MSAN 601: Linear Regression Analysis
Quiz 3

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There are 3 questions on this quiz. Each question is weighted equally. Grades will be out of 100 points. You have until 9:45. Good luck!

1. Let μ be some unknown parameter for which from data you have an estimator $\widehat{\mu}$. Furthermore, you know that $\widehat{\mu} \sim N(\mu, 0.5)$.

   (a) Construct a 95% confidence interval for μ. Give a clear step-by-step description of how to construct this confidence interval. At what value of $\widehat{\mu}$ will we be 95 % confident that μ is not 0?

   (b) Develop a decision rule for the α - level hypothesis test $H_o : \mu = 2$ versus $H_1 : \mu > 2$. Give a clear step-by-step description of how to construct this decision rule.

2. Recall that the covariance between the random variables X and Y is defined by $\text{Cov}(X, Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$.

   (a) Use the above definition to prove (justifying steps as necessary) that $\text{Cov}(aX + b, bY + d) = ac\text{Cov}(X, Y)$ for constants a, b, c, d.

   (b) Suppose that an engineer is interested in the height of a skyscraper in New York. She first measures the skypscraper in feet and determines, from quantifying her error in measurement from past measurements, that the height is a random variable with mean 1800 feet and variance 81. She hands these measurements over to another engineer, who asks that she report these numbers in yards (there are 3 feet in 1 yard). What is the mean and variance of the skyscraper in yards?

3. Let X and Y be two random variables.

   (a) Define the *correlation coefficient* between X and Y ($\rho(X, Y)$) and give its range of possible values.

   (b) Describe how to interpret $\rho(X, Y)$. (Be as specific as possible.)

   (c) What specifically can you say about two random variables such that $\rho(X, Y) = 1$?

   (d) If $Y = aX^2$ for some $a > 0$, what do you expect $\rho(X, Y)$ to look like?