

## Tutorial-04

Ques1

$$T(n) = 3T(n/2) + n^2$$

$$\rightarrow a=3, b=2, f(n)=n^2$$

$$n^{\log_a b} = n^{\log_2 3}$$

$$\therefore n^{\log_2 3} < f(n)$$

$$\therefore T(n) = O(n^2)$$

Ques2

$$T(n) = 4T(n/2) + n^2$$

$$a=4, b=2, f(n)=n^2$$

$$n^{\log_a b} = n^{\log_2 4} = n^2$$

$$n^{\log_a b} = f(n)$$

$$\therefore T(n) = O(n^2 \log n)$$

Ques3

$$T(n) = T(n/2) + 2^n$$

$$\rightarrow a=1, b=2, f(n)=2^n$$

$$n^{\log_a b} = n^{\log_2 1} = n^0 = 1$$

$$n^{\log_a b} < f(n)$$

$$T(n) = O(2^n)$$

Ques4

$$T(n) = 2^n + T(n/2) + n$$

$\therefore a$  is a function

Master theorem is not possible.

Ques5

$$T(n) = 16T(n/4) + n$$

$$a=16, b=4, f(n)=n$$

$$n^{\log_a b} = n^{\log_4 16} = n^2$$

$$n^{\log_a b} > f(n)$$

$$T(n) = O(n^2) \text{ Ans}$$

Ques 6

$$T(n) = 2T(n/2) + n \log n$$

$$\rightarrow a=2, b=2, f(n) = n \log n$$

$$n \log_b^a = n \log_2^2 = n$$

$$n \log_b^a < f(n)$$

$$T(n) = \Theta(f(n)) = \Theta(n \log n)$$

Ques 7

$$T(n) = 2T(n/2) + n/\log n$$

$$\rightarrow a=2, b=2, f(n) = n/\log n$$

$$n \log_b^a = n \log_2^2 = n$$

$$n \log_b^a > f(n)$$

$$T(n) = \Theta(n)$$

Ques 8

$$T(n) = 2T(n/4) + n^{0.51}$$

$$a=2, b=4, f(n) = n^{0.51}$$

$$n \log_b^a = n \log_4^2 = n^{1/2} = \sqrt{n}$$

$$n \log_b^a < f(n)$$

$$T(n) = \Theta(f(n)) = \Theta(n^{0.51}) \text{ Ans}$$

Ques 9

$$T(n) = 0.5T(n/2) + 1/n$$

$a < 1$ , master theorem is not applicable

Ques 10

$$T(n) = 16T(n/4) + n!$$

$$a=16, b=4, f(n) = n!$$

$$n \log_b^a = n \log_4^{16} = n^2$$

$$n \log_b^a < f(n)$$

$$T(n) = \Theta(n^2) \text{ Ans}$$

Ques 11

$$T(n) = 4T(n/2) + \log n$$

$$a=4, b=2, f(n) = \log n$$

$$n \log_b^a = n \log_2^4 = n^2$$

$$T(n) = \Theta(n^2)$$



Ques 12

$$T(n) = \log(n) T(n/2) + \log n$$

$\therefore a$  is not constant

master theorem is not applicable.

Q.13)

$$T(n) = 3T(n/2) + n$$

$$a=3, b=2, f(n)=n$$

$$n^{\log_2 3} = n^{1.58}$$

$$n^{\log_2 3} > f(n)$$

$$T(n) = O(n^{1.58})$$

Ques 14

$$T(n) = 3T(n/3) + \sqrt{n}$$

$$\rightarrow a=3, b=3, f(n)=\sqrt{n}$$

$$n^{\log_3 3} = n$$

$$f(n) = O(n)$$

Ques 15

$$T(n) = 4T(n/2) + cn$$

$$\rightarrow a=4, b=2, f(n)=cn$$

$$n^{\log_2 4} = n^{\log_2 4} = n^2$$

$$n^{\log_2 4} \gg f(n)$$

$$T(n) = O(n^2)$$

Q.ues 16

$$T(n) = 3T(n/4) + n \log n$$

$$a=3, b=4, f(n)=n \log n$$

$$n^{\log_4 3} = n^{0.79}$$

$$n^{\log_4 3} < f(n)$$

$$T(n) = O(n \log n)$$

Ques 17

$$T(n) = 3T(n/3) + n/2$$

$$a=3, b=3, f(n)=n/2$$

$$n^{\log_3 3} = n^{\log_3 3} = n$$

$$n^{\log_3 3} > f(n)$$

$$T(n) = O(n)$$

Ques 18

$$T(n) = 6T(n/3) + n^2 \log n$$

$$\rightarrow a=6, b=3, f(n) = n^2 \log n$$

$$n^{\log_b a} = n^{1.63}$$

$$\underline{T(n) = O(n^2 \log n)}$$

Ques 19

$$T(n) = 4T(n/2) + n/\log n$$

$$a=4, b=2, f(n) = n/\log n$$

$$n^{\log_b a} = n^2$$

$$\underline{T(n) = O(n^2)}$$

Ques 20

$$T(n) = 64T(n/8) - n^2 \log n$$

$f(n)$  is not increasing function so master's theorem is not applicable.

Ques 21

$$T(n) = 7T(n/3) + n^2$$

$$a=7, b=3, f(n) = n^2$$

$$n^{\log_b a} = n^{\log_3 7} = n^{1.3}$$

$$n^{\log_b a} < f(n)$$

$$\underline{T(n) = O(n^2)}$$

Ques 22

$$T(n) = T(n/2) + n(2 - \log n)$$

Master's theorem is not applicable.

$\therefore$  regularity condition is isolated in case 3.