Quantum superpositions and Schrödinger cat states in quantum optics

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We describe the properties of a quantum system prepared in superpositions of classically distinguishable states. These states, often called Schrödinger cat states, are of great interest at present. We discuss how they may be realized in quantum optics using nonlinear interactions and cavity quantum electrodynamics. We first describe the quantum properties of field states in a cavity, and demonstrate the interference properties which characterize superposition states and discuss how fragile they are in dissipative environments. Finally, we review current experimental approaches which may realize these states.

I. INTRODUCTION

It has often been said that the principle of superposition is at the heart of quantum mechanics. In classical physics, we do not speak of superpositions of possible states of a system, rather, we assume that the physical attributes of a system objectively exist even if unknown. As Einstein might say, the moon really is there when nobody looks. But in quantum mechanics it appears necessary to abandon the notion of an objective local reality.[1] Instead, a quantum system is described by a state vector which may be expanded into a coherent superposition of the eigenstates of some observable,

$$|\Psi\rangle=\sum_i c_i|\psi_i\rangle, \tag{1}$$

where the coefficients c_i are probability amplitudes. The probability that a measurement of that observable finds the system in state $|\psi_i\rangle$ is $|c_i|^2$. But the state vector of Eq. (1) is not merely a reflection of our ignorance of the true state of the system before a measurement but rather of its objective indefiniteness: The system has no objectively definite state prior to a measurement. The act of measurement ''collapses'' the state vector to one of the eigenstates. The basic feature of the superposition principle is that probability amplitudes can interfere: a feature that has no analog in classical physics.

The above lines conform to the Copenhagen interpretation (some would say dogma) of quantum mechanics.[2] It is certainly the case that such superposition states are not observable in the everyday world of classical physics. We do not observe macroscopic objects in coherent superposition states and therefore it may be comforting to conclude that the superposition principle operates only on the microscopic scale, at a level inaccessible to everyday experience. But this begs the question as to where one ought to place the border between what is classical and what is quantum mechanical. Is it merely a matter of size? And if so, what are the delimiting dimensions? The answer does not appear to be straightforward. One approach is to realize that macroscopic systems are generally not closed systems, they interact dissipatively with their environment.[3] These interactions are complex, involving entanglement between the system and the environment, and produce an irreversible evolution. If the system is initially in a coherent superposition state, the dissipative, irreversible interaction decoheres the initial superposition into a statistical mixture which can be characterized by a density operator of the form

$$\hat{\rho}_{\text{mixture}}=\sum_i p_i|\psi_i\rangle\langle\psi_i|, \tag{2}$$

where p_i is the probability of the system being in the state $|\psi_i\rangle$. Unlike for the state of Eq. (1), probability amplitudes do not enter in this case, so interference does not occur. This is easily seen by writing down the density operator for the pure state of Eq. (1):

$$\hat{\rho}_{|\psi\rangle}=|\psi\rangle\langle\psi|=\sum_i\sum_j c_ic_j^*|\psi_i\rangle\langle\psi_j|. \tag{3}$$

If $\hat{A}$ is some observable for which $|\psi_i\rangle$ are not eigenstates, then

$$\langle\hat{A}\rangle_{|\psi\rangle}=\langle\psi|\hat{A}|\psi\rangle=\mathrm{Tr}(\hat{\rho}_{|\psi\rangle}\hat{A}) \tag{4}$$

$$=\sum_i\sum_j c_ic_j^*\langle\psi_j|\hat{A}|\psi_i\rangle, \tag{5}$$

whereas

$$\langle\hat{A}\rangle_{\text{mixture}}=\mathrm{Tr}(\hat{\rho}_{\text{mixture}}\hat{A})=\sum_i p_i\langle\psi_i|\hat{A}|\psi_i\rangle. \tag{6}$$

Clearly, the former exhibits interference as the off-diagonal elements of $\hat{A}$ contribute to the expectation value, these elements not being present for the mixture. According to the work of Zurek and others,[3] an initially pure state, with density operator of the form of Eq. (3), upon interacting with the environment, evolves in such a way that the off-diagonal elements rapidly vanish and the density operator evolves into the form of Eq. (2) for a statistical mixture, i.e., the initial pure state decoheres into a mixture. This is the result of tracing over the unobserved states of the environment. The characteristic time for this decoherence to occur depends on the size of the system: The more distinctly different the components are in a superposition, the more rapid the decay of coherence. The density matrix becomes diagonal in a preferred basis determined by the coupling of the system to the environment.

For macroscopic objects, the decoherence time is almost instantaneous. On the other hand, there might be situations for which systems, on the classical-quantum boundary, could be isolated long enough from dissipative interactions so that superpositions of distinguishable states[4] could be realized on a fairly large scale—perhaps at least on a mesoscopic scale, before decoherence sets in. However, the prospect that such superpositions could exist on a large scale causes us to recall the famous thought experiment of Schrödinger formulated in 1935 and commonly known as the Schrödinger cat paradox.[5] As originally formulated, a cat (obviously macroscopic) is placed inside a closed steel box along with a bit of radioactive substance with a decay rate such that after one hour an atom may have decayed. If the atom decays, a Geiger counter discharge releases a hammer which shatters a flask containing a poisonous gas, subsequently killing the cat. According to the Copenhagen interpretation, the atom and cat are in an entangled state of the form

$$|\psi\rangle = \frac{1}{\sqrt{2}}\,[|\text{atom not decayed}\rangle|\text{cat alive}\rangle + |\text{atom decayed}\rangle|\text{cat dead}\rangle]. \tag{7}$$

Although, as far as we know, the word entanglement was used first by Schrödinger to describe states of this sort, the concept certainly appears in the paper of Einstein, Podolsky, and Rosen (EPR),[6] the paper that motivated Schrödinger's remarks. In any event, upon opening the box, the state vector collapses to one state or the other in the superposition. Schrödinger refers to this as a ''quite ridiculous case,'' intending to show the absurdity of the Copenhagen interpretation.

The ''paradox'' has often been dismissed as having no observable consequences and this is certainly true of the original formulation. A single atom decay requires amplification, an irreversible process, in order to be detected. However, with recent developments in technology, the paradox has become harder to ignore. For example, there has been a great deal of effort to obtain superpositions of macroscopically distinct states of flux in superconducting Josephson junctions.[7] In the field of quantum optics, the subject of this paper, there is likewise considerable effort being expended on generating superpositions of macroscopically (or mesoscopically) distinct states of the quantized electromagnetic field,[8] and of the vibrational motion of an ion in a Paul trap.[9] We wish to make the point here that these kinds of states, which are referred to as Schrödinger cat states (or simply cats) in the literature, are not the entangled states of the type of Eq. (7) as originally discussed by Schrödinger, but rather are coherent superpositions of distinguishable states of a single system. In most cases, the superpositions contain only two distinguishable states, these states being coherent states of the single-mode field, with a relative phase difference of 180°. As is well known, the coherent states are quantum states of a single-mode field, described as a harmonic oscillator, which can contain a large average photon number and which are as close as possible to classical states, containing the noise of the vacuum. Properties of the coherent states were reviewed in this Journal some time ago by Howard and Roy.[10]

In this paper we present a pedagogical review of recent developments in the area of quantum optics regarding the properties and possible experimental generation schemes of Schrödinger cat states. In Sec. II we begin with a brief review of the harmonic oscillator coherent states, paying particular attention to their classical-like nature. In Sec. III we discuss the Schrödinger cat states given as superpositions of coherent states shifted in phase by 180°. Specific examples are given. In Sec. IV we address the issue of dissipation and in Sec. V we give two examples of how such states may be generated. Section VI gives our conclusions.

II. COHERENT STATES

A. Single-mode quantized fields

If we consider a free field of frequency ω linearly polarized in the x direction in a cavity of length L, then the Maxwell equations plus the boundary condition that the electric field vector vanishes at $z=0$ and L leads to the following expression for the cavity field:[11]

$$E_x(z,t) = \left(\frac{2\omega^2}{\epsilon_0 V}\right)^{1/2} q(t)\sin kz, \tag{8}$$

where the wave vector $\mathbf{k}$ and the frequency ω are related by $\mathbf{k}=(\omega/c)\mathbf{z}$, V is the volume of the cavity, and ϵ_0 the permittivity of free space. The amplitude of the electric field is governed by the time-dependent factor $q(t)$, which has the dimensions of length, so that the electric field can be regarded as a kind of canonical position. The magnetic field $H_y(z,t)$ can similarly be expressed as

$$H_y(z,t) = \left(\frac{\epsilon_0}{k}\right)\left(\frac{2\omega^2}{\epsilon_0 V}\right)^{1/2} \dot{q}(t)\cos kz \tag{9}$$

and its amplitude is governed by a kind of canonical momentum $\dot{q}(t)$.

The field energy H can be expressed as the sum of electric and magnetic field energies in the cavity:

$$H = \left(\frac{1}{2}\right)\int dV(\epsilon_0 E_x^2(z,t) + \mu_0 H_y^2(z,t)), \tag{10}$$

which, on using Eqs. (8) and (9), becomes

$$H = \tfrac{1}{2}(p^2 + \omega^2 q^2). \tag{11}$$

In other words, the field mode energy is precisely that of a unit mass harmonic oscillator, with the electric and magnetic fields playing the role of canonical position and momentum.

B. Quantization of the single-mode field

Field quantization is straightforward once the canonical variables have been identified.[11] We simply take the correspondence rule that the variables q and p above are replaced by their operator equivalents $\hat{q},\hat{p}$ (operators will be distinguished from c numbers by a caret superscript), satisfying the commutation rule

$$[\hat{q},\hat{p}] = i\hbar, \tag{12}$$

so that the electric field mode operator is written as

$$\hat{E}_x(z,t) = \left(\frac{2\omega^2}{\epsilon_0 V}\right)^{1/2} \hat{q}(t)\sin kz, \tag{13}$$

and the magnetic field mode operator as

$$\hat{H}_y(z,t) = \left(\frac{\epsilon_0}{k}\right)\left(\frac{2\omega^2}{\epsilon_0 V}\right)^{1/2} \hat{p}(t)\cos kz, \tag{14}$$

and the energy as

$$\hat{H} = \tfrac{1}{2}(\hat{p}^2 + \omega^2 \hat{q}^2). \tag{15}$$

The mode structure in quantum theory is *identical* to that in classical theory, so diffraction and interference phenomena will have the same spatial dependence in both theories. The key points are the lack of commutability of electric and magnetic field operators, and the *discreteness* of the field energy, with the field state with n excitations having the energy

$$E_n = (n + \tfrac{1}{2})\hbar\omega. \tag{16}$$

The state with no excitations is the ground state or vacuum, and in quantum theory possesses a residual zero-point energy of $(1/2)\hbar\omega$.

C. Annihilation and creation operators

We have characterized the electric and magnetic fields by operators $\hat{p}$ and $\hat{q}$, which are Hermitian. But it is traditional to introduce the non-Hermitian (and therefore nonobservable) annihilation ($\hat{a}$) and creation ($\hat{a}^\dagger$) operators through the combinations

$$\hat{a} = (2\hbar\omega)^{-1/2}(\omega\hat{q} + i\hat{p}), \tag{17}$$

$$\hat{a}^\dagger = (2\hbar\omega)^{-1/2}(\omega\hat{q} - i\hat{p}). \tag{18}$$

In terms of these new operators we can write the field energy operator as

$$\hat{H} = (\hat{a}^\dagger\hat{a} + \tfrac{1}{2})\hbar\omega. \tag{19}$$

The basic commutation rule—Eq. (6)—becomes

$$[\hat{a}, \hat{a}^\dagger] = 1 \tag{20}$$

and the electric field operator is

$$\hat{E}_x(z,t) = \mathscr{E}_0(\hat{a} + \hat{a}^\dagger)\sin kz, \tag{21}$$

where the constants have been lumped together as

$$\mathscr{E}_0 = \left(\frac{\hbar\omega}{\epsilon_0 V}\right)^{1/2}, \tag{22}$$

which represents the ''electric field per photon.''[11]

The time dependence of the annihilation operator can be derived from the Heisenberg equation of motion

$$\frac{d\hat{a}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{a}], \tag{23}$$

so that

$$\hat{a}(t) = \hat{a}(0)\exp(-i\omega t), \tag{24}$$

$$\hat{a}^\dagger(t) = \hat{a}^\dagger(0)\exp(i\omega t). \tag{25}$$

D. Number operators and number states

Although the annihilation and creation operators do not themselves describe physical variables, their ''normal order'' product

$$\hat{n} = \hat{a}^\dagger\hat{a} \tag{26}$$

describes the number of excitations n in a single-mode field state $|n\rangle$,

$$\hat{n}|n\rangle = n|n\rangle. \tag{27}$$

The number state $|n\rangle$ is the energy eigenstate of the Hamiltonian Eq. (19) with eigenvalue E_n,

$$\hat{H}|n\rangle = \hbar\omega(\hat{a}^\dagger\hat{a} + \tfrac{1}{2})|n\rangle = E_n|n\rangle. \tag{28}$$

The lowest level $|0\rangle$ is defined through

$$\hat{a}|0\rangle = 0 \tag{29}$$

and the other states are given by

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \tag{30}$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \tag{31}$$

$$|n\rangle = (n!)^{-1/2}(\hat{a}^\dagger)^n|0\rangle. \tag{32}$$

The annihilation and creation operators decrease or increase the excitation of the mode by one quanta, and have only off-diagonal matrix elements between number states:

$$\langle n-1|\hat{a}|n\rangle = \sqrt{n}, \tag{33}$$

$$\langle n+1|\hat{a}^\dagger|n\rangle = \sqrt{n+1}. \tag{34}$$

E. Quantum fluctuations of a single-mode field

Although the number state $|n\rangle$ describes a state of precisely defined energy, it does not describe a state of well-defined *field*:

$$\langle n|\hat{E}_x(z,t)|n\rangle = \mathscr{E}_0 \sin kz(\langle n|\hat{a}^\dagger|n\rangle + \text{h.c.}) = 0. \tag{35}$$

The mean field is zero, but the mean square is not, for it is, of course, one component of the field mode energy in Eq. (10):

$$\langle n|\hat{E}_x^2(z,t)|n\rangle = \mathscr{E}_0^2 \sin^2 kz(\langle n|\hat{a}^\dagger\hat{a}^\dagger + \hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger + \hat{a}\hat{a}|n\rangle) = 2\mathscr{E}_0^2 \sin^2 kz(n + \tfrac{1}{2}). \tag{36}$$

The vacuum state is not an empty void but represents a field of rms magnitude $\mathscr{E}_0 \sin kz$. If we average over the spatial variation of the single-mode field, we find again that the ''electric field per photon'' is $(\hbar\omega/\epsilon_0 V)^{1/2}$. To see how large these ''vacuum fluctuations'' can be, consider a box of size 0.01 cm supporting visible-frequency radiation. Then, $\mathscr{E}_0 \approx 1$ V/cm, and this corresponds to an intensity of 1 mW/cm^2. These are not negligible quantities!

F. Coherent states—phase and superpositions

Now, as the expectation value of the field operator is zero for any number state, it is clear that number states, even for large n, do not in any way describe a field with classical-like properties. Classical fields have well-defined amplitudes and phases. However, a classical-like field can be obtained from a particular type of superposition, called coherent states, of all the number states. These states, denoted $|\alpha\rangle$, are given by

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{37}$$

where α is a complex number. They are defined in such a way as to be right eigenstates of the annihilation operator:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \tag{38}$$

$$\langle\alpha|\hat{a}^\dagger = \alpha^*\langle\alpha|. \tag{39}$$

As

$$\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle = |\alpha|^2 \tag{40}$$

we may interpret $|\alpha|^2$ as the average photon number $\overline{n}$. Further, since α is complex, it can be written in the polar form as $\alpha = \sqrt{\overline{n}}\exp(i\theta)$, where θ is the phase of the coherent state.

Thus the coherent state expectation value of the field operator is

$$\langle\alpha|\hat{E}_x(z,t)|\alpha\rangle=\mathscr{E}_0\sin kz\{\alpha\exp(-i\omega t)+\alpha^*\exp(i\omega t)\}$$
$$=2\mathscr{E}_0\sqrt{\bar{n}}\sin kz\cos(\omega t+\theta). \tag{41}$$

This has the form of a classical electric field as expected for a monochromatic wave. The fluctuations in the field at $t=0$ are given this time by

$$(\langle\hat{E}_x^2(z,0)\rangle_\alpha-\langle\hat{E}_x(z,0)\rangle_\alpha^2)=\mathscr{E}_0^2(\sin kz)^2 \tag{42}$$

and by comparing with Eq. (36) we see that the coherent states have only the fluctuations of the vacuum. Thus the coherent states are classical-like in the sense that they yield classical-like field expectation values and have fluctuations identical to that of the vacuum.

From Eq. (40), the probability of finding n photons in the field is the Poisson distribution

$$p(n)=\frac{\bar{n}^n}{n!}\exp(-\bar{n}). \tag{43}$$

The variance of the photon number is

$$\langle(\Delta\hat{n})^2\rangle_\alpha=\langle\alpha|(\hat{a}^\dagger\hat{a})^2|\alpha\rangle-(\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle)^2=|\alpha|^2=\bar{n}, \tag{44}$$

as expected for a Poisson distribution.

The coherent states are actually overcomplete and, in general, two different coherent states are not orthogonal. For two coherent states $|\alpha\rangle$ and $|\beta\rangle$, the inner product is

$$\langle\alpha|\beta\rangle=\exp\left\{-\frac{1}{2}|\alpha|^2-\frac{1}{2}|\beta|^2\right\}\sum_{n=0}^{\infty}\frac{(\alpha^*\beta)^n}{n!} \tag{45}$$

$$=\exp\{-\tfrac{1}{2}|\alpha|^2-\tfrac{1}{2}|\beta|^2+\alpha^*\beta\}. \tag{46}$$

But for $|\alpha-\beta|$ large, the states are nearly orthogonal as

$$|\langle\alpha|\beta\rangle|^2=\exp(-|\alpha-\beta|^2)\approx 0. \tag{47}$$

As said earlier, a classical monochromatic field is well defined in phase and amplitude. The concept of phase in quantum mechanics is problematic, however, and has had a long and contentious history. Here, we shall show that the phase properties of quantum states are well described by phase distributions obtained from the phase states[12] defined as

$$|\theta\rangle=\sum_{n=0}^{\infty}e^{in\theta}|n\rangle,\quad -\pi<\theta\leqslant\pi. \tag{48}$$

These states are not orthogonal but are complete. The phase distribution for an arbitrary field state $|\psi\rangle$ is given by[12,13]

$$P(\theta)=\frac{1}{2\pi}|\langle\theta|\psi\rangle|^2 \tag{49}$$

and is normalized according to

$$\int_{-\pi}^{\pi}P(\theta)d\theta=1. \tag{50}$$

Averages of any function of the phase θ, $f(\theta)$, are determined as

$$\langle f(\theta)\rangle=\int_{-\pi}^{\pi}f(\theta)P(\theta)d\theta. \tag{51}$$

For example, for a number state $|\psi\rangle=|n\rangle$ we have $P(\theta)=1/2\pi$, from which it follows that $\langle\theta\rangle=0$ and $\langle(\Delta\theta)^2\rangle=\pi^2/3$, meaning, as expected, that for a number state there are random fluctuations on the phase. But, for a coherent state $|\alpha\rangle$, we find that

$$P(\theta)=\frac{1}{2\pi}\left|\sum_{n=0}^{\infty}\frac{e^{in(\theta-\theta_0)}|\alpha|^n}{\sqrt{n!}}\exp\left(-\frac{1}{2}|\alpha|^2\right)\right|^2. \tag{52}$$

For large α we can approximate the Poisson distribution by a Gaussian:

$$\frac{|\alpha|^{2n}}{\sqrt{n!}}\exp\left(-\frac{1}{2}|\alpha|^2\right)\approx\frac{1}{\sqrt{2\pi|\alpha|^2}}\exp\left[-\frac{(n-|\alpha|^2)^2}{2|\alpha|^2}\right]. \tag{53}$$

Then, converting the sum into an integral, we obtain

$$P(\theta)\approx\left(\frac{2\bar{n}}{\pi}\right)^{1/2}\exp[-2\bar{n}(\theta-\theta_0)^2], \tag{54}$$

which is peaked at $\theta=\theta_0$. Furthermore, the larger $\bar{n}$ (hence the greater the fluctuations in $\hat{a}^\dagger\hat{a}$), the narrower the phase distribution. In other words, the phase becomes more well defined for larger field excitation, as we might expect for a classical field. In fact, there is a heuristic uncertainty relation between the number operator fluctuations and the phase fluctuations:

$$\langle(\Delta\hat{n})^2\rangle\langle(\Delta\theta)^2\rangle\geqslant\tfrac{1}{4}. \tag{55}$$

Using Eq. (56), we obtain $\langle\theta\rangle\approx\theta_0$ and $\langle(\Delta\theta)^2\rangle\approx 1/4\bar{n}$ and since $\langle(\Delta\bar{n})^2\rangle=\bar{n}$ then the above uncertainty relation is approximately satisfied with the equality.

The coherent states may be generated from the vacuum by the action of the displacement operator[14]

$$\hat{D}(\alpha)=\exp(\alpha\hat{a}^\dagger-\alpha^*\hat{a}). \tag{56}$$

That is,

$$|\alpha\rangle=\hat{D}(\alpha)|0\rangle, \tag{57}$$

as may be verified by using the operator theorem

$$e^{\hat{A}+\hat{B}}=e^{\hat{A}}e^{\hat{B}}e^{-[\hat{A},\hat{B}]/2}, \tag{58}$$

valid if $[\hat{A},[\hat{A},\hat{B}]]=[\hat{B},[\hat{A},\hat{B}]]=0$. Then

$$\hat{D}(\alpha)=\exp\left(-\frac{|\alpha|^2}{2}\right)e^{\alpha\hat{a}^\dagger}e^{\alpha^*\hat{a}} \tag{59}$$

and, as $e^{\alpha^*\hat{a}}|0\rangle=0$, upon expanding $e^{\alpha\hat{a}^\dagger}$ we arrive at Eq. (56).

A coherent state may be generated by a classically oscillating current.[11] Consider a quantum electromagnetic vector potential $\mathbf{A}(\mathbf{r},t)$ that is interacting with a classical current described by the vector current density $\mathbf{j}(\mathbf{r},t)$. Then according to electromagnetic theory the interaction energy V is given by

$$\hat{V}(t)=-\int d^3\mathbf{r}\,\mathbf{j}(\mathbf{r},t)\cdot\hat{\mathbf{A}}(\mathbf{r},t). \tag{60}$$

For a single-mode field, we take $\mathbf{A}(\mathbf{r},t)$ to be

$$\hat{\mathbf{A}}(\mathbf{r},t)=\boldsymbol{\epsilon}\left(\frac{\hbar}{2\omega\epsilon_0 V}\right)^{1/2}[\hat{a}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}+\text{h.c.}], \tag{61}$$

where $\boldsymbol{\epsilon}$ is the polarization unit vector. Then

$$\hat{V}(t) = -\left(\frac{\hbar}{2\omega\epsilon_0 V}\right)^{1/2}[\hat{a}\boldsymbol{\epsilon}\cdot\mathbf{J}(\mathbf{k},t)e^{-i\omega t}+\text{h.c.}], \tag{62}$$

where

$$\mathbf{J}(\mathbf{k},t) = \int d^3\mathbf{r}\mathbf{j}(\mathbf{r},t)e^{i\mathbf{k}\cdot\mathbf{r}} \tag{63}$$

is the Fourier transform of the current density. The infinitesimal time evolution operator in the interaction picture for the time interval t to $t+\Delta t$ is

$$\hat{U}(t+\Delta t,t) = \exp[-i\hat{V}(t)\Delta t/\hbar] = \exp\{-\Delta t[u(t)\hat{a} - u^*(t)\hat{a}^\dagger]\} = \hat{D}[u(t)\Delta t], \tag{64}$$

where

$$u(t) = -\left(\frac{\hbar}{2\omega\epsilon_0 V}\right)^{1/2}\boldsymbol{\epsilon}\cdot\mathbf{J}(\mathbf{k},t)e^{-i\omega t}. \tag{65}$$

Given the displacement operator satisfies the property

$$D(\alpha_2)D(\alpha_1) = \exp[(\alpha_1\alpha_2^* - \alpha_1^*\alpha_2)/2]D(\alpha_1+\alpha_2), \tag{66}$$

it is evident that repeated application of the short-time evolution operator on an initial vacuum state leads to a coherent state $|\alpha\rangle$, with

$$\alpha = \int_0^t dt' u(t'), \tag{67}$$

apart from an irrelevant overall phase.

In classical mechanics, the state of a system in phase space may be represented by a well-defined point if both position and momentum are accurately known, or by a distribution of points if all that is known is a range of likelihoods. In quantum mechanics it is impossible to specify joint values of position and momentum: The uncertainty principle shows that the product of intrinsic uncertainties $\Delta p \Delta x \geqslant \hbar/2$ and nothing we can do will enable us to study dynamics at a finer level than this minimum phase-space area of $\hbar/2$. Nevertheless, there are joint phase-space distribution functions in quantum mechanics: the best known is that introduced by Wigner in 1932 (Ref. 15); the price we have to pay is that these joint distributions are not probability distributions and, in particular, need not be positive (for this reason they are known as quasiprobabilities). The Wigner function $W_\Psi(q,p)$ of a system described by the pure-state wave function $\Psi^*(q)$ is given by

$$W_\Psi(q,p) = \int \frac{dq'}{2\pi\hbar}\Psi^*(q+q'/2)\Psi(q-q'/2)e^{ipq'/\hbar}. \tag{68}$$

It is a bilinear function of the wave function and is the simplest quasiprobability capable of generating the correct measurable marginal distributions $|\Psi(q)|^2$ and $|\Psi(p)|^2$ for position and momentum. A coherent state has a Gaussian Wigner function, one which occupies a special place in quantum mechanics: It is the only pure state Wigner function which is positive everywhere (Ref. 16). Given that coherent states are often regarded as quasiclassical, a frequently used criterion for nonclassicality is the size of any negativities in Wigner functions. As we shall see, the interference terms characteristic of Schrödinger cat superpositions result in substantial negativities in Wigner distributions in phase space.

III. SCHRÖDINGER CATS

Whether dead or alive, Schrödinger's cat is clearly a macroscopic object. Furthermore, the state of being alive is clearly distinguishable from the state of being dead.[17] Therefore, we require a superposition of two macroscopically distinguishable quantum states in order to realize a Schrödinger cat.

As we have seen, the coherent state $|\alpha\rangle$ can have any amplitude field since $|\alpha|^2$ can be arbitrarily large. Let us assume that $|\alpha|^2$ is "macroscopic." As our distinguishable state we choose a coherent state of the same amplitude but with a phase shift of 180°: $|-\alpha\rangle$. Both of these are classical-like states but the superposition of the two has many nonclassical properties. For definiteness, we take as our Schrödinger cat state the superposition

$$|\Psi\rangle = \mathcal{N}[|\alpha\rangle + e^{i\phi}|-\alpha\rangle], \tag{69}$$

where the normalization factor is given by

$$\mathcal{N} = [2+2\cos\phi\exp(-2\alpha^2)]^{-1/2} \tag{70}$$

and where we have assumed α real for simplicity. Note that for α large, $|\alpha\rangle$ and $|-\alpha\rangle$ are essentially orthogonal and $\mathcal{N}$ reduces to $1/\sqrt{2}$. The relative phase ϕ can, of course, be arbitrary, but we shall consider three special cases. For $\phi=0$ we obtain the even coherent states

$$|\Psi_e\rangle = \mathcal{N}_e[|\alpha\rangle + |-\alpha\rangle] \tag{71}$$

and, for $\phi=\pi$, the odd states

$$|\Psi_o\rangle = \mathcal{N}_o[|\alpha\rangle - |-\alpha\rangle] \tag{72}$$

and for $\phi=\pi/2$, we find

$$|\Psi_{\text{YS}}\rangle = \frac{1}{\sqrt{2}}[|\alpha\rangle + i|-\alpha\rangle], \tag{73}$$

which we call the Yurke–Stoler states.[18] Finally, we note that all superposition states of the form of Eqs. (71)–(73) are eigenstates of the square of the annihilation operator with α^2 as the eigenvalue:

$$\hat{a}^2|\alpha\rangle = \alpha^2|\alpha\rangle. \tag{74}$$

The superpositions of the form of Eq. (69) must be distinguished from a mere statistical mixture given by the density operator

$$\hat{\rho} = \tfrac{1}{2}(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|). \tag{75}$$

(Note that $\hat{a}^2\hat{\rho} = \alpha^2\hat{\rho}$.) One possible way to make this distinction would be to consider the respective phase distributions. Using the phase states and the earlier large field approximation, we must have for a state of the form of Eq. (69) that

$$P_\Psi(\theta) \approx \left(\frac{2\bar{n}}{\pi}\right)^{1/2}|\mathcal{N}|^2\{\exp[-2\bar{n}(\theta-\theta_0)^2] + \exp[-2\bar{n}(\theta-\theta_0-\pi)^2] + 2\cos(\bar{n}\pi-\phi) \times \exp[-\bar{n}(\theta-\theta_0)^2 - \bar{n}(\theta-\theta_0-\pi)^2]\}, \tag{76}$$

where $\bar{n}=|\alpha|^2$. For the density operator of Eq. (75) we have

$$P_\rho(\theta) \approx \frac{1}{2}\left(\frac{2\bar{n}}{\pi}\right)^{1/2}\{\exp[-\bar{n}(\theta-\theta_0)^2 - \bar{n}(\theta-\theta_0-\pi)^2]\}. \tag{77}$$

The distributions are both peaked at $\theta-\theta_0=0$ and π, as expected, but the first has an interference term. However, the interference term is essentially zero as the Gaussians in the product have very little overlap. Thus the phase distributions, at high field strengths $|\alpha|$, do not exhibit strong quantum interferences and hence do not exhibit the nonclassical nature of superpositions. The simplest way, however, to distinguish a cat state from a statistical mixture is to look for interference fringes in the quadrature phase distributions measured in homodyne detection. If one knows the phase of the superposed coherent states, it is only necessary to measure the quadrature phase operator along the axis orthogonal to the line through the coherent amplitudes of the corresponding states. Cat states would then generate fringes in the homodyne output, which are absent for statistical mixtures.[18]

However, a full signature that nonclassical properties exist may be obtained from examining the corresponding Wigner functions. Consider first an even coherent state. The Wigner function for this is

$$W_e(x,y)=\frac{1}{\pi[1+\exp(-2\alpha^2)]}\{\exp[-2(x-\alpha)^2-2y^2]+\exp[-2(x+\alpha)^2-2y^2]+2\times\exp[-2x^2-2y^2]\cos(4y\alpha)\}. \tag{78}$$

The last term on the right-hand side arises as a result of the quantum interference between $|\alpha\rangle$ and $|-\alpha\rangle$ and is responsible for the nonclassical properties of the even coherent states. Essentially, the last term can cause the Wigner function to be negative in some regions of phase space and thus it cannot be interpreted as a probability distribution. For contrast, we now consider the Wigner function corresponding to a statistical mixture of the states $|\alpha\rangle$ and $|-\alpha\rangle$ given by the density operator

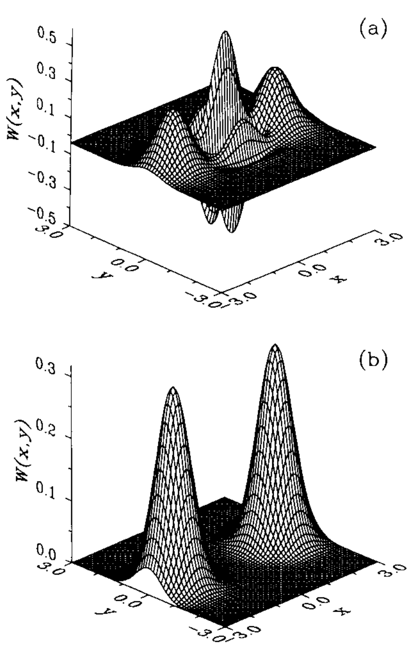

Fig. 1. Phase-space variation of the Wigner function of (a) the even coherent superposition given by Eq. (78), compared with (b) that of the statistical mixture of coherent states given by Eq. (80). In both cases $\alpha=2$.

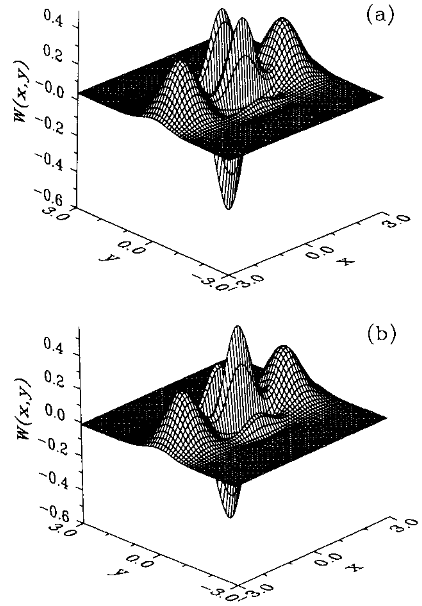

Fig. 2. As in Fig. 1, but (a) for the odd coherent state and (b) the Yurke–Stoler coherent state superpositions given by Eqs. (81) and (82).

$$\hat{\rho}=\tfrac{1}{2}(|\alpha\rangle\langle\alpha|+|-\alpha\rangle\langle-\alpha|). \tag{79}$$

We find

$$W_M(x,y)=\frac{1}{\pi}\{\exp[-2(x-\alpha)^2-2y^2]+\exp[-2(x+\alpha)^2-2y^2]\}, \tag{80}$$

which obviously contains no oscillating term. In Fig. 1 we plot $W_e(x,y)$ for the even coherent state and W_M for the statistical mixture where it is clear that the function for the even coherent state has ''interference fringes'' which take on negative values, whereas that for the mixture exhibits no such fringes and is always positive. For the odd coherent state we have

$$W_o(x,y)=\frac{1}{\pi[1+\exp(-2\alpha^2)]}\{\exp[-2(x-\alpha)^2-2y^2]+\exp[-2(x+\alpha)^2-2y^2]-2\times\exp[-2x^2-2y^2]\cos(4y\alpha)\}, \tag{81}$$

which is obviously similar to $W_e(x,y)$, but notice that the fringes are shifted in sign as compared to the even coherent state. For the Yurke–Stoler state we have

$$W_{\mathrm{YS}}(x,y)=\frac{1}{\pi}\{\exp[-2(x-\alpha)^2-2y^2]+\exp[-2(x+\alpha)^2-2y^2]-2\exp[-2x^2-2y^2]\sin(4y\alpha)\}. \tag{82}$$

These functions are plotted in Fig. 2 and clearly show the expected interference fringes.

We now examine these states for specific nonclassical properties. For single-mode field states there are essentially two possible ways for nonclassical effects to manifest themselves. These are (i) the occurrence of sub-Poissonian statistics, also called amplitude squeezing, and (ii) quadrature squeezing. (There are, in fact, various forms of higher order squeezing which we shall ignore here, but see Ref. 8.) We begin by looking at the photon number distributions for the states $|\Psi_e\rangle$ and $|\Psi_o\rangle$. For the former, from Eq. (71), we have

$$p(n)=\frac{2\exp(\alpha^2)}{1+\exp(-\alpha^2)}\frac{\alpha^{2n}}{n!},\quad n \text{ even}, \tag{83}$$

$$p(n)=0,\quad n \text{ odd} \tag{84}$$

and for the latter, from Eq. (72),

$$p(n)=0,\quad n \text{ even}, \tag{85}$$

$$p(n)=\frac{2\exp(\alpha^2)}{1+\exp(-\alpha^2)}\frac{\alpha^{2n}}{n!},\quad n \text{ odd}. \tag{86}$$

For the Yurke–Stoler state of Eq. (73), $p(n)$ is identical to that of the coherent state, Eq. (43). To characterize the width of the distribution, it is convenient to use Mandel's Q-parameter, defined as[19]

$$Q=\frac{\langle(\Delta\hat{n})^2\rangle-\langle\hat{n}\rangle}{\langle\hat{n}\rangle}. \tag{87}$$

For $-1\leqslant Q<0$ the distribution is sub-Poissonian. For a coherent state and a Yurke–Stoler state, $Q=0$. Now for the even coherent state Q has the form

$$Q=\frac{4\alpha^2\exp(\alpha^2)}{1+\exp(-4\alpha^2)}>0 \tag{88}$$

for all α. Thus this state is, in fact, super-Poissonian rather than sub-Poissonian. But for the odd coherent state we have

$$Q=-\frac{4\alpha^2\exp(\alpha^2)}{1+\exp(-4\alpha^2)}<0, \tag{89}$$

which is clearly sub-Poissonian. It approaches -1 as $\alpha\to 0$.

We next look at squeezing.[20] To do this, we introduce the two quadrature operators

$$\hat{X}_1=\frac{1}{2}(\hat{a}+\hat{a}^\dagger),\quad \hat{X}_2=\frac{1}{2i}(\hat{a}-\hat{a}^\dagger) \tag{90}$$

(essentially dimensionless position and momentum operators for a harmonic oscillator), satisfying the commutation relation

$$[\hat{X}_1,\hat{X}_2]=\frac{i}{2}, \tag{91}$$

from which follows the uncertainty relation

$$\langle(\Delta\hat{X}_1)^2\rangle\langle(\Delta\hat{X}_2)^2\rangle\geqslant\tfrac{1}{16}. \tag{92}$$

For the coherent state it is easy to show that

$$\langle(\Delta\hat{X}_1)^2\rangle=\langle(\Delta\hat{X}_2)^2\rangle=\tfrac{1}{4} \tag{93}$$

and thus the uncertainty relation becomes an equality with equal uncertainties in both variables (these being the same as the vacuum, in fact). The variance of $\frac{1}{4}$ in the above is known as the standard quantum limit. It is possible to reduce the noise below this limit, and states for which $\langle(\Delta\hat{X}_1)^2\rangle$ or $\langle(\Delta\hat{X}_2)^2\rangle$ is less than $\frac{1}{4}$ are called squeezed, and squeezing is a distinctly nonclassical effect.[20]

Now, for the even coherent state, we find that

$$\langle(\Delta\hat{X}_1)^2\rangle=\frac{1}{4}+\frac{\alpha^2}{1+\exp(-2\alpha^2)}, \tag{94}$$

$$\langle(\Delta\hat{X}_2)^2\rangle=\frac{1}{4}-\frac{\alpha^2\exp(-2\alpha^2)}{1+\exp(-2\alpha^2)}, \tag{95}$$

so that reduced fluctuations appear in the $\hat{X}_2$ quadrature. (Note that the uncertainty relation is still satisfied, of course.) For the odd coherent state we have

$$\langle(\Delta\hat{X}_1)^2\rangle=\frac{1}{4}+\frac{\alpha^2}{1-\exp(-2\alpha^2)}, \tag{96}$$

$$\langle(\Delta\hat{X}_2)^2\rangle=\frac{1}{4}+\frac{\alpha^2\exp(-2\alpha^2)}{1-\exp(-2\alpha^2)}, \tag{97}$$

and thus no squeezing is evident. Note that the even state shows squeezing but does not have sub-Poissonian statistics while for the odd state it is vice versa. For the Yurke–Stoler state we have

$$\langle(\Delta\hat{X}_1)^2\rangle=\tfrac{1}{4}+\alpha^2, \tag{98}$$

$$\langle(\Delta\hat{X}_2)^2\rangle=\tfrac{1}{4}-\alpha^2\exp(-4\alpha^2), \tag{99}$$

and it is apparent that squeezing appears in $\hat{X}_2$. The maximum degree of squeezing is less than for the even coherent state. Finally, for the statistical mixture of Eq. (79) we find

$$\langle(\Delta\hat{X}_1)^2\rangle=\tfrac{1}{4}+\alpha^2, \tag{100}$$

$$\langle(\Delta\hat{X}_2)^2\rangle=\tfrac{1}{4}, \tag{101}$$

so that the fluctuations of $\hat{X}_2$ are those of a coherent state (or the vacuum), those of $\hat{X}_1$ are enhanced over those of a coherent state and so no squeezing occurs, as expected.

IV. DISSIPATIVE ENVIRONMENTS AND THE DESTRUCTION OF QUANTUM COHERENCE

Quantum coherence is an extraordinarily fragile property: The slightest coupling of the quantum system of interest to the wider world outside leads to the destruction of the superpositions which are so interesting in quantum physics. Any system we may be interested in will be connected to the outer environment: Radiation fields in high Q cavities will be resistively damped by the resonator walls, trapped laser cooled ions can emit photons spontaneously, and so on. The environment does more than extract energy from the quantum system. It also randomly disturbs the phases of the components in the superposition. Imagine, for example, that our quantum system is a harmonic oscillator (a field mode, or the motional state of a trapped ion, for instance) and is described by annihilation and creation operators of excitation in the oscillator, $\hat{a}$ and $\hat{a}^\dagger$. Any dissipative coupling to the outer world will result in the loss of at least one excitation and this is described by the application of the annihilation operator.[21] Suppose we were able to create an even Schrödinger cat state by some means in such an oscillator, so that the state was $|\Psi_e\rangle=\mathcal{N}_e[|\alpha\rangle+|-\alpha\rangle]$. The environmental effect of losing one excitation then results in the state after such a loss given by $|\Psi_o\rangle=\mathcal{N}'[|\alpha\rangle-|-\alpha\rangle]$, where $\mathcal{N}'$ is a normalization factor. That is, at some random time, dissipation transforms even cat states to odd cat states (and vice versa).[3] The probability of such a loss in a time interval Δt is

$$\Delta p=\gamma\Delta t\langle\Psi(t)|\hat{a}^\dagger\hat{a}|\Psi(t)\rangle, \tag{102}$$

where γ is the rate of energy dissipation. We immediately note that the environment destroys the integrity of superpositions at a rate $\overline{n}\gamma$, where $\overline{n}$ is the mean excitation of the state. In other words, decoherence is much faster than energy decay for superpositions involving substantial excitation numbers.[3] This is the reason why quantum superpositions of macroscopically different states are essentially impossible to realize (or put in a different way, why the world we see is essentially classical). Many papers have been devoted to the problem of the creation or destruction of macroscopic coherence within dissipative systems (see, e.g., the references cited in Ref. 22). Milburn and Holmes[22] solve for the dynamics of Schrödinger cat generation in the presence of dissipation in an exactly solvable model of light in a strongly nonlinear Kerr medium. This nonlinear dissipative model demonstrates not only how dissipation destroys coherence but also makes it difficult to generate such coherence in the first place.

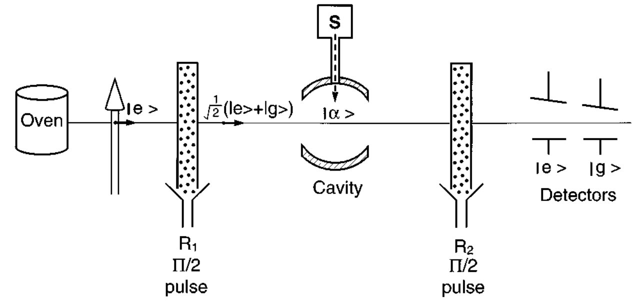

Fig. 3. Schematic outline of cavity QED setup proposed by Haroche *et al.* in (Ref. 24) to generate cat states in a microwave cavity through the interaction of Rydberg atoms, selective microwave excitations in two Ramsey zones R_1 and R_2, and state-selective field-ionization detection of Rydberg atoms in excited states $|e\rangle$ and ground states $|g\rangle$. A classical current drives the high Q cavity to initiate a coherent state $|\alpha\rangle$.

V. GENERATION OF CAT STATES

We now turn to the practical problem of how to generate Schrödinger cat states of the type previously described. Actually a rather large number of schemes have been developed, but here, in the context of quantum optics, we shall focus on only two; one that produces the Yurke–Stoler state and another that can produce the even and the odd coherent states.

We begin with the Yurke–Stoler states. These states may be produced by the interaction of a Kerr-like nonlinear medium with low dissipation with an initial coherent state. The medium is modeled by the interaction Hamiltonian

$$\hat{H}=\hbar\gamma(\hat{a}^\dagger\hat{a})^2, \tag{103}$$

where γ is proportional to the nonlinear susceptibility. An initial coherent state evolves into

$$|\Psi(t)\rangle=e^{-i\hat{H}_I t/\hbar}|\alpha\rangle \tag{104}$$

$$=\exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\,e^{-i\gamma n^2 t}|n\rangle. \tag{105}$$

Since n^2 is integer, this state is periodic with period $T=2\pi/\gamma$. If $t=\pi/\gamma$, we find, since $e^{-i\gamma n^2 t}=e^{-i\pi n^2}=(-1)^n$, that $|\Psi(\pi/\gamma)\rangle=|-\alpha\rangle$. But, for $t=\pi/2\gamma$, we find that $e^{-i\gamma n^2 t}=e^{-i\pi n^2/2}=1$ for even n, $e^{-i\pi n^2/2}=-i$ for odd n. Thus we have

$$\left|\Psi\left(\frac{\pi}{2\gamma}\right)\right\rangle=\frac{1}{\sqrt{2}}\left[\exp\left(-i\,\frac{\pi}{4}\right)\left|\alpha\right\rangle+\exp\left(i\,\frac{\pi}{4}\right)|-\alpha\rangle\right. \tag{106}$$

$$=\frac{1}{\sqrt{2}}\exp\left(-i\,\frac{\pi}{4}\right)[|\alpha\rangle+i|-\alpha\rangle], \tag{107}$$

which, apart from an irrelevant overall phase factor, is obviously the Yurke–Stoler state. Generating the Yurke–Stoler states will require appreciable nonlinearity, and low dissipation losses. Silica glass fiber has been much studied in guided-wave nonlinear optics, due to its very low losses. The required Kerr-like nonlinearity needed for producing such states found in normal SiO_2 fibers is, however, extremely small, hence long lengths would be required. At present these lengths are not realizable without significant losses leading to a decoherence of the superposition.

The Yurke–Stoler state is produced by a unitary transformation (time evolution) on a coherent state $|\alpha\rangle$. It is not possible to produce the even and odd coherent states by a unitary transformation. However, these states can be produced through the interaction of a field with another system, such as an atom, followed by state selective quantum measurements. Such processes involve the inherently nonclassical concepts of entanglement and the projection postulate (collapse of the state vector) of von Neumann.[23] Here we shall discuss a particular example, in the context of cavity quantum electrodynamics (CQED), proposed, and realized experimentally, by Haroche and collaborators[24] in Paris.

The proposed experimental setup is pictured in Fig. 3. The cavity is made of superconducting niobium cooled to a temperature of about 1 K, has dimensions typically of 10^{-2} m, and has a very high $Q(10^8)$. It supports a single mode of the quantized field of frequency $\nu_c=50$ GHz (submillimeter microwave). The cavity decay time is, therefore, $T_{\rm cav}=Q/\nu_c\approx 10^{-2}$ s. The cavity mode is prepared in coherent state $|\alpha\rangle$ by a classical source attached to the cavity. On the left, an oven (along with a number of other manipulating devices) produces velocity selected circular Rydberg atoms (atoms with principal quantum number $n\geqslant 30$ and with $|m|=n-1$). In the proposed experiment, three circular Rydberg levels are required. We label these levels as $|g\rangle$, $|e\rangle$, and $|f\rangle$ for $n=50,51,52$, respectively. The resonant frequency between states $|e\rangle$ and $|f\rangle$ is somewhat detuned from the cavity frequency ν_c while that between states $|g\rangle$ and $|e\rangle$ is highly detuned from the cavity (see Fig. 4). The atom–cavity field interaction is, therefore, a dispersive one described by the effective interaction Hamiltonian

$$\hat{H}_I=\hbar\xi\hat{a}^\dagger\hat{a}\sigma_3, \tag{108}$$

where $\sigma_3=|f\rangle\langle f|-|e\rangle\langle e|$, $\xi=2d^2/\Delta$ and where d is the atomic dipole moment and Δ the detuning. No real transitions occur between levels $|e\rangle$ and $|f\rangle$. However, outside the cavity on either side—in regions marked R_1 and R_2, classical microwave fields resonantly drive transitions between $|g\rangle$ and $|e\rangle$. These regions are referred to as Ramsey zones and the transition frequency between $|g\rangle$ and $|e\rangle$ is about 50G Hz. These Ramsey zones are set to produce $\pi/2$ pulses

$$|e\rangle\rightarrow\frac{1}{\sqrt{2}}\,(|e\rangle+|g\rangle), \tag{109}$$

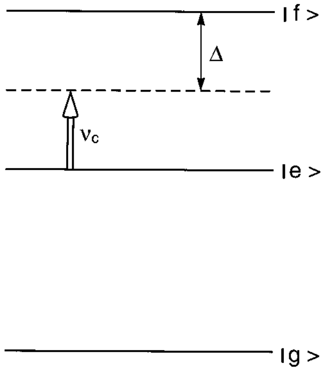

Fig. 4. Rydberg atom level scheme relevant for the method outlined in Fig. 3 to generate cat states. The cavity field mode frequency ν_c is chosen to be detuned by an amount Δ from resonance with transitions $|e\rangle$ to some level $|f\rangle$. The Ramsey zones in Fig. 3 drive transitions between $|e\rangle$ and $|g\rangle$.

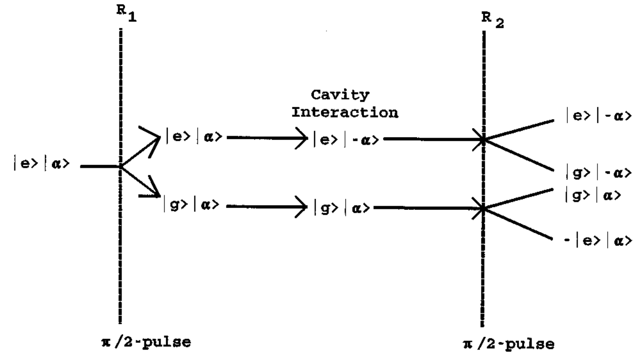

Fig. 5. Interfering pathways in the scheme shown in Fig. 3, demonstrating the potential outcomes after Ramsey microwave interaction in zones R_1 and R_2 and cavity interaction in the center of the figure.

$$|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle). \tag{110}$$

Finally, the atom, after passing through R_1, the cavity, and R_2, is selectively ionized, field ionization for circular Rydberg atoms being nearly 100% efficient. Now when an atom emerges from the oven it is laser excited to state $|e\rangle$. The first Ramsey zone creates the atomic superposition

$$|\Psi_A\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle). \tag{111}$$

Then the initial state of the atom–cavity field system is $|\Psi_{\mathrm{AF}}(0)\rangle = |\Psi_A\rangle|\alpha\rangle$. With the atom in the cavity, at time t, the system state vector is

$$|\Psi_{\mathrm{AF}}(t)\rangle = e^{-i\hat{H}_I t/\hbar}|\Psi_{\mathrm{F}}(0)\rangle \tag{112}$$

$$= \frac{1}{\sqrt{2}}(|e\rangle|\alpha e^{i\xi t}\rangle + |g\rangle|\alpha\rangle), \tag{113}$$

where we have used the relation

$$e^{-i\xi t\hat{a}^\dagger\hat{a}\sigma_3}|e\rangle|\alpha\rangle = e^{i\xi t\hat{a}^\dagger\hat{a}}|\alpha\rangle|e\rangle \tag{114}$$

$$= |\alpha e^{i\xi t}\rangle|e\rangle \tag{115}$$

(the term containing $|g\rangle$ being unaffected). We now suppose that the atomic velocity is selected such that at the time the atom leaves the cavity $\xi t = \pi$. (The required velocity for a cavity of width 10^{-2} m is about 100 m/s, the interaction time being 10^{-4} s$\ll T_{\mathrm{cav}}$.) Then our state vector has the form

$$\left|\Psi_{\mathrm{AF}}\left(\frac{\xi}{\pi}\right)\right\rangle = \frac{1}{\sqrt{2}}(|e\rangle|-\alpha\rangle + |g\rangle|\alpha\rangle). \tag{116}$$

This state is an entanglement between the cavity field and the atom, an EPR–Bohm type of state.[6,25] In fact, this is precisely the sort of state Schrödinger had in mind, in his paper of 1935, in the sense that the alive or dead cat is correlated with the atom not decayed or decayed, respectively. The correlations between atom and field in Eq. (116) are maintained even after the atom leaves the cavity. Selective ionization detecting $|g\rangle$ projects the cavity field into $|\alpha\rangle$, while detecting $|e\rangle$ gives $|-\alpha\rangle$. To obtain superpositions of $|\alpha\rangle$ and $|-\alpha\rangle$ we must apply the second Ramsey field pulse to obtain

$$\left|\Psi_{\mathrm{AF}}\left(\frac{\pi}{\xi}\right)\right\rangle \rightarrow \left|\Psi'_{\mathrm{AF}}\left(\frac{\pi}{\xi}\right)\right\rangle \tag{117}$$

$$= \tfrac{1}{2}[(|e\rangle + |g\rangle)|-\alpha\rangle + (|g\rangle - |e\rangle)|g\rangle] \tag{118}$$

$$= \tfrac{1}{2}[|g\rangle(|\alpha\rangle + |-\alpha\rangle) + |e\rangle(|\alpha\rangle - |-\alpha\rangle)]. \tag{119}$$

Now, selectively ionizing and detecting $|g\rangle$ projects the cavity field into state $|\Psi_e\rangle = \mathcal{N}_e(|\alpha\rangle + |-\alpha\rangle)$, while for $|e\rangle$ into $|\Psi_o\rangle = \mathcal{N}_o(|\alpha\rangle - |-\alpha\rangle)$. The application of the second Ramsey field has caused the original correlated terms of Eq. (116) to interfere as is illustrated in Fig. 5.

The procedure discussed above amounts to a measurement of the parity of the cavity field. To see this, suppose the cavity contains only a number state. Then preparing the atomic state exactly as before, the interaction with the cavity field results in

$$\frac{1}{\sqrt{2}}(e^{in\xi t}|e\rangle| + |g\rangle)|n\rangle. \tag{120}$$

The second $\pi/2$ pulse produces

$$\tfrac{1}{2}\{|e\rangle[e^{in\xi t} - 1] + |g\rangle[e^{in\xi t} + 1]\}|n\rangle, \tag{121}$$

which, for $\xi t = \pi$, becomes $|g\rangle|n\rangle$, n even, or $|g\rangle|n\rangle$ for n odd. Thus, having established an even or odd coherent state, a second atom sent through the cavity should always be detected in the same state as the first. This amounts to a detection of the Schrödinger cat states.

Losses through the cavity walls will, of course, cause the superpositions to decohere. The decoherence time is given by $T_{\mathrm{dec}} = T_{\mathrm{cav}}/|\alpha|^2$. In order that T_{dec} not be too short, this effectively limits the size of $|\alpha|^2$ to about 10 (this could be improved for higher Q cavities). If, for example, an even coherent state is produced (detection of $|g\rangle$), decoherence will cause some of the odd number states to be populated. The decoherence is detectable since it is apparent that the second atom could be found in the excited state. The decoherence rate could be detected by time delaying the second atom. For very large $|\alpha|^2$, of course, the decoherence will be essentially instantaneous. But for the intermediate $|\alpha|^2$ this is detectable and detection of this decoherence was one of the key achievements of the Paris experiment.[24]

VI. CONCLUSIONS

The paradigm of Schrödinger's cat is, like that of Einstein, Podolsky, and Rosen (EPR),[6] often presented as though it were a paradox (particularly so in some of the popular literature on the subject[26]). But Schrödinger's cat ''paradox'' is no paradox at all; it is a phenomenon. There are no true paradoxes in nature. Historically, this alleged paradox has often been dismissed as having no observable consequences. Such a position can no longer be maintained.

In this paper, we have given a pedagogical presentation of theoretical ideas behind attempts to experimentally realize Schrödinger's cat states in quantum optics. The particular states we have discussed are those formed as superpositions of two coherent states, shifted in phase by 180°, and we have discussed two ways by which they may be generated. The cavity-QED experiments are of particular interest as they have the potential to elucidate the border between the classical and the quantum through the observation of the decoherence of a superposition of at least mesoscopically distinguishable states. However, we have far from exhausted the various types of cat states and methods of their generation that have been considered in quantum optics. Many of these have recently been reviewed by Bužek and Knight.[8]

Finally, we must also mention that catlike states have been generated in different contexts. Noel and Stroud[27] have generated radial Schrödinger's cat states in a Rydberg atom with average principal quantum number 65. Two radial wave packets are created that can be separated by as much as 0.4 μm. More recently, Monroe *et al.*[9] have generated the even and odd coherent states as well as Yurke–Stoler states for the quantized vibrational motion of a single trapped ion. With an average of about nine vibrational quanta, wave packets of maximal spatial separation of about 83 nm, significantly larger than the size of a single-component wave function in the superposition, about 7 nm, were obtained. These superpositions are obviously not macroscopic but do approach being mesoscopic and therefore may be taken as a realization of the Schrödinger cat phenomenon.

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