

Tutorial - 06

Ques 1

Minimum spanning tree :- It is a tree subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles and with minimum possible edge weighted.

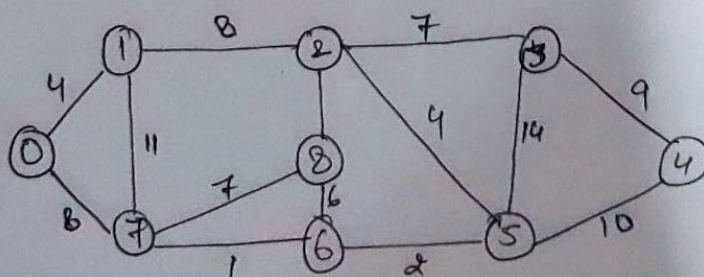
APPLICATION:-

- 1) Consider n stations are to be linked using a communication network and lying of link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.
- 2) Designing LAN.
- 3) Laying pipeline connecting offshore drilling sites, refineries and consumer markets.

Ques 2

Algorithms	Time complexity	Space complexity
1) Prim's Algorithm	$O(E \log V)$	$O(V)$
2) Kruskal Algorithm	$O(E \log E)$	$O(V)$
3) Dijkstra Algorithm	$O(V^2)$	$O(V^2)$
4) Bell-man Ford	$O(VE)$	$O(E)$

Ques 3

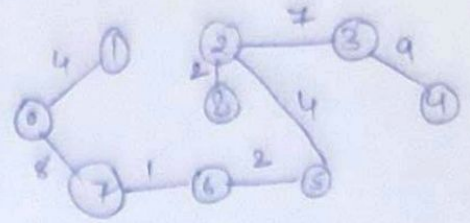


Prim's Algorithm :-

$$\begin{aligned}
 \text{weight} &= 4 + 8 + 2 + 4 + 2 + 7 + 9 + 3 \\
 &= \underline{37}
 \end{aligned}$$

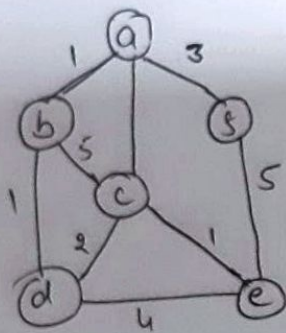
Kruskal's Algorithm :-

	V	W	
0	✓	1	✓
6	✓	2	✓
5	6	2	✓
2	8	4	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	✓
3	5	14	X



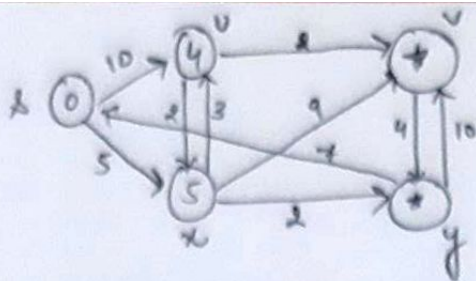
$$\begin{aligned} \text{weight} &= 1+2+2+4+4+7 \\ &\quad +6+9 \\ &= \underline{\underline{37}} \end{aligned}$$

Ques 4



- 1) The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'. for eg- let the shortest path of weight is and has edge. let there be another path with 2 edges & total weights. The weight of shortest path is increased by 5×10 and becomes $15+50$ weight of other path is increased by 2×10 & becomes $20+20$ so the shortest path changes to other path with weight as 40.
- 2) If we multiply all the edges weight by 10, the shortest path cannot change. The reason is that weights of all path from 's' to 't' gets multiplied by some const. The no. of edges or path doesn't matter.

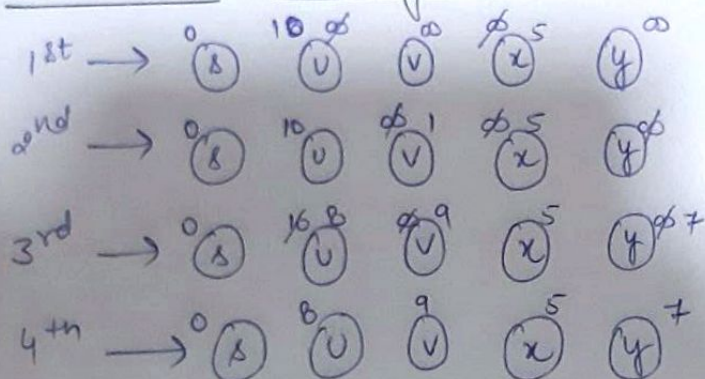
Ques 5



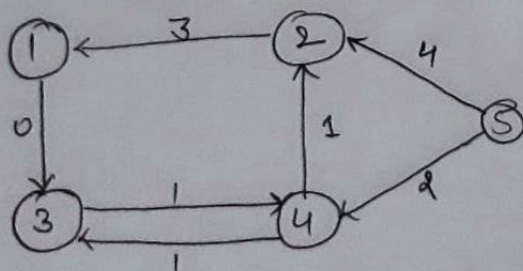
→ Dijkstra's Algorithm :-

Node	shortest distance from source node
u	8
x	5
v	9
y	7

→ Bellman Ford Algorithm :-



Ques 6



	1	2	3	4	5
1	0	∞	6	3	∞
2	2	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & \infty & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix}
 \end{matrix}$$

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & \infty & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix}
 \end{matrix}$$

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & \infty & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix}
 \end{matrix}$$

$$\left. \begin{array}{l} \text{T.C.} \rightarrow O(|V|^3) \\ \text{S.C.} \rightarrow O(|V|^2) \end{array} \right\}$$