cos n\theta)$

$n\log r = \log a^n + \log\cos n\theta$

$n\log r = n\log a + \log\cos n\theta$

diff w.r.to θ

$$n\cdot\frac{1}{r}\frac{dr}{d\theta} = n(0) + \frac{1}{\cos n\theta}\cdot[-\sin n\theta\cdot n]$$

$\cancel{n}\cot\phi = -n\tan n\theta$

$\cot\phi = -\tan n\theta$

$\cot\phi = \cot(\pi/2 + n\theta)$

$\boxed{\therefore \phi = \pi/2 + n\theta}$

(iii) $r^n = a^n \sin n\theta$

$\rightarrow$ Taking log on both sides

$\log r^n = \log(a^n \sin n\theta)$

$n\log r = \log a^n + \log\sin n\theta$

$n\log r = n\log a + \log\sin n\theta$

diff w.r.to θ

$$n \cdot \frac{1}{r} \frac{dr}{d\theta} = n(0) + \frac{1}{\sin n\theta}[\cos n\theta \cdot n]$$

$$n \cdot \cot \phi = \cot n\theta \cdot n$$

$$\cot \phi = \cot n\theta$$

$$\boxed{\phi = n\theta}$$

(iv) $r = a(1+\cos\theta)$

→ Taking log on both sides.

$$\log r = \log[a(1+\cos\theta)]$$

$$\log r = \log a + \log(1+\cos\theta)$$

Diff w.r.to θ.

$$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(0-\sin\theta)$$

$$\cot \phi = \frac{-\sin\theta}{1+\cos\theta}$$

$$\cot \phi = \frac{-\cancel{2}\sin\theta/2 \, \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$

$$\cot \phi = \frac{-\sin\theta/2}{\cos\theta/2}$$

$$\cot \phi = -\tan\theta/2$$

$$\cot \phi = \cot(\pi/2 + \theta/2)$$

$$\boxed{\phi = \pi/2 + \theta/2}$$

(v) $r^2 \cos 2\theta = a^2$

→ Taking log on both sides.

$$\log[r^2 \cos 2\theta] = \log a^2$$

$$\log r^2 + \log \cos 2\theta = 2\log a$$

$$2\log r + \log \cos 2\theta = 2\log a.$$

Diff w.r.to θ.

$$2 \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} + \frac{1}{\cos 2\theta}[-\sin 2\theta \cdot 2] = 2(0)$$

$$2\cot\phi - \frac{2 \cdot \sin 2\theta}{\quad} = 0$$

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$2\cot\phi - 2\tan 2\theta = 0.$

$\cancel{2}\cot\phi = \cancel{2}\tan 2\theta.$

$\cot\phi = \cot\{\pi/2 - 2\theta\}.$

$\boxed{\therefore \phi = \pi/2 - 2\theta}$

(vi) $r^m = a^m[\cos m\theta + \sin m\theta]$

→ Taking log on both sides.

$\log r^m = \log\{a^m(\cos m\theta + \sin m\theta)\}$

$m\log r = m\log a + \log(\cos m\theta + \sin m\theta)$

Diff w.r.to θ.

$m \cdot \frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta + \sin m\theta}(-\sin m\theta + \cos m\theta)$

$m\cot\phi = m\frac{[\cos m\theta - \sin m\theta]}{[\cos m\theta + \sin m\theta]}.$

$\cot\phi = \frac{\frac{\cos m\theta}{\cos m\theta} - \frac{\sin m\theta}{\cos m\theta}}{\frac{\cos m\theta}{\cos m\theta} + \frac{\sin m\theta}{\cos m\theta}}$

$\cot\phi = \frac{1 - \tan m\theta}{1 + \tan m\theta} = \cot[\pi/4 + m\theta]$

$\boxed{\phi = \pi/4 + m\theta.}$

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* Find the angle b/w the radius vector & tangent to the cardioide, $r = a(1+\cos\theta)$ hence show that the tangent is parallel to the initial line $\theta = \pi/3$.

(1)

$\Rightarrow r = a(1+\cos\theta)$

Taking log on b.s

$\log r = \log a[1+\cos\theta]$

$\log r = \log a + \log\{1+\cos\theta\}$

Diff w.r to θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}\cdot[0-\sin\theta]$$

$$\cot\phi = \frac{-\sin\theta}{1+\cos\theta}$$

$$\cot\phi = \frac{-2\sin\theta/2\ \cos\theta/2}{2\cos^2\theta/2}$$

$$\cot\phi = -\tan\theta/2$$

$$\cot\phi = \cot(\pi/2+\theta/2)$$

$$\boxed{\phi = \pi/2 + \theta/2} \rightarrow (1)$$

we have $\psi = \phi + \theta$

$$\psi = \pi/2 + \frac{\theta}{2} + \theta$$

$$= \pi/2 + \frac{\theta + 2\theta}{2}$$

$$\boxed{\psi = \pi/2 + \frac{3\theta}{2}} \rightarrow (2)$$

at $\theta = \pi/3$

(2) $\Rightarrow \psi = \pi/2 + 3/2\,(\pi/3)$

$$\psi = \pi/2 + \pi/2$$

$$\boxed{\psi = \pi}$$

we know that slope of the tangent
$\tan \psi = \tan \pi = 0$.
since $\tan \psi = 0$, the slope of the tangent is parallel to the initial line
at $\theta = \pi/3$

(2) find the angle b/w radius vector and tangent to the curve $r^2 = a^2 \cos 2\theta$.

A→ $r^2 = a^2 \cos 2\theta$.
Taking log on b.s
$\log r^2 = \log a^2 \cos 2\theta$.
$2 \log r = 2 \log a + \log \cos 2\theta$.
diff. w.r.to θ.

$$2 \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos 2\theta} (-\sin 2\theta) 2$$

$$2 \cot \phi = \frac{-2 \sin 2\theta}{\cos 2\theta}$$

$$\cot \phi = \frac{-\sin 2\theta}{\cos 2\theta}$$

$$\cot \phi = -\tan 2\theta$$

$$\cot \phi = \cot (\pi/2 + 2\theta).$$

$$\boxed{\phi = \pi/2 + 2\theta}$$

(3) find the slope of the tangent to the curve $r^2 = a^2 \cos 2\theta$.

→ Taking log on b.s.
$\log r^2 = \log [a^2 (\cos 2\theta)]$
$2 \log r = 2 \log a + \log \cos 2\theta$.
diff. w.r.t θ.

$$2 \cdot \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos 2\theta} (-\sin 2\theta) . 2.$$

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$$\cancel{2} \cdot \cot\phi = \frac{-\cancel{2} \cdot \sin 2\theta}{\cos 2\theta}$$

$$\cot\phi = \frac{-\sin 2\theta}{\cos 2\theta}$$

$$\cot\phi = -\tan 2\theta$$

$$\cot\phi = \cot(\pi/2 + 2\theta)$$

$$\boxed{\phi = \pi/2 + 2\theta}$$

$$\psi = \phi + \theta$$

$$= \pi/2 + 2\theta + \theta$$

$$\boxed{\psi = \pi/2 + 3\theta}$$

$$\tan\psi = \tan(\pi/2 + 3\theta)$$ //

★ find the angle of intersection of the following pair $r = \sin\theta + \cos\theta$, $r = 2\sin\theta$.

★ $\log r = \log(\sin\theta + \cos\theta)$

Diff w.r.to θ.

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{\sin\theta + \cos\theta}(\cos\theta - \sin\theta)$$

$$\cot\phi_1 = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$\cot\phi_1 = \frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$\cot\phi_1 = \frac{1 - \tan\theta}{1 + \tan\theta}$$

$$\cot\phi_1 = \cot[\pi/4 + \theta]$$

$$\boxed{\phi_1 = \pi/4 + \theta}$$

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consider ϕ_2

$r = 2\sin\theta$

$\log r = \log[2\sin\theta]$

$= \log 2 + \log\sin\theta$

Diff wr to θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sin\theta}[\cos\theta]$

$\cot\phi_2 = \cot\theta$

$\boxed{\phi_2 = \theta}$

we have $\phi_1 - \phi_2$

$\therefore |\phi_1 - \phi_2| = |\pi/4 + \cancel{\theta} - \cancel{\theta}$

$= \pi/4$

(2) $r = a(1+\cos\theta)$ $r = a(1-\cos\theta)$

Ans $\log r = \log[a(1+\cos\theta)]$

$\log r = \log a + \log(1+\cos\theta)$

Diff w.r.to θ

$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{(1+\cos\theta)}(0 - \sin\theta$

$\cot\phi_1 = \frac{-\sin\theta}{1+\cos\theta}$

$\cot\phi_1 = \frac{-\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$

$\cot\phi_1 = -\tan\theta/2$

$\cot[\pi/2 + \theta/2]$

$\phi_1 = $

$\phi_1 = [\pi/2 + \theta/2] \rightarrow (1)$

$r = a(1-\cos\theta)$

$\log r = \log a + (1-\cos\theta)]$

$\log r = \log a + \log(1-\cos\theta)$

Diff w.r. to θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(0+\sin\theta)$$

$$\cot\phi = \frac{\sin\theta}{1-\cos\theta}$$

$$\cot\phi = \frac{2\sin\theta/2\,\cos\theta/2}{2\sin^2\theta/2}$$

$$\cot\phi = \cot\theta/2$$

$$\boxed{\phi = \theta/2} \rightarrow (2)$$

$$|\phi_1 - \phi_2| = |\pi/2 + \cancel{\theta/2} - \cancel{\theta/2}|$$

$$= \pi/2$$

(3) $r = a\cos\theta \quad 2r = a.$

taking log on b.s.

$\log r = \log[a\cos\theta]$.

$\log r = \log a + \log\cos\theta$.

Diff w.r. to θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\cos\theta}\cdot(-\sin\theta)$$

$$\cot\phi_1 = \frac{-\sin\theta}{\cos\theta}.$$

$\cot\phi_1 = -\tan\theta$.

$\cot\phi_1 = \cot[\pi/2 + \theta]$

$\phi_1 = [\pi/2 + \theta]$

$2r = a$

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$\log(2r) = \log a$

$\log 2 + \log r = \log a$

diff w.r.to θ.

$0 + \frac{1}{r} \cdot \frac{dr}{d\theta} = 0$

$\cot \phi_2 = 0$

$\phi_2 = \cot^{-1}(0) - \pi/2$

$\therefore$ consider $|\phi_1 - \phi_2| = [\pi/2 + \theta - [\pi/2]$

$|\phi_1 - \phi| = \theta$

$r = a\cos\theta \rightarrow (1)$, $2r = a$

$\Rightarrow r = a/2 \rightarrow (2)$

equation the eq^n (1) & (2) we get

$a\cos\theta = a/2$

$\cos\theta = 1/2$

$\theta = \cos^{-1}[1/3] = \pi/3$

$\theta = \pi/3$

$|\phi_1 - \phi_2| = \theta = \pi/3$

~~(4)~~ $r = a(1-\cos\theta)$ $r = 2a\cos\theta$

$\rightarrow \log r = \log[a(1-\cos\theta)]$

$\log r = \log a + \log(1-\cos\theta)$

diff w.r.to θ.

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{(1-\cos\theta)}(0 + \sin\theta)$

$\cot\phi_1 = \frac{\sin\theta}{(1-\cos\theta)}$

$\cot\phi_1 = \frac{2\sin\theta/2 \cos\theta/2}{2\sin^2\theta/2}$

$\cot\phi_1 = \cot\theta/2$ $[\phi_1 = \theta/2]$

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$r = 2a\cos\theta$.

$\log r = \log [2a\cos\theta]$

$\log r = \log 2 + \log a + \log\cos\theta$

Diff w.r.t θ.

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + 0 + \frac{1}{\cos\theta}(-\sin\theta)$

$\cot\phi_2 = \frac{\sin\theta}{\cos\theta}$.

$\cot\phi_2 = -\tan\theta$

$\cot\phi_2 = \cot(\pi/2 + \theta)$.

$\boxed{\phi_2 = \pi/2 + \theta}$

$|\phi_1 - \phi_2| = |\theta/2 - (\pi/2 + \theta)|$

$= |\theta/2 - \pi/2 - \theta|$

$= \left|\frac{\theta - \pi - 2\theta}{2}\right|$

$= \left|\frac{\theta - \pi}{2}\right|$

$= \left|-\left(\frac{\pi + \theta}{2}\right)\right|$

$|\phi_1 - \phi_2| = \frac{\pi + \theta}{2}$

(Q) $r = a\theta$ & $r = a/\theta$.

Ans $\log r = \log a\theta$

$\log r = \log a + \log\theta$.

Diff w.r.t. to θ.

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\theta}(1)$.

$\cot\phi_1 = \frac{1}{\theta} \Rightarrow \frac{1}{\cot\phi_1} = \frac{1}{1/\theta}$

$\boxed{\tan\phi_1 = \theta}$

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$r = a/\theta$

$\log r = \log(a/\theta)$

$\log r = \log a - \log \theta$

Diff w.r.to θ

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\theta} \cdot (1)$

$\cot \phi_1 = \frac{1}{\theta} \Rightarrow \frac{1}{\cot \phi_2} = \frac{1}{(-1/\theta)}$

$= \boxed{\tan \phi_2 = -\theta}$

consider

$r = a\theta$ & $r = a/\theta$

$\cancel{a} \cdot \theta = \frac{\cancel{a}}{\theta}$

$\theta^2 = 1$

$\theta = \sqrt{1}$

$\theta = \pm 1$

$\tan \phi_1 = \theta \quad \tan \phi_2 = -\theta$

when $\theta = 1$

$\tan \phi_1 = 1 \cdot \tan \phi_2 = -1$

$\tan \phi_1 - \tan \phi_2 = 1(-1) = -1$

$\therefore$ the given curves intersect each other orthogonaly (or) the angle b/w the given curves is $\pi/2$ when

$\theta = -1$

$\tan \phi_1 = 1 \quad \tan \phi_2 = -1$

$\tan \phi_1 - \tan \phi_2 = (-1)(1)$

$= -1$

$\therefore$ The given curves intersect each others orthogonally.

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(6) $r = \frac{a\theta}{1+\theta}$, $r = \frac{a}{1+\theta_2}$

A $\log r = \log\left\{\frac{a\theta}{1+\theta}\right\}$.

$\log r = \log a\theta - \log(1+\theta)$

$\log r = \log a + \log \theta - \log(1+\theta)$

Diff w.r to θ

$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\theta}(1) - \frac{1}{1+\theta}(0+1)$

$\cot\phi_1 = \frac{1}{\theta} - \frac{1}{1+\theta}$.

$\cot\phi_1 = \frac{1+\phi-\phi}{(1+\theta)}$.

$\cot\phi_1 = \frac{1}{\theta(1+\theta)}$.

$\boxed{\tan\phi = \theta(1+\theta)}$

$r = \frac{a}{1+\theta^2}$

$\to \log r = \log(a/1+\theta)$.

$= \log a - \log(1+\theta^2)$.

Diff w.r to θ.

$\frac{1}{r}\cdot\frac{dr}{d\theta} = \theta - \frac{1}{1+\theta^2}(2\theta)(1)$.

$\cot\phi_2 = \frac{-2\theta}{1+\theta^2}$

$\boxed{\tan\phi_2 = \frac{1+\theta^2}{-2\theta}}$

considers both the curves.

$r = \frac{a\theta}{1+\theta}$. $r = \frac{a}{1+\theta^2}$

$$\frac{a\theta}{1+\theta} = \frac{a}{1+\theta^2}$$

$\theta(1+\theta^2) = 1+\theta$

$\theta+\theta^3 = 1+\cancel{\theta}$

$\theta = \sqrt[3]{1}$

$\boxed{\theta = 1}$

we have

$\tan\phi_1 = \theta(1+\theta)$

$= 1(1+1)$

$\boxed{\tan\phi_1 = 2}$

$$\tan\phi_2 = \frac{1+\theta^2}{(1-2\theta)}$$

$$\tan\phi_2 = \frac{1+1}{-2}$$

$\boxed{\tan\phi_2 = -1}$

Hence $\tan|\phi_1-\phi_2| = \left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}\right|$

$$= \left|\frac{2-(-1)}{1+(2)(-1)}\right| = \left|\frac{3}{1-2}\right| = \left(\frac{3}{-1}\right)$$

$= |-3|$

$\tan|\phi_1-\phi_2| = 3$

$|\phi_1-\phi_2| = \tan^{-1}(3)$ //

(8) $r^2\sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$.

consider log on b.s.

$\log r^2\sin 2\theta = \log 4$

$\log r^2 + \log\sin 2\theta = \log 4$

diff w.r.to θ.

$$2\cdot\frac{1}{r}\cdot\frac{dr}{d\theta} + \frac{1}{\sin 2\theta}\cos 2\theta(2) = 0.$$

$$\cancel{2}\cot\phi_1 + \cancel{2}\frac{\cos 2\theta}{\sin 2\theta} = 0.$$

$$\cot\phi_1 = \frac{-\cos 2\theta}{\sin 2\theta}.$$

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$\cot \phi_1 = \dfrac{-\cos 2\theta}{\sin 2\theta}$

$\cot \phi_1 = -\cot 2\theta$

$\boxed{\phi_1 = -2\theta}$

consider 2nd curve

$r^2 = 16 \sin 2\theta$

$\log r^2 = \log 16 + \log \sin 2\theta$

diff w.r.to θ

$2 \dfrac{1}{r} \dfrac{dr}{d\theta} = 0 + \dfrac{1}{\sin 2\theta} \cos 2\theta (2)$

$\cot \phi_2 = \dfrac{\cos 2\theta}{\sin 2\theta}$

$\boxed{\phi_2 = 2\theta}$

$\therefore |\phi_1 - \phi_2| = |-2\theta - 2\theta|$

$= 4\theta$

consider both the curves

$r^2 \sin 2\theta = 4$

$r^2 = \dfrac{4}{\sin 2\theta} \rightarrow (1)$

$\dfrac{4}{\sin 2\theta} = 16 \sin 2\theta$

$4 = 16 \sin^2 2\theta$

$1 = \dfrac{\cancel{16}^{4}}{\cancel{4}_{1}} \sin^2 2\theta$

$\sin^2 2\theta = 1/4$

$\sin 2\theta = \pm \sqrt{1/4}$

$\sin 2\theta = 1/2$

$2\theta = \sin^{-1} [1/2]$

$2\theta = \pi/6$

$\boxed{\theta = \pi/12}$

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substitute value of θ.

$$|\phi_1 - \phi_2| = |\pi/2 - \pi/3|$$

$$= \pi/3$$

(8) $r = 2\cos\theta$ and $r = 2\sin\theta$.

Ans log on b.s.

$\log r = \log [2 \cdot \cos\theta]$

$\log r = \log 2 + \log \cos\theta$.

diff w.r.t θ.

$$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cos\theta}(-\sin\theta]$$

$\cot\phi_1 = -\tan\theta$.

$\cot\phi_1 = \cot(\pi/2 + \theta]$.

$$\boxed{\phi_1 = \pi/2 + \theta}$$

consider 2nd curve

$r = 2\sin\theta$.

$\log r = \log 2 + \log \sin\theta$.

diff w.r to θ.

$$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\sin\theta}(\cos\theta]$$

$$\boxed{\cot\phi_2 = \cot\theta}$$

$\therefore |\phi_1 - \phi_2| = |\pi/2 + \cancel{\theta} - \cancel{\theta}|$

$$= \pi/2$$

(9) $r = 6\cos\theta$ and $r = 2(1 + \cos\theta]$

A log on b.s.

$\log r = \log (6\cos\theta]$

$\log r = \log 6 + \log \cos\theta$.

diff w.r.to θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\cos\theta}(-\sin\theta)$$

$$\cot\phi_1 = -\tan\theta$$

$$\cot\phi_1 = \cot(\pi/2 + \theta)$$

$$\boxed{\phi_1 = \pi/2 + \theta}$$

consider 2nd curves

$$r = a(1+\cos\theta)$$

$$\log r = \log(a(1+\cos\theta))$$

$$\log r = \log a + \log(1+\cos\theta)$$

diff w.r.to θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(0 - \sin\theta)$$

$$\cot\phi_2 = \frac{-\sin\theta}{1+\cos\theta}$$

$$\cot\phi_2 = \frac{-\cancel{2}\sin\theta/2\cos\theta/2}{\cancel{2}\cos^2\theta/2}$$

$$\cot\phi_2 = -\tan\theta/2$$

$$\cot\phi_2 = \cot[\pi/2 + \theta/2]$$

$$\boxed{\phi_2 = \pi/2 + \theta/2}$$

~~whe~~ we have $|\phi_1 - \phi_2| = \pi/2 + \theta - \pi/2 - \theta/2$

$$= \theta - \theta/2$$

$$= \frac{2\theta - \theta}{2}$$

$$= \theta/2$$

consider Both the curves.

$r = 6\cos\theta \to ①$ $\quad r = 2(1+\cos\theta) \to ②$

from ① & ②

$6\cos\theta = 2(1+\cos\theta)$

$6\cos\theta = 2 + 2\cos\theta$

$6\cos\theta - 2\cos\theta = 2$

$4\cos\theta = 2$

$\cos\theta = \cancel{2}/\cancel{4}2$

$\cos\theta = 1/2$

$\theta = \cos^{-1}[1/2]$

$$\boxed{\theta = \pi/3}$$

Hence $|\phi_1 - \phi_2| = \pi/3$ 2

$$\boxed{|\phi_1 - \phi_2| = \pi/6}$$ //

show that the following pair of curves intersect each other orthogonally:-

① $r = a[1+\cos\theta]$ $\cdot$ $r = b(1-\cos\theta)$.

$\log r = \log(a(1+\cos\theta))$.

$\log r = \log a + \log(1+\cos\theta)$

Diff wor to θ

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(0 - \sin\theta)$

$\cot\phi_1 = \frac{+\sin\theta}{1+\cos\theta}$

$\cot\phi_1 = \frac{1+\cos\theta}{-\sin\theta}$

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$r = b(1-\cos\theta)$

$\log r = \log [b(1-\cos\theta)]$

$= \log b + \log (1-\cos\theta)$

Diff w.r.t θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$$

$$\cot\phi_2 = \frac{\sin\theta}{1-\cos\theta}$$

$$\cot\phi_2 = \frac{1-\cos\theta}{\sin\theta}$$

consider

$$\tan\phi_1 \tan\phi_2 = \left[\frac{1+\cos\theta}{-\sin\theta}\right]\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= \frac{1-\cos^2\theta}{-\sin^2\theta}$$

$$= \frac{\sin^2\theta}{-\sin^2\theta}$$

$$= -1$$

Thus the given curves intersect each other orthogonally.

(2) $r = a[1+\sin\theta]$ & $r = a(1-\sin\theta)$

Ans $\to \log r = \log [a(1+\sin\theta)]$

$\log r = \log a + \log (1+\sin\theta)$

Diff w.r.t θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{(1+\sin\theta)}(0+\cos\theta)$$

$$\cot\phi_1 = \frac{\cos\theta}{1+\sin\theta}$$

$$\cot\phi = \frac{1+\sin\theta}{\cos\theta}$$

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$r = a[1-\sin\theta]$
$\log r = \log a\,[1-\sin\theta]$
$\log r = \log a + \log[1-\sin\theta]$
Diff w.r.to θ.
$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{(1-\sin\theta)}\{0-\cos\theta\}.$$
$$\cot\phi_2 = \frac{-\cos\theta}{1-\sin\theta}$$
$$\cot\phi_2 = \frac{1-\sin\theta}{-\cos\theta}$$
consider,
$$\tan\phi_1 + \tan\phi_2 = \left(\frac{1+\sin\theta}{\cos\theta}\right)\left(\frac{1-\sin\theta}{-\cos\theta}\right)$$
$$= \frac{1-\sin^2\theta}{-\cos^2\theta}$$
$$= \frac{+\cos^2\theta}{-\cos^2\theta}$$
$$= -1$$

③ $r^n = a^n\cos n\theta$ & $r^n = b^n\sin n\theta$.
Ans $\log r^n = \log a^n\cdot\cos n\theta$
$n\log r = n\log a + \log\cos n\theta$
diff w.r.to θ.
$$n\cdot\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}\,-\sin n\theta\, n.$$
$$\cancel{n}\cot\phi_1 = -\cancel{n}\frac{\sin n\theta}{\cos n\theta}$$
$\cot\phi_1 = -\tan n\theta$.
$\cot\phi_1 = \cot(\pi/2 + n\theta)$
$\boxed{\phi = (\pi/2 + n\theta)}$
$r^n = b^n\sin n\theta$

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$\log r^n = \log b^n \sin n\theta$

$n \log r = n \log b + \log \sin n\theta$

diff w.r to θ

$n \cdot \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\sin n\theta} (\cos n\theta \cdot n)$

$n \cdot \cot \phi_2 = \frac{\cos n\theta \cdot n}{\sin n\theta}$

$\cancel{n} \cdot \cot \phi_2 = \cancel{n} \cdot \frac{\cos n\theta}{\sin n\theta}$

$\cot \phi_1 = \cot n\theta$

$\boxed{\phi_2 = n\theta}$

consider

$|\phi_1 - \phi_2| = |\pi/2 + \cancel{n\theta} - \cancel{n\theta}|$

$= \pi/2$

(4) $r = a(1+\cos\theta)$ & $r^2 = a^2 = a^2 \cos 2\theta)$

$\rightarrow \log r = \log [a(1+\cos\theta)]$.

$\log r = \log a + \log (1+\cos\theta)$.

Diff w.r.t to θ

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta} \cdot (a - \sin\theta)$

$\cot \phi_1 = \frac{-\sin\theta}{1+\cos\theta}$

$\cot \phi_1 = \frac{-2\sin^2 \theta/2 \cos \theta/2}{2\cos^2 \theta/2}$

$\cot \phi_1 = -\tan \theta/2$

$\cot \phi_1 = \cot(\pi/2 + \theta/2)$

$\phi_1 = \pi/2 + \theta/2$ //

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(5) $r = ae^{\theta}$; $re^{\theta} = b$

As log on b.s.

$\log r = \log (ae^{\theta})$

$\log r = \log a + \log e^{\theta}$

$\log r = \log_e a + \theta \log_e e$

$= \log a + \theta(1)$

$\log r = \log a + \theta$

diff w.r.to θ

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + 1$

$\cot \phi_1 = 1$

$\tan \phi_1 = 1$

$re^{\theta} = b$

$\log (re^{\theta}) = \log b$

$\log r + \log e^{\theta} = \log b$

$\log_e r + \theta \log e = \log b$

$\log_e r + \theta \log e = \log b$

$\log r + \theta(1) = \log b$

$\log r + \theta = \log b$

Diff w.r.to θ

$\frac{1}{r} \cdot \frac{dr}{d\theta} + 1 = 0$

$\cot \phi_2 + 1 = 0$

$\cot \phi_2 = -1$

$\boxed{\tan \phi_2 = -1}$

consider,

$\tan \phi_1 \cdot \tan \phi_2 = (1)(-1)$

$= -1$

(6) $r = \frac{a}{1 + \cos\theta}$; $r = \frac{b}{1 - \cos\theta}$

$\Rightarrow$ log on b.s.

$$\log r = \log\left[\frac{a}{1+\cos\theta}\right]$$

$$\log r = \log a \log 1+\cos\theta$$

Diff w.r.to θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 = \frac{1}{1+\cos\theta}(-\sin\theta)$$

$$\cot\phi_1 = \frac{-\sin\theta}{1+\cos\theta}$$

$$\tan\phi_1 = \frac{1+\cos\theta}{-\sin\theta}$$

$$r = \frac{b}{1-\cos\theta}$$

$$r = \frac{b}{1-\cos\theta}$$

$$\Rightarrow \log r = \log\left[\frac{b}{1-\cos\theta}\right].$$

$$\log r = \log b - \log(1-\cos\theta).$$

diff wr to θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \theta - \frac{1}{1-\cos\theta}(\sin\theta).$$

$$\cot\phi_2 = \frac{-\sin\theta}{1-\cos\theta}.$$

$$\tan\phi_2 = \frac{1-\cos\theta}{-\sin\theta}$$

consider.

$$\tan\phi_1 \times \tan\phi_2 = \frac{1+\cos\theta}{\sin\theta}\cdot\frac{1-\cos\theta}{-\sin\theta}$$

$$= \frac{1-\cos^2\theta}{-\sin^2\theta} = \frac{\sin^2\theta}{-\sin^2\theta} = -1 //$$

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(1) $r^2 \sin 2\theta = a^2$

Ans $\log r^2 + \log 2\theta = \log a^2$

$2 \log r + \log \sin 2\theta = 2 \log a$

diff w.r.t θ

$$2 \frac{1}{r} \cdot \frac{dr}{d\theta} + \frac{1}{\sin 2\theta}(\cos 2\theta \cdot 2) = 0$$

$$\cancel{2} \cot \phi_1 + \cancel{2} \frac{\cos 2\theta}{\sin 2\theta} = 0$$

$$\cot \phi_1 + \frac{\cos 2\theta}{\sin 2\theta} = 0$$

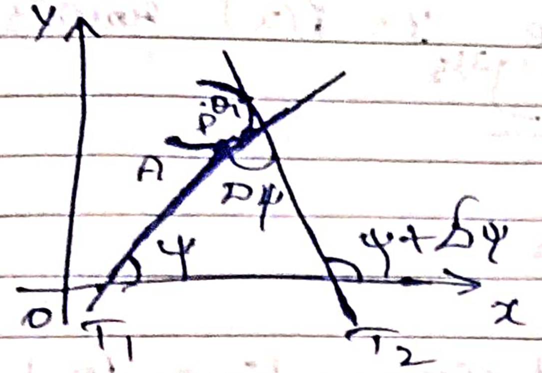
$$\cot \phi_1 = -\cot 2\theta$$

$$\phi_1 = -2\theta$$

after this having 1st 3 page

turn next

* curvature



δ Δ

let us consider A curve In the xoy plain let a be the fixat point, P and Q or nibouring points on the curve then are ~~Q~~ ark P is S ark Q = S + ΔS so that RPQ = S + S = ΔS − S

sis and $\Delta\phi$ be the angle mate by the tangent at P & Q with the x axis

$\Delta\psi$ ~~is called~~ b/w the tangent is called the bending of the curve witch depends on ~~delta~~ δS

$\Delta\psi$ upon ΔS is called mean curvetur of RPQ.

the amount of bending of curve at P. is called curveture of curve.

mathametically It is difined as limit ΔS is cuting ~~0~~ ~~0~~ $\frac{\Delta\psi}{\Delta s}$ = ~~$\rho = \frac{1}{K} = \frac{1}{d\psi/ds} = \frac{ds}{d\psi}$~~

The reciprocal curvatur is called as the raidus of curvetur and is given by

$$\rho = \frac{1}{K} = \frac{1}{\frac{d\psi}{ds}} = \frac{ds}{d\psi}$$

expresion for raidus of curvetur In case of cortion curve

$y = f(x)$

$\rho =$

For cartesian curve $y = f(x)$ radius is given by

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

Note: If the tangent perpendicular to the x axis parallel to the y axis $\psi = \pi/2$ $\therefore \tan\psi = \tan \pi/2 = \infty$. In this case we use.

$$\rho = \frac{\left[1+\left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}} \quad / \quad \rho = \frac{(1+x_1^2)^{3/2}}{x_2}$$

expression radius of curvature in case of pedal curve.

$$\boxed{\rho = r \cdot \frac{dr}{dp}}$$

(1) Show that for the catenary. $y = c\cosh(x/c)$ the radius of curvature is equal to y^2/c

Sol:- catenary

$y = c \cosh\left(\frac{x}{c}\right)$.

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$y = c\cosh(x/c)$

Diff w.r.t x.

$$\frac{dy}{dx} = y_1 = \cancel{c}\sinh(x/c) \cdot 1/\cancel{c}$$

$$\boxed{y_1 = \sinh(x/c)}$$

Diff w.r.t x.

$$\frac{d^2y}{dx^2} = y_2 = \cosh\left(\frac{x}{c}\right)\frac{1}{c}$$

we have

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{\left\{1+\left(\sinh\left(\frac{x}{c}\right)\right)^2\right\}^{3/2}}{\cosh\left(\frac{x}{c}\right)\cdot\frac{1}{c}}$$

$$\rho = \frac{c\left\{1+\sinh^2\left(\frac{x}{c}\right)\right\}^{3/2}}{\cosh\left(\frac{x}{c}\right)}$$

$$\rho = \frac{c\left(\cosh^2\left(\frac{x}{c}\right)\right)^{3/2}}{\cosh\left(\frac{x}{c}\right)}$$

$$\therefore \rho = \frac{c\cosh^{\cancel{3}\,2}\left(\frac{x}{c}\right)}{\cancel{\cosh\left(\frac{x}{c}\right)}}$$

$$\boxed{\rho = c\cosh^2\left(\frac{x}{c}\right)} \rightarrow (2)$$

from eqn (1)

$$\cosh\left(\frac{x}{c}\right) = \frac{y}{c}.$$

$$(2) \Rightarrow \rho = c\left(\frac{y}{c}\right)^2$$

$$= \cancel{c}\left(\frac{y^2}{c^{\cancel{2}}}\right)$$

$$\boxed{\rho = \frac{y^2}{c}}$$

(2) Show that the radius curvature at the point
(*) $(a/4, a/4)$ on the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is
$a/\sqrt{2}$

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Sol:- $\sqrt{x}+\sqrt{y}=\sqrt{a} \to ①$

Diff w.r.t 'x'

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\cdot y_1=0$$

$$\frac{y_1}{2\sqrt{y}}=-\frac{1}{2\sqrt{x}}$$

$$y_1=\frac{-\cancel{2}\sqrt{y}}{\cancel{2}\sqrt{x}}$$

$$\therefore \boxed{y_1=\frac{-\sqrt{y}}{\sqrt{x}}}$$

At $(a/4, a/4)$

$$y_1=-\frac{\sqrt{\cancel{a/4}}}{\sqrt{\cancel{a/4}}}$$

$$\boxed{y_1=-1}$$

Diff ② w.r.t 'x'

$$y_2=-\left\{\frac{\sqrt{x}\,\frac{1}{2}\sqrt{y}^{\,y_1}-\sqrt{y}\,\frac{1}{2}\sqrt{x}}{(\sqrt{x})^2}\right\}$$

$$y_2=-\left\{\frac{\frac{\sqrt{x}\,y_1}{2\sqrt{y}}-\frac{\sqrt{y}}{2\sqrt{x}}}{x}\right\}$$

At $(a/4, a/4)$

$$y_2=-\left\{\frac{\frac{\sqrt{\cancel{a/4}}(-1)}{2\sqrt{\cancel{a/4}}}-\frac{\sqrt{\cancel{a/4}}}{2\sqrt{\cancel{a/4}}}}{a/4}\right\}$$

$$y_2=-\left\{\frac{-\frac{1}{2}-\frac{1}{2}}{a/4}\right\}$$

$$=-\frac{(-1)4}{a}$$

$$\boxed{y_2=4/a}$$

We have

$$\rho=\frac{(1+y_1^2)^{3/2}}{y_2}$$

$$=\frac{\left\{1+(-1)^2\right\}^{3/2}}{4/a}$$

$$=\frac{1+1^{3/2}}{4}a$$

$$\rho=\frac{2^{3/2}a}{4}$$

$$=\frac{\sqrt{2}^{\,3}}{4}a$$

$$=\frac{a\times\sqrt{8}}{4}$$

$$=\frac{a\times\sqrt{4\times2}}{4}$$

$$=\frac{a\times\sqrt{4\times2}}{4}$$

$$=\frac{a\times\cancel{2}\sqrt{2}}{\cancel{4}_2}$$

$$=\frac{a\sqrt{\cancel{2}}}{\sqrt{2}\sqrt{\cancel{2}}}$$

$$\boxed{\rho=a/2}$$

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(2) find the radius of curvature for the curve $x^4+y^4=2$ at $(1,1)$

sol: Diff w.r.t 'x'

$\cancel{\frac{1}{2\sqrt{x^4}}+\frac{1}{2\sqrt{y^4}}\cdot y_1=2}$

$4x^3+4y^3y_1=0$

$4y^3y_1=-4x^3$

$y_1=\frac{-4x^3}{4y^3}$

$\boxed{y_1=-\frac{x^3}{y^3}} \rightarrow (2)$

$y_1=-1/1$

$\boxed{y_1=-1}$

diff (2) w.r.t x

$$y_2=-\left\{\frac{y^3\cdot 3x^2-x^3\cdot 3y^2\cdot y_1}{(y^3)^2}\right\}$$

$$=-\left\{\frac{3x^2y^3-3x^3y^2y_1}{y^6}\right\}$$

At $(1,1)$

$$y_2=-\left\{\frac{3(1)(1)-3(1)(1)(-1)}{1}\right\}$$

$$=-\{3+3\}$$

$\boxed{y_2=-6}$

$$\rho=\frac{(1+y_1^2)^{3/2}}{y_2}$$

$$=\frac{\{1+(-1)^2\}^{3/2}}{(-6)}$$

$$\rho=\frac{2^{3/2}}{-6}$$

$$\rho=\frac{\sqrt{2^3}}{(-6)}$$

$$=\frac{\sqrt{4\times 2}}{(-6)}$$

$$=\frac{\cancel{2}\sqrt{2}}{-\cancel{6}\,3}$$

$$\therefore \rho=\frac{-\sqrt{2}}{3}$$

Imp Q-

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(4) Find the radius of curvature for follium $x^3+y^3 = 3axy$ at the point $(3a/2, 3a/2)$ on it

Sol:- $x^3+y^3 = 3axy$.

diff w.r.t 'x'

$3x^2+3y^2y_1 = 3a[xy_1+y.1]$.

$3y^2y_1 - 3axy_1 = 3ay - 3x^2$

$y_1(3y^2-3ax) = 3ay - 3x^2$

$\therefore y_1 = \frac{3ay-3x^2}{3y^3-3ax}$

$= \frac{\cancel{3}(ay-x^2)}{3(y^2-ax)}$.

$y_1 = \frac{ay-x^2}{y^2-ax} \rightarrow (1)$

At ~~(3a)~~ $(3a/2, 3a/2)$

$y_1 = \frac{a\cdot 3(a/2) - (3a/2)^2}{3(a/2)^2 - a(3a/2)}$

$= \frac{3a^2/2 - 9a^2/4}{9a^2/4 - 3a^2/2}$

$= -\left(\frac{\cancel{9a^2/4 - 3a^2/2}}{\cancel{9a^2/4 - 3a^2/2}}\right)$

$\boxed{y_1 = -1}$

Diff w.r.t 'x'

$y_2 = \frac{(y^2-ax)(ay_1-2x) - (ay-x^2)(2yy_1-a.1)}{(y^2-ax)^2}$

At $(3a/2, 3a/2)$

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$$y_2 = \frac{\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)(a\cdot(-1) - 2\cdot\frac{3a}{2}) - \left(a\frac{3a}{2} - \left(\frac{3a}{2}\right)^2\right)\left[2\cdot\frac{3a}{2}(-1) + a\right]}{\left[\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)\right]^2}$$

$$y_2 = \frac{\boxed{\frac{9a^2}{2} - \frac{3a^2}{2}}\,[-a - 3a] - \left[\frac{3a^2}{2} - \frac{9a^2}{4}\right][-3a - a]}{\left(\frac{9a^2}{4} - \frac{3a^2}{2}\right)^2}$$

$$y_2 = \frac{(-4a)\left(\frac{9a^2}{4} - \frac{3a^2}{2}\right) - \left(-4a\left[-\frac{9a^2}{4} - \frac{3a^2}{2}\right]\right)}{\left(\frac{9a^2}{4} - \frac{3a^2}{2}\right)^2}$$

$$= \frac{(-4a)\left(\frac{9a^2}{4} - \frac{3a^2}{2}\right)(1+1)}{\left(\frac{9a^2}{4} - \frac{3a^2}{2}\right)^2}$$

$$= \left(\frac{-2(4a)}{\frac{9a^2 - 6a^2}{4}}\right)$$

$$y_2 = \frac{-8a}{\frac{3a^2}{4}}$$

$$\boxed{y_2 = \frac{-32}{3a}}$$

$y_1 = 1$, $y_2 = -32/3a$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$\rho = \frac{[1 + (-1)^2]^{3/2}}{-32/3a}$$

$$= \frac{3a(2)^{3/2}}{(-32)}$$

$$\frac{3a\sqrt{2^3}}{(-32)}$$

$$\rho = \frac{3a \times \sqrt{8}}{(-32)}$$

$$= \frac{3a \times \sqrt{4\times 2}}{(-32)}$$

$$= \frac{3a \times 2\sqrt{2}}{\frac{(-32)}{16}}$$

$$= \frac{3\sqrt{2}a}{-8\times 2}$$

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$y_1 = \infty \Rightarrow y_2 =$

$$\rho = \frac{3\sqrt{2}\, a}{(-8)\sqrt{2}\sqrt{2}}$$

$$|\rho| = \left|\frac{-3a}{8\sqrt{2}}\right|$$

$$|\rho| = \frac{3a}{8\sqrt{2}}$$

* Show that radius of curvature at $a, 0$ on the curve $y^2 = a^2 \frac{(a-x)}{x}$ is $a/2$.

Sol: $(a, 0)$

$$y^2 = a^2 \frac{(a-x)}{x}$$

$$y^2 = \frac{a^3}{x} - \frac{a^2 x}{x}$$

$$y^2 = \frac{a^3}{x} - a^2$$

Diff w.r.t 'x'

$$2y\, y_1 = -\frac{a^3}{x^2} - 0$$

$$\boxed{y_1 = \frac{-a^3}{2x^2 y} = \frac{dy}{dx}}$$

At $(a, 0)$

$$\boxed{y_1 = \infty}$$

Since y_1 is ∞ we have to consider x_1.

$$\therefore x_1 = \frac{dx}{dy} = \frac{-2x^2 y}{a^3} \rightarrow (1)$$

At $(a, 0)$.

$$x_1 = \frac{-2(a^2)(0)}{a^3}$$

$$\boxed{x_1 = 0}$$

Diff (1) w.r.t 'y'

$$\frac{d^2x}{dy^2} = \frac{-2}{a^3}\left(x^2 \cdot 1 + y \cdot 2x x_1\right)$$

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$$\frac{d^2x}{dy^2} = -\frac{2}{a^3}(x^2 + 2x\, x_1 y) = x_2$$

At $(a, 0)$.

$$x_2 = \frac{-2}{a^3}(a^2 + 0)$$

$$x_2 = \frac{-2a^2}{a^3} = -2/a$$

$$\rho = \frac{(1 + x_1^2)^{3/2}}{x_2}$$

$$= \frac{(1+0)^{3/2}}{-2/a}$$

$$= 1^{3/2}\left(\frac{-a}{2}\right)$$

$$\therefore \rho = -a/2$$

$$|\rho| = \left|\frac{-a}{2}\right| = \frac{a}{2}$$

* find the radius of curvature for the curve $x^2 y = a\{x^2 + y^2\}$ at the point $-2a, 2a$.

Sol. $x^2 y = a(x^2 + y^2)$

diff. wrt. x

$$x^2 y_1 + y(2x) = a(2x + 2y y_1)$$

$$x^2 y_1 - 2ay y_1 = 2ax - 2xy.$$

$$y_1 (x^2 - 2ay) = 2ax - 2xy.$$

$$y_1 = \frac{2ax - 2xy}{x^2 - 2ay}.$$

At $(-2a, 2a)$

$$y_1 = \frac{2a(-2a) - 2(-2a)(2a)}{(2a)^2 - 2a(2a)}$$

$$y_1 = \frac{-4a^2 + 8a^2}{4a^2 - 4a^2}$$

$$y_1 = \frac{4a^2}{0}$$

$\boxed{y_1 = \infty}$

$$x_1 = \frac{x^2 - 2ay}{2ax - 2xy} \rightarrow (1)$$

At $(-2a, 2a)$

$$x_1 = \frac{1}{\infty} = 0$$

Diff (1) wrt y.

$$\frac{d^2x}{dy^2}$$

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$$x_2 = \frac{2ax - 2xy(2xx_1 - 2a) - (x^2 - 2ay)(2ax_1 - 2(x_1)^2)}{(2ax - 2xy^2)}$$

At $(-2a, 2a)$.

$$x_2 = \frac{(2a(-2a) - 2(-2a)(2a)\{0 - 2a\} - ((-2a)^2 - 2a)[0 - 2[(-2a)+0]]}{(2a(-2a) - 2(-2a)(2a))^2}$$

$$x_2 = \frac{(-4a^2 + 8a^2)(-2a) - (4a^2 - 4a^2)(4a)}{(-4a^2 + 8a^2)^2}$$

$$x_2 = \frac{4a^2(-2a)}{(4a^2)^2}$$

$$x_2 = \frac{-2a}{(4a^2)}$$

$$\boxed{x_2 = -\frac{1}{2a}}$$

$$\rho = \frac{(1 + x_1^2)^{3/2}}{x_2}$$

$$\rho = \frac{(1+0)^{3/2}}{-1/2a}$$

$$\rho = (-2a)(1)^{3/2}$$

$\rho = -2a$

$|\rho| = |-2a|$

$|\rho| = 2a$

Find the radius of curvature for the curve $x = a(t + \sin t)$, $y = a(1 - \cos t)$

Sol:- $x = a(t + \sin t)$.

$x = x(t)$; $y = (t)$

Parametric

$x = a(t + \sin t)$

diff wrt t

$$\frac{dx}{dt} = a(1 + \cos t)$$

$y = a(1-\cos t)$

Diff wrt $\cancel{t}$

$\frac{dy}{dt} = a(0+\sin t)$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{\cancel{a}\sin t}{\cancel{a}(1+\cos t)}$

$y_1 = \frac{2\sin t/2 \cos t/2}{2\cos^2 t/2}$

$\boxed{y_1 = \tan t/2}$

Diff wrt x

$y_2 = \sec^2 t/2 \cdot \frac{1}{2} \cdot \frac{dt}{dx}$

$y_2 = \frac{1}{2}\sec^2 t/2 \left(\frac{1}{a(1+\cos t)}\right)$

$= \frac{\sec^2 t/2}{2a(2\cos^2 t/2)}$

$y_2 = \frac{1}{4a}(\sec^4 t/2)$

$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

$= \frac{(1+\tan^2 t/2)^{3/2}}{\frac{1}{4a}\sec^4 t/2}$

$\rho = \frac{4a(\sec^{\cancel{2}} t/2)^{3/2}}{\sec^4 t/2}$

$\rho = \frac{4a\cancel{\sec^3 t/2}}{\sec^{\cancel{4}} t/2}$

$\therefore \rho = \frac{4a}{\sec t/2}$

$\boxed{\rho = 4a\cos t/2}$

* find radius of curvature $x = a\log(\sec t + \tan t)$
$y = a\sec t$

Sol parametric

Diff w.r.t.

$$\frac{dx}{dt} = a\frac{1}{\sec t + \tan t}(\sec t\tan t + \sin t).$$

$$= \frac{a\sec t\,\cancel{(\tan t + \sec t)}}{\cancel{(\sec t + \tan t)}} \Rightarrow \frac{dx}{dt} = a\sec t$$

$$\frac{dy}{dt} = a\sec t\tan t$$

$$\frac{dy}{dx} = y_1 = \frac{dy/dt}{dx/dt}$$

$$= \frac{\cancel{a\sec t}\tan t}{\cancel{a\sec t}}$$

$$\frac{dx}{dy} = \tan t = y_1$$

diff w.r.t x

$$\frac{d^2y}{dx^2} = y_2 = \sec^2 t\cdot\frac{dt}{dx}$$

$$y_2 = \sec^2 t\cdot\frac{1}{a\sec t}$$

$$\boxed{y_2 = \frac{1}{a}\sec t}$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1+\tan^2 t)^{3/2}}{\frac{1}{a}\sec t}$$

$$\rho = \frac{(\sec^2 t)^{3/2}\cdot a}{\sec t}$$

$$\rho = \frac{a\sec^3 t}{\cancel{\sec t}}$$

$$\therefore \rho = a\sec^2 t$$

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(a) find radius of curvature for asteroid. $x = a\cos^3 t$
$y = a\sin^3 t$ at $t = \pi/4$.

Ans

$\frac{dx}{dt} = a\, 3\sin^2 t\,(\cos t)$.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{3a\,\sin^2 t\,\cos t}{-3a\,\cos^2 t\,\sin t}$

$y_2 = -\sec^2 t$

$\rho = \frac{[1+\tan^2 t)^2]^{3/2}}{-\sec^2 t}$

$= \frac{(1+\tan^2 t)^{3/2}}{\sec^2 t}$

$= \frac{\sec^3 t}{-\sec^2 t}$

$\boxed{\rho = -\sec t}$

$|\rho| = |-\sec t|$

$|\rho| = \sec t$

$\therefore \sec \pi/4 = \sqrt{2}$

$\frac{dx}{dt} = 3a\cos^2 t(-\sin t)$.

$= -3a\cos^2 t(-\sin t)$

$= -3a\sin t\cos^2 t$

$\frac{dy}{dt} = 3a\sin^2 t\cos t$

$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{3a\sin^2 t\cos t}{-3a\sin t\cos^2 t}$

$t = \pi/4$

$\boxed{y_1 = -\tan t}$

Diff w.r.t x'

$y_2 = -\sec^2 t\, dt/dx$

$= \frac{+\sec^2 t}{+3a\sin t\cos^2 t}$

$= \frac{1}{3a}\operatorname{cosec} t\sec^4 t$

→ Show that the radius of curvature (or, curve, curve)
$r^n = a^n \cos n\theta$ varies inversely as r^{n-1}

A → $\log r^n = \log[a^n \cos n\theta]$

$n \log r = \log a^n + \log \cos n\theta$

$= n \log a + \log \cos n\theta$

Diff w.r.t. θ

$n \cdot \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta} (-\sin n\theta \cdot n)$

(÷n)

$\cancel{n} \cot \phi = -\cancel{n} \tan n\theta$

$\cot \phi = \cot (\pi/2 + n\theta)$

$\boxed{\phi = \pi/2 + n\theta}$

W.K.T.

$p = r \sin \phi$

$p = r \sin(\pi/2 + n\theta)$

$p = r \sin(\pi/2 + n\theta)$

$p = r \cos n\theta \rightarrow (2)$

$(1) \Rightarrow \cos n\theta = r^n / a^n$

$(2) \Rightarrow p = r \cdot \frac{r^n}{a^n}$

$\boxed{p = \frac{r^{n+1}}{a^n}}$

Diff. w.r.t. 'r'

$\frac{dp}{dr} = \frac{1}{a^n} \{(n+1) r^{n+\cancel{1}-\cancel{1}}\}$

$\frac{dp}{dr} = \frac{(n+1)}{a^n} r^n$

W.K.T., $\rho = r \frac{dr}{dp}$

$= \frac{r \cdot a^n}{(n+1) r^n}$

$\rho = r \frac{dr}{dp}$

$\rho = \left(\frac{a^n}{n+1}\right) \frac{1}{r^{-1} r^n}$

$\rho = \left(\frac{a^n}{n+1}\right) \cdot \frac{1}{r^{n-1}}$

$\boxed{\rho \propto \frac{1}{r^{n-1}}}$

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* for the cardioid $r = a(1 \pm \cos\theta)$ prove that $\frac{\rho^2}{r}$ is a constant

Ans $r = a(1+\cos\theta) \rightarrow ①$

$\log r = \log a(\log 1 + \cos\theta)$.

diff wrt θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta).$$

$$\cot\phi = \frac{-\cancel{2}\sin\theta/_2 \cos\theta/_2}{\cancel{2}\cos^2\theta/_2}$$

$$\cot\phi = -\tan\theta/_2$$
$$= \cot(\pi/_2 + \theta/_2)$$

$$\therefore \boxed{\phi = \pi/_2 + \theta/_2}$$

we have.

$$p = r\sin\phi.$$
$$= r\sin(\pi/_2 + \theta/_2)$$
$$p = r\cos\theta/_2 \rightarrow ②$$

from ① $\Rightarrow 1+\cos\theta = r/a$

$$2\cos^2\theta/_2 = r/a.$$

$$\cos^2\theta/_2 = \frac{r}{2a}$$

$$\cos\theta/_2 = \sqrt{\frac{r}{2a}}$$

② $\Rightarrow$

$$p = r\cdot\sqrt{\frac{r}{2a}}$$

$$p = \frac{r\cdot\sqrt{r}}{\sqrt{2a}}$$

$$p = \frac{r\cdot r^{1/2}}{\sqrt{2a}}$$

$$\boxed{p = \frac{r^{3/2}}{\sqrt{2a}}}$$

diff wrt 'r'.

$$\frac{dp}{dr} = \frac{1}{\sqrt{2a}}\cdot\frac{3}{2}r^{1/2}$$

$$\frac{dp}{dr} = \frac{3\sqrt{r}}{2\sqrt{2a}}$$

WKT $\rho = r\frac{dr}{dp}$

$$\rho = \frac{r\cdot 2\sqrt{2a}}{3\sqrt{r}}$$

$$\rho^2 = \frac{r^{\cancel{2}}\cdot 4(2a)}{9\cancel{r}}$$

$$\frac{\rho^2}{r} = \frac{8a}{9}.$$

$\therefore$ ρ^2/r is a constant

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* find the radius of curvature for the curve

$r = a(1-\cos\theta) \rightarrow ①$

Ans $\log r = \log \{a(1-\cos\theta)\}$

$= \log a + \log(1-\cos\theta)$

Diff. w.r.t θ

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$

$\cot\phi = \frac{2\sin\theta/2 \, \cos\theta/2}{2\sin^2\theta/2}$

$\cot\phi = \cot\theta/2$

$\therefore \phi = \theta/2$

we have

$p = r\sin\phi$

$p = r\sin\theta/2 \rightarrow ②$

from ① $\Rightarrow$

$1-\cos\theta = r/a$

$2\sin^2\theta/2 = r/a$

$\sin^2\theta/2 = \frac{r}{2a}$

$\sin\theta/2 = \sqrt{\frac{r}{2a}}$

② $\Rightarrow$

$p = r\frac{\sqrt{r}}{\sqrt{2a}}$

$\boxed{p = \frac{r^{3/2}}{\sqrt{2a}}}$

Diff w.r.t 'r'

$\frac{dp}{dr} = \frac{3r^{3/2-1}}{2\sqrt{2a}}$

$\frac{dp}{dr} = \frac{3r^{1/2}}{2\sqrt{2a}}$

$= \frac{3\sqrt{r}}{2\sqrt{2a}}$

$\rho = r \cdot \frac{dr}{dp}$

$= r \cdot \frac{2\sqrt{2a}}{3\sqrt{r}}$

$\rho^2 = \frac{4r^2 \, 2a}{9r}$

$\boxed{\rho^2 = \frac{8ar}{9}}$

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Show that for the curve $r^2 \sec 2\theta = a^2$; $\rho = \frac{a^2}{3r}$

Ans $r^2 = \frac{a^2}{\sec 2\theta}$.

$r^2 = a^2 \cos 2\theta \rightarrow (1)$

$\log r^2 = \log (a^2 \cos 2\theta)$.

$2\log r = \log a^2 + \log \cos 2\theta$

$2\log r = 2\log a + \log + \log \cos 2\theta$.

Diff wrt θ.

$2 \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos 2\theta} (-\sin 2\theta \cdot 2)$

$\cot \phi = -\tan 2\theta$.

$\cot \phi = \cot (\pi/2 + 2\theta)$

$\boxed{\phi = \pi/2 + 2\theta}$.

WKT : $p = r \sin \phi$

$= r \sin (\pi/2 + 2\theta)$

$p = r \cos 2\theta \rightarrow (2)$

from (1) $\Rightarrow$

$\cos 2\theta = r^2/a^2$

(2) $\Rightarrow$ $p = r \cdot r^2/a^2$

$\boxed{p = \frac{r^3}{a^2}}$

Diff wrt r

$\frac{dp}{dr} = \frac{3r^2}{a^2}$

we have

$\rho = r \frac{dr}{dp}$.

$= r \cdot \frac{a^2}{3r^2}$

$\boxed{\rho = \frac{a^2}{3r}}$

* Find the radius of curvature for the curve
$r^3 = 2ap^2$

Ans $p^2 = \frac{r^3}{2a}$

Diff w.r.t. r'

$2p \cdot \frac{dp}{dr} = \frac{1}{2a}(3r^2)$

$\frac{dp}{dr} = \frac{3r^2}{(2p)(2a)} = \frac{3r^2}{4ap}$

$\rho = r\frac{dr}{dp}$

$= r \cdot \frac{4ap}{3r^2}$

$\rho = \frac{4ap}{3r}$

$\rho^2 = \frac{16a^2p^2}{9r^2}$

$\rho^2 = \frac{\cancel{16}^{8}a}{9\cancel{r^2}} \cdot \left(\frac{r^3}{\cancel{2a}}\right)$

$\boxed{\rho^2 = \frac{8ar}{9}}$

(*) ① $r^n = a^n \sin n\theta$.
② $r^2 \cos 2\theta = a^2$
③ $p^2 = ar$

① $p = r\sin\phi \Rightarrow r\sin n\theta$
$r^n = a^n \sin n\theta \Rightarrow \sin n\theta = \frac{r^n}{a^n}$

$p = r\left(\frac{r^n}{a^n}\right) \quad f = \frac{r^{n+1}}{a^n}$

$\frac{dp}{dr} = \frac{n+1 r^{n+\cancel{r}-\cancel{r}}}{a^n}$

$\frac{dp}{dr} = \frac{(n+1)r^n}{a^n}$

$\rho = r\frac{dr}{dp} \qquad \rho = r\left(\frac{a^n}{(n+1)r^p}\right.$

$\rho = \frac{a^n}{(n+1)r^{-1} r^n}$

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$\Rightarrow \boxed{\rho = \frac{a^n}{(n+1)} \cdot \frac{1}{r^{n-1}}}$

2) $r^2 \cos 2\theta = a^2$

$\Rightarrow \log r^2 + \log \cos 2\theta = \log a^2$

$\Rightarrow 2 \log r + \frac{2(-\sin 2\theta)}{\cos 2\theta} = 0$

$\cot \phi = -\tan 2\theta$

$\cot \phi = \cot(\pi/2 - 2\theta)$

Ans $\rho = \frac{r^3}{a^2}$

$\boxed{\phi = \pi/2 - 2\theta}$

3) $p = r\sin\phi \Rightarrow p = r\sin(\pi/2 - 2\theta)$

$\Rightarrow p = r\cos 2\theta$

$\cos 2\theta = \frac{a^2}{r^2}$

$p = r\left(\frac{a^2}{r^2}\right)$

$\boxed{p = \frac{a^2}{r}}$

4) $\cancel{p = a r}$ =

Ans $\cancel{r^n = a^n \sin}$

3) $p^2 = ar$.

diff w.r. to x

$2p \cdot \frac{dp}{dr} = a$

$\frac{dp}{dr} = \frac{a}{2p}$

W.K.T. $\rho = r\frac{dr}{dp}$

$= r\frac{2p}{a}$

$= r \cdot \frac{2p}{a}$

$\rho^2 = \frac{r^2 4p^2}{a^2}$

$\rho^2 = \frac{r^2 4(ar)}{a^2}$

$\boxed{\rho^2 = \frac{4r^3}{a}}$