

Solution of 6 persons at a party problem.

Using the method of case by case;

Let $(X, Y, Z, P1, P2, P3)$ to be a random arrangement of the six people.

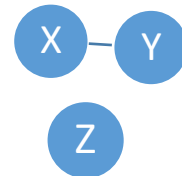
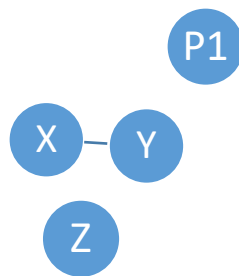
Let's suppose that we don't have 3 people none of whom knows either of the other two,

So that means that for each group of 3 people among the six we find at least one relation between two members (Hypothesis), this brings us to 3 cases:

1. Only one connection :

In this case we consider another person P1,

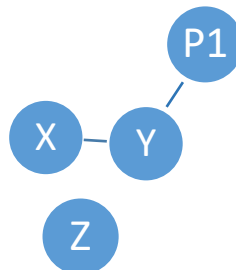
So we get this shape:



So the triplet $(P1, Y, Z)$ should have at least one relation if we follow what the hypothesis says,

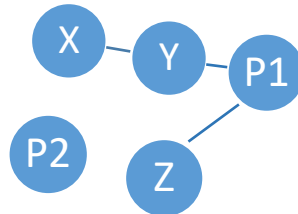
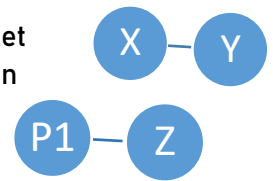
We get to these two cases:

- Y knows P1 or both Y and Z knows P1, so we get a triangle of two connections in this case



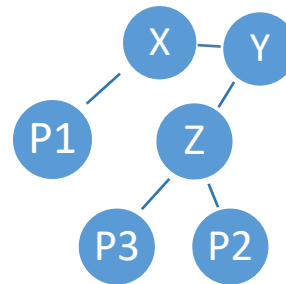
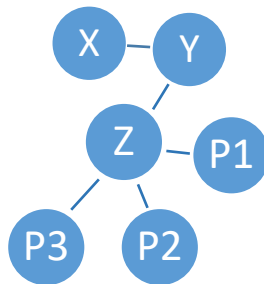
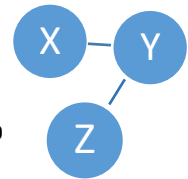
- P1 knows only Z in the triplet (P1, Z, Y) :

- In this case we add another person to the graph P2, so the triplet (P1, P2, Y) should have at least one connection, each connection will lead us to form a group of two connections in this case (P1, Y, Z):



2. Two connections :

- For each person of the rest we consider the triangle (Pi, X, Z) such that $i \in \{1, 2, 3\}$, for that we have two cases, X or Z knows all the three at Least, or X or Z knows two of the persons, and the other one knows the remaining person, all the possibilities will lead us to this two shapes (if the triangle (Pi, X, Z) has only one connection):



In both cases we can get 3 connections by considering the triplet (Y, P1, P2) in both figures and apply the hypothesis to find 3 connections which means 3 people who know all each other.

No matter how many relations are there in the triangle (Pi, X, Z) we get to the same result because the triangle (P1, P2, Y) is independent of the count of relations that P1 and P2 have.

3. Three connections :

We found the set of 3 people that all know each other.

