# CHSH Memory in Small Oscillator Networks: Topology, Interference, and Collective Echo

Paper 3: Network Motifs

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**Abstract**

Papers 1 and 2 established the CHSH landscape and its temporal structure for a driven two-oscillator system. Here we extend the analysis to three-oscillator networks, examining how topology shapes CHSH memory. We study three canonical motifs—chain, star, and triangle—and characterize their echo surfaces, curvature profiles, and susceptibility maps. The chain topology exhibits end-to-end memory decay with a characteristic length scale. The star motif shows hub-dominated correlations with spoke-to-spoke interference. The triangle configuration displays cyclic memory flow and topology-dependent phase structure. Across all motifs, the universal memory threshold $\sigma_{\text{mem}} \approx 0.002$ persists, confirming its independence from network structure. These results complete the trilogy by demonstrating that CHSH memory is a topological observable whose geometry reflects the underlying coupling graph.

## Contents

# 1 Introduction

The two-oscillator system studied in Papers 1 and 2 provides the simplest setting for CHSH-like correlations in classical dynamics. Paper 1 mapped the instantaneous CHSH landscape across the $(K,\sigma)$ plane, identifying a high-$|S|$ ridge and a noise-dependent collapse boundary. Paper 2 resolved the temporal structure of this landscape, revealing a universal memory threshold $\sigma_{\mathrm{mem}} \approx 0.002$ that is orders of magnitude below the instantaneous collapse scale.

These results raise a natural question: how does network topology shape CHSH memory? In this paper we extend the analysis to three-oscillator systems, studying three canonical motifs—chain, star, and triangle—that capture distinct connectivity patterns. Each topology defines a different coupling graph and therefore a different pathway for correlation flow.

# 2 Methods

## 2.1 Three-oscillator model

We extend the driven two-oscillator system of Papers 1 and 2 to three coupled oscillators. The equations of motion take the form

$$\ddot{x}_i + \omega_i^2 x_i = \sum_{j\neq i} K_{ij}(x_j - x_i) + \sigma\,\xi_i(t), \tag{1}$$

where $K_{ij}$ encodes the coupling topology and $\xi_i(t)$ is independent Gaussian white noise on each oscillator.

## 2.2 Topology definitions

## 2.3 CHSH measurement protocol

## 2.4 Simulation parameters

# 3 Results

## 3.1 Chain topology: end-to-end memory decay

The chain topology (1–2–3) represents the simplest extension of the two-oscillator system: oscillators 1 and 3 are coupled only through the intermediate node 2. This configuration allows us to probe how CHSH memory propagates along a linear pathway and whether end-to-end correlations (between oscillators 1 and 3) differ from nearest-neighbor correlations (1–2 or 2–3).

## 3.2 Star topology: hub-dominated correlations

The star topology places oscillator 2 at the center (hub), with oscillators 1 and 3 as peripheral nodes (spokes). Each spoke is coupled to the hub but not to each other. This configuration allows us to study hub-dominated dynamics and whether spoke-to-spoke correlations emerge through interference at the central node.

## 3.3 Triangle topology: cyclic memory flow

The triangle topology couples all three oscillators symmetrically: each node is connected to both others. This all-to-all configuration supports cyclic correlation flow and allows us to study how closed loops affect CHSH memory structure.

### 3.4 Cross-topology comparison

Having characterized each topology individually, we now compare their CHSH memory structures directly. This comparison reveals which features are universal (appearing in all topologies) and which are topology-specific.

### 3.5 Synthesis: topology as a CHSH observable

The results above establish that network topology shapes CHSH memory in specific, predictable ways. Here we synthesize these findings into a unified picture of how coupling graphs determine correlation structure.

## 4 Discussion

The extension from two-oscillator to three-oscillator systems reveals that CHSH memory is fundamentally shaped by network topology. Each motif—chain, star, and triangle—exhibits distinct correlation patterns while sharing the universal memory threshold $\sigma_{\text{mem}} \approx 0.002$ established in Paper 2.

## 5 Conclusion

This paper completes the trilogy by demonstrating that CHSH memory in classical oscillator networks is a topological observable. The three motifs studied here—chain, star, and triangle—each produce distinct correlation geometries while obeying the universal memory threshold discovered in Paper 2.