Q1. Which of the following represent simple harmonic motion?

a) $y = Ae^{i\omega t}$

b) $y = A^{\omega t}$

→ $$\frac{d^2y}{dt^2} + \omega^2 y = 0.$$

a) $y = Ae^{I\omega t}$

$$\frac{dy}{dt} = A \cdot i\omega e^{i\omega t}.$$

$$\frac{d^2y}{dt^2} = A \cdot (i\omega)^2 e^{\omega t} = -A\omega^2 e^{i\omega t}.$$

$$-A\omega^2 e^{i\omega t} + \omega^2 (Ae^{i\omega t}) = 0.$$

∴ This satistifies the SHM condition.

b) $Y = -Ae^{\omega t}$.

$$\frac{dy}{dt} = -A\omega e^{-\omega t}.$$

$$\frac{d^2y}{dt} = A\omega^2 e^{-\omega t}.$$

$$A\omega^2 e^{-\omega t} + \omega^2 (Ae^{-\omega t}) \neq 0.$$

∴ This does not satisfy the SHM condition.

∴ (A) $y = Ae^{i\omega t}$ represent simple harmonic motion.

Q.2) Calculate the amplitude, angular frequency, frequency, time period and initial phase for the simple harmonic oscillation given below.

a) $y = 2 \cos \pi t$

b) $y = 3 \sin (2\pi t - 1.5)$

c) $y = 3 \sin (20\pi - 1.1) + 3 \cos (20\pi - 1.1)$

→ a = $y = 2 \cos (\pi t)$

• Amplitude :- the amplitude of the cosine function $y = A \cos (\omega t + \phi)$ is

Here, $A = \underline{\underline{2}}$

• Angular frequency :- The angular frequency ω is the coefficient of t in the argument the cosine function.

for $y = 2 \cos (\pi t)$,

$\omega = \underline{\underline{\pi}}$

• Frequency :- frequency f is realeted to the angular frequency by $\omega = 2\pi f$.

Therefore $f = \frac{\omega}{2\pi} = \frac{\cancel{\pi}}{2\cancel{\pi}} = \underline{\underline{\frac{1}{2}}}$ Hz

• Time period :- The time period T is the reciprocal of the frequency.

$T = \frac{1}{f} = \frac{1}{\frac{1}{2}} = \underline{\underline{2}}$ second

• Initial phase :- The initial phase ϕ is the phase shift in the cosine function, Here, there is no phase shift, so $\phi = \underline{\underline{0}}$ radians.

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b). $y = 3 \sin(2\pi t - 1.5)$

→ • Amplitude :- The amplitude is the coefficient in front of the sine function. Here, $A = 3$.

• Angular frequency :- the angular frequency, ω is the coefficient of t in the argument of the sine function.

$$\therefore \omega = 2\pi$$

• Frequency :- using the relation $\omega = 2\pi f$.

we find $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1\,Hz$

• Time period :- The time period T is the reciprocal of the frequency,

$$T = \frac{1}{f}$$

$$= \frac{1}{1}$$

$$= 1 \text{ second.}$$

• Initial phase :- The initial phase ϕ is given by the phase shift in the sine function.

$y = 3 \sin(2\pi t - 1.5)$

$\phi = -1.5$ radian.

c) $y = 3\sin(20\pi t - 1.1) + 3\cos(20\pi t - 1.1)$

→ To simplify and find the amplitude, angular frequency, and phase for this combined function, we use trigonometric identities.

Rewrite the function as: $y = 3(\sin(20\pi t - 1.1) + \cos(20\pi t - 1.1))$

We can use the trigonometric identity to combine sine and cosine:

$$\sin(x) + \cos(x) = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

Here $x = 20\pi t - 1.1$

Thus: $y = 3\sqrt{2}\sin(20\pi t - 1.1 + \pi/4)$

- Amplitude: The amplitude is $3\sqrt{2}$
- Angular frequency: $y = 3\sqrt{2}\sin\left(20\pi t - 1.1 + \frac{\pi}{4}\right)$

$$\therefore \omega = 20\pi$$

- Frequency: $\omega = 2\pi f$.

$$f = \frac{\omega}{2\pi} = \frac{20\pi}{2\pi} = 10\ Hz$$

- Time period $= T = \frac{1}{f} = \frac{1}{10}$ seconds.

- Initial phase $= -1.1 + \frac{\pi}{4}$ $\left(\frac{\pi}{4} = 0.785\right)$

$$= -1.1 + 0.785$$
$$\approx -0.315 \text{ radians}$$

Q.3) A nurse measured the heart beat of the patient in terms of number of beats measured per minute is 75. What will be the average heart beats of a patient in terms of time period?

→

- Heart rate in beats per minute : 75 bets per min

- Convert beats per minute to frequency.

frequency f in (Hz) is the number of beats per second.

$$f = \frac{\text{beats per minute}}{60} = \frac{75}{60} = 1.25 \text{ Hz}$$

- Time period :- Time period (T) is reciporal of the frequency

$$T = \frac{1}{f} = \frac{1}{1.25} = 0.8 \text{ second.}$$