Inhaltsverzeichnis

1 Technische Mathematik

1.2 Grundrechnungsarten

1.2.1 Zahlen, Addition und Subtraktion

Lösungen zu 1.2.1

10/1. a) $(3+7)+1 = 3+(7+1) = (3+1)+7 = 3+(1+7) =$
$10 \quad +1 = 3+ \quad 8 \quad = \quad 4 \quad +7 = 3+ \quad 8 \quad = \mathbf{11}$

b) $(11+9)-5 = 11+(9-5) = (11-5)+9 = 11+(-5+9) =$
$20 \quad -5 = 11+ \quad 4 \quad = \quad 6 \quad +9 = 11+ \quad 4 \quad = \mathbf{15}$

c) $2+3+4 = (2+3)+4 = 2+(3+4) = (2+4)+3 = 2+(4+3) =$
$5 \quad +4 = 2+ \quad 7 \quad = \quad 6 \quad +3 = 2+ \quad 7 \quad = \mathbf{9}$

d) $8+2+4 = (8+2)+4 = 8+(2+4) = (8+4)+2 = 8+(4+2) =$
$10 \quad +4 = 8+ \quad 6 \quad = \quad 12 \quad +2 = 8+ \quad 6 \quad = \mathbf{14}$

e) $(11+14)+(16+19) = 11+(14+16)+19 =$
$(11+19)+(14+16) = (19+14)+(11+16) = \mathbf{60}$

10/2. a) $400-(46+18-120+14+52-16) = 400-(46+14+52+18-120-16) =$
$= 400-(-6) = \mathbf{406}$

$400-46-18+120-14-52+16 = 400+120+16-(46+14+18+52) = 536-130 =$
$= \mathbf{406}$

b) $647-123-(79-68+37+21-67+20) = 647-123-(79+21+37+20-68-67) =$
$= 524-22 = \mathbf{502}$

$647-123-79+68-37-21+67-20 = 647+68+67-(123+37+79+21+20) =$
$= 782-280 = \mathbf{502}$

c) $288-(50-12+88)-12-90-180 = 288-126-(12+90+180) =$
$= 288-126-282 = \mathbf{-120}$

$288-50+12-88-12-90-180 = 288+12-(50+88+12+90+180) =$
$= 300-420 = \mathbf{-120}$

d) $368-(152-32-77)-(28+103-120) = 368-43-11 = 368-(43+11) = \mathbf{314}$
$368-152+32+77-28-103+120 = (368+32+77+120)-(152+28+103) =$
$= 597-283 = \mathbf{314}$

10/3. a) $5a+6x+4a+3b+4x = 5a+4a+3b+6x+4x = \mathbf{9a+3b+10x}$
b) $9x+3y+2+5x+7y+4 = 9x+5x+3y+7y+2+4 = \mathbf{6+14x+10y}$
c) $8{,}7a+21{,}2n+5{,}3a+12{,}4n = 8{,}7a+5{,}3a+21{,}2n+12{,}4n = \mathbf{14a+33{,}6n}$

1.2.2 Multiplikation und Division

Lösungen zu 1.2.2

11/1. a) $3a \cdot 5b = \mathbf{15ab}$; b) $8c \cdot 3ab = \mathbf{24abc}$; c) $3 \cdot 4{,}5a \cdot 3bc + 4ac \cdot 3b = 40{,}5abc + 12abc = \mathbf{52{,}5abc}$;
d) $4{,}5ab \cdot 8x - 2{,}5ax \cdot 9b + 5bx \cdot 3a = 36abx - 22{,}5abx + 15abx = \mathbf{28{,}5abx}$

11/2. a) $8 \cdot (-5b) = \mathbf{-40b}$; b) $4b \cdot (-e) = \mathbf{-4be}$; c) $(-10a) \cdot (-12x) = \mathbf{120ax}$;
d) $(-n) \cdot (-m) \cdot (-x) = \mathbf{-nmx}$; e) $(-2x) \cdot 3y \cdot (-4z) = \mathbf{24xyz}$; f) $0{,}5x \cdot (-0{,}3y) \cdot 4 = \mathbf{-0{,}6xy}$;
g) $40 : (-8) = \mathbf{-5}$; h) $(-63c) : (-9) = \mathbf{7c}$ i) $(24 : 4) : 2 = \mathbf{3}$;
j) $[24 : (-4)] : 2 = \mathbf{-3}$; k) $[(-24) : (-4)] : 2 = \mathbf{3}$; l) $[(-24) : (-4)] : (-2) = \mathbf{-3}$

11/3. a) $(a+3) \cdot 6 = \mathbf{18+6a}$; b) $(a-b) \cdot 7 = \mathbf{7a-7b}$;
c) $8 \cdot (2a-5b+6) = \mathbf{48+16a-40b}$; d) $(8+4x-a) \cdot (-4) = \mathbf{-32+4a-16x}$;
e) $(a+b) \cdot 5 + 4 \cdot (a-b) = \mathbf{9a+b}$; f) $(2a+3b) \cdot 2c + 4bc = \mathbf{4ac+10bc}$;
g) $(y-9) \cdot (x-3) = \mathbf{27+xy-9x-3y}$; h) $(n-3) \cdot (a+6) = \mathbf{an+6n-3a-18}$

11/4. a) $25 \cdot 12 + 15 \cdot 25 - 2 \cdot 25 = 25 \cdot (12 + 15 - 2) = 25 \cdot 25 = \mathbf{625}$;
b) $ax - 4az + 7ay = \mathbf{a \cdot (x + 7y - 4z)}$;
c) $24ab - 12by + 48ab = 12b \cdot (2a - y + 4a) = \mathbf{12b \cdot (6a - y)}$;
d) $25ab + 125ac + 100ax = \mathbf{25a \cdot (b + 5c + 4x)}$;
e) $5bx - 2bx - 15bx = bx \cdot (5 - 2 - 15) = \mathbf{-12bx}$;
f) $am + bm - cm + zm = \mathbf{m \cdot (a + b - c + z)}$;
g) $(a + b) \cdot x + (a + b) \cdot y = \mathbf{(a + b) \cdot (x + y)}$;
h) $(b - c) \cdot y + b - c = \mathbf{(b - c) \cdot (y + 1)}$;
i) $(a - b) \cdot x + (a - b) \cdot y = \mathbf{(a - b) \cdot (x + y)}$

1.3 Rechnen mit Brüchen

Lösungen zu 1.3

12/1. a) $\frac{1}{4} - \frac{3}{14} - \frac{3}{35} = \frac{1 \cdot 35}{4 \cdot 35} - \frac{3 \cdot 10}{14 \cdot 10} - \frac{3 \cdot 4}{35 \cdot 4} = \frac{35 - 30 - 12}{140} = -\frac{7}{140} = -\frac{1}{20} = \mathbf{-0{,}05}$

b) $\frac{9}{14} - \frac{1}{42} - \frac{17}{28} + \frac{2}{7} = \frac{9 \cdot 6}{14 \cdot 6} - \frac{1 \cdot 2}{42 \cdot 2} - \frac{17 \cdot 3}{28 \cdot 3} + \frac{2 \cdot 12}{7 \cdot 12} = \frac{54 - 2 - 51 + 24}{84} = \frac{25}{84} = \mathbf{0{,}298}$

c) $\frac{5}{6} \cdot \frac{9}{35} = \frac{3}{14} = \mathbf{0{,}214}$ d) $18 : \frac{24}{35} = \frac{105}{4} = \mathbf{26{,}25}$ e) $\frac{121}{27} : \frac{66}{45} = \frac{55}{18} = \mathbf{3{,}056}$

12/2. a) $\frac{7}{\frac{1}{3} + \frac{1}{4}} = \frac{7}{\frac{4 + 3}{12}} = \frac{7 \cdot 12}{7} = \mathbf{12}$; b) $\frac{1}{\frac{3}{4} - \frac{2}{5}} = \frac{1}{\frac{15 - 8}{20}} = \frac{20}{7} = \mathbf{2{,}857}$

c) $\frac{-22}{\frac{1}{8} - \frac{5}{18}} = \frac{-22}{\frac{9 - 20}{72}} = \frac{-22 \cdot 72}{-11} = \mathbf{144}$ d) $\frac{104glm}{130gm} = \frac{4l}{5}$

e) $\frac{28ef}{-84ef} = -\frac{1}{3}$ f) $\frac{-68kmr}{-102kr} = \frac{2m}{3}$

12/3. a) $\frac{1}{d} + \frac{1}{e} = \frac{e + d}{e \cdot d}$ b) $\frac{6}{t} - \frac{1}{s} = \frac{6s - t}{st}$ c) $\frac{3}{ab} + \frac{2}{bc} = \frac{3c + 2a}{abc}$

d) $\frac{15}{k} - 3 + \frac{7}{l = \frac{15l - 3kl + 7k}{kl}}$ e) $\frac{3}{uv} + \frac{12}{uw} - 15 = \frac{3w + 12v - 15uvw}{uvw} = \frac{3(w + 4v - 5uvw)}{uvw}$

12/4. a) $\frac{2f}{3r} + \frac{g}{2s} - \frac{5h}{rs} = \frac{4fs + 3gr - 30h}{6rs}$

b) $\frac{5l}{6a} - k + \frac{h}{12ab} + \frac{5l}{18a} = \frac{30bl - 36abk + 3h + 10bl}{36ab} = \frac{40bl - 36abk + 3h}{36ab}$

c) $\frac{6ab}{38cd} \cdot \frac{57}{48a} = \frac{3b}{16cd}$ d) $\frac{32b}{21cd} : \frac{20ab}{49d} = \frac{32b \cdot 49d}{21cd \cdot 20ab} = \frac{56}{15ac}$

12/5. a) $\frac{6x - 30}{8} : \frac{5x - 25}{20y - 4} = \frac{(6x - 30) \cdot (20y - 4)}{8 \cdot (5x - 25)} = \frac{24 \cdot (x - 5) \cdot (5y - 1)}{40 \cdot (x - 5)} = \frac{3 \cdot (5y - 1)}{5}$

b) $\frac{1 - 6v}{14s - 2} : \frac{36v - 6}{8 - 56s} = \frac{(1 - 6v) \cdot (8 - 56s)}{(14s - 2) \cdot (36v - 6)} = \frac{-(6v - 1) \cdot 8 \cdot (1 - 7s)}{-2(1 - 7s) \cdot 6 \cdot (6v - 1)} = \frac{2}{3}$

c) $\frac{1}{\frac{2}{m} + \frac{3}{n}} = \frac{1}{\frac{2n + 3m}{mn}} = \frac{mn}{2n + 3m}$ d) $\frac{15a + 10}{\frac{3}{2} + \frac{1}{a}} = \frac{5 \cdot (3a + 2)}{\frac{3a + 2}{2a}} = 10a$

1.4 Potenzen und Wurzeln

1.4.1 Potenzen

Lösungen zu 1.4.1

13/1. a) x^6 b) b^{2x+2} c) 10^{3x} d) $7d^{-2}$ e) $3x^{-3} \cdot y^6$ f) 2^{-2x}

13/2. a) $a^2 + 2a + 1$ b) $16y^2 - 40y + 25$ c) $9 + 12b + 4b^2$
d) $x^2 + 2xy + y^2$ e) $x^2 - 2xy + y^2$ f) $4r^2 + 12rs + 9s^2$

13/3. a) $(a + b) \cdot (a + b) = \mathbf{a^2 + 2ab + b^2}$; b) $(a - b) \cdot (a - b) = \mathbf{a^2 - 2ab + b^2}$;
c) $(a + b) \cdot (a + b) \cdot (a + b) = \mathbf{a^3 + 3a^2b + 3ab^2 + b^3}$;
d) $(a - b) \cdot (a - b) \cdot (a - b) = \mathbf{a^3 - 3a^2b + 3ab^2 - b^3}$;
e) $(a + b) \cdot (a + b) \cdot (a + b) \cdot (a + b) = \mathbf{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}$;
f) $(a - b) \cdot (a - b) \cdot (a - b) \cdot (a - b) = \mathbf{a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4}$

13/4. a) $\frac{4t^2}{9}$ b) $\frac{125}{8b^3}$ c) $\frac{16y^2}{49x^2}$ d) $\frac{4}{x^2 + 2x + 1}$ e) $\frac{25 - 10x + x^2}{25 + 10x + x^2}$

13/5. a) x^{10} b) y^6 c) 10^{2x} d) 2^{5y} e) 10^{2x+2} f) 2^{3y-6}

13/6. a) 9; 900; 90000| 64; 6400; 640000; 64000000| 49; 0,49; 0,0049| 81; 0,81; 0,0081; 0,000081
b) 8; 8000; 8000000| 0,125; 125; 125000; 125000000| −1000; −0,001| −64; −0,064; −0,000064

13/7. a) $4 \cdot 10^{-3} \cdot 5 \cdot 10^2 = 20 \cdot 10^{-1} = \mathbf{2}$; $35 \cdot 10^{-3} \cdot 6 \cdot 10^4 = 210 \cdot 10^1 = 2100 = \mathbf{2{,}1 \cdot 10^3}$;
$48 \cdot 10^{-5} \cdot 75 \cdot 10^4 = 3600 \cdot 10^{-1} = \mathbf{360}$; $24 \cdot 10^{-6} \cdot 15 \cdot 10^2 = 360 \cdot 10^{-4} = \mathbf{36 \cdot 10^{-3}}$;
$16 \cdot 10^{-5} \cdot 45 \cdot 10^3 = 720 \cdot 10^{-2} = \mathbf{7{,}2}$
b) $6 \cdot 10^2/(12 \cdot 10^3) = 0{,}5 \cdot 10^{-1} = \mathbf{50 \cdot 10^{-3}}$; $48 \cdot 10^1/(16 \cdot 10^4) = \mathbf{3 \cdot 10^{-3}}$;
$20 \cdot 10^{-3}/(5{,}5 \cdot 10^3) = \mathbf{3{,}6364 \cdot 10^{-6}}$; $72 \cdot 10^{-4}/(36 \cdot 10^2) = \mathbf{2 \cdot 10^{-6}}$;
$42 \cdot 10^{-5}/(35 \cdot 10^3) = \mathbf{12 \cdot 10^{-9}}$

13/8. a) $10^{1+7+3-6} = \mathbf{10^5}$; b) $10^{1-6+3} = \mathbf{10^{-2}}$; c) $10^{4-12+9+2} = \mathbf{10^3}$;
d) $10^{3-4+12-9} = \mathbf{10^2}$

13/9. a) $\frac{42000 \cdot 500}{0{,}06} = \frac{42 \cdot 10^3 \cdot 5 \cdot 10^2}{6 \cdot 10^{-2}} = \mathbf{35 \cdot 10^7}$

b) $\frac{46000 \cdot 0{,}5}{50000} = \frac{46 \cdot 10^3 \cdot 5 \cdot 10^{-1}}{5 \cdot 10^4} = 46 \cdot 10^{-2} = \mathbf{0{,}46}$

c) $\frac{0{,}0065 \cdot 0{,}025}{13000 \cdot 0{,}0005} = \frac{65 \cdot 10^{-4} \cdot 25 \cdot 10^{-3}}{13 \cdot 10^3 \cdot 5 \cdot 10^{-4}} = \mathbf{25 \cdot 10^{-6}}$

d) $\frac{4200 \cdot 0{,}007}{35000} = \frac{42 \cdot 10^2 \cdot 7 \cdot 10^{-3}}{35 \cdot 10^3} = \mathbf{8{,}4 \cdot 10^{-4}}$

1.4.2 Wurzeln

Lösungen zu 1.4.2

14/1. a) a^3 b) b^{2n} c) x^{2s} d) k^{2y} e) $9 \cdot \sqrt{z}$ f) $8 \cdot b^{\frac{3}{2}}$ g) $4 \cdot a^{\frac{2}{3}}$

14/2. a) $\frac{4a}{7c^2}$ b) $\frac{2a \cdot c^2}{5b}$ c) $\frac{16q}{25s^2t}$ d) $\frac{7m^2}{6n}$ e) $\frac{3d^2}{5f}$ f) $\frac{4x}{7z^2}$

14/3. a) 2; 6,32; 20; 63,2; 200 b) 2,65; 8,37; 26,5; 83,7; 265
c) 30; 9,49; 3; 0,949; 0,3; 0,0949 d) 22,4; 7,07; 2,24; 0,707; 0,224; 0,0707

14/4. a) 8; 25; 32; 4; 9 b) 300; $(64 \cdot 10^6)^{1/3} = 400$; $(169 \cdot 10^{-6})^{1/2} = 0{,}013$

14/5. a) $\sqrt{u^2 + v^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$
b) $\sqrt{10^2 + 7{,}5^2} = \sqrt{100 + 56{,}25} = \sqrt{156{,}25} = 12{,}5$
c) $\sqrt{0{,}48^2 + 0{,}36^2} = \sqrt{0{,}2304 + 0{,}1296} = \sqrt{0{,}3600} = 0{,}60$

14/6. a) $18 \cdot \sqrt{ab}$; b) $2 \cdot \sqrt{2 \cdot 25x} = 10 \cdot \sqrt{2x}$;
c) $90 \cdot \sqrt{\frac{2}{3} \cdot \frac{8}{9} \cdot \frac{3}{2}} = 90 \cdot \sqrt{\frac{48}{54}} = 90 \cdot \sqrt{\frac{16 \cdot 3}{6 \cdot 9}} = 120 \cdot \sqrt{\frac{1}{2}}$;
d) $11 \cdot \sqrt{x + y}$

14/7. a) $\frac{4}{7} \cdot \sqrt[3]{\frac{c}{d}}$; b) $3 \cdot \frac{2 \cdot 4}{3} \cdot \sqrt[3]{\frac{nab}{x}} = 8 \cdot \sqrt[3]{\frac{abn}{x}}$;
c) $\sqrt{\frac{5xy}{60} \cdot \frac{30}{10x}} = \frac{1}{2} \cdot \sqrt{y}$; d) $\sqrt{\frac{5x}{6} \cdot \frac{12x}{20}} = x \cdot \sqrt{\frac{1}{2}}$

14/8. a) $\sqrt{b}$; b) $\sqrt[3]{y}$; c) $a + b$; d) $4\sqrt{m}$

14/9. a) $\frac{a^2b}{c^3}$; b) $\frac{27m}{8n}$; c) $\frac{4}{5}\sqrt{\frac{x + 2y}{a - 2b}}$; d) $\frac{5n}{3m}$

1.5 Logarithmen

1.5.1 Rechnen mit Logarithmen

Lösungen zu 1.5.1

15/1. a) 5; 4; 3; 2; 1; 0; −1 b) 4,7; 3,7; 2,7; 1,7; 0,7; −0,3; −1,3; −2,3
c) 4,4; 3,4; 2,4; 1,4; 0,4; −0,6; −1,6 d) 4,1; 3,1; 2,1; 1,1; 0,1; −0,9; −1,9

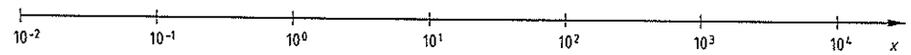
15/2. a) $\lg 250 + \lg 320 = 4{,}903$; $\lg(250 \cdot 320) = \lg 80000 = 4{,}903$; $10^{4{,}903} = 80000$;
$\ln 250 + \ln 320 = 11{,}29$; $\ln(250 \cdot 320) = \ln 80000 = 11{,}29$; $e^{11{,}29} = 80000$
b) $\lg 25 + \lg 32 = 2{,}903$; $\lg(25 \cdot 32) = \lg 800 = 2{,}903$; $10^{2{,}903} = 800$;
$\ln 25 + \ln 32 = 6{,}685$; $\ln(25 \cdot 32) = \ln 800 = 6{,}685$; $e^{6{,}685} = 800$
c) $\lg 4{,}5 + \lg 80 = 2{,}556$; $\lg(4{,}5 \cdot 80) = \lg 360 = 2{,}556$; $10^{2{,}556} = 360$;
$\ln 4{,}5 + \ln 80 = 5{,}886$; $\ln(4{,}5 \cdot 80) = \ln 360 = 5{,}886$; $e^{5{,}886} = 360$
d) $\lg 0{,}45 + \lg 8 = 0{,}556$; $\lg(0{,}45 \cdot 8) = \lg 3{,}6 = 0{,}556$; $10^{0{,}556} = 3{,}6$;
$\ln 0{,}45 + \ln 8 = 1{,}281$; $\ln(0{,}45 \cdot 8) = \ln 3{,}6 = 1{,}281$; $e^{1{,}281} = 3{,}6$

15/3. a) $\lg 18{,}52 = 1{,}2676$; $\sqrt[3]{\lg 7} = 1{,}2676$; $10^{1{,}2676} = 18{,}52$
b) $\lg 4{,}6416 = 0{,}6667$; $\sqrt[3]{\lg 10} = \frac{2}{3} = 0{,}6667$; $10^{0{,}6667} = 4{,}6419$
c) $\ln 63{,}00 = 4{,}143$; $\sqrt[3]{\ln 500} = 4{,}143$; $e^{4{,}143} = 2{,}718^{4{,}143} = 62{,}99$
d) $\ln 23{,}68 = 3{,}1646$; $\sqrt[3]{\ln 68} = 3{,}1646$; $e^{3{,}1646} = 2{,}718^{3{,}1646} = 23{,}68$
e) $\lg 0{,}6817 = -0{,}1664$; $\sqrt[3]{\lg 0{,}6} = -0{,}1664$; $10^{-0{,}1664} = 0{,}6817$
f) $\ln 0{,}1597 = -1{,}8346$; $\sqrt[3]{\ln 0{,}047} = -1{,}8346$; $e^{-1{,}8346} = 0{,}1597$

1.5.2 Logarithmische Maßstäbe

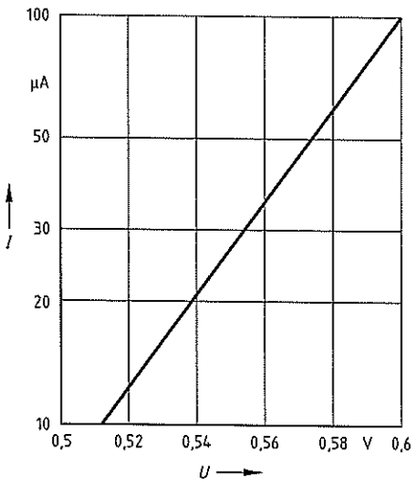
Lösungen zu 1.5.2

16/1.



16/2. I = 10 µA bis 100 µA

I in µA	10	20	30	50	100
U in V	0,51	0,54	0,56	0,58	0,6



zu 16/2.

16/3. a) 70 A; b) 120 A; c) 150 A; d) 180 A

1.6 Gleichungen und Formeln

1.6.1 Arbeiten mit Gleichungen

Lösungen zu 1.6.1

19/1.o. a) $x = 44 - 17 = 27$; $x = 27 + 5 = 32$; b) $3x = 12 \Rightarrow x = 4$; $7x = 14 \Rightarrow x = 2$
c) $2x = 2 \Rightarrow x = 1$; $5x = -15 \Rightarrow x = -3$

19/2.o. a) $3y = 45 \Rightarrow y = 15$; $5y = 13 \cdot 15 \Rightarrow y = 39$
b) $\frac{3y}{1} = \frac{5}{4} \Rightarrow y = \frac{5}{12}$; $y = \frac{5}{4}$; c) $\frac{5y}{3} = \frac{7}{2} \Rightarrow y = \frac{21}{10}$; $\frac{4y}{3} = \frac{2}{5} \Rightarrow y = \frac{3}{10}$

19./3.o. a) $x + 7 = 10 \Rightarrow x = 3$; $8 - x = 6 \Rightarrow x = 2$
b) $5x + 4 = 24 \Rightarrow x = 4$; $\frac{7 + 2x}{5} = 7 \Rightarrow 7 + 2x = 35 \Rightarrow x = 14$
c) $\frac{2}{5} = \frac{2x + 3}{6} \Rightarrow 12 = 10x + 15 \Rightarrow x = -\frac{3}{10}$; $\frac{5}{3} = \frac{7 - 2x}{5} \Rightarrow 25 = 21 - 6x \Rightarrow x = -\frac{2}{3}$

19/4.o. a) $z^2 = 16 \Rightarrow z = \pm 4$; $\frac{4z^2}{9} = 16 \Rightarrow z = \pm 6$
b) $z^2 = 25 \Rightarrow z = \pm 5$; $\frac{4z^2}{3} = 27 \Rightarrow z = \pm\frac{9}{2}$
c) $75z^2 = 180 \Rightarrow 5z^2 = 12 \Rightarrow z = \pm\sqrt{\frac{12}{5}}$; $\frac{3}{4z} = \frac{z}{3} \Rightarrow z = \pm\frac{3}{2}$

19/5.o. a) $\frac{1}{x} = \frac{2 + 3}{6} \Rightarrow x = \frac{6}{5}$; $\frac{1}{y} = \frac{1 + 2}{6} \Rightarrow y = 2$
b) $\frac{1}{y} = \frac{6 + 4}{15} \Rightarrow y = \frac{3}{2}$; $\frac{1}{x} = \frac{10 - 9}{24} \Rightarrow x = 24$
c) $\frac{2}{z} = \frac{5 - 3}{15} \Rightarrow z = 15$; $\frac{11}{z} = \frac{15 - 4}{10} \Rightarrow z = 10$
d) $\frac{1}{x} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20} \Rightarrow x = -20$; $\frac{1}{x} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10} \Rightarrow x = -10$
e) $\frac{1}{x} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6} \Rightarrow x = 6$; $\frac{1}{x} = \frac{1}{7} - \frac{1}{2} = -\frac{5}{14} \Rightarrow x = -\frac{14}{5}$
f) $\frac{3}{2x} = \frac{2}{5} - \frac{3}{10} = \frac{1}{10} \Rightarrow x = 15$; $\frac{4}{5x} = \frac{2}{5} - \frac{5}{4} = -\frac{17}{20} \Rightarrow x = -\frac{16}{17}$

19/6.o. a) $36 - 15z = 100 - 25z \Rightarrow 10z = 64 \Rightarrow z = 6{,}4$
b) $5z + 105 = 63 - 9z \Rightarrow 14z = -42 \Rightarrow z = -3$
c) $5z + 120 = 126 - 7z \Rightarrow 12z = 6 \Rightarrow z = 0{,}5$

19/7.o. a) $x^2 = 16 \Rightarrow x = \pm 4$; b) $x^2 = 100 \Rightarrow x = \pm 10$
c) $4x^2 = 64 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$; d) $27x^2 = 12 \Rightarrow 9x^2 = 4 \Rightarrow x = \pm\frac{2}{3}$

19/8.o. a) $x = \ln 50 = 3{,}91$; $x = \ln 5 = 1{,}609$; b) $x = \ln 2 = 0{,}693$; $x = \ln 0{,}2 = -1{,}61$
c) $x = \ln 10 = 2{,}30$; $2x = \ln 10 \Rightarrow x = 0{,}5 \cdot \ln 10 = 1{,}15$
d) $0{,}2x = \ln 8 \Rightarrow x = 5 \cdot \ln 8 = 10{,}4$; $0{,}4x = \ln 8 \Rightarrow x = 2{,}5 \cdot \ln 8 = 5{,}20$

19/9.o. a) $-x = \ln 4 \Rightarrow x = -\ln 4 = -1{,}39$; $-x = \ln 16 \Rightarrow x = -\ln 16 = -2{,}77$
b) $-x/2 = \ln 3 \Rightarrow x = -2 \cdot \ln 3 = -2{,}20$; $-x/2 = \ln 9 \Rightarrow x = -2 \cdot \ln 9 = -4{,}39$
c) $2 = 4 \cdot (1 - e^{-x/8}) \Rightarrow 1 - e^{-x/8} = 0{,}5 \Rightarrow -x/8 = \ln 0{,}5 \Rightarrow x = 5{,}55$
d) $6{,}3 = 10 \cdot (1 - e^{-x/3}) \Rightarrow 1 - e^{-x/3} = 0{,}63 \Rightarrow -x/3 = \ln 0{,}37 \Rightarrow x = 2{,}98$

1.6.2 Arbeiten mit Formeln

Lösungen zu 1.6.2

19/1.u. a) $v = \frac{P}{F}$ b) $F = \frac{M}{r}$ c) $P = \frac{W}{t}$ d) $R = \frac{U}{I}$
e) $v = \frac{s}{t}$ f) $\omega = \frac{P}{M}$ g) $\varrho = \frac{m}{V}$ h) $d = \frac{u}{\pi}$

19/2.u. a) $h = \frac{V}{l \cdot b}$ b) $n = \frac{v}{d \cdot \pi}$ c) $B = \frac{U}{v \cdot l}$; d) $L = \frac{X_L}{2\pi \cdot f}$

19/3.u. a) $U_2 = U - U_1$ b) $U_0 = U + U_i$ c) $t_0 = t_1 - \Delta t$ d) $R_i = R - R_v$

19/4.u. a) $Q = I \cdot t$ b) $U = R \cdot I$ c) $P_1 = \frac{P_2}{\eta}$ d) $F = \frac{P \cdot t}{s}$
e) $A = \frac{l}{\gamma \cdot R}$ f) $Q = \frac{F \cdot s}{U}$ g) $R = \frac{\omega \cdot L}{Q}$ h) $A = \frac{2I \cdot l}{\gamma \cdot \Delta U}$

19/5.u. a) $U = \sqrt{\frac{2 \cdot W}{C}}$ b) $L = \frac{U^2}{\omega \cdot Q_{bL}}$ c) $I = \sqrt{\frac{Q}{X}}$ d) $U = \sqrt{\frac{Q_{bC}}{\omega \cdot C}}$

19/6.u. a) $X_L = \sqrt{Z^2 - R^2}$ b) $L = \frac{T^2}{4\pi^2 \cdot C}$ c) $I_{bL} = \sqrt{I^2 - I_w^2}$

19/7.u. a) $R_i = \frac{U_0 - U}{I}$ b) $n = \frac{R_v + R_m}{R_m} = \frac{R_v}{R_m} + 1$ c) $F_1 = \frac{F_2 \cdot v - P}{v} = F_2 - \frac{P}{v}$

19/8.u. a) $C_1 = \frac{1}{\frac{1}{C} - \frac{1}{C_2}} = \frac{C_2 \cdot C}{C_2 - C}$ b) $R_1 = \frac{R_2 \cdot R}{R_2 - R}$ c) $R_1 = \frac{(U - U_2) \cdot R_2}{U_2} = R_2\left(\frac{U}{U_2} - 1\right)$
d) $R_m = R_p \cdot (n - 1)$ e) $U = U_0 - R_i \cdot I$ f) $U = \frac{U_{20} \cdot (R_1 + R_2)}{R_2} = U_{20}\left(\frac{R_1}{R_2} + 1\right)$

19/9.u. a) $e^{t/\tau} = \frac{U_0}{u_c} \Rightarrow t = \tau \cdot \ln\left(\frac{U_0}{u_c}\right) = -\tau \cdot \ln\left(\frac{u_c}{U_0}\right)$ b) $\frac{I_0}{i_L} = e^{t/\tau} \Rightarrow \tau = \frac{t}{\ln(I_0/i_L)}$
c) $I_0 \cdot e^{-t/\tau} = I_0 - i_L \Rightarrow e^{t/\tau} = \frac{I_0}{I_0 - i_L} \Rightarrow t/\tau = \ln\left(\frac{I_0}{I_0 - i_L}\right) \Rightarrow t = \tau \cdot \ln\left(\frac{I_0}{I_0 - i_L}\right)$

1.6.3 Verhältnisgleichungen, Dreisatzrechnen

Lösungen zu 1.6.3

20/1.o. 25 Stück ≙ 12 € ⇒ 6 Stück ≙ 2,88 €

20/2.o. 2,25 kg ≙ 50 m ⇒ 1,44 kg ≙ 32 m

20/3.o. 5 Leuchten ≙ 240 h ⇒ 3 Leuchten ≙ 400 h

1.6.4 Verhältnisgleichungen, Prozentrechnen

Lösungen zu 1.6.4

20/1.u. (100 – 25) % ≙ 54 € ⇒ 100 % = 72 €

20/2.u. 100 % ≙ 400 V ⇒ ±2,5 % ≙ ±10 V

20/3.u. 80 % ≙ 84 Ah ⇒ 100 % ≙ 105 Ah

1.7 Funktionen

Lösungen zu 1.7

22/1. a) $t_1 = 16\,h \Rightarrow U_1 = 11{,}5\,V$
b) $U_2 = 12\,V \Rightarrow t_2 = 12\,h$

22/2. a) $P_1 = 50\,W \Rightarrow \eta_1 \approx 0{,}5$
b) $\eta_m = 0{,}75 \Rightarrow P_m = 150\,W$
c) $\eta_2 = 0{,}7 \Rightarrow P_{21} = 100\,W$
$P_{22} = 200\,W$

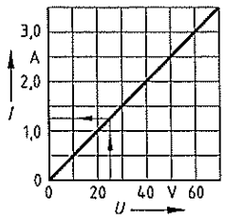
22/3. a) $I_0 = 0 \Rightarrow U_0 = 4{,}5\,V$
$I_1 = 0{,}5\,A \Rightarrow U_1 = 3{,}5\,V$
b) $U_2 = 3{,}7\,V \Rightarrow I_2 = 0{,}4\,A$
$U_3 = 2{,}5\,V \Rightarrow I_3 = 0{,}96\,A$
c) $R = \frac{\Delta U}{\Delta I} = \frac{2{,}5\,V - 4{,}5\,V}{0{,}96\,A - 0\,A}$
$R = \left|-2{,}1\,\frac{V}{A}\right| = 2{,}1\,\frac{V}{A}$

22/4. a) $t_1 = 10\,s \Rightarrow U_{C1} = 30\,V$
$t_2 = 20\,s \Rightarrow U_{C2} = 43\,V$
b) $U_{C3} = 0{,}63 \cdot 60\,V = 37{,}8\,V$
$U_{C3} = 37{,}8\,V \Rightarrow t_3 = 15\,s$

22/5. a) b)

U in A	0	10	20	30	40	50	60
I in A	0	0,5	1,0	1,5	2,0	2,5	3,0

c) d)



zu 22/5. c) d)

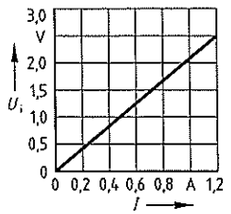
e) $U_1 = 25\,V \Rightarrow I_1 = 1{,}25\,A$; $I_1 = \frac{25\,V}{20\,\Omega} = 1{,}25\,A$

f) Steigung $= \frac{\Delta I}{\Delta U} = \frac{3{,}0\,A}{60\,V} = 0{,}05\,\frac{A}{V}$

22/6. a)

I in A	0	0,20	0,40	0,60	0,80	1,0	1,2
U_i in V	0	0,42	0,85	1,25	1,65	2,05	2,50

b) c)



zu 22/6. b) c)

d) Steigung $= \frac{\Delta U_i}{\Delta I} = \frac{2{,}05\,V}{1{,}0\,A} = 2{,}05\,\frac{V}{A} = 2{,}05\,\Omega$

1.8 Rechnen am Dreieck

1.8.1 Satz des Pythagoras

Lösungen zu 1.8.1

23/1. a) $c = \sqrt{(34\,cm)^2 + (47\,cm)^2} = 58{,}0\,cm$
b) $a = \sqrt{(76\,cm)^2 - (53\,cm)^2} = 54{,}5\,cm$
c) $b = \sqrt{(41\,cm)^2 - (27\,cm)^2} = 30{,}9\,cm$
d) $a = \sqrt{(2{,}3\,m)^2 - (0{,}92\,m)^2} = 2{,}11\,m$
e) $b = \sqrt{(3{,}4\,cm)^2 - (0{,}86\,cm)^2} = 3{,}29\,cm$

23/2. $l = \sqrt{(320\,mm)^2 + (290\,mm)^2} = 432\,mm$

23/3. $s_1 = \sqrt{(650\,mm)^2 + (500\,mm)^2} = 820\,mm$; $s_2 = \sqrt{(500\,mm)^2 - (400\,mm)^2} = 300\,mm$

23/4. $s = \sqrt{(D/2)^2 + (D/2)^2} = \frac{D}{2} \cdot \sqrt{2} = 35\,mm \cdot 1{,}414 = 49{,}5\,mm$

23/5. $\frac{s}{2} = \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{D}{4}\right)^2} = \sqrt{\frac{3D^2}{16}} = \frac{D}{4} \cdot \sqrt{3} \Rightarrow s = \frac{D}{2} \cdot \sqrt{3} = 24\,mm \cdot 1{,}732 = 41{,}6\,mm$

23/6. $U = \sqrt{U_W^2 + U_{bL}^2} = \sqrt{(208V)^2 + (98V)^2} = 230\,V$

23/7. $I_{bL} = \sqrt{I^2 - I_W^2} = \sqrt{3{,}6^2A^2 - 2^2A^2} = 3\,A$

1.8.2 Winkelfunktionen

Lösungen zu 1.8.2

24/1.

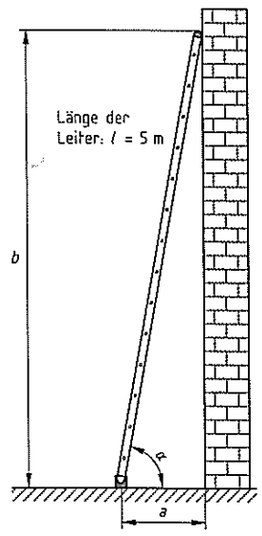
Teil	a	b	c	d	e	f	g	h	i
b in mm	79,5	26,5	86,6	45	32	21	44	63	50
a in mm	42,3	33,9	50,0	77,9	24	66,8	33	84	50
c in mm	90	43	100	90	40	70	55	105	70,7
β in °	62	38	60	30	53,1	17,5	53,1	36,9	45
α in °	28	52	30	60	36,9	72,5	36,9	53,1	45

24/2. $s = 4{,}5\,m / \cos 25° = 4{,}97\,m$; $h = 4{,}5\,m \cdot \tan 25° = 2{,}1\,m$

24/3. $a = s/\tan\alpha = 2{,}7\,mm/\tan 25° = 5{,}79\,mm$

24/4. $\tan\alpha = s/a = 3{,}5\,mm/8\,mm = 0{,}438 \Rightarrow \alpha = 23{,}63° = 23°37'$

24/5.



zu 24/5

a) $\cos\alpha = \frac{a}{l} \Rightarrow a = l \cdot \cos\alpha = 5\,m \cdot \cos 70° = 1{,}71\,m$

b) $\sin\alpha = \frac{b}{l} \Rightarrow b = l \cdot \sin\alpha = 5\,m \cdot \sin 70° = 4{,}7\,m$

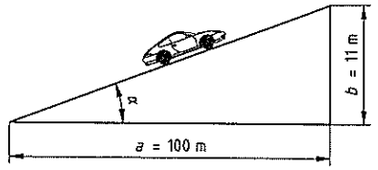
24/6. a) $\tan\alpha = \frac{b}{a} = \frac{11\,m}{100\,m} = 0{,}11$

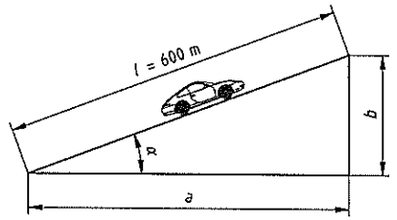
$\alpha = 6{,}28°$

b) $\cos\alpha = \frac{a}{l} \Rightarrow$

$a = l \cdot \cos\alpha = 600\,m \cdot \cos 6{,}28° = \mathbf{596{,}4\,m}$

c) $b = \sqrt{l^2 - a^2} = \sqrt{600^2 m^2 - 596{,}4^2 m^2} = \mathbf{65{,}6\,m}$





zu 24/6.

1.8.3 Winkel im Grad- und Bogenmaß

Lösungen zu 1.8.3

25/1.

Teil	a	b	c	d	e	f	g	h	i
β_G in °	16	48	70	140	20,1	43,0	77,3	178	258
β_B in rad	0,279	0,834	1,222	2,443	0,351	0,75	1,35	3,11	4,5

25/2.

Teil	a	b	c	d	e	f	g	h	i
φ in °	360	270	180	90	120	60	30	45	15
φ in rad	$2\cdot\pi$	$3\cdot\pi/2$	π	$\pi/2$	$2\cdot\pi/3$	$\pi/3$	$\pi/6$	$\pi/4$	$\pi/12$

25/3. a) $5{,}5° = 5° + 0{,}5° = 5° + 0{,}5° \cdot \frac{60'}{1°} = 5° + 30' = \mathbf{5°30'}$

b) $3{,}28° = 3° + 0{,}28° = 3° + 0{,}28° \cdot \frac{60'}{1°} = 3° + 16{,}8' = 3° + 16' + 0{,}8' = 3° + 16' + 0{,}8' \cdot \frac{60''}{1'} =$

$= 3° + 16' + 48'' = \mathbf{3°16'48''}$

c) $15{,}53° = 15° + 0{,}53° = 15° + 0{,}53° \cdot \frac{60'}{1°} = 15° + 31{,}8' = 15° + 31' + 0{,}8' =$

$= 15° + 31' + 0{,}8' \cdot \frac{60''}{1'} = 15° + 31' + 48'' = \mathbf{15°31'48''}$

25/4. a) $10°24' = 10° + 24' \cdot \frac{1°}{60'} = 10° + 0{,}4° = \mathbf{10{,}4°}$

b) $7°5'24'' = 7° + 5' + 24'' = 7° + 5' + 24'' \cdot \frac{1'}{60''} = 7° + 5' + 0{,}4' = 7° + 5{,}4' =$

$= 7° + 5{,}4' \cdot \frac{1°}{60'} = 7° + 0{,}09° = \mathbf{7{,}09°}$

c) $50°25'30'' = 50° + 25' + 30'' = 50° + 25' + 30'' \cdot \frac{1°}{60''} = 50° + 25' + 0{,}5' = 50° + 25{,}5' =$

$= 50° + 25{,}5' \cdot \frac{1°}{60'} = 50° + 0{,}425° = \mathbf{50{,}425°}$

1.8.4 Rechnen am beliebigen Dreieck

Lösungen zu 1.8.4

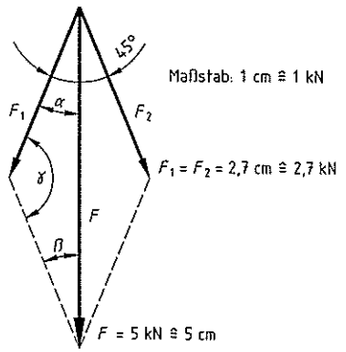
26/1.

Teil	a	b	c	d	e	f	g
a	40 mm	84,3 mm	73,4 mm	3,5 m	55 cm	40 cm	6,0 cm
b	75,2 mm	55 mm	82,9 mm	5,5 m	35 cm	70 cm	4,0 cm
c	78,8 mm	74,1 mm	90 mm	5,98 m	78,6 cm	50 cm	4,8 cm
α	30°	80°	50°	35,2°	37,3°	34,04°	85,5°
β	70°	40°	60°	64,8°	22,7°	101,5°	41,6°
γ	80°	60°	70°	80°	120°	44,4°	52,9°

26/2. $\alpha = \beta = \frac{45°}{2} = 22{,}5°$

$\gamma = 180° - \alpha - \beta = 180° - 22{,}5° - 22{,}5° = 135°$

$\frac{F}{F_1} = \frac{\sin\gamma}{\sin\beta} \Rightarrow F_1 = F \cdot \frac{\sin\beta}{\sin\gamma} = 5\,kN \cdot \frac{\sin 22{,}5°}{\sin 135°} = \mathbf{2{,}7\,kN}$

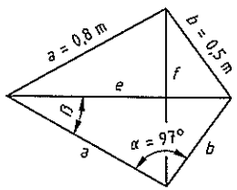


zu 26/2.

26/3. $a = \sqrt{(2{,}0\,m)^2 + (1{,}5\,m)^2 - 2 \cdot 2{,}0\,m \cdot 1{,}5\,m \cdot \cos 120°} = \mathbf{3{,}04\,m}$

$h = 2{,}0\,m \cdot \sin(180° - 120°) = 2{,}0\,m \cdot \sin 60° = \mathbf{1{,}73\,m}$

26/4.



zu 26/4.

$e = \sqrt{a^2 + b^2 - 2ab \cdot \cos\alpha} = \sqrt{(0{,}8\,m)^2 + (0{,}5\,m)^2 - 2 \cdot 0{,}8\,m \cdot 0{,}5\,m \cdot \cos 97°} = \mathbf{0{,}99\,m}$

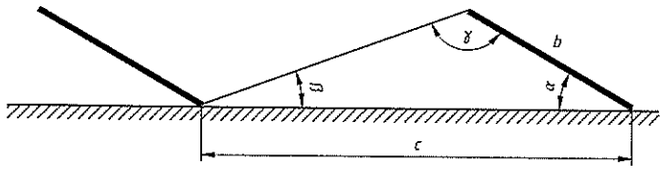
$\frac{\sin\alpha}{\sin\beta} = \frac{e}{b} \Rightarrow \sin\beta = \frac{b}{e} \cdot \sin\alpha = \frac{0{,}5\,m}{0{,}99\,m} \cdot \sin 97° = 0{,}5013 \Rightarrow \beta = 30°$

$\sin\beta = \frac{\frac{f}{2}}{a} \Rightarrow f = 2a \cdot \sin\beta = 2 \cdot 0{,}8\,m \cdot \sin 30° = \mathbf{0{,}8\,m}$

26/5. $b = \frac{1{,}6\,m}{\cos(90° - 25° - 40°)} = \frac{1{,}6\,m}{\cos 25°} = 1{,}77\,m$; $c = \frac{\sin 40°}{\sin 25°} \cdot b = \mathbf{2{,}69\,m}$

$c = 1{,}6\,m \cdot \tan(90° - 25°) - 1{,}6\,m \cdot \tan(90° - 25° - 40°) = 1{,}6\,m \cdot (\tan 65° - \tan 25°) = \mathbf{2{,}69\,m}$

26/6. ●



zu 26/6.

$\gamma = 180° - \alpha - \beta = 180° - 30° - 20° = 130°$

Module im Querformat angeordnet:

$\frac{\sin\beta}{\sin\gamma} = \frac{b}{c} \Rightarrow c = \frac{b \cdot \sin\gamma}{\sin\beta} = \frac{560\,mm \cdot \sin 130°}{\sin 20°} = 1254\,mm$

$n_B = \frac{8\,m}{a} = \frac{8000\,mm}{1200\,mm} = 6{,}6 \Rightarrow$ 6 Module

$d = \frac{10\,m}{c} = \frac{10\,000\,mm}{1254\,mm} = 7{,}97$ Abstände $\Rightarrow n_L = 8$ Reihen

$n_M = n_B \cdot n_L = \mathbf{6 \cdot 8 = 48}$ **Module**

$A = n_M \cdot a \cdot b = 54 \cdot 1{,}2\,m \cdot 0{,}56\,m = \mathbf{32{,}26\,m^2}$

Module im Hochformat angeordnet:

$\frac{\sin\beta}{\sin\gamma} = \frac{a}{c} \Rightarrow c = \frac{a \cdot \sin\gamma}{\sin\beta} = \frac{1200\,mm \cdot \sin 130°}{\sin 20°} = 2687{,}72\,mm$

$n_B = \frac{8\,m}{b} = \frac{8000\,mm}{560\,mm} = 14{,}3 \Rightarrow$ 14 Module

$d = \frac{10\,m}{c} = \frac{10\,000\,mm}{2687{,}72\,mm} = 3{,}72$ Abstände $\Rightarrow n_L = 4$ Reihen

$n_M = n_B \cdot n_L = \mathbf{14 \cdot 4 = 56}$ **Module**

$A = n_M \cdot a \cdot b = 56 \cdot 1{,}2\,m \cdot 0{,}56\,m = \mathbf{37{,}63\,m^2}$

Die Module müssen im Hochformat montiert werden.

1.9 Runden

Lösungen zu 1.9

27/1. a) 655,837 ⇒ 655,84; 655,8; 656; $0{,}66 \cdot 10^3$; $0{,}7 \cdot 10^3$
b) 2174,95 ⇒ 2175,0; 2175; $2{,}17 \cdot 10^3$; $2{,}2 \cdot 10^3$; $2 \cdot 10^3$
c) 18,7484 ⇒ 18,748; 18,75; 18,7; 19; $0{,}02 \cdot 10^3$
d) 5,68458 ⇒ 5,6846; 5,685; 5,68; 5,7; 6
e) 0,963493 ⇒ 0,96349; 0,9635; 0,963; 0,96; 1
f) 0,748396 ⇒ 0,74840; 0,7484; 0,748; 0,75; 0,7
g) 98,3185 ⇒ 98,319; 98,32; 98,3; 98; $0{,}1 \cdot 10^3$
h) 8,97946 ⇒ 8,9795; 8,979; 8,98; 9,0; 9
i) 0,479658 ⇒ 0,47966; 0,4797; 0,480; 0,48; 0,5
k) 0,0845853 ⇒ 0,084585; 0,08459; 0,0846; 0,085; 0,08

27/2. $F = \frac{k \cdot M}{100\%} = \frac{1\% \cdot 23{,}86\,V}{100\%} = 0{,}2386\,V$

$w_o = M + F = 23{,}86\,V + 0{,}2386\,V = 24{,}0986\,V$

$w_u = M - F = 23{,}86\,V - 0{,}2386\,V = 23{,}621\,V$

$w_u \leq$ gerundeter $M \leq w_o$

$23{,}621\,V \leq 24\,V \leq 24{,}0986\,V$

27/3. a) n = 2

$x_{min} = 10$; o = 10,49; u = 9,50 $\Rightarrow e_{max} = 0{,}5$

$f_{max} = \frac{e_{max}}{x_{min}} \cdot 100\% = \frac{0{,}5}{10} \cdot 100\% = 5\%$

$x_{max} = 99$; o = 99,49; u = 98,50 $\Rightarrow e_{max} = 0{,}5$

$f_{min} = \frac{e_{max}}{x_{max}} \cdot 100\% = \frac{0{,}5}{99} \cdot 100\% = 0{,}5\%$

b) n = 3

$x_{min} = 100$; o = 100,49; u = 99,50 $\Rightarrow e_{max} = 0{,}5$

$f_{max} = \frac{e_{max}}{x_{min}} \cdot 100\% = \frac{0{,}5}{100} \cdot 100\% = 0{,}5\%$

$x_{max} = 999$; o = 999,49; u = 998,50 $\Rightarrow e_{max} = 0{,}5$

$f_{min} = \frac{e_{max}}{x_{max}} \cdot 100\% = \frac{0{,}5}{999} \cdot 100\% = 0{,}05\%$

c) n = 4

$x_{min} = 1000$; o = 1000,49; u = 999,50 $\Rightarrow e_{max} = 0{,}5$

$f_{max} = \frac{e_{max}}{x_{min}} \cdot 100\% = \frac{0{,}5}{1000} \cdot 100\% = 0{,}05\%$

$x_{max} = 9999$; o = 9999,49; u = 9998,50 $\Rightarrow e_{max} = 0{,}5$

$f_{min} = \frac{e_{max}}{x_{max}} \cdot 100\% = \frac{0{,}5}{9999} \cdot 100\% = 0{,}005\%$

2 Physikalische Grundlagen

2.1 Vorsätze

Lösungen zu 2.1

28/1. a) 1,0 m; 0,075 m; 1,2 m; 6500 m
b) 200 dm; 18,8 dm; 0,14 dm; 0,342 dm
c) 300 cm; 812 cm; 1,72 cm; 2,4 cm
d) 1200 mm; 410 mm; 890 mm; 0,08 mm
e) 405 µm; 5000 µm; 1625 µm
f) 5,3 km; 0,625 km; 28,3 km

28/2. a) 1,8 m^2; 0,12 m^2; 0,00089 m^2
b) 190 dm^2; 0,0845 dm^2; 9,41 dm^2
c) 8700 cm^2; 1200 cm^2; 4,73 cm^2
d) 1050000 mm^2; 160000 mm^2; 18400 mm^2

28/3. a) $115 \cdot 10^{-6}\ m^3$; $61 \cdot 10^{-9}\ m^3$; $12{,}4 \cdot 10^{-3}\ m^3$
b) $130 \cdot 10^{-3}\ dm^3$; $10753\ dm^3$; $4{,}2 \cdot 10^{-3}\ dm^3$
c) $10^7\ cm^3$; $28{,}4 \cdot 10^3\ cm^3$; $5 \cdot 10^{-3}\ cm^3$
d) $2000\ mm^3$; $15 \cdot 10^6\ mm^3$; $127 \cdot 10^9\ mm^3$

28/4. a) 201,6 mil b) 295,3 mil c) 393,7 mil

28/5. a) 43,18 cm b) 53,34 cm c) 68,58 cm

28/6. a) Abstände: 4,233 mm; 3,175 mm
Längen: 88,9 mm; 139,7 mm; 152,4 mm; 177,8 mm; 215,9 mm; 279,4 mm; 304,8 mm; 355,6 mm
b) 297 mm = 11,69 in ≈ 12 in

2.2 Kreisumfang, gestreckte Länge

Lösungen zu 2.2

29/1. a) 56,55 mm b) 5,0 mm c) 138,2 mm d) 2,70 cm
e) 80,11 cm f) 0,205 m g) 1,21 m

29/2. $u_m = \pi \cdot d_m = \pi \cdot (140\ mm + 16\ mm) = \mathbf{490{,}1\ mm}$

29/3. $d_m = \frac{D + d}{2} = \frac{250\ mm + 205\ mm}{2} = 227{,}5\ mm$
$u_m = d_m \cdot \pi = 227{,}5\ mm \cdot \pi = 714{,}7\ mm \approx \mathbf{715\ mm}$

29/4. a) $u_i = \pi \cdot d = \pi \cdot 750\ mm = \mathbf{2356\ mm}$; $u_a = \pi \cdot D = \pi \cdot 1600\ mm = \mathbf{5027\ mm}$
b) $h = \frac{D - d}{2} = \frac{1600\ mm - 750\ mm}{2} = \mathbf{425\ mm}$
c) $d_m = \frac{D + d}{2} = \frac{1600\ mm + 750\ mm}{2} = \mathbf{1175\ mm}$

29/5. $L = \frac{\pi \cdot d_{m1}}{2} + 2 \cdot l_1 + \frac{\pi \cdot d_{m2}}{2} + 2 \cdot l_2 = \frac{\pi \cdot 45\ mm}{2} + 2 \cdot 10\ mm + \frac{\pi \cdot 15\ mm}{2} + 2 \cdot 10\ mm = \mathbf{134\ mm}$

29/6. a) $d_m = \frac{D + d}{2} = \frac{90\ mm + 80\ mm}{2} = 85\ mm$; $l = \pi \cdot d_m = \pi \cdot 85\ mm = \mathbf{267\ mm}$
b) $L = 12 \cdot l + 11 \cdot 1\ mm = 12 \cdot 267\ mm + 11\ mm = \mathbf{3215\ mm}$

29/7. $L = 2 \cdot l + 2 \cdot b - 8 \cdot r + \pi \cdot d_m = 2 \cdot 90\ mm + 2 \cdot 35\ mm - 8 \cdot 3\ mm + \pi \cdot 26\ mm = \mathbf{308\ mm}$

29/8. $u_{max} = \pi \cdot 2(r + 3\ mm) = \pi \cdot 2(282\ mm + 3\ mm) = \mathbf{1791\ mm}$
$u_{min} = \pi \cdot 2(r - 3\ mm) = \pi \cdot 2(282\ mm - 3\ mm) = \mathbf{1753\ mm}$

29/9. $u = \frac{s}{n_{Umdr}} = \frac{s}{n_{Imp} \cdot \frac{1}{72}} = \frac{15\ m}{9000 \cdot \frac{1}{72}} = 0{,}12\ m = 12\ cm$
$r = \frac{u}{2\pi} = \frac{12\ cm}{2\pi} = \mathbf{1{,}91\ cm}$

29/10. $u = \pi \cdot d$; $u_1 = \pi \cdot 570\ mm = 1791\ mm$; $u_2 = \pi \cdot 564\ mm = 1772\ mm$
$n_1 = \frac{s}{u_1} = \frac{560\ km}{1791\ mm} = 312674$; $n_2 = \frac{s}{u_2} = \frac{560\ km}{1772\ mm} = 316027$
$s' = \frac{s \cdot n_2}{n_1} = \frac{560\ km \cdot 316027}{312674} = \mathbf{566\ km}$

2.3 Flächen

Lösungen zu 2.3

30/1. $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(44^2\ mm^2 - 25^2\ mm^2) = \mathbf{1030\ mm^2}$
$b = \frac{D - d}{2} = \frac{44\ mm - 25\ mm}{2} = \mathbf{9{,}5\ mm}$

30/2. $A = A_1 + A_2$
$A_1 = l \cdot b - 2 \cdot l_1 \cdot l_2 = 6\ cm \cdot 4\ cm - 2 \cdot 3\ cm \cdot 1\ cm = 18\ cm^2$; $A_2 = l \cdot b_2 = 6\ cm \cdot 1\ cm = 6\ cm^2$
$A = 18\ cm^2 + 6\ cm^2 = \mathbf{24\ cm^2}$

30/3. $A = 12 \cdot (A_1 - A_\square - A_\triangle) = 12 \cdot \left(4\ cm \cdot 5\ cm - 2{,}5\ cm \cdot 2\ cm - \frac{2{,}5\ cm \cdot 2\ cm}{2}\right) = 150\ cm^2$
Werkstoffbedarf $= A \cdot 1{,}65 = 150\ cm^2 \cdot 1{,}65 = \mathbf{247{,}5\ cm^2}$

30/4. $A = 2 \cdot A_1 = 2 \cdot \frac{\pi \cdot 0{,}8^2\ mm^2}{4} = \mathbf{1{,}0\ mm^2}$

30/5. a) $A_1 = \frac{A}{2} = \frac{1\ mm^2}{2} = 0{,}5\ mm^2$; $d_1 = \sqrt{\frac{A_1 \cdot 4}{\pi}} = \sqrt{\frac{0{,}5\ mm^2 \cdot 4}{\pi}} = \mathbf{0{,}80\ mm}$;
b) $A_1 = \frac{A}{3} = \frac{1\ mm^2}{3} = 0{,}33\ mm^2$; $d_1 = \sqrt{\frac{A_1 \cdot 4}{\pi}} = \sqrt{\frac{0{,}33\ mm^2 \cdot 4}{\pi}} = \mathbf{0{,}65\ mm}$

30/6. a) $A_1 = 26 \cdot \frac{\pi}{4} \cdot d_1^2 = 26 \cdot \frac{\pi}{4} \cdot 3^2\ mm^2 = \mathbf{183{,}8\ mm^2}$
b) $A_2 = 7 \cdot \frac{\pi}{4} \cdot d_2^2 = 7 \cdot \frac{\pi}{4} \cdot 2{,}33^2\ mm^2 = \mathbf{29{,}8\ mm^2}$
c) Al : St $= 183{,}8\ mm^2 : 29{,}8\ mm^2 = \mathbf{6{,}16 : 1}$

30/7. a) $e = \sqrt{b^2 + h^2} \Rightarrow h = \sqrt{e^2 - b^2}$

$\frac{b}{h} = \frac{16}{10} \Rightarrow b = \frac{16}{10} \cdot h = 1{,}6 \cdot h$

$h = \sqrt{e^2 - (1{,}6 \cdot h)^2} = \sqrt{e^2 - 2{,}56 \cdot h^2} \Rightarrow h^2 = e^2 - 2{,}56 \cdot h^2 \Rightarrow$

$h = \sqrt{\frac{e^2}{3{,}56}} = \sqrt{\frac{(27\,\text{in} \cdot 2{,}54\,\frac{\text{cm}}{\text{in}})^2}{3{,}56}} = \mathbf{36{,}35\,cm}$

$b = 1{,}6 \cdot h = 1{,}6 \cdot 36{,}35\,\text{cm} = \mathbf{58{,}16\,cm}$

b) $A = b \cdot h = 58{,}16\,\text{cm} \cdot 36{,}35\,\text{cm} = 2114{,}12\,\text{cm}^2 \approx \mathbf{21{,}14\,dm^2}$

2.4 Rauminhalt und Masse

Lösungen zu 2.4

31/1. $V = l \cdot b \cdot h = 8{,}8\,\text{dm} \cdot 4{,}5\,\text{dm} \cdot 4{,}3\,\text{dm} = \mathbf{170\,dm^3 = 170\,l}$

31/2. $V = A \cdot l = 0{,}8 \cdot 0{,}1\,\text{dm}^2 \cdot 24\,\text{dm} = 1{,}92\,\text{dm}^3$; $m = V \cdot \varrho = 1{,}92\,\text{dm}^3 \cdot 2{,}7\,\frac{\text{kg}}{\text{dm}^3} = \mathbf{5{,}18\,kg}$

31/3. $V = V_1 + 2 \cdot V_2 = 10\,\text{cm} \cdot 9\,\text{cm}^2 + 2 \cdot 7{,}5\,\text{cm} \cdot 9\,\text{cm}^2 = 225\,\text{cm}^3$

$\varrho = \frac{m}{V} = \frac{1766\,\text{g}}{225\,\text{cm}^3} = 7{,}85\,\frac{\text{g}}{\text{cm}^3} = \mathbf{7{,}85\,\frac{kg}{dm^3}} \Rightarrow$ **Stahl**

31/4. $m_{Cu} = 3700\,\text{g} - 200\,\text{g} = 3500\,\text{g}$; $V_{Cu} = \frac{m_{Cu}}{\varrho} = \frac{3500\,\text{g}}{8{,}9\,\frac{\text{g}}{\text{cm}^3}} = 393{,}26\,\text{cm}^3$

$l = \frac{V_{Cu}}{A} = \frac{393{,}26\,\text{cm}^3}{\frac{\pi}{4} \cdot 0{,}04^2 \cdot \text{cm}^2} = 312945\,\text{cm} \approx \mathbf{3130\,m}$

31/5. a) $V = \frac{A_1 \cdot h_1}{3} + A_2 \cdot h_2 = \frac{\pi \cdot 5^2\,\text{mm}^2 \cdot 1{,}2\,\text{mm}}{4 \cdot 3} + \frac{\pi \cdot 2^2\,\text{mm}^2}{4} \cdot 2{,}5\,\text{mm} = 15{,}7\,\text{mm}^3$

$m = V \cdot \varrho \cdot 100 = 0{,}0157\,\text{cm}^3 \cdot 8{,}9\,\frac{\text{g}}{\text{cm}^3} \cdot 100 = \mathbf{14\,g}$

b) $V_1 = A_1 \cdot h_1 = \frac{\pi \cdot 15^2\,\text{mm}^2}{4} \cdot 2\,\text{mm} = 353\,\text{mm}^3$

$V_2 = V_1 + A_2 \cdot h_2 = 353\,\text{mm}^3 + \frac{\pi \cdot 8^2\,\text{mm}^2}{4} \cdot 7\,\text{mm} = 705\,\text{mm}^3$

$m = \left(0{,}353\,\text{cm}^3 \cdot 19{,}3\,\frac{\text{g}}{\text{cm}^3} + 0{,}705\,\text{cm}^3 \cdot 8{,}9\,\frac{\text{g}}{\text{cm}^3}\right) \cdot 60 = \mathbf{785\,g}$

31/6. $\text{Kupferzahl} = \frac{\varrho_{Cu} \cdot V}{1\,\text{km}} = \frac{\varrho_{Cu} \cdot 3 \cdot A \cdot l}{1\,\text{km}} = \frac{8{,}9\,\frac{\text{kg}}{\text{dm}^3} \cdot 3 \cdot 1{,}5 \cdot 10^{-4}\,\text{dm}^2 \cdot 1000 \cdot 10\,\text{dm}}{1\,\text{km}} = \mathbf{40{,}05\,\frac{kg}{km}}$

2.5 Berechnung von Spulen

Lösungen zu 2.5

32/1. $d_2 = d_1 + 2 \cdot s = 1{,}2\,\text{mm} + 2 \cdot 0{,}04\,\text{mm} = 1{,}28\,\text{mm}$; $d_2 = \mathbf{1{,}58\,mm;\ 1{,}88\,mm;\ 2{,}08\,mm}$

32/2. a) $d_2 = d_1 + 2 \cdot s = 0{,}5\,\text{mm} + 2 \cdot 0{,}09\,\text{mm} = \mathbf{0{,}68\,mm}$; $d_2 = \mathbf{0{,}78\,mm;\ 0{,}93\,mm;\ 1{,}68\,mm}$

b) $A_1 = \frac{d_1^2 \cdot \pi}{4} = \frac{0{,}5^2\,\text{mm}^2 \cdot \pi}{4} = \mathbf{0{,}196\,mm^2}$; $A_1 = \mathbf{0{,}283\,mm^2;\ 0{,}442\,mm^2;\ 1{,}767\,mm^2}$

32/3. a) $l_m = 2 \cdot a' + 2 \cdot b' + \pi \cdot h = 2 \cdot 10\,\text{mm} + 2 \cdot 4\,\text{mm} + \pi \cdot 5\,\text{mm} = \mathbf{43{,}7\,mm}$

b) $z = \frac{h}{d_2} = \frac{5\,\text{mm}}{0{,}12\,\text{mm}} = 41{,}7 \approx \mathbf{41}$ c) $N_1 = \frac{b}{d_2} = \frac{50\,\text{mm}}{0{,}12\,\text{mm}} = 416{,}7 \approx \mathbf{416}$

d) $N = z \cdot N_1 = 41 \cdot 416 = 17056 \approx \mathbf{17050}$ e) $l = l_m \cdot N = 43{,}7\,\text{mm} \cdot 17050 = \mathbf{745\,m}$

33/4. a) $N = \frac{b}{d_2} = \frac{90\,\text{mm}}{0{,}6\,\text{mm}} = \mathbf{150}$ b) $d_m = d + d_2 = 30\,\text{mm} + 0{,}6\,\text{mm} = \mathbf{30{,}6\,mm}$

c) $l = \pi \cdot d_m \cdot N = \pi \cdot 30{,}6\,\text{mm} \cdot 150 = 14420\,\text{mm} \approx \mathbf{14{,}5\,m}$

33/5. a) $N = \frac{b}{d_2} = \frac{440\,\text{mm}}{0{,}2\,\text{mm}} = \mathbf{2200}$

b) $d_m = d + d_2 = 60\,\text{mm} + 0{,}2\,\text{mm} = 60{,}2\,\text{mm}$; $l = \pi \cdot d_m \cdot N = \pi \cdot 60{,}2\,\text{mm} \cdot 2200 = \mathbf{416\,m}$

c) $m = \frac{\pi}{4} \cdot 0{,}02^2\,\text{cm}^2 \cdot 41600\,\text{cm} \cdot 8{,}3\,\text{g/cm}^3 = \mathbf{108\,g}$

33/6. $A_w = b \cdot h = 60\,\text{mm} \cdot 10\,\text{mm} = 600\,\text{mm}^2$; $A_D = A_w \cdot f = 600\,\text{mm}^2 \cdot 0{,}65 = 390\,\text{mm}^2$

$N = \frac{A_D}{A_1} = \frac{390\,\text{mm}^2}{\frac{\pi \cdot 0{,}4^2\,\text{mm}^2}{4}} = 3104 \approx \mathbf{3100}$; $l = N \cdot l_m = N \cdot \pi \cdot d_m = 3100 \cdot \pi \cdot 50\,\text{mm} = \mathbf{487\,m}$

33/7. a) $z = \frac{h}{d_2} = \frac{12\,\text{mm}}{0{,}8\,\text{mm}} = 15$; $N_1 = \frac{b}{d_2} = \frac{45\,\text{mm}}{0{,}8\,\text{mm}} = 56$; $N = z \cdot N_1 = 15 \cdot 56 = \mathbf{840}$

b) $l_m = 2 \cdot a' + 2 \cdot b' + \pi \cdot h = 2 \cdot 36\,\text{mm} + 2 \cdot 35\,\text{mm} + \pi \cdot 12\,\text{mm} = 180\,\text{mm}$

$l = l_m \cdot N = 0{,}180\,\text{m} \cdot 840 = 151{,}20\,\text{m}$; $l' = l + l \cdot 4\% = 151{,}20\,\text{m} + 6{,}05\,\text{m} = \mathbf{157{,}25\,m}$

33/8. $d_m = d + d_2 = 30\,\text{mm} + 1{,}25\,\text{mm} = 31{,}25\,\text{mm}$; $N = \frac{l}{\pi \cdot d_m} = \frac{11000\,\text{mm}}{\pi \cdot 31{,}25\,\text{mm}} = 112$

$b = N \cdot d_2 = 112 \cdot 1{,}25\,\text{mm} = \mathbf{140\,mm}$

33/9. $z = \frac{h}{d_2} = \frac{220\,\text{mm}}{31\,\text{mm}} = 7{,}1$; 7 Lagen; $N_1 = \frac{b}{d_2} = \frac{500\,\text{mm}}{31\,\text{mm}} = 16{,}13$; 16 Wdg. je Lage

$N = N_1 \cdot z = 16 \cdot 7 = 112$; $d_m = d + h = 500\,\text{mm} + 220\,\text{mm} = 720\,\text{mm}$

$l = \pi \cdot d_m \cdot N = \pi \cdot 720\,\text{mm} \cdot 112 = \mathbf{253\,m}$

33/10. a) $h = \frac{D - d}{2} = \frac{22\,\text{mm} - 9\,\text{mm}}{2} = \mathbf{6{,}5\,mm}$ b) $N_1 = \frac{b}{d_2} = \frac{40\,\text{mm}}{0{,}44\,\text{mm}} = \mathbf{90}$

c) $z = \frac{h}{d_2} = \frac{6{,}5\,\text{mm}}{0{,}44\,\text{mm}} = 14{,}8$; **14 Lagen** d) $N = N_1 \cdot z = 90 \cdot 14 = \mathbf{1260}$

e) $d_m = \frac{D + d}{2} = \frac{22\,\text{mm} + 9\,\text{mm}}{2} = 15{,}5\,\text{mm}$; $l = \pi \cdot d_m \cdot N = \pi \cdot 15{,}5\,\text{mm} \cdot 1260 = \mathbf{61{,}4\,m}$

f) $m = \frac{\pi}{4} \cdot d_1^2 \cdot l \cdot \varrho = \frac{\pi}{4} \cdot 0{,}04^2\,\text{cm}^2 \cdot 6140\,\text{cm} \cdot 8{,}9\,\frac{\text{g}}{\text{cm}^3} = 68{,}7\,\text{g} \approx \mathbf{69\,g}$

g) $f = \frac{N \cdot A_1}{b \cdot h} = \frac{1260 \cdot \frac{\pi}{4} \cdot 0{,}4^2\,\text{mm}^2}{40\,\text{mm} \cdot 6{,}5\,\text{mm}} = \mathbf{0{,}61}$

33/11. a) $h = \frac{D - d}{2} = \frac{19\,\text{mm} - 7{,}6\,\text{mm}}{2} = 5{,}7\,\text{mm}$; $z = \frac{h}{d_2} = \frac{5{,}7\,\text{mm}}{0{,}32\,\text{mm}} = 17{,}8$; 17 Lagen

$N_1 = \frac{b}{d_2} = \frac{36\,\text{mm}}{0{,}32\,\text{mm}} = 112{,}5$; 112 Windungen; $N = z \cdot N_1 = 17 \cdot 112 = 1904 \approx \mathbf{1900}$

b) $d_m = \frac{D + d}{2} = \frac{19\,\text{mm} + 7{,}6\,\text{mm}}{2} = 13{,}3\,\text{mm}$

$l = \pi \cdot d_m \cdot N = \pi \cdot 13{,}3\,\text{mm} \cdot 1900 = 79388\,\text{mm} \approx \mathbf{80\,m}$

33/12. a) $l_m = 2 \cdot a' + 2 \cdot b' + \pi \cdot h = 2 \cdot 20\,mm + 2 \cdot 5\,mm + \pi \cdot 12\,mm = \mathbf{87{,}7\,mm}$

b) $z = \frac{h}{d_2} = \frac{12\,mm}{0{,}222\,mm} = \mathbf{54\ Lagen}$

c) $N_1 = \frac{b}{d_2} = \frac{45\,mm}{0{,}222\,mm} = \mathbf{202}$; $N = N_1 \cdot z = 202 \cdot 54 = 10\,908 \approx \mathbf{10\,900}$

d) $l = l_m \cdot N = 87{,}7\,mm \cdot 10\,900 = \mathbf{956\,m}$

33/13. $z = \frac{h}{d_2} = \frac{5\,mm}{0{,}09\,mm} = 55{,}6$; 55 Lagen; $N_1 = \frac{b}{d_2} = \frac{40\,mm}{0{,}09\,mm} = 444{,}4$; 444 Windungen/Lage

$N = N_1 \cdot z = 444 \cdot 55 = \mathbf{24\,420}$

$l_m = 2 \cdot a' + 2 \cdot b' + \pi \cdot h = 2 \cdot 8\,mm + 2 \cdot 5\,mm + \pi \cdot 5\,mm = 41{,}7\,mm$

$l = N \cdot l_m = 24\,420 \cdot 41{,}7\,mm = \mathbf{1\,018\,m}$

2.6 Bewegung mit konstanter Geschwindigkeit

2.6.1 Gleichförmige Bewegung

Lösungen zu 2.6.1

34/1. $v = \frac{s}{t} = \frac{32\,m}{24\,s} = 1{,}33\frac{m}{s} \cdot 60\frac{s}{min} = \mathbf{80\,\frac{m}{min}}$

34/2. $v = \frac{s}{t} = \frac{450\,m}{72\,s} = \mathbf{6{,}25\,\frac{m}{s}}$

34/3. $s = v \cdot t = 2{,}339\,\frac{cm}{s} \cdot 240\,min = 2{,}339\,\frac{cm}{s} \cdot 240 \cdot 60\,s = 33\,681{,}6\,cm \approx \mathbf{336{,}82\,m}$

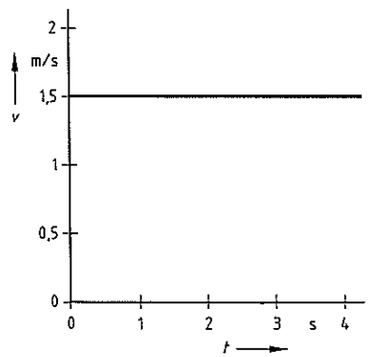
34/4. a) $s_{LP} = v_{LP} \cdot t_{LP} = 12{,}5\,\frac{mm}{s} \cdot \frac{1\,m}{1000\,mm} \cdot 90\,min\,\frac{60\,s}{1\,min} = \mathbf{67{,}5\,m}$

b) $t_{SP} = \frac{s_{LP}}{v_{SP}} = \frac{67{,}5\,m}{18{,}8\frac{mm}{s} \cdot \frac{1\,m}{1000\,mm}} = 3590{,}4\,s = 59{,}84\,min \approx \mathbf{60\,min}$

34/5. $t = \frac{2 \cdot s}{v} = \frac{2 \cdot 384\,400\,km}{299\,800\,km/s} = \mathbf{2{,}564\,s}$

34/6. a) $v = \frac{\Delta s}{\Delta t} = \frac{3\,m}{2\,s} = \mathbf{1{,}5\,\frac{m}{s}}$; $1{,}5\frac{m}{s} \cdot 3600\frac{s}{h} : 1000\frac{m}{km} = \mathbf{5{,}4\,\frac{km}{h}}$

b)



zu 34/6.

34/7. $n = \frac{b}{1{,}2\,mm} = \frac{1\,140\,mm}{1{,}2\,mm} = 950$ Arbeitshübe

$t_1 = \frac{s}{v_1} = \frac{950 \cdot (2{,}50\,m + 0{,}08\,m)}{18\,m/min} = 136{,}17\,min$; $t_2 = \frac{s}{v_2} = \frac{950\,(2{,}50\,m + 0{,}08\,m)}{25\,m/min} = 98{,}04\,min$

$t = t_1 + t_2 = 136{,}17\,min + 98{,}04\,min = 234{,}21\,min = \mathbf{3\,h\ 54\,min}$

34/8. a) Rechteck mit den Seiten v_H und v_W; daraus Diagonale v.

$v^2 = v_H^2 + v_W^2 \Rightarrow v = \sqrt{v_H^2 + v_W^2}$

$= \sqrt{6{,}3^2\left(\frac{m}{min}\right)^2 + 19^2\left(\frac{m}{min}\right)^2} = \sqrt{400{,}69\left(\frac{m}{min}\right)^2} = 20{,}02\,\frac{m}{min} \approx \mathbf{20\,\frac{m}{min}}$

b) $s = v \cdot t = 20\,\frac{m}{min} \cdot \frac{1}{60}\,\frac{min}{s} \cdot 16\,s = \mathbf{5{,}33\,m}$

2.6.2 Kreisförmige Bewegung mit konstanter Bahngeschwindigkeit

Lösungen zu 2.6.2

35/1. $v = \pi \cdot d \cdot n = \pi \cdot 4\,mm \cdot 1\,440\,min^{-1} = 18{,}096\,\frac{m}{min} = \mathbf{0{,}3\,\frac{m}{s}}$

35/2. $v = \pi \cdot d \cdot n = \pi \cdot 0{,}18\,m \cdot 8500\,min^{-1} = 4806{,}6\,\frac{m}{min} \approx \mathbf{80\,\frac{m}{s}}$

35/3. $d = \frac{v}{\pi \cdot n} = \frac{30\,m/s}{\pi \cdot 18\,000\,min^{-1}} = \frac{30\,000\,mm/s}{\pi \cdot 300\,s^{-1}} \approx \mathbf{31{,}8\,mm}$

35/4. $d = \frac{v}{\pi \cdot n} = \frac{45\,m/s}{\pi \cdot 43\,000\,min^{-1}} = \frac{45\,000\,mm/s}{\pi \cdot 716{,}7\,s^{-1}} = \mathbf{20\,mm}$

35/5. $v = \pi \cdot d \cdot n \Rightarrow n = \frac{v}{\pi \cdot d} = \frac{7{,}85\,m/s}{\pi \cdot 0{,}025\,m} = 100\,s^{-1} = \mathbf{6000\,min^{-1}}$

35/6. $n = \frac{v}{\pi \cdot d} = \frac{2{,}339\,cm/s}{\pi \cdot 0{,}352\,cm} = 2{,}115\,s^{-1} = \mathbf{126{,}91\,min^{-1}}$

35/7. a) $s = u \cdot n = d \cdot \pi \cdot n = 27 \cdot 0{,}0254\,m \cdot \pi \cdot 4640 = \mathbf{9996{,}9\,m}$

b) $n = \frac{v \cdot t}{d \cdot \pi} = \frac{25\,000\,\frac{m}{h} \cdot \frac{1}{60}\,h}{27 \cdot 0{,}0254\,m \cdot \pi} = \mathbf{193{,}4}$

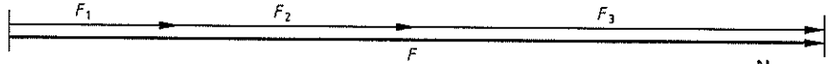
c) $v = d \cdot \pi \cdot n = 27 \cdot 0{,}0254\,m \cdot \pi \cdot 5\,\frac{1}{s} = 10{,}77\,\frac{m}{s} = \mathbf{38{,}8\,\frac{km}{h}}$

2.7 Kräfte

Lösungen zu 2.7

36/1. Kräftemaßstab: 100 N ≙ 5 mm

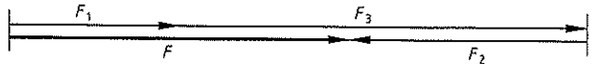
a)



120 mm ≙ F; $F = 120\ mm \cdot 20\ \frac{N}{mm} = 2400\ N$

$F = F_1 + F_2 + F_3 = 500\ N + 700\ N + 1200\ N = 2400\ N$

b)

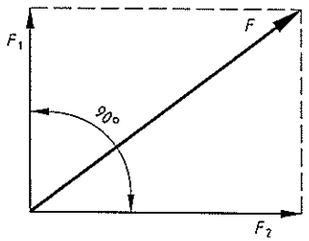


50 mm ≙ F = 1000 N

$F = F_1 + F_3 - F_2 = 500\ N + 1200\ N - 700\ N = 1000\ N$

36/2. a) $F = \sqrt{F_1^2 + F_2^2} = \sqrt{150^2\,N^2 + 200^2\,N^2} = 250\ N$

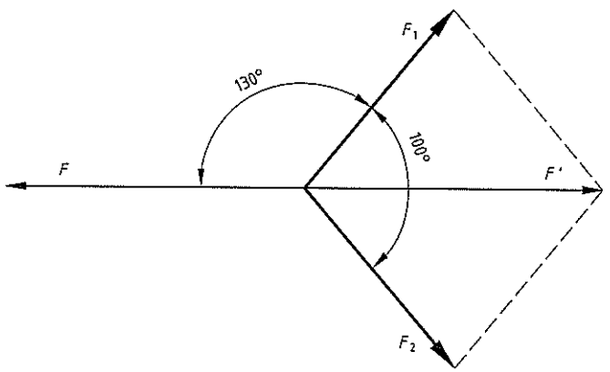
b) Kräftemaßstab: 100 N ≙ 20 mm



zu 36/2.b)

50 mm ≙ F = 250 N

36/3. Kräftemaßstab: 1 kN ≙ 2 mm



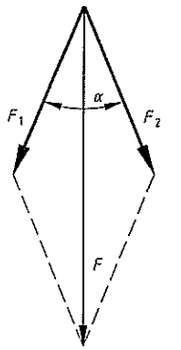
zu 36/3.

F = F'

34 mm ≙ $F_1 = F_2$

$F_1 = F_2 = 34\ mm \cdot 0{,}5\ \frac{kN}{mm} = 17\ kN$

36/4. Kräftemaßstab: 100 N ≙ 1 mm

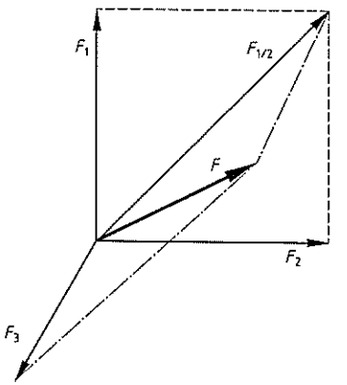


zu 36/4.

27 mm ≙ $F_1 = F_2$

$F_1 = F_2 = 27\ mm \cdot 100\ \frac{N}{mm} = 2{,}7\ kN$

36/5. Kräftemaßstab: 1 kN ≙ 2 mm



zu 36/5.

26 mm ≙ F

$F = 26\ mm \cdot 0{,}5\ \frac{kN}{mm} = 13\ kN$

2.8 Moment und Hebel

Lösungen zu 2.8

37/1. $F = \frac{M}{r} = \frac{23\ Nm}{0{,}04\ m} = 575\ N$

37/2. $F_2 = \frac{F_1 \cdot r_1}{r_2} = \frac{120\ N \cdot 100\ mm}{80\ mm} = 150\ N$

37/3. a) $F_2 = \frac{F_1 \cdot r_1}{r_2} = \frac{650\ N \cdot 85\ mm}{240\ mm} = 230\ N$

b) $r_2 = \frac{F_1 \cdot r_1}{F_2} = \frac{650\ N \cdot 85\ mm}{100\ N} = 553\ mm$

37/4. $M = F \cdot r = 200\ N \cdot 3{,}4\ m = 680\ Nm$

37/5. $F = \frac{M}{r} = \frac{159\ Nm}{0{,}24\ m} = 662{,}5\ N$

37/6. $F_1 = \frac{F_2 \cdot r_2}{r_1} = \frac{700\ N \cdot 12\ mm}{84\ mm} = 100\ N$

2.9 Mechanische Arbeit

Lösungen zu 2.9

38/1. a) $F_G = m \cdot g = 5{,}4\,\text{kg} \cdot 9{,}81\,\frac{\text{m}}{\text{s}^2} = \mathbf{53\,N}$

b) $F_G = m \cdot g = 48\,\text{kg} \cdot 9{,}81\,\frac{\text{m}}{\text{s}^2} = \mathbf{471\,N}$

38/2. $W = F \cdot s = 1{,}3\,\text{kN} \cdot 15\,\text{m} = \mathbf{19{,}5\,kJ}$

38/3. $h = \frac{W_h}{F_G} = \frac{180\,\text{kJ}}{9\,\text{kN}} = \mathbf{20\,m}$

38/4. a) $F_G = \frac{W_h}{h} = \frac{30\,\text{kJ}}{2{,}5\,\text{m}} = \mathbf{12\,kN}$

b) $m = \frac{F_G}{g} = \frac{12000\,\text{N}}{9{,}81\,\text{m/s}^2} = \mathbf{1223\,kg}$

38/5. $V = l \cdot b \cdot h = 320\,\text{m} \cdot 85\,\text{m} \cdot 16{,}5\,\text{m} = 448{,}8 \cdot 10^3\,\text{m}^3$;
$F_G = \varrho \cdot V \cdot g = 1\,\frac{\text{kg}}{\text{dm}^3} \cdot 448{,}8 \cdot 10^6\,\text{dm}^3 \cdot 9{,}81\,\frac{\text{m}}{\text{s}^2} = 4{,}40 \cdot 10^9\,\text{N} = 4{,}40\,\text{GN}$;
$W_p = F_G \cdot h = 4{,}40\,\text{GN} \cdot 283\,\text{m} = \mathbf{1246\,GNm = 1246\,GJ}$

38/6. $W = F \cdot s = 420\,\text{N} \cdot 2{,}5\,\text{m} = 1050\,\text{Nm} = \mathbf{1{,}05\,kJ}$

38/7. $\varphi = 30° \Rightarrow \cos\varphi = 0{,}866$:
a) $F_w = F \cdot \cos\varphi = 280\,\text{N} \cdot 0{,}866 = \mathbf{242{,}5\,N}$ b) $W = F_w \cdot s = 242{,}5\,\text{N} \cdot 2{,}5\,\text{m} = \mathbf{606\,J}$

2.10 Mechanische Leistung

Lösungen zu 2.10

39/1. $P = \frac{F \cdot s}{t} = \frac{100\,\text{kN} \cdot 6{,}3\,\text{m}}{30\,\text{s}} = 21\,\frac{\text{kNm}}{\text{s}} = \mathbf{21\,kW}$

39/2. a) Kl 1: $P = \frac{\Delta W}{\Delta t} = \frac{400\,\text{Nm}}{1\,\text{s}} = 400\,\frac{\text{Nm}}{\text{s}} = \mathbf{400\,W}$; Kl 2: **120 W**; Kl 3: **57,1 W**; Kl 4: **14,3 W**

b) Leistung $= \frac{\Delta W}{\Delta t}$ = Steigung der W(t)-Kennlinie.
kleine Steigung ⇒ kleine Leistung

39/3. $P = \frac{F \cdot s}{t} = \frac{\varrho \cdot V \cdot g \cdot s}{t} = \frac{1\,\text{kg} \cdot 120\,\text{dm}^3 \cdot 9{,}81\,\text{m} \cdot 51\,\text{m}}{\text{dm}^3 \cdot \text{s}^2 \cdot 60\,\text{s}} = 1000{,}62\,\text{W} = \mathbf{1\,kW}$

39/4. $P = \frac{F \cdot s}{t} = \frac{m \cdot g \cdot s}{t} = \frac{75\,\text{kg} \cdot 9{,}81\,\text{m} \cdot 5{,}35\,\text{m}}{\text{s}^2 \cdot 1{,}2\,\text{s}} = 3280\,\text{W} = \mathbf{3{,}28\,kW}$

39/5. $P = F \cdot v; \Rightarrow F = \frac{P}{v} = \frac{20\,\text{kW}}{1{,}25\,\text{m/s}} = \frac{20\,\text{kNm/s}}{1{,}25\,\text{m/s}} = \mathbf{16\,kN}$

39/6. $P = F \cdot v \Rightarrow v = \pi \cdot d \cdot n$; $P = F \cdot \pi \cdot d \cdot n = 0{,}275\,\text{kN} \cdot \pi \cdot 0{,}355\,\text{m} \cdot \frac{1450}{\text{min}} \cdot \frac{1}{60}\,\frac{\text{min}}{\text{s}} = \mathbf{7{,}41\,kW}$

39/7. $P = \frac{F \cdot s}{t} = \frac{m \cdot g \cdot s}{t} = \frac{1250\,\text{kg} \cdot 9{,}81\,\text{m} \cdot 1{,}8\,\text{m}}{\text{s}^2 \cdot 5{,}5\,\text{s}} = 4013\,\text{W} \approx \mathbf{4\,kW}$

39/8. $F = \frac{P}{v} = \frac{160 \cdot 10^6\,\text{W}}{850\,\text{km/h}} = \frac{160 \cdot 10^6\,\text{W}}{236{,}11\,\text{m/s}} = \mathbf{678\,kN}$

39/9. $v = \frac{P}{F} = \frac{3\,\text{kW}}{1{,}3\,\text{kN}} = 2{,}3\,\frac{\text{m}}{\text{s}} = \mathbf{138\,\frac{m}{min}}$

39/10. $F = 50\,\text{kN} - 38\,\text{kN} = 12\,\text{kN}$; $v = \frac{P}{F} = \frac{33\,\text{kW}}{12\,\text{kN}} = \mathbf{2{,}75\,\frac{m}{s}}$

3 Elektrotechnische Grundlagen

3.1 Umrechnen von Einheiten

Lösungen zu 3.1

40 a) 1 230 000 µV; 400 000 mV; 0,25 V; 0,4 kV
b) 0,12 mA; 23 400 µA; 0,32 A; 6 kA
c) 0,22 MΩ; 1200 mΩ; 0,0025 Ω
d) 850 W; 32 mW; 4,5 kW
e) 0,1 kWh; 36 000 Ws; 7 200 000 Ws; 0,33 Wh

3.2 Stromstärke und Ladung

Lösungen zu 3.2

40/1. $I = \frac{Q}{t} = \frac{4{,}4\ \text{mAs}}{1\ \text{s}} = 4{,}4\ \text{mA}$

40/2. $I = \frac{Q}{t} = \frac{6\ \text{mAs}}{4\ \text{s}} = 1{,}5\ \text{mA}$

40/3. $Q = I \cdot t = 0{,}4\ \text{A} \cdot 0{,}001\ \text{s} = 0{,}4 \cdot 10^{-3}\ \text{As} = 0{,}4\ \text{mAs}$

40/4. a) $Q = n \cdot e = 1{,}87 \cdot 10^{19} \cdot 1{,}6021 \cdot 10^{-19}\ \text{C} = 2{,}996\ \text{C} \approx 3\ \text{As}$ b) $I = \frac{Q}{t} = \frac{3\ \text{As}}{2\ \text{s}} = 1{,}5\ \text{A}$

3.3 Elektrische Spannung

Lösungen zu 3.3

41/1. a) $W = F \cdot s = 6\ \text{mN} \cdot 0{,}06\ \text{m} = 0{,}36\ \text{mNm} = 0{,}36\ \text{mJ}$ b) $U = \frac{W}{Q} = \frac{0{,}36\ \text{mJ}}{0{,}03\ \text{mC}} = 12\ \text{V}$

41/2. a) $U = \frac{W}{Q} = \frac{0{,}36\ \text{mNm}}{30\ \mu\text{C}} = \frac{0{,}36\ \text{mVAs}}{0{,}03\ \text{mAs}} = 12\ \text{V}$

41/3. a) $s = \frac{W}{F} = \frac{2{,}88\ \text{mJ}}{220\ \text{mN}} = \frac{2{,}88\ \text{mNm}}{220\ \text{mN}} = 0{,}013\ \text{m} = 13\ \text{mm}$ b) $U = \frac{W}{Q} = \frac{2{,}88\ \text{mJ}}{0{,}12\ \text{mC}} = 24\ \text{V}$

3.4 Widerstand und Leitwert

Lösungen zu 3.4

41/1. a) 5 Ω b) 2 Ω c) 1,25 Ω

41/2. a) $G = \frac{1}{R} = \frac{1}{12\ \Omega} = 0{,}083\ \text{S} = 83\ \text{mS}$ b) 2,22 S c) 4 S d) 0,8 mS e) 16,7 mS

3.5 Ohmsches Gesetz

Lösungen zu 3.5

42/1. $I = \frac{U}{R} = \frac{230\ \text{V}}{40\ \Omega} = 5{,}75\ \text{A}$

42/2. $U = R \cdot I = 8\ \text{m}\Omega \cdot 16\ \text{A} = 128\ \text{mV}$

42/3. $R = \frac{U_1}{I_1} = \frac{230\ \text{V}}{4{,}35\ \text{A}} = 52{,}87\ \Omega$

$U_2 = U_1 \cdot 1{,}05 = 230\ \text{V} \cdot 1{,}05 = 241{,}5\ \text{V}$

$I_2 = \frac{U_2}{R} = \frac{241{,}5\ \text{V}}{52{,}87\ \Omega} = 4{,}57\ \text{A}$

42/4. a) $R = \frac{U}{I} = \frac{6\ \text{V}}{0{,}05\ \text{A}} = 120\ \Omega$ b) $G = \frac{1}{R} = \frac{1}{120\ \Omega} = 8{,}33\ \text{mS}$

c) $I = \frac{U}{R} = \frac{9\ \text{V}}{120\ \Omega} = 75\ \text{mA}$

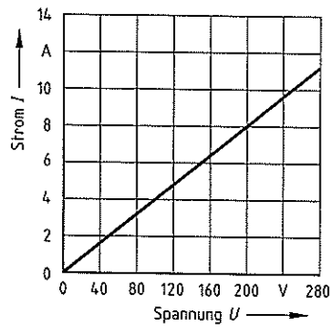
42/5. a) $R = \frac{U}{I} = \frac{10\ \text{kV}}{0{,}12\ \text{A}} = 83{,}3\ \text{k}\Omega$ b) $G = \frac{1}{R} = \frac{1}{83{,}3\ \text{k}\Omega} = 12 \cdot 10^{-6}\ \text{S} = 12\ \mu\text{S}$

42/6. a) Aus U-I-Diagramm bei U = 6 V: $I_1 = 12\ \text{A}$; $I_2 = 6\ \text{A}$; $I_3 = 3\ \text{A}$

b) $R_1 = \frac{U}{I_1} = \frac{6\ \text{V}}{12\ \text{A}} = 0{,}5\ \Omega$; $R_2 = 1{,}0\ \Omega$; $R_3 = 2{,}0\ \Omega$

42/7. a) Aus Diagramm: $I_1 = 3\ \text{A}$; $I_2 = 1{,}5\ \text{A}$

b) $U = R_1 \cdot I_1 = R_2 \cdot I_2 = 20\ \Omega \cdot 3\ \text{A} = 40\ \Omega \cdot 1{,}5\ \text{A} = 60\ \text{V}$



zu 42/8.b

42/8. a) $R = \frac{1}{G} = \frac{1}{40\ \text{mS}} = 25\ \Omega$;

$I = \frac{U}{R} = \frac{230\ \text{V}}{25\ \Omega} = 9{,}2\ \text{A}$

b) $U = 0\ \text{V} \Rightarrow I = 0\ \text{A}$

z.B. $U = 200\ \text{V} \Rightarrow I = \frac{U}{R} = \frac{200\ \text{V}}{25\ \Omega} = 8\ \text{A}$

c) Aus Diagramm: $U \approx 150\ \text{V}$

42/9. $R = \frac{U}{I} = \frac{13\ \text{V}}{0{,}04\ \text{A}} = 325\ \Omega$

42/10. $R = \frac{\Delta U}{\Delta I} = \frac{230\ \text{V} - 180\ \text{V}}{0{,}1\ \text{A}} = 500\ \Omega$; $G = \frac{1}{R} = \frac{1}{0{,}5\ \text{k}\Omega} = 2\ \text{mS}$

42/11. $R_{max} = \frac{U}{I_{min}} = \frac{240\ \text{mV}}{190{,}5\ \mu\text{A}} = 1{,}26\ \text{k}\Omega$

$R_{min} = \frac{U}{I_{max}} = \frac{240\ \text{mV}}{210{,}5\ \mu\text{A}} = 1{,}14\ \text{k}\Omega \Rightarrow R = 1{,}2\ \text{k}\Omega$

$\Delta R_{max} = R_{max} - R = 1{,}26\ \text{k}\Omega - 1{,}2\ \text{k}\Omega = 60\ \Omega$

$\Delta R_{min} = R_{min} - R = 1{,}14\ \text{k}\Omega - 1{,}2\ \text{k}\Omega = -60\ \Omega$

$\Delta R\% = \frac{\Delta R_{min/max}}{R} \cdot 100\% = \frac{\pm 60\ \Omega}{1{,}2\ \text{k}\Omega} \cdot 100\% = \pm 5\%$

$\Rightarrow$ E-Reihe ≙ E24

42/12. a) $R_1 = \frac{U_{R1}}{I_{R1}} = \frac{8\,V}{40\,mA} = 200\,\Omega$

b) $I_{R3} = I_{R4} = \frac{U_4}{R_4} = \frac{1{,}8\,V}{100\,\Omega} = 18\,mA$

$R_3 = \frac{U_3}{I_{R3}} = \frac{1{,}6\,V}{18\,mA} = 88{,}89\,\Omega$

c) Farbkennzeichnung Braun, Grün, Braun, Gold ≙ 150 Ω, Toleranz ±5%

$U_{R2max} = R_{2max} \cdot I = 150\,\Omega \cdot 1{,}05 \cdot 22\,mA = 3{,}465\,V$

$U_{R2min} = R_{2min} \cdot I = 150\,\Omega \cdot 0{,}95 \cdot 22\,mA = 3{,}135\,V$

3.6 Stromdichte

Lösungen zu 3.6

43/1. $A = a \cdot b = 30\,mm \cdot 10\,mm = 300\,mm^2$

$J = \frac{I}{A} = \frac{630\,A}{300\,mm^2} = 2{,}1\,\frac{A}{mm^2}$

43/2. $I = \frac{U}{R} = \frac{24\,V}{100\,\Omega} = 0{,}24\,A$; $A = \frac{\pi \cdot d^2}{4} = \frac{\pi \cdot (0{,}35\,mm)^2}{4} = 0{,}0962\,mm^2$;

$J = \frac{I}{A} = \frac{0{,}24\,A}{0{,}0962\,mm^2} = 2{,}49\,\frac{A}{mm^2} = 2{,}5\,\frac{A}{mm^2}$

43/3. a) Aus Rechenbuch Elektrotechnik, Tabelle 1, Seite 269

bei $A_1 = 1{,}5\,mm^2$: $I_{r1} = 19{,}5\,A$

bei $A_2 = 6\,mm^2$: $I_{r2} = 46{,}0\,A$

b) $J_1 = \frac{I_{r1}}{A_1} = \frac{19{,}5\,A}{1{,}5\,mm^2} = 13\,\frac{A}{mm^2}$

$J_2 = \frac{I_{r2}}{A_2} = \frac{46{,}0\,A}{6\,mm^2} = 7{,}67\,\frac{A}{mm^2}$

c) Der Querschnitt hat sich vervierfacht, die Oberfläche jedoch nur verdoppelt. Die zulässige Stromdichte sinkt bei steigendem Querschnitt, da die Wärmeabfuhr schlechter wird.

43/4. a) $J = \frac{I}{A} \Rightarrow A = \frac{I}{J} = \frac{1{,}35\,A}{3{,}5\,\frac{A}{mm^2}} = 0{,}386\,mm^2$

b) $A = \frac{\pi \cdot d^2}{4} \Rightarrow d = \sqrt{\frac{4 \cdot A}{\pi}} = \sqrt{\frac{4 \cdot 0{,}386\,mm^2}{\pi}} = 0{,}70\,mm$

43/5. a) $U = I \cdot R = 4{,}35\,A \cdot 52{,}9\,\Omega = 230\,V$

b) $J_{Zul} = \frac{I}{A_1} = \frac{4{,}35\,A}{1\,mm^2} = 4{,}35\,\frac{A}{mm^2}$

c) $J_{Heiz} = \frac{I}{A_2} = \frac{4{,}35\,A}{0{,}246\,mm^2} = 17{,}7\,\frac{A}{mm^2}$

43/6. $A_1 = \frac{\pi \cdot d^2}{4} = \frac{\pi \cdot (0{,}45\,mm)^2}{4} = 0{,}159\,mm^2$; $A_2 = \frac{\pi \cdot d^2}{4} = \frac{\pi \cdot (0{,}35\,mm)^2}{4} = 0{,}0962\,mm^2$

$J_1 = \frac{I}{A_1} = \frac{4{,}35\,A}{0{,}159\,mm^2} = 27{,}4\,\frac{A}{mm^2}$; $J_2 = \frac{I}{A_2} = \frac{4{,}35\,A}{0{,}0962\,mm^2} = 45{,}2\,\frac{A}{mm^2}$

3.7 Elektrischer Widerstand

3.7.1 Leiterwiderstand

Lösungen zu 3.7.1

44/1. $R = \frac{\varrho \cdot l}{A} = \frac{0{,}49\,(\Omega \cdot mm^2)/m \cdot 150\,m}{0{,}196\,mm^2} = 375\,\Omega$

44/2. $R = \frac{l}{\gamma \cdot A}$; $l = R \cdot \gamma \cdot A = 0{,}6\,\Omega \cdot 56\,m/(\Omega \cdot mm^2) \cdot 1{,}5\,mm^2 = 50{,}4\,m$

44/3. a) $A = \frac{\varrho \cdot l}{R} = \frac{0{,}01786\,(\Omega \cdot mm^2)/m \cdot 100\,m}{0{,}3\,\Omega} = 5{,}95\,mm^2$

b) $A = \frac{\varrho \cdot l}{R} = \frac{0{,}0286\,(\Omega \cdot mm^2)/m \cdot 100\,m}{0{,}3\,\Omega} = 9{,}53\,mm^2$

44/4. a) $R = \frac{l}{\gamma \cdot A} = \frac{10\,m}{56\,\frac{m}{\Omega \cdot mm^2} \cdot 2{,}5\,mm^2} = 0{,}071\,\Omega \Rightarrow R = 71{,}43\,m\Omega$

$U = R \cdot I = 71{,}43\,m\Omega \cdot 8\,A = 0{,}57\,V$

44/5. a) $N = \frac{l}{s} = \frac{230\,mm}{0{,}5\,mm} = 460$ b) $l = \pi \cdot d_m \cdot N = \pi \cdot 0{,}0565\,m \cdot 460 = 81{,}6\,m$

c) $A = \frac{(0{,}5\,mm)^2 \cdot \pi}{4} = 0{,}1963\,mm^2$; $R = \frac{\varrho \cdot l}{A} = \frac{0{,}49\,(\Omega \cdot mm^2)/m \cdot 81{,}6\,m}{0{,}1963\,mm^2} = 204\,\Omega$

44/6. $R = \frac{2 \cdot \varrho \cdot l}{A}$; $l = \frac{R \cdot A}{2 \cdot \varrho} = \frac{0{,}775\,\Omega \cdot 70\,mm^2}{2 \cdot 0{,}0286\,\frac{\Omega \cdot mm^2}{m}} = 948\,m$

44/7. a) $d_m = \frac{10\,mm + 23\,mm}{2} = 16{,}5\,mm$; $U_m = \pi \cdot 16{,}5\,mm = 51{,}8\,mm$;

$l = N \cdot U_m = 19900 \cdot 0{,}0518\,m = 1030\,m$

$A = \frac{l}{\gamma \cdot R} = \frac{1030\,m}{56\,m/(\Omega \cdot mm^2) \cdot 8800\,\Omega} = 2{,}09 \cdot 10^{-3}\,mm^2$

$A = \pi \cdot \frac{d^2}{4} \Rightarrow d = \sqrt{\frac{A \cdot 4}{\pi}} = \sqrt{\frac{2{,}09 \cdot 10^{-3}\,mm^2 \cdot 4}{\pi}} = 0{,}0516\,mm$

b) $d_2 = 0{,}0516\,mm + 0{,}007\,mm = 0{,}0586\,mm$; $N_L = \frac{13\,mm}{0{,}0586\,mm} = 222$;

$n_L = \frac{19900}{222} = 89\,Lagen$ c) $I = \frac{U}{R} = \frac{230\,V}{8800\,\Omega} = 26{,}1\,mA$

3.7.2 Widerstand und Temperatur

Lösungen zu 3.7.2

45/1. a) $\Delta R = \alpha \cdot R_{20} \cdot \Delta\vartheta = 0{,}00015\,\frac{1}{K} \cdot 104\,\Omega \cdot 35\,K = 0{,}546\,\Omega$

b) $R_\vartheta = R_{20} + \Delta R = 104\,\Omega + 0{,}5\,\Omega = 104{,}5\,\Omega$

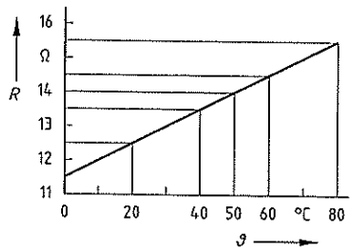
45/2. a) $\Delta R = \alpha \cdot R_{20} \cdot \Delta\vartheta = 0{,}00004\,\frac{1}{K} \cdot 4200\,\Omega \cdot 10\,K = 1{,}68\,\Omega$

b) $R_\vartheta = R_{20} + \Delta R = 4200\,\Omega + 1{,}68\,\Omega = 4201{,}68\,\Omega \approx 4202\,\Omega$

45/3. a) $\Delta R = \alpha \cdot R_{20} \cdot \Delta\vartheta$

$= 0{,}0039\,\frac{1}{K} \cdot 12{,}5\,\Omega \cdot 60\,K = \mathbf{2{,}93\,\Omega}$

b)



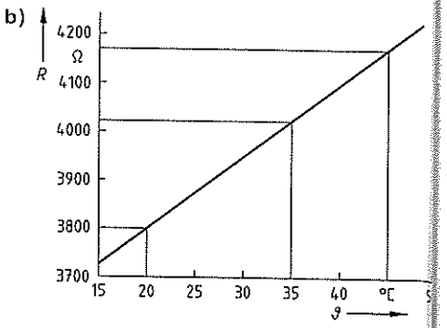
(Abbildung verkleinert)

c) Abgelesen aus Zeichnung
für 40 °C → **13,5 Ω**
für 50 °C → **14,0 Ω**
für 60 °C → **14,5 Ω**

45/4. a) $\Delta R = \alpha \cdot R_{20} \cdot \Delta\vartheta$

$= 0{,}0039\,\frac{1}{K} \cdot 3{,}8\,k\Omega \cdot 7\,K = \mathbf{0{,}104\,k\Omega}$

b)



(Abbildung verkleinert)

c) Abgelesen aus Zeichnung
für 35 °C → **4,02 kΩ**
für 45 °C → **4,17 kΩ**

45/5. a) $\Delta\vartheta = \frac{R_\vartheta - R_{20}}{\alpha \cdot R_{20}} = \frac{9\,\Omega - 8{,}5\,\Omega}{0{,}0055\,^1/K \cdot 8{,}5\,\Omega} = \mathbf{10{,}7\,K}$;

b) $\vartheta = 293{,}15\,K + 10{,}7\,K = 303{,}85\,K$

$303{,}85\,K \triangleq \mathbf{30{,}7\,°C}$

45/6. $\Delta\vartheta = \frac{R_\vartheta - R_{20}}{\alpha \cdot R_{20}} = \frac{0{,}185\,\Omega - 0{,}170\,\Omega}{0{,}004\,^1/K \cdot 0{,}17\,\Omega} = 22{,}1\,K$;

$\vartheta = 293{,}15\,K + 22{,}1\,K = 315{,}25\,K$

$315{,}25\,K \triangleq \mathbf{42{,}1\,°C}$

45/7. $R_{20} = \frac{\Delta R}{\alpha \cdot \Delta\vartheta} = \frac{-3{,}3\,\Omega}{-0{,}0013\,^1/K \cdot 80\,K} = \mathbf{31{,}7\,\Omega}$

45/8. $\alpha = \frac{R_\vartheta - R_{20}}{\Delta\vartheta \cdot R_{20}} = \frac{150\,\Omega - 110\,\Omega}{130\,K \cdot 110\,\Omega}$; $\quad \alpha = \mathbf{0{,}0028\,\frac{1}{K}}$

46/9. $R_{20} = \frac{\Delta R}{\alpha \cdot \Delta\vartheta} = \frac{5{,}5\,\Omega}{0{,}0039\,^1/K \cdot 47\,K} = \mathbf{30{,}0\,\Omega}$

46/10. $l = \pi \cdot d_m \cdot N = \pi \cdot 20{,}5\,mm \cdot 400 = 25760\,mm = 25{,}8\,m$

$R_{20} = \frac{\varrho \cdot l}{A} = \frac{0{,}49\,\Omega \cdot mm^2/m \cdot 25{,}8\,m}{0{,}1963\,mm^2} = \mathbf{64{,}4\,\Omega}$;

$R_\vartheta = R_{20} \cdot (1 + \alpha \cdot \Delta\vartheta) = 64{,}4\,\Omega \cdot (1 + 0{,}00004\,\frac{1}{K} \cdot 80\,K) = \mathbf{64{,}6\,\Omega}$

46/11. a) $l = \pi \cdot d_m \cdot N = \pi \cdot 0{,}06\,m \cdot 500 = 94{,}2\,m$; $\quad A = \left(\pi \cdot \frac{d^2}{4}\right) = 0{,}3848\,mm^2$

$R_{20} = \frac{l}{\gamma \cdot A} = \frac{94{,}2\,m}{56\,\Omega \cdot mm^2/m \cdot 0{,}385\,mm^2} = \mathbf{4{,}37\,\Omega}$

b) $R_\vartheta = R_{20} \cdot (1 + \alpha \cdot \Delta\vartheta) = 4{,}37\,\Omega \cdot (1 + 0{,}0039\,\frac{1}{K} \cdot 38\,K) = \mathbf{5{,}02\,\Omega}$

c) $I_{20} = \frac{U}{R_{20}} = \frac{22\,V}{4{,}37\,\Omega} = \mathbf{5{,}03\,A}$ $\qquad$ d) $I_\vartheta = \frac{U}{R_\vartheta} = \frac{22\,V}{5{,}02\,\Omega} = \mathbf{4{,}38\,A}$

46/12. a) α = relative Widerstandszunahme je $K = \frac{0{,}4}{61\,K} = \mathbf{0{,}00656\,\frac{1}{K}}$; $\qquad$ b) z. B. um reines Eisen

3.7.3 Übertemperatur

Lösungen zu 3.7.3

46/1. a) $\Delta\vartheta = \frac{R_2 - R_1}{R_1} \cdot (235\,K + \vartheta_1) + \vartheta_1 - \vartheta_a =$

$= \frac{1{,}98\,\Omega - 1{,}76\,\Omega}{1{,}76\,\Omega} \cdot (235\,K + 17\,°C) + 17\,°C - 19\,°C = \frac{0{,}22\,\Omega}{1{,}76\,\Omega} \cdot 252\,K - 2\,K = \mathbf{29{,}5\,K}$

b) $\Delta\vartheta = \frac{1{,}21\,\Omega - 0{,}98\,\Omega}{0{,}98\,\Omega} \cdot (235\,K + 18\,°C) + 18\,°C - 24\,°C = \frac{0{,}23\,\Omega}{0{,}89\,\Omega} \cdot 253\,K - 6\,K = \mathbf{53{,}4\,K}$

c) $\Delta\vartheta = \frac{1{,}52\,\Omega - 1{,}30\,\Omega}{1{,}30\,\Omega} \cdot (235\,K + 22\,°C) + 22\,°C - 23\,°C = \frac{0{,}22\,\Omega}{1{,}30\,\Omega} \cdot 257\,K - 1\,K = \mathbf{42{,}5\,K}$

d) $\Delta\vartheta = \frac{5{,}2\,\Omega - 4{,}7\,\Omega}{4{,}7\,\Omega} \cdot (235\,K + 23\,°C) + 23\,°C - 28\,°C = \frac{0{,}5\,\Omega}{4{,}7\,\Omega} \cdot 258\,K - 5\,K = \mathbf{22{,}4\,K}$

46/2. a) $\Delta\vartheta = \frac{0{,}82\,\Omega - 0{,}65\,\Omega}{0{,}65\,\Omega} \cdot (235\,K + 20\,°C) + 20\,°C - 23\,°C = \frac{0{,}17\,\Omega}{0{,}65\,\Omega} \cdot 255\,K - 3\,K = \mathbf{63{,}7\,K}$

b) Die Übertemperatur von 63,7 K ist **zulässig.**

46/3. $\Delta\vartheta = \frac{2{,}15\,\Omega - 1{,}63\,\Omega}{1{,}63\,\Omega} \cdot (225\,K + 15\,°C) + 15\,°C - 17\,°C = \frac{0{,}52\,\Omega}{1{,}63\,\Omega} \cdot 240\,K - 2\,K = \mathbf{74{,}6\,K}$

3.8 Schaltung von Widerständen

3.8.1 Reihenschaltung von Widerständen

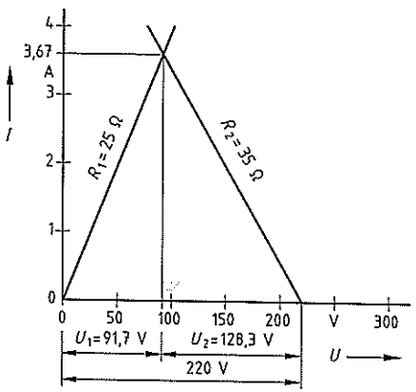
Lösungen zu 3.8.1

47/1. a) $R = R_1 + R_2 = 25\,\Omega + 35\,\Omega = \mathbf{60\,\Omega}$

b) $I = \frac{U}{R} = \frac{220\,V}{60\,\Omega} = \mathbf{3{,}67\,A}$

c) $U_1 = I \cdot R_1 = 3{,}67\,A \cdot 25\,\Omega = \mathbf{91{,}7\,V}$

$U_2 = U - U_1 = 220\,V - 91{,}7\,V = \mathbf{128{,}3\,V}$



Graphische Lösung zu 47/1.

47/2. a) $R = R_1 + R_2 + R_3 =$

$= 1{,}5\,k\Omega + 2{,}2\,k\Omega + 6{,}8\,k\Omega = \mathbf{10{,}5\,k\Omega}$

b) $I = \frac{U}{R} = \frac{9{,}0\,V}{10{,}5\,k\Omega} = \mathbf{0{,}857\,mA}$

c) $U_1 = I \cdot R_1 = 0{,}857\,mA \cdot 1{,}5\,k\Omega = \mathbf{1{,}29\,V}$

$U_2 = I \cdot R_2 = 0{,}857\,mA \cdot 2{,}2\,k\Omega = \mathbf{1{,}89\,V}$

$U_3 = I \cdot R_3 = 0{,}857\,mA \cdot 6{,}8\,k\Omega = \mathbf{5{,}83\,V}$

47/3. a) $I = \frac{U_2}{R_2} = \frac{75\,V}{150\,\Omega} = \mathbf{0{,}5\,A}$

b) $U_1 = I \cdot R_1 = 0{,}5\,A \cdot 50\,\Omega = \mathbf{25\,V}$; $\quad U_3 = I \cdot R_3 = 0{,}5\,A \cdot 200\,\Omega = \mathbf{100\,V}$

c) $U = U_1 + U_2 + U_3 = 25\,V + 75\,V + 100\,V = \mathbf{200\,V}$

d) $R = \frac{U}{I} = \frac{200\,V}{0{,}5\,A} = \mathbf{400\,\Omega}$ $\quad$ oder $\quad R = R_1 + R_2 + R_3 = 50\,\Omega + 150\,\Omega + 200\,\Omega = \mathbf{400\,\Omega}$

47/4. a) $R_2 = R - R_1 = 140\,\Omega - 50\,\Omega = \mathbf{90\,\Omega}$ $\qquad$ b) $U_1 = I \cdot R_1 = 2\,A \cdot 50\,\Omega = \mathbf{100\,V}$;

c) $U = I \cdot R = 2\,A \cdot 140\,\Omega = \mathbf{280\,V}$

48/5. $R_1 = \frac{U_1}{I} = \frac{10\,V}{0{,}1\,A} = 100\,\Omega$

$U_3 = I \cdot R_3 = 0{,}1\,A \cdot 56\,\Omega = 5{,}6\,V$; $U_2 = U - (U_1 + U_3) = 24\,V - 10\,V - 5{,}6\,V = 8{,}4\,V$

$R = \frac{U}{I} = \frac{24\,V}{0{,}1\,A} = 240\,\Omega$; $R_2 = R - R_1 - R_3 = 240\,\Omega - 100\,\Omega - 56\,\Omega = 84\,\Omega$

48/6. a) $\Sigma U_0 = 4 \cdot U_{01} = 4 \cdot 1{,}56\,V = 6{,}24\,V$; $I = \frac{\Sigma U_0}{R} = \frac{6{,}24\,V}{37\,\Omega} = 0{,}169\,A$

b) $\Sigma U_0 = 2 \cdot U_{01} = 2 \cdot 1{,}56\,V = 3{,}12\,V$; $I = \frac{\Sigma U_0}{R} = \frac{3{,}12\,V}{37\,\Omega} = 0{,}0843\,A = 84{,}3\,mA$

48/7. a) $\Sigma U_0 = U_{01} + U_{02} = 12\,V + 6\,V = 18\,V$

$R = R_1 + R_2 + R_3 = 270\,\Omega + 120\,\Omega + 180\,\Omega = 570\,\Omega$; $I = \frac{\Sigma U_0}{R} = \frac{18\,V}{570\,\Omega} = 0{,}0316\,A = 31{,}6\,mA$

b) $\Sigma U_0 = U_{01} + U_{02} = 12\,V - 6\,V = 6\,V$; $I = \frac{\Sigma U_0}{R} = \frac{6\,V}{570\,\Omega} = 0{,}0105\,A = 10{,}5\,mA$

48/8. $U_v = U - U_{Lampe} = 24\,V - 6\,V = 18\,V$; $R_v = \frac{U_v}{I} = \frac{18\,V}{0{,}35\,A} = 51{,}4\,\Omega$

48/9. a) $U_{Lampe} = \frac{U_n}{n} = \frac{48\,V}{6} = 8\,V$

b) Spannung je Lampe bei 8 Lampen: $U_{Lampe} = \frac{48\,V}{8} = 6\,V \Rightarrow 6\,V \triangleq 100\%$

Spannungserhöhung $\Delta U = 8\,V - 6\,V = 2\,V \Rightarrow 2\,V \triangleq \frac{2\,V \cdot 100\%}{6\,V} = 33{,}3\%$

48/10. Index I ≙ große Leistung; Index II ≙ kleine Leistung

$I_I = \frac{U}{R_{Lötkolben}} = \frac{230\,V}{1610\,\Omega} = 0{,}143\,A$; $\frac{U_I}{U_{II}} = \frac{I_I}{I_{II}} \Rightarrow I_{II} = \frac{U_{II} \cdot I_I}{U_I} = \frac{126\,V \cdot 0{,}143\,A}{230\,V} = 0{,}0783\,A$

$U_v = U_I - U_{II} = 230\,V - 126\,V = 104\,V$; $R_v = \frac{U_v}{I_{II}} = \frac{104\,V}{0{,}0783\,A} = 1330\,\Omega = 1{,}3\,k\Omega$

48/11. a) Schalter S1, S2 und S3 geschlossen: $I = 100\,mA$; $U = U_4 = I \cdot R_4 = 0{,}1\,A \cdot 100\,\Omega = 10\,V$

Schalter S1 und S2 geschlossen: $I = \frac{100\,mA \cdot 100\%}{120\%} = 83{,}3\,mA$

Schalter S1 geschlossen: $I = \frac{83{,}3\,mA \cdot 100\%}{120\%} = 69{,}4\,mA$

Alle Schalter offen: $I = \frac{69{,}4\,mA \cdot 100\%}{120\%} = 57{,}8\,mA$

$R = \frac{U}{I} = \frac{10\,V}{57{,}8\,mA} = 173\,\Omega$; $R_1 + R_2 + R_3 = R - R_4 = 173\,\Omega - 100\,\Omega = 73\,\Omega$

S1 geschlossen: $R = \frac{U}{I} = \frac{10\,V}{69{,}4\,mA} = 144\,\Omega$; $R_1 = 173\,\Omega - 144\,\Omega = 29\,\Omega$

S1 und S2 geschlossen: $R = \frac{U}{I} = \frac{10\,V}{83{,}3\,mA} = 120\,\Omega$; $R_2 = 144\,\Omega - 120\,\Omega = 24\,\Omega$; $R_3 = 120\,\Omega - 100\,\Omega = 20\,\Omega$

b) S1 und S2 geschlossen: $R = R_4 + R_3 = 100\,\Omega + 20\,\Omega = 120\,\Omega$; $I = 83{,}3\,mA$

S1 und S3 geschlossen: $R = R_4 + R_2 = 100\,\Omega + 24\,\Omega = 124\,\Omega$; $I = 80{,}6\,mA$

S2 und S3 geschlossen: $R = R_4 + R_1 = 100\,\Omega + 29\,\Omega = 129\,\Omega$; $I = 77{,}5\,mA$

48/12. $R = R_1 + R_2 + R_3 + R_4 =$
$= 1{,}2\,k\Omega + 2{,}2\,k\Omega + 3{,}9\,k\Omega + 4{,}7\,k\Omega = 12\,k\Omega$

$I = \frac{U}{R} = \frac{24\,V}{12\,k\Omega} = 2\,mA$

$U_1 = R_1 \cdot I = 1{,}2\,k\Omega \cdot 2\,mA = 2{,}4\,V$

$U_2 = R_2 \cdot I = 2{,}2\,k\Omega \cdot 2\,mA = 4{,}4\,V$

$U_3 = R_3 \cdot I = 3{,}9\,k\Omega \cdot 2\,mA = 7{,}8\,V$

$U_4 = R_4 \cdot I = 4{,}7\,k\Omega \cdot 2\,mA = 9{,}4\,V$

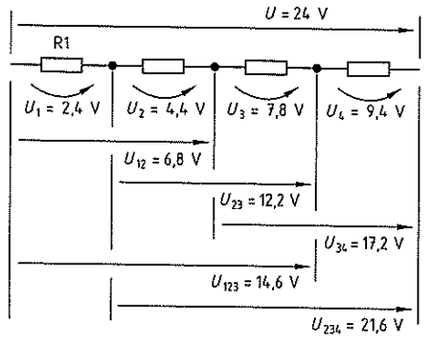
$U_{12} = U_1 + U_2 = 2{,}4\,V + 4{,}4\,V = 6{,}8\,V$

$U_{23} = U_2 + U_3 = 4{,}4\,V + 7{,}8\,V = 12{,}2\,V$

$U_{34} = U_3 + U_4 = 7{,}8\,V + 9{,}4\,V = 17{,}2\,V$

$U_{123} = U_1 + U_2 + U_3 = 2{,}4\,V + 4{,}4\,V + 7{,}8\,V = 14{,}6\,V$

$U_{234} = U_2 + U_3 + U_4 = 4{,}4\,V + 7{,}8\,V + 9{,}4\,V = 21{,}6\,V$



zu 48/12.

48/13. a) $I_m = \frac{U_m}{R_m} = \frac{100\,mV}{100\,\Omega} = 1\,mA$; $R_v = \frac{U - U_m}{I_m} = \frac{150\,mV - 100\,mV}{1\,mA} = 50\,\Omega$

b) $R_v = \frac{U - U_m}{I_m} = \frac{3\,V - 0{,}1\,V}{0{,}001\,A} = 2900\,\Omega$

c) $R_v = \frac{U - U_m}{I_m} = \frac{600\,V - 0{,}1\,V}{0{,}001\,A} = 599900\,\Omega = 599{,}9\,k\Omega$

48/14. a) $R_{Spule} = \frac{U}{I} = \frac{48\,V}{0{,}029\,A} = 1655\,\Omega$; $R = \frac{U}{I_{Betr}} = \frac{48\,V}{0{,}022\,A} = 2182\,\Omega$

$R_v = R - R_{Spule} = 2182\,\Omega - 1655\,\Omega = 527\,\Omega$

b) $U = R_{Spule} \cdot I_{Betr} = 1655\,\Omega \cdot 0{,}022\,A = 36{,}4\,V$

48/15. Stufe 1: $R = \frac{U - U_F}{I} = \frac{6\,V - 2\,V}{0{,}08\,A} = 50\,\Omega$; Stufe 2: $R = \frac{U - U_F}{I} = \frac{6\,V - 2{,}2\,V}{0{,}18\,A} = 21{,}1\,\Omega$

Stufe 3: $R = \frac{U - U_F}{I} = \frac{6\,V - 2{,}4\,V}{0{,}3\,A} = 12\,\Omega$; Stufe 4: $R = \frac{U - U_F}{I} = \frac{6\,V - 2{,}6\,V}{0{,}44\,A} = 7{,}73\,\Omega$

3.8.2 Parallelschaltung von Widerständen

Lösungen zu 3.8.2

49/1. $R = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{24\,\Omega \cdot 36\,\Omega}{24\,\Omega + 36\,\Omega} = 14{,}4\,\Omega$

49/2. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{10\,k\Omega} + \frac{1}{20\,k\Omega} + \frac{1}{30\,k\Omega} + \frac{1}{40\,k\Omega} = 0{,}208\,\frac{1}{k\Omega}$; $R = 4{,}8\,k\Omega$

49/3. $R = \frac{R_1 \cdot R_2}{R_1 + R_2} \Rightarrow R_2 = \frac{R_1 \cdot R}{R_1 - R} = \frac{44\,\Omega \cdot 33\,\Omega}{44\,\Omega - 33\,\Omega} = \frac{1452\,\Omega}{11\,\Omega} = 132\,\Omega$

49/4. $\frac{1}{R_4} = \frac{1}{R} - \frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3} = \frac{1}{4\,\Omega} - \frac{1}{27\,\Omega} - \frac{1}{33\,\Omega} - \frac{1}{47\,\Omega} = 0{,}1614\,\frac{1}{\Omega}$; $R_4 = 6{,}2\,\Omega$

50/5. $\frac{1}{R_3} = \frac{1}{R} - \frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_4} = \frac{1}{1\,\Omega} - \frac{1}{5{,}6\,\Omega} - \frac{1}{15\,\Omega} - \frac{1}{39\,\Omega} = 0{,}729\,\frac{1}{\Omega}$; $R_3 = 1{,}37\,\Omega$

50/6. a) $\frac{I_1}{I_4} = \frac{R_4}{R_1} \Rightarrow I_1 = \frac{I_4 \cdot R_4}{R_1} = \frac{2\,\text{mA} \cdot 2{,}7\,\text{k}\Omega}{6{,}8\,\text{k}\Omega} = 0{,}794\,\text{mA}$

$I_2 = \frac{I_4 \cdot R_4}{R_2} = \frac{2\,\text{mA} \cdot 2{,}7\,\text{k}\Omega}{2{,}2\,\text{k}\Omega} = 2{,}45\,\text{mA}$; $I_3 = \frac{I_4 \cdot R_4}{R_3} = \frac{2\,\text{mA} \cdot 2{,}7\,\text{k}\Omega}{5{,}6\,\text{k}\Omega} = 0{,}964\,\text{mA}$

b) $I = I_1 + I_2 + I_3 + I_4 = 0{,}794\,\text{mA} + 2{,}45\,\text{mA} + 0{,}964\,\text{mA} + 2\,\text{mA} = 6{,}21\,\text{mA}$

c) $U = I_4 \cdot R_4 = 2\,\text{mA} \cdot 2{,}7\,\text{k}\Omega = 5{,}4\,\text{V}$ d) $R = \frac{U}{I} = \frac{5{,}4\,\text{V}}{6{,}21\,\text{mA}} = 870\,\Omega$

50/7. a) $I_1 = \frac{U_1}{R_1} = \frac{220\,\text{V}}{120\,\Omega} = 1{,}83\,\text{A}$; $I_2 = I - I_1 = 5\,\text{A} - 1{,}83\,\text{A} = 3{,}17\,\text{A}$

b) $R_2 = \frac{U}{I_2} = \frac{220\,\text{V}}{3{,}17\,\text{A}} = 69{,}4\,\Omega$ c) $R = \frac{U}{I} = \frac{220\,\text{V}}{5\,\text{A}} = 44\,\Omega$

50/8. $U = I_2 \cdot R_2 = 2\,\text{A} \cdot 7\,\Omega = 14\,\text{V}$; $R_1 = \frac{U}{I_1} = \frac{14\,\text{V}}{5\,\text{A}} = 2{,}8\,\Omega$

$I = I_1 + I_2 = 5\,\text{A} + 2\,\text{A} = 7\,\text{A}$; $R = \frac{U}{I} = \frac{14\,\text{V}}{7\,\text{A}} = 2\,\Omega$

50/9. $I_2 = \frac{U}{R_2} = \frac{100\,\text{V}}{150\,\Omega} = 0{,}67\,\text{A}$; $I_3 = \frac{U}{R_3} = \frac{100\,\text{V}}{200\,\Omega} = 0{,}50\,\text{A}$

$I_1 = I - I_2 - I_3 = 3{,}50\,\text{A} - 0{,}67\,\text{A} - 0{,}50\,\text{A} = 2{,}33\,\text{A}$

50/10. $U = I_1 \cdot R_1 = 6\,\text{A} \cdot 30\,\Omega = 180\,\text{V}$; $I_2 = \frac{U}{R_2} = \frac{180\,\text{V}}{60\,\Omega} = 3\,\text{A}$; $R_3 = \frac{U}{I_3} = \frac{180\,\text{V}}{4\,\text{A}} = 45\,\Omega$;

$I = I_1 + I_2 + I_3 = 6\,\text{A} + 3\,\text{A} + 4\,\text{A} = 13\,\text{A}$; $R = \frac{U}{I} = \frac{180\,\text{V}}{13\,\text{A}} = 13{,}8\,\Omega$

50/11. a) $I_1 = \frac{U_1}{R_1} = \frac{230\,\text{V}}{882\,\Omega} = 0{,}261\,\text{A}$ b) $I = 6 \cdot I_1 = 6 \cdot 0{,}261\ \text{A} = 1{,}56\,\text{A}$

c) $R = \frac{U}{I} = \frac{230\,\text{V}}{1{,}56\,\text{A}} = 147\,\Omega$

50/12. a) $U = R_m \cdot I_m = 2{,}4\,\Omega \cdot 25\,\text{mA} = 0{,}06\,\text{V}$

$I_p = I - I_m = 50\,\text{mA} - 25\,\text{mA} = 25\,\text{mA}$

$R_p = \frac{U}{I_p} = \frac{0{,}06\,\text{V}}{25\,\text{mA}} = 2{,}4\,\Omega$

b) $I_p = I - I_m = 1{,}5\,\text{A} - 25\,\text{mA} = 1{,}475\,\text{A}$

$R_p = \frac{U}{I_p} = \frac{0{,}06\,\text{V}}{1{,}475\,\text{A}} = 40{,}7\,\text{m}\Omega$

50/13. a) $I_1 = \frac{I}{n} = \frac{13{,}0\,\text{A}}{4} = 3{,}25\,\text{A}$; $R_1 = \frac{U_1}{I_1} = \frac{230\,\text{V}}{3{,}25\,\text{A}} = 70{,}8\,\Omega$

b) Stufe 1: 2 Widerstände parallel $R = \frac{R_1}{n} = \frac{70{,}8\,\Omega}{2} = 35{,}4\,\Omega$

Stufe 2: 3 Widerstände parallel $R = \frac{R_1}{n} = \frac{70{,}8\,\Omega}{3} = 23{,}6\,\Omega$

Stufe 3: 4 Widerstände parallel $R = \frac{R_1}{n} = \frac{70{,}8\,\Omega}{4} = 17{,}7\,\Omega$

c) Stufe 1: $I = I_1 \cdot n = 3{,}25\,\text{A} \cdot 2 = 6{,}50\,\text{A}$; Stufe 2: $I = I_1 \cdot n = 3{,}25\,\text{A} \cdot 3 = 9{,}75\,\text{A}$

Stufe 3: $I = I_1 \cdot n = 3{,}25\,\text{A} \cdot 4 = 13{,}0\,\text{A}$

50/14. a) $R_{\text{Girlande}} = \frac{R_1}{n_1} = \frac{2116\,\Omega}{24} = 88{,}2\,\Omega$ b) $R = \frac{R_{\text{Girlande}}}{n_2} = \frac{88{,}2\,\Omega}{4} = 22{,}05\,\Omega$

c) $I = \frac{U}{R} = \frac{230\,\text{V}}{22{,}05\,\Omega} = 10{,}43\,\text{A}$

50/15. a) Widerstandswert von 6 parallelen Widerständen $= \frac{R_1}{6}$

Widerstandswert von 4 parallelen Widerständen $= \frac{R_1}{4}$

$\frac{R_1}{4} - \frac{R_1}{6} = 5\,\Omega$; $\frac{3R_1 - 2R_1}{12} = \frac{R_1}{12} = 5\,\Omega$; $R_1 = 5\,\Omega \cdot 12 = 60\,\Omega$

b) $n = 6$; $R = \frac{R_1}{6} = \frac{60\,\Omega}{6} = 10\,\Omega$ c) $n = 4$; $R = \frac{R_1}{4} = \frac{60\,\Omega}{4} = 15\,\Omega$

50/16. $I_3 = I - I_1 - I_2 - I_4 = 15{,}0\,\text{mA} - 5{,}25\,\text{mA} - 3{,}5\,\text{mA} - 4{,}2\,\text{mA} = 2{,}05\,\text{mA}$

$U_3 = U = I_3 \cdot R_3 = 2{,}05\,\text{mA} \cdot 6{,}8\,\text{k}\Omega = 13{,}9\,\text{V}$

$R_1 = \frac{U_1}{I_1} = \frac{13{,}9\,\text{V}}{5{,}25\,\text{mA}} = 2{,}65\,\text{k}\Omega$; $R_2 = \frac{U_2}{I_2} = \frac{13{,}9\,\text{V}}{3{,}5\,\text{mA}} = 3{,}97\,\text{k}\Omega$

$R_4 = \frac{U_4}{I_4} = \frac{13{,}9\,\text{V}}{4{,}20\,\text{mA}} = 3{,}31\,\text{k}\Omega$; $R_{1-4} = \frac{U}{I} = \frac{13{,}9\,\text{V}}{15{,}0\,\text{mA}} = 927\,\Omega$

$R_{1-5} = R_{1-4} - 40\,\Omega = 927\,\Omega - 40\,\Omega = 887\,\Omega$; $I_{1-5} = \frac{U}{R_{1-5}} = \frac{13{,}9\,\text{V}}{887\,\Omega} = 15{,}67\,\text{mA}$

$I_5 = I_{1-5} - I_{1-4} = 15{,}67\,\text{mA} - 15{,}0\,\text{mA} = 0{,}67\,\text{mA}$; $R_5 = \frac{U}{I_5} = \frac{13{,}9\,\text{V}}{0{,}67\,\text{mA}} = 20{,}7\,\text{k}\Omega$

50/17. $R_1 = R_1$; $R_2 = 2R_1$; $R_3 = 3R_1$; $R_4 = 4R_1$

$R = \frac{1}{\frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{3R_1} + \frac{1}{4R_1}} = \frac{1}{\frac{25}{12R_1}} = \frac{12}{25}R_1$; $I = \frac{U}{R} = \frac{9\,\text{V}}{\frac{12}{25}R_1} = \frac{75\,\text{V}}{4R_1}$; $I' = I + 18\,\text{mA}$

$R' = R - 4{,}2\,\Omega \Rightarrow \frac{U}{I'} = \frac{12}{25}R_1 - 4{,}2\,\Omega$

$\frac{9\,\text{V}}{\frac{75\,\text{V}}{4R_1} + 18\,\text{mA}} = \frac{12}{25}R_1 - 4{,}2\,\Omega$

$9\,\text{V} = \left(\frac{75\,\text{V}}{4R_1} + 18\,\text{mA}\right) \cdot \left(\frac{12}{25}R_1 - 4{,}2\,\Omega\right)$

$9\,\text{V} = 9\,\text{V} - \frac{78{,}75\,\text{V}\Omega}{R_1} + 8{,}64\,\text{mA} \cdot R_1 - 75{,}6\,\text{mA} \cdot \Omega \quad | \cdot R_1$

$8{,}64\,\text{mA} \cdot R_1^2 - 75{,}6\,\text{mV} \cdot R_1 - 78{,}75\,\text{V} \cdot \Omega = 0$

$R_{1(1/2)} = \frac{75{,}6\,\text{mV} \pm \sqrt{(75{,}6\,\text{mV})^2 + 4 \cdot 8{,}64\,\text{mA} \cdot 78{,}75\,\text{V}\Omega}}{17{,}28\,\text{mA}}$

$R_1 = \frac{75{,}6\,\text{mV} + \sqrt{(-75{,}6\,\text{mV})^2 + 2{,}7216\,\text{V}^2}}{17{,}28\,\text{mA}} = \frac{1{,}727\,\text{V}}{17{,}28\,\text{mA}} = 99{,}95\,\Omega \approx 100\,\Omega$

$R_2 = 2 \cdot 100\,\Omega = 200\,\Omega$; $R_3 = 3 \cdot 100\,\Omega = 300\,\Omega$; $R_4 = 4 \cdot 100\,\Omega = 400\,\Omega$

$R = \frac{12}{25}R_1 = \frac{12}{25} \cdot 100\,\Omega = 48\,\Omega$

$I_1 = \frac{U}{R_1} = \frac{9\,\text{V}}{100\,\Omega} = 90\,\text{mA}$; $I_2 = \frac{U}{R_2} = \frac{9\,\text{V}}{200\,\Omega} = 45\,\text{mA}$

$I_3 = \frac{U}{R_3} = \frac{9\,\text{V}}{300\,\Omega} = 30\,\text{mA}$; $I_4 = \frac{U}{R_4} = \frac{9\,\text{V}}{400\,\Omega} = 22{,}5\,\text{mA}$ $I_5 = 18\,\text{mA}$

$I = \frac{U}{R} = \frac{9\,V}{48\,\Omega} = 187{,}5\,mA$; $I' = I + I_5 = 187{,}5\,mA + 18\,mA = 205{,}5\,mA$

$R_5 = \frac{U}{I_5} = \frac{9\,V}{18\,mA} = 500\,\Omega$; $R' = \frac{U}{I'} = \frac{9\,V}{205{,}5\,mA} = 43{,}8\,\Omega$

3.8.3 Gemischte Schaltungen (Gruppenschaltungen)

Lösungen zu 3.8.3

51/1. a) $R_t = R_1 + R_2 = 60\,\Omega + 40\,\Omega = 100\,\Omega$; $R = \frac{R_t \cdot R_3}{R_t + R_3} = \frac{100\,\Omega \cdot 80\,\Omega}{100\,\Omega + 80\,\Omega} = 44{,}4\,\Omega$

b) $I_1 = \frac{U_2}{R_2} = \frac{60\,V}{40\,\Omega} = 1{,}5\,A$; $\frac{I_3}{I_1} = \frac{R_t}{R_3} \Rightarrow I_3 = I_1 \cdot \frac{R_t}{R_3} = 1{,}5\,A \cdot \frac{100\,\Omega}{80\,\Omega} = 1{,}88\,A$

c) $I = I_1 + I_3 = 1{,}5\,A + 1{,}88\,A = 3{,}38\,A$ d) $U_1 = R_1 \cdot I_1 = 60\,\Omega \cdot 1{,}5\,A = 90\,V$

e) $U = U_1 + U_2 = 90\,V + 60\,V = 150\,V$

51/2. a) $R_t = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100\,\Omega \cdot 25\,\Omega}{100\,\Omega + 25\,\Omega} = 20\,\Omega$; $R = R_t + R_1 = 20\,\Omega + 70\,\Omega = 90\,\Omega$

b) $I_2 = \frac{R_3 \cdot I_3}{R_2} = \frac{25\,\Omega \cdot 2\,A}{100\,\Omega} = 0{,}5\,A$; $I_1 = I_2 + I_3 = 0{,}5\,A + 2\,A = 2{,}5\,A$

c) $U_1 = I_1 \cdot R_1 = 2{,}5\,A \cdot 70\,\Omega = 175\,V$; $U_2 = R_3 \cdot I_3 = 25\,\Omega \cdot 2\,A = 50\,V$

d) $U = U_1 + U_2 = 175\,V + 50\,V = 225\,V$

52/3. a) $U_3 = R_3 \cdot I = 40\,\Omega \cdot 3{,}5\,A = 140\,V$; $U = U_1 + U_2 = 100\,V + 140\,V = 240\,V$

b) $R = \frac{U}{I} = \frac{240\,V}{3{,}5\,A} = 68{,}57\,\Omega$; $R' = R - R_3 = 68{,}57\,\Omega - 40\,\Omega = 28{,}57\,\Omega$

$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_1} = \frac{1}{R'} - \frac{1}{R_2} = \frac{1}{28{,}57\,\Omega} - \frac{1}{50\,\Omega} = 0{,}035\frac{1}{\Omega} - 0{,}02\frac{1}{\Omega} = 0{,}015\frac{1}{\Omega}$

$R_1 = \frac{1}{0{,}015}\,\Omega = 66{,}7\,\Omega$

52/4. b) $R_t = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{40\,\Omega \cdot 120\,\Omega}{40\,\Omega + 120\,\Omega} = 30\,\Omega$

$R = R_t + R_3 = 30\,\Omega + 18\,\Omega = 48\,\Omega$

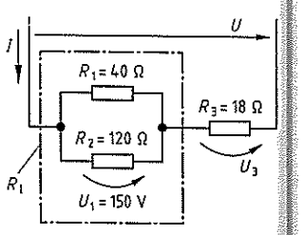
c) $I_1 = \frac{U_1}{R_1} = \frac{150\,V}{40\,\Omega} = 3{,}75\,A$

$I_2 = \frac{U_1}{R_2} = \frac{150\,V}{120\,\Omega} = 1{,}25\,A$

d) $I = I_1 + I_2 = 3{,}75\,A + 1{,}25\,A = 5\,A$

e) $U_3 = I \cdot R_3 = 5\,A \cdot 18\,\Omega = 90\,V$

f) $U = U_1 + U_3 = 150\,V + 90\,V = 240\,V$



zu 52/4.a)

52/5. b) $R_t = R_2 + R_3 = 24\,\Omega + 36\,\Omega = 60\,\Omega$

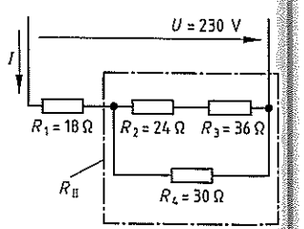
$R_{II} = \frac{R_t \cdot R_4}{R_t + R_4} = \frac{60\,\Omega \cdot 30\,\Omega}{60\,\Omega + 30\,\Omega} = 20\,\Omega$

$R = R_1 + R_{II} = 18\,\Omega + 20\,\Omega = 38\,\Omega$

c) Spannung an R_4: $\frac{U_{II}}{U} = \frac{R_{II}}{R}$

$U_4 = U_{II} = \frac{U \cdot R_{II}}{R} = \frac{230\,V \cdot 20\,\Omega}{38\,\Omega} = 121\,V$

d) Strom durch R_2: $I_2 = \frac{U_{II}}{R_t} = \frac{121\,V}{60\,\Omega} = 2{,}02\,A$



zu 52/5.a)

52/6. a) $U_1 = R_1 \cdot I_1 = 150\,\Omega \cdot 0{,}2\,A = 30\,V$; $U = U_1 + U_2 = 30\,V + 20\,V = 50\,V$

b) $R_2 = \frac{U_2}{I_1} = \frac{20\,V}{0{,}2\,A} = 100\,\Omega$;

$\frac{1}{R} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{150\,\Omega + 100\,\Omega} + \frac{1}{220\,\Omega} + \frac{1}{470\,\Omega} = 0{,}01067\frac{1}{\Omega}$;

$R = \frac{1}{0{,}01067}\,\Omega = 93{,}69\,\Omega$

c) $I = \frac{U}{R} = \frac{50\,V}{93{,}69\,\Omega} = 0{,}5337\,A = 533{,}7\,mA$; $I_{34} = I - I_1 = 533{,}7\,mA - 200\,mA = 333{,}7\,mA$

$I_3 = \frac{U}{R_3} = \frac{50\,V}{220\,\Omega} = 227{,}3\,mA$; $I_4 = \frac{U}{R_4} = \frac{50\,V}{470\,\Omega} = 106{,}4\,mA$

52/7. $U_1 = R_1 \cdot I_1 = 560\,\Omega \cdot 34{,}8\,mA = 19{,}49\,V$; $I_4 = I - I_1 = 50\,mA - 34{,}8\,mA = 15{,}2\,mA$

$R_4 = \frac{U}{I_4} = \frac{24\,V}{0{,}0152\,A} = 1579\,\Omega$; $R = \frac{U}{I} = \frac{24\,V}{0{,}05\,A} = 480\,\Omega$

$U_2 = U - U_1 = 24\,V - 19{,}49\,V = 4{,}51\,V$; $I_3 = \frac{U_2}{R_3} = \frac{4{,}51\,V}{180\,\Omega} = 25{,}1\,mA$

$I_2 = I_1 - I_3 = 34{,}8\,mA - 25{,}1\,mA = 9{,}7\,mA$; $R_2 = \frac{U_2}{I_2} = \frac{4{,}51\,V}{9{,}7\,mA} = 465\,\Omega$

52/8. b) $R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{5{,}6\,k\Omega \cdot 4{,}7\,k\Omega}{5{,}6\,k\Omega + 4{,}7\,k\Omega} = 2{,}555\,k\Omega$

$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4} = \frac{1{,}2\,k\Omega \cdot 1{,}8\,k\Omega}{1{,}2\,k\Omega + 1{,}8\,k\Omega} = 0{,}72\,k\Omega$

$R = R_{12} + R_{34} = 2{,}555\,k\Omega + 0{,}72\,k\Omega = 3{,}275\,k\Omega$

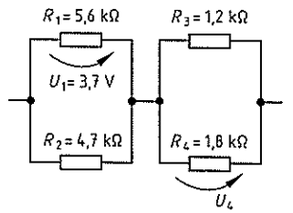
c) $\frac{U_4}{U_1} = \frac{R_{34}}{R_{12}} \Rightarrow U_4 = U_1 \cdot \frac{R_{34}}{R_{12}} = 3{,}7\,V \cdot \frac{0{,}72\,k\Omega}{2{,}555\,k\Omega} = 1{,}043\,V$

d) $I_1 = \frac{U_1}{R_1} = \frac{3{,}7\,V}{5{,}6\,k\Omega} = 0{,}661\,mA$; $I_3 = \frac{U_4}{R_3} = \frac{1{,}043\,V}{1{,}2\,k\Omega} = 0{,}869\,mA$

$I_2 = \frac{U_1}{R_2} = \frac{3{,}7\,V}{4{,}7\,k\Omega} = 0{,}787\,mA$; $I_4 = \frac{U_4}{R_4} = \frac{1{,}043\,V}{1{,}8\,k\Omega} = 0{,}579\,mA$

e) $I = I_1 + I_2 = 0{,}661\,mA + 0{,}787\,mA = 1{,}448\,mA$

f) $U = I \cdot R = 1{,}448\,mA \cdot 3{,}275\,k\Omega = 4{,}74\,V$



zu 52/8.a)

52/9. b) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3 + R_4} = \frac{1}{680\,\Omega} + \frac{1}{390\,\Omega} + \frac{1}{560\,\Omega + 1200\,\Omega}$

$= 4{,}60 \cdot 10^{-3}\frac{1}{\Omega}$; $R = 217{,}3\,\Omega$

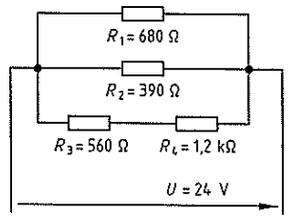
c) $I = \frac{U}{R} = \frac{24\,V}{217{,}3\,\Omega} = 110{,}4\,mA$

d) $I_1 = \frac{U_1}{R_1} = \frac{24\,V}{680\,\Omega} = 35{,}29\,mA$; $I_2 = \frac{U_2}{R_2} = \frac{24\,V}{390\,\Omega} = 61{,}54\,mA$

$I_3 = I_4 = \frac{U}{R_3 + R_4} = \frac{24\,V}{560\,\Omega + 1200\,\Omega} = 13{,}64\,mA$

e) $U_1 = U_2 = U_{34} = 24\,V$

$U_3 = R_3 \cdot I_3 = 560\,\Omega \cdot 13{,}64\,mA = 7{,}64\,V$; $U_4 = R_4 \cdot I_4 = 1{,}2\,k\Omega \cdot 13{,}64\,mA = 16{,}37\,V$



zu 52/9.a)

52/10. R_3, R_4, R_5 und R_6 sind zueinander parallel geschaltet.

a) $$R = R_1 + R_2 + \frac{1}{\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_6}} = 120\,\Omega + 150\,\Omega + \frac{1}{\frac{1}{180\,\Omega}+\frac{1}{270\,\Omega}+\frac{1}{100\,\Omega}+\frac{1}{120\,\Omega}}$$

$$R = 270\,\Omega + \frac{1}{0{,}02759\,\frac{1}{\Omega}} = 270\,\Omega + 36{,}24\,\Omega = \mathbf{306{,}24\,\Omega}$$

b) $$I = \frac{U}{R} = \frac{6\,V}{306{,}24\,\Omega} = 19{,}6\,mA$$

$$U_3 = U - (R_1 + R_2)\cdot I = 6\,V - (120\,\Omega + 150\,\Omega)\cdot 0{,}0196\,A = 6\,V - 5{,}29\,V = 0{,}71\,V$$

$$\mathbf{I_5} = \frac{U_3}{R_5} = \frac{0{,}71\,V}{100\,\Omega} = \mathbf{7{,}1\ mA}$$

c) $$\mathbf{U_2} = R_2 \cdot I = 150\,\Omega \cdot 0{,}0196\,A = \mathbf{2{,}94\,V}$$

52/11. a) $R_I = R_1 + R_2 = 10\,\Omega + 20\,\Omega = 30\,\Omega$; $R_{II} = \frac{R_I \cdot R_3}{R_I + R_3} = \frac{30\,\Omega \cdot 30\,\Omega}{30\,\Omega + 30\,\Omega} = 15\,\Omega$

$R_{III} = R_{II} + R_4 = 15\,\Omega + 40\,\Omega = 55\,\Omega$

$$R = \frac{R_{III}\cdot R_5}{R_{III}+R_5} = \frac{55\,\Omega \cdot 50\,\Omega}{55\,\Omega + 50\,\Omega} = \frac{2750\,\Omega^2}{105\,\Omega} = \mathbf{26{,}19\,\Omega}$$

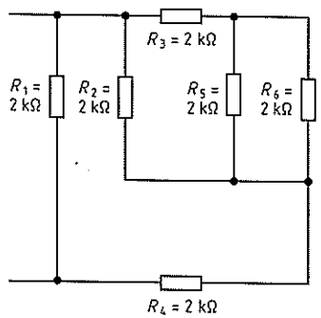
b) $I_2 = \frac{U_2}{R_2} = \frac{3\,V}{20\,\Omega} = 0{,}15\,A$; $U_{12} = U_3 = R_{12}\cdot I_2 = 30\,\Omega \cdot 0{,}15\,A = 4{,}5\,V$

$I_3 = \frac{U_3}{R_3} = \frac{4{,}5\,V}{30\,\Omega} = 0{,}15\,A$; $I_4 = I_2 + I_3 = 0{,}15\,A + 0{,}15\,A = \mathbf{0{,}3\,A}$

c) $U_4 = R_4 \cdot I_4 = 40\,\Omega \cdot 0{,}3\,A = 12\,V$; $U = U_5$

$U_5 = U_3 + U_4 = 4{,}5\,V + 12\,V = 16{,}5\,V$

$$\mathbf{I} = \frac{U_5}{R} = \frac{16{,}5\,V}{26{,}19\,\Omega} = \mathbf{0{,}63\,A}$$



zu 52/12.

52/12. $$R_I = \frac{R_5 \cdot R_6}{R_5 + R_6} = \frac{2\,k\Omega \cdot 2\,k\Omega}{2\,k\Omega + 2\,k\Omega} = 1\,k\Omega$$

$R_{II} = R_I + R_3 = 1\,k\Omega + 2\,k\Omega = 3\,k\Omega$

$$R_{III} = \frac{R_{II}\cdot R_2}{R_{II}+R_2} = \frac{3\,k\Omega \cdot 2\,k\Omega}{3\,k\Omega + 2\,k\Omega} = 1{,}2\,k\Omega$$

$R_{IV} = R_{III} + R_4 = 1{,}2\,k\Omega + 2\,k\Omega = 3{,}2\,k\Omega$

$$R = \frac{R_{IV}\cdot R_1}{R_{IV}+R_1} = \frac{3{,}2\,k\Omega \cdot 2\,k\Omega}{3{,}2\,k\Omega + 2\,k\Omega} = \mathbf{1{,}23\,k\Omega}$$

52/13. a) $$\frac{1}{R_I} = \frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_6} = \frac{1}{80\,\Omega}+\frac{1}{100\,\Omega}+\frac{1}{120\,\Omega} = 0{,}03083\,\frac{1}{\Omega};\quad R_I = 32{,}4\,\Omega$$

$$R_{II} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{200\,\Omega \cdot 50\,\Omega}{200\,\Omega + 50\,\Omega} = 40\,\Omega$$

$R = R_1 + R_I + R_{II} = 60\,\Omega + 32{,}4\,\Omega + 40\,\Omega = 132{,}4\,\Omega$

$\mathbf{U} = I \cdot R = 1{,}5\,A \cdot 132{,}4\,\Omega = \mathbf{198{,}6\,V}$

b) $\mathbf{U}$ an $R_5 = U_I = I \cdot R_I = 1{,}5\,A \cdot 32{,}4\,\Omega = \mathbf{48{,}6\,V}$

c) $$I_4 = \frac{U_I}{R_4} = \frac{48{,}6\,V}{80\,\Omega} = \mathbf{0{,}608\,A}$$

53/14. a) $R_I = R_6 + R_7 = 40\,\Omega + 50\,\Omega = 90\,\Omega$

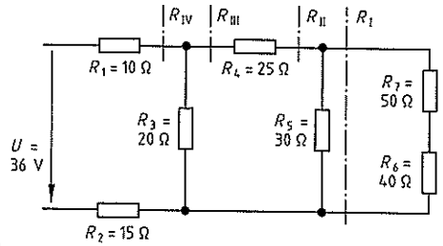
$$R_{II} = \frac{R_I \cdot R_5}{R_I + R_5} = \frac{90\,\Omega \cdot 30\,\Omega}{90\,\Omega + 30\,\Omega} = 22{,}5\,\Omega$$

$R_{III} = R_4 + R_{II} = 25\,\Omega + 22{,}5\,\Omega = 47{,}5\,\Omega$

$$R_{IV} = \frac{R_3 \cdot R_{III}}{R_3 + R_{III}} = \frac{20\,\Omega \cdot 47{,}5\,\Omega}{20\,\Omega + 47{,}5\,\Omega} = 14{,}07\,\Omega$$

$R = R_1 + R_2 + R_{IV} = 10\,\Omega + 15\,\Omega + 14{,}07\,\Omega$

$= 39{,}07\,\Omega = \mathbf{39{,}1\,\Omega}$



zu 53/14.

b) $\frac{U_3}{U} = \frac{R_{IV}}{R}$; $U_3 = U_{IV} = \frac{U \cdot R_{IV}}{R} = \frac{36\,V \cdot 14{,}07\,\Omega}{39{,}1\,\Omega} = 12{,}96\,V$

$U_2 = \frac{U \cdot R_2}{R} = \frac{36\,V \cdot 15\,\Omega}{39{,}07\,\Omega} = 13{,}82\,V$; $U_1 = \frac{U \cdot R_1}{R} = \frac{36\,V \cdot 10\,\Omega}{39{,}07\,\Omega} = 9{,}21\,V$

$\frac{U_4}{U_3} = \frac{R_4}{R_{III}}$; $U_4 = \frac{U_3 \cdot R_4}{R_{III}} = \frac{12{,}96\,V \cdot 25\,\Omega}{47{,}5\,\Omega} = 6{,}82\,V$

Probe: $U_3 + U_2 + U_1 = 12{,}96\,V + 13{,}82\,V + 9{,}21\,V = 35{,}99\,V \approx 36\,V$

$U_5 = U_3 - U_4 = 12{,}96\,V - 6{,}82\,V = \mathbf{6{,}14\,V}$; $U_6 = \frac{U_5 \cdot R_6}{R_I} = \frac{6{,}14\,V \cdot 40\,\Omega}{90\,\Omega} = 2{,}73\,V$

$U_7 = U_5 - U_6 = 6{,}14\,V - 2{,}73\,V = 3{,}41\,V$

c) $I_6 = I_7 = \frac{U_6}{R_6} = \frac{2{,}73\,V}{40\,\Omega} = \mathbf{68{,}3\,mA}$; $I_5 = \frac{U_5}{R_5} = \frac{6{,}14\,V}{30\,\Omega} = \mathbf{204{,}7\,mA}$

$I_4 = I_6 + I_5 = 68{,}3\,mA + 204{,}7\,mA = \mathbf{273\,mA}$; $I_3 = \frac{U_3}{R_3} = \frac{12{,}96\,V}{20\,\Omega} = \mathbf{0{,}648\,A}$

$I = I_4 + I_3 = 0{,}273\,A + 0{,}648\,A = \mathbf{0{,}921\,A}$; Probe: $I = \frac{U}{R} = \frac{36\,V}{39{,}07\,\Omega} = \mathbf{0{,}921\,A}$

53/15. $$\frac{1}{R_{ges}} = \frac{1}{R} + \frac{1}{2R} = \frac{1+2}{2R} = \frac{3}{2R} \Rightarrow R_{ges} = \frac{3}{2}\cdot R = \frac{3}{2}\cdot 10\,\Omega = \mathbf{15\,\Omega}$$

53/16. b) $U_4 = R_4 \cdot I_4 = 120\,\Omega \cdot 0{,}03\,A = 3{,}6\,V$;

$$I_{45} = \frac{U_4}{\frac{R_4 \cdot R_5}{R_4 + R_5}} = \frac{U_4(R_4 + R_5)}{R_4 \cdot R_5} = \frac{3{,}6\,V \cdot (120\,\Omega + 220\,\Omega)}{120\,\Omega \cdot 220\,\Omega}$$

$I_{45} = 0{,}04636\,A = 46{,}36\,mA$

$$\frac{1}{R_{123}} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} = \frac{1}{270\,\Omega}+\frac{1}{390\,\Omega}+\frac{1}{470\,\Omega}$$

$\frac{1}{R_{123}} = 0{,}008395\,\frac{1}{\Omega}$; $R_{123} = 119{,}1\,\Omega$

$U_1 = R_{123} \cdot I_{45} = 119{,}1\,\Omega \cdot 0{,}04636\,A = 5{,}52\,V$

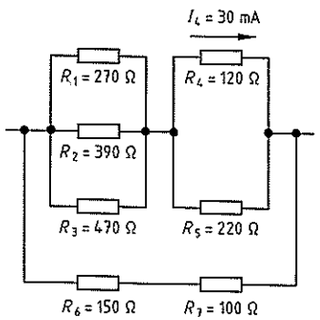
c) $I_2 = \frac{U_2}{R_2} = \frac{5{,}52\,V}{390\,\Omega} = 0{,}014154\,A = \mathbf{14{,}15\,mA}$

d) $R_{67} = R_6 + R_7 = 150\,\Omega + 100\,\Omega = 250\,\Omega$

$I_6 = \frac{U_1 + U_4}{R_{67}} = \frac{5{,}52\,V + 3{,}6\,V}{250\,\Omega} = \frac{9{,}12\,V}{250\,\Omega} = 0{,}03648\,A = 36{,}5\,mA$

$I = I_{45} + I_6 = 46{,}36\,mA + 36{,}48\,mA = \mathbf{82{,}84\,mA}$

e) $\mathbf{U} = U_1 + U_4 = 5{,}52\,V + 3{,}6\,V = \mathbf{9{,}12\,V}$



zu 53/16.a)

53/17. a) A–C: $R_{23} = R_2 + R_3 = 35\,\Omega + 35\,\Omega = 70\,\Omega$

$R_{56} = R_5 + R_6 = 120\,\Omega + 120\,\Omega = 240\,\Omega$

$$R = R_1 + \frac{R_{23} \cdot R_{56}}{R_{23} + R_{56}} + R_7 = 480\,\Omega + \frac{70\,\Omega \cdot 240\,\Omega}{70\,\Omega + 240\,\Omega} + 460\,\Omega$$

$= 480\,\Omega + 54{,}2\,\Omega + 460\,\Omega = 994\,\Omega$

$$I = \frac{U}{R} = \frac{50\,V}{994\,\Omega} = 50{,}3\,mA$$ (I ≥ 50 mA ⇒ gefährlich, siehe **Rechenbuch Elektrotechnik, Seite 191, Bild 2**)

b) B–D: $R_{25} = R_2 + R_5 = 35\,\Omega + 120\,\Omega = 155\,\Omega$; $R_{25} = R_{36}$

$$R = R_4 + \frac{R_{25}}{2} + R_8 + R_{10} = 40\,\Omega + \frac{155\,\Omega}{2} + 20\,\Omega + 850\,\Omega = 987{,}5\,\Omega$$

$$I = \frac{U}{R} = \frac{50\,V}{987{,}5\,\Omega} = 50{,}6\,mA$$ (I ≥ 50 mA ⇒ gefährlich, siehe **Rechenbuch Elektrotechnik, Seite 191, Bild 2**)

c) A–D: $R_{236} = R_2 + R_3 + R_6 = 35\,\Omega + 35\,\Omega + 120\,\Omega = 190\,\Omega$

$$R_I = \frac{R_5 \cdot R_{236}}{R_5 + R_{236}} = \frac{120\,\Omega \cdot 190\,\Omega}{120\,\Omega + 190\,\Omega} = 73{,}55\,\Omega$$

$R = R_1 + R_I + R_8 + R_9 = 480\,\Omega + 73{,}55\,\Omega + 20\,\Omega + 850\,\Omega = 1423{,}55\,\Omega$

$$I = \frac{U}{R} = \frac{50\,V}{1{,}4236\,k\Omega} = 35{,}1\,mA$$ (nicht ungefährlich, siehe **Rechenbuch Elektrotechnik, Seite 191, Bild 2**)

d) A–B: $R_{356} = R_3 + R_5 + R_6 = 35\,\Omega + 120\,\Omega + 120\,\Omega = 275\,\Omega$

$$R_I = \frac{R_2 \cdot R_{356}}{R_2 + R_{356}} = \frac{35\,\Omega \cdot 275\,\Omega}{35\,\Omega + 275\,\Omega} = 31{,}05\,\Omega$$

$R = R_1 + R_I + R_4 = 480\,\Omega + 31{,}05\,\Omega + 40\,\Omega = 551{,}05\,\Omega$

$$I = \frac{U}{R} = \frac{50\,V}{551{,}05\,\Omega} = 90{,}7\,mA$$ (lebensgefährlich, siehe **Rechenbuch Elektrotechnik, Seite 191, Bild 2**)

53/18. a) Taster geöffnet: $U_{rel} = R_{rel} \cdot I = 3\,k\Omega \cdot 8\,mA = 24\,V$

$U_1 = U - U_{rel} = 48\,V - 24\,V = 24\,V$

$$R_1 = \frac{U_1}{I} = \frac{24\,V}{8\,mA} = 3\,k\Omega$$

Taster geschlossen: $U_{rel} = 24\,V - 8\,V = 16\,V$

$$I_1 = \frac{U_{rel}}{R_{rel}} = \frac{16\,V}{3\,k\Omega} = 5{,}33\,mA$$

$$I = \frac{U - U_{rel}}{R_1} = \frac{48\,V - 16\,V}{3\,k\Omega} = \frac{32\,V}{3\,k\Omega} = 10{,}67\,mA$$

$I_2 = I - I_1 = 10{,}67\,mA - 5{,}33\,mA = 5{,}34\,mA$

$$R_2 = \frac{U_{rel}}{I_2} = \frac{16\,V}{5{,}34\,mA} = 3\,k\Omega$$

b) $$I_{Rel} = \frac{U_{Rel}}{R_{Rel}} = \frac{16\,V}{3\,k\Omega} = 5{,}34\,mA$$

53/19. für n = 10: $R_m = R_p \cdot (n-1) \Rightarrow R_m = (R_{p1} + R_{p2} + R_{p3}) \cdot 9$

$$\Rightarrow \frac{R_m}{9} = R_{p1} + R_{p2} + R_{p3} \Rightarrow \frac{R_m}{9} - R_{p1} = R_{p2} + R_{p3} \quad (1)$$

für n = 50: $49(R_{p2} + R_{p3}) = (R_{p1} + R_m) \Rightarrow R_{p2} + R_{p3} = \frac{R_{p1}}{49} + \frac{R_m}{49}$ (2)

für n = 250: $249\,R_{p3} = R_{p1} + R_{p2} + R_m$ (3)

(1) = (2): $$\frac{R_m}{9} - R_{p1} = \frac{R_{p1}}{49} + \frac{R_m}{49} \Rightarrow \frac{R_m}{9} - \frac{R_m}{49} = \frac{50}{49} R_{p1}$$

$$R_m\left(\frac{1}{9} - \frac{1}{49}\right) = \frac{50}{49} R_{p1} \Rightarrow R_{p1} = 10\,\Omega \left(\frac{1}{11{,}025}\right) \cdot \frac{49}{50} = 0{,}89\,\Omega$$

(1) = (3): $$249\,R_{p3} - R_m = \frac{R_m}{9} - R_{p3}$$

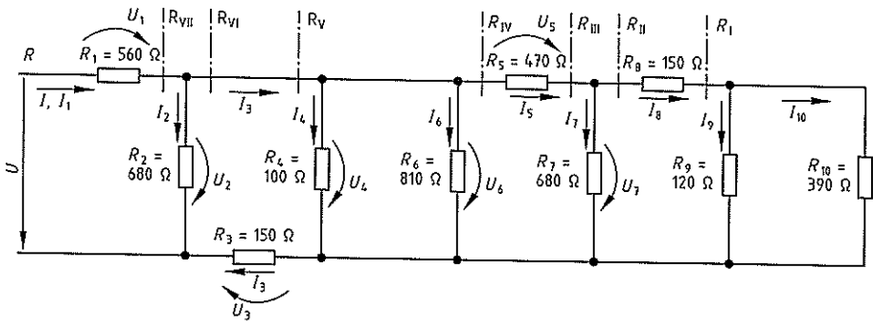
$$\frac{R_m}{9} + R_m = 249\,R_{p3} + R_{p3}$$

$$\frac{10}{9} R_m = 250\,R_{p3}$$

$$R_{p3} = 0{,}044\,\Omega$$

$$R_{p2} = \frac{R_m}{9} - R_{p1} - R_{p3} = \frac{10}{9}\,\Omega - 0{,}89\,\Omega - 0{,}044\,\Omega = 0{,}18\,\Omega$$

53/20.



zu 53/20.

a) $$R_I = \frac{R_9 \cdot R_{10}}{R_9 + R_{10}} = \frac{120\,\Omega \cdot 390\,\Omega}{120\,\Omega + 390\,\Omega} = 91{,}8\,\Omega; \quad R_{II} = R_8 + R_I = 150\,\Omega + 91{,}8\,\Omega = 241{,}8\,\Omega$$

$$R_{III} = \frac{R_{II} \cdot R_7}{R_{II} + R_7} = \frac{241{,}8\,\Omega \cdot 680\,\Omega}{241{,}8\,\Omega + 680\,\Omega} = 178{,}4\,\Omega; \quad R_{IV} = R_5 + R_{III} = 470\,\Omega + 178{,}4\,\Omega = 648{,}4\,\Omega$$

$$R_V = \frac{1}{\frac{1}{R_4} + \frac{1}{R_6} + \frac{1}{R_{IV}}} = \frac{1}{\frac{1}{100\,\Omega} + \frac{1}{810\,\Omega} + \frac{1}{648{,}4\,\Omega}} = \frac{1}{0{,}01277\,\frac{1}{\Omega}} = 78{,}3\,\Omega$$

$R_{VI} = R_V + R_3 = 78{,}3\,\Omega + 150\,\Omega = 228{,}3\,\Omega$

$$R_{VII} = \frac{R_{VI} \cdot R_2}{R_{VI} + R_2} = \frac{228{,}3\,\Omega \cdot 680\,\Omega}{228{,}3\,\Omega + 680\,\Omega} = 170{,}9\,\Omega$$

$R = R_1 + R_{VII} = 170{,}9\,\Omega + 560\,\Omega = 730{,}9\,\Omega$

b) $$I = \frac{U}{R} = \frac{220\,V}{730{,}9\,\Omega} = 0{,}3010\,A = 301{,}0\,mA; \quad U_1 = R_1 \cdot I_1 = 560\,\Omega \cdot 0{,}301\,A = 168{,}6\,V$$

$U_2 = U - U_1 = 220\,V - 168{,}6\,V = 51{,}4\,V$; $$I_2 = \frac{U_2}{R_2} = \frac{51{,}4\,V}{680\,\Omega} = 0{,}0756\,A = 75{,}6\,mA$$

$I_3 = I - I_2 = 301{,}0\,mA - 75{,}6\,mA = 225{,}4\,mA$

$U_3 = R_3 \cdot I_3 = 150\,\Omega \cdot 0{,}2254\,A = 33{,}81\,V$ $U_4 = U_6 = U_2 - U_3 = 51{,}44\,V - 33{,}81\,V = 17{,}63\,V$

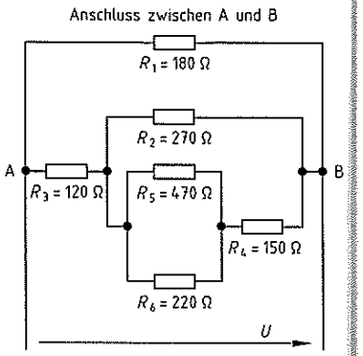
c) $$I_4 = \frac{U_4}{R_4} = \frac{17{,}63\,V}{100\,\Omega} = 0{,}1763\,A = 176{,}3\,mA; \quad I_6 = \frac{U_6}{R_6} = \frac{17{,}63\,V}{810\,\Omega} = 0{,}0218\,A = 21{,}8\,mA$$

$I_5 = I_3 - I_4 - I_6 = 225{,}4\,mA - 176{,}3\,mA - 21{,}8\,mA = 27{,}3\,mA$

$U_5 = R_5 \cdot I_5 = 470\,\Omega \cdot 0{,}0273\,A = 12{,}83\,V$; $U_7 = U_6 - U_5 = 17{,}63\,V - 12{,}83\,V = 4{,}80\,V$

$$I_7 = \frac{U_7}{R_7} = \frac{4{,}80\,V}{680\,\Omega} = 7{,}1\,mA$$

53/21. a) Anschluss zwischen A und B: $U = 9\ V$



zu 53/21.a)

$$R_I = \frac{R_5 \cdot R_6}{R_5 + R_6} = \frac{470\ \Omega \cdot 220\ \Omega}{470\ \Omega + 220\ \Omega} = 149{,}86\ \Omega$$

$$R_{II} = R_I + R_4 = 149{,}86\ \Omega + 150\ \Omega = 299{,}86\ \Omega$$

$$R_{III} = \frac{R_2 \cdot R_{II}}{R_2 + R_{II}} = \frac{270\ \Omega \cdot 299{,}86\ \Omega}{270\ \Omega + 299{,}86\ \Omega} = 142{,}07\ \Omega$$

$$R_{VI} = R_3 + R_{III} = 120\ \Omega + 142{,}07\ \Omega = 262{,}07\ \Omega$$

$$R = \frac{R_1 \cdot R_{IV}}{R_1 + R_{IV}} = \frac{180\ \Omega \cdot 262{,}07\ \Omega}{180\ \Omega + 262{,}07\ \Omega} = \mathbf{106{,}71\ \Omega}$$

$$U_1 = U = 9\ V;\quad I_1 = \frac{U_1}{R_1} = \frac{9\ V}{180\ \Omega} = \mathbf{50\ mA}$$

$$I_3 = \frac{U}{R_{IV}} = \frac{9\ V}{262{,}07\ \Omega} = \mathbf{34{,}34\ mA}$$

$$U_3 = R_3 \cdot I_3 = 120\ \Omega \cdot 34{,}34\ mA = \mathbf{4{,}121\ V};$$

$$U_2 = U - U_3 = 9\ V - 4{,}121\ V = \mathbf{4{,}879\ V}$$

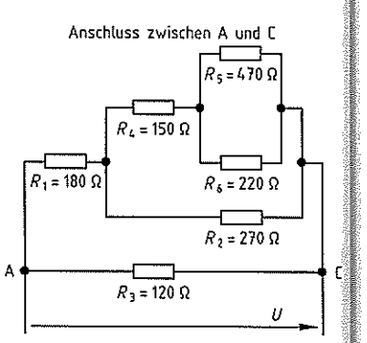
$$I_2 = \frac{U_2}{R_2} = \frac{4{,}879\ V}{270\ \Omega} = \mathbf{18{,}07\ mA};\quad I_4 = I_3 - I_2 = 34{,}34\ mA - 18{,}07\ mA = \mathbf{16{,}27\ mA}$$

$$U_4 = R_4 \cdot I_4 = 150\ \Omega \cdot 16{,}27\ mA = \mathbf{2{,}441\ V}$$

$$U_5 = U_6 = U_2 - U_4 = 4{,}879\ V - 2{,}441\ V = \mathbf{2{,}438\ V}$$

$$I_5 = \frac{U_5}{R_5} = \frac{2{,}438\ V}{470\ \Omega} = \mathbf{5{,}187\ mA};\quad I_6 = \frac{U_6}{R_6} = \frac{2{,}438\ V}{220\ \Omega} = \mathbf{11{,}08\ mA}$$

53/21. b) Anschluss zwischen A und C:



zu 53/21.b)

$$R_I = \frac{R_5 \cdot R_6}{R_5 + R_6} = \frac{470\ \Omega \cdot 220\ \Omega}{470\ \Omega + 220\ \Omega} = 149{,}86\ \Omega$$

$$R_{II} = R_I + R_4 = 149{,}86\ \Omega + 150\ \Omega = 299{,}86\ \Omega$$

$$R_{III} = \frac{R_{II} \cdot R_2}{R_{II} + R_2} = \frac{299{,}86\ \Omega \cdot 270\ \Omega}{299{,}86\ \Omega + 270\ \Omega} = 142{,}07\ \Omega$$

$$R_{IV} = R_1 + R_{III} = 180\ \Omega + 142{,}07\ \Omega = 322{,}07\ \Omega$$

$$R = \frac{R_3 \cdot R_{IV}}{R_3 + R_{IV}} = \frac{120\ \Omega \cdot 322{,}07\ \Omega}{120\ \Omega + 322{,}07\ \Omega} = \mathbf{87{,}43\ \Omega}$$

$$I = \frac{U}{R} = \frac{9\ V}{87{,}43\ \Omega} = \mathbf{102{,}94\ mA}$$

$$I_3 = \frac{U}{R_3} = \frac{9\ V}{120\ \Omega} = \mathbf{75\ mA}$$

$$I_1 = I - I_3 = 102{,}94\ mA - 75\ mA = \mathbf{27{,}94\ mA};\quad U_1 = I_1 \cdot R_1 = 27{,}94\ mA \cdot 180\ \Omega = \mathbf{5{,}03\ V}$$

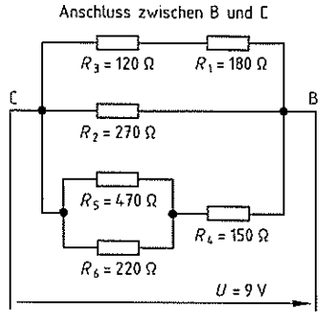
$$U_2 = U - U_1 = 9\ V - 5{,}03\ V = \mathbf{3{,}97\ V};\quad I_2 = \frac{U_2}{R_2} = \frac{3{,}97\ V}{270\ \Omega} = \mathbf{14{,}7\ mA}$$

$$I_4 = I_1 - I_2 = 27{,}94\ mA - 14{,}7\ mA = \mathbf{13{,}24\ mA};\quad U_4 = I_4 \cdot R_4 = 13{,}24\ mA \cdot 150\ \Omega = \mathbf{1{,}986\ V}$$

$$U_5 = U_6 = U_2 - U_4 = 3{,}97\ V - 1{,}986\ V = \mathbf{1{,}984\ V}$$

$$I_5 = \frac{U_5}{R_5} = \frac{1{,}984\ V}{470\ \Omega} = \mathbf{4{,}22\ mA};\quad I_6 = \frac{U_6}{R_6} = \frac{1{,}984\ V}{220\ \Omega} = \mathbf{9{,}02\ mA}$$

53/21. c) Anschluss zwischen B und C:



zu 53/21.c)

$$R_I = \frac{R_5 \cdot R_6}{R_5 + R_6} = \frac{470\ \Omega \cdot 220\ \Omega}{470\ \Omega + 220\ \Omega} = 149{,}86\ \Omega$$

$$R_{II} = R_I + R_4 = 149{,}86\ \Omega + 150\ \Omega = 299{,}86\ \Omega$$

$$R_{III} = R_1 + R_3 = 180\ \Omega + 120\ \Omega = 300\ \Omega$$

$$R = \frac{1}{\frac{1}{R_{II}} + \frac{1}{R_2} + \frac{1}{R_{III}}} = \frac{1}{\frac{1}{299{,}86\ \Omega} + \frac{1}{270\ \Omega} + \frac{1}{300\ \Omega}} = \frac{1}{0{,}01037\ \frac{1}{\Omega}} = \mathbf{96{,}4\ \Omega}$$

$$I_1 = \frac{U}{R_{III}} = \frac{9\ V}{300\ \Omega} = \mathbf{30\ mA};\quad U_2 = U = \mathbf{9\ V};$$

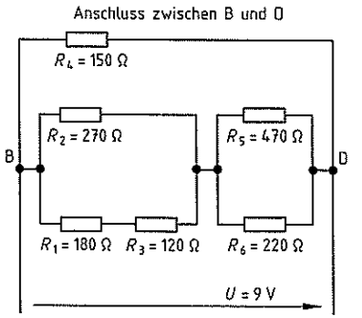
$$U_1 = I_1 \cdot R_1 = 30\ mA \cdot 180\ \Omega = \mathbf{5{,}4\ V};\quad U_3 = I_1 \cdot R_3 = 30\ mA \cdot 120\ \Omega = \mathbf{3{,}6\ V}$$

$$I_2 = \frac{U}{R_2} = \frac{9\ V}{270\ \Omega} = \mathbf{33{,}3\ mA};\quad I_4 = \frac{U}{R_{II}} = \frac{9\ V}{299{,}86\ \Omega} = \mathbf{30\ mA}$$

$$U_4 = R_4 \cdot I_4 = 150\ \Omega \cdot 30\ mA = \mathbf{4{,}5\ V};\quad U_5 = U - U_4 = 9\ V - 4{,}50\ V = \mathbf{4{,}50\ V}$$

$$I_5 = \frac{U_5}{R_5} = \frac{4{,}50\ V}{470\ \Omega} = \mathbf{9{,}57\ mA};\quad I_6 = \frac{U_5}{R_6} = \frac{4{,}50\ V}{220\ \Omega} = \mathbf{20{,}5\ mA}$$

53/21. d) Anschluss zwischen B und D:



zu 53/21.d)

$$R_I = R_1 + R_3 = 180\ \Omega + 120\ \Omega = 300\ \Omega$$

$$R_{II} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{300\ \Omega \cdot 270\ \Omega}{300\ \Omega + 270\ \Omega} = 142{,}11\ \Omega$$

$$R_{III} = \frac{R_5 \cdot R_6}{R_5 + R_6} = \frac{470\ \Omega \cdot 220\ \Omega}{470\ \Omega + 220\ \Omega} = 149{,}86\ \Omega$$

$$R_{IV} = R_{II} + R_{III} = 142{,}11\ \Omega + 149{,}86\ \Omega = 291{,}97\ \Omega$$

$$R = \frac{R_4 \cdot R_{III}}{R_4 + R_{III}} = \frac{150\ \Omega \cdot 291{,}86\ \Omega}{150\ \Omega + 291{,}86\ \Omega} = \mathbf{99{,}09\ \Omega}$$

$$U_4 = U = 9\ V;\quad I_4 = \frac{U_4}{R_4} = \frac{9\ V}{150\ \Omega} = \mathbf{60\ mA}$$

$$I_{5+6} = \frac{U}{R_{IV}} = \frac{9\ V}{291{,}86\ \Omega} = \mathbf{30{,}84\ mA}$$

$$U_5 = U_6 = R_{III} \cdot I_{5+6} = 149{,}86\ \Omega \cdot 0{,}03084\ A = \mathbf{4{,}62\ V}$$

$$I_5 = \frac{U_5}{R_5} = \frac{4{,}62\ V}{470\ \Omega} = \mathbf{9{,}83\ mA};$$

$$I_6 = \frac{U_6}{R_6} = \frac{4{,}62\ V}{220\ \Omega} = \mathbf{21\ mA}$$

$$U_2 = U - U_5 = 9{,}00\ V - 4{,}62\ V = \mathbf{4{,}38\ V}$$

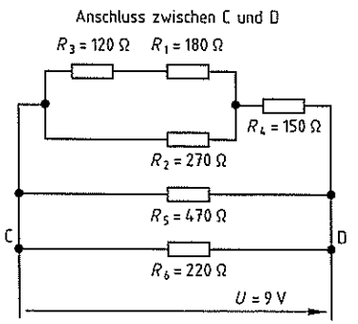
$$I_2 = \frac{U_2}{R_2} = \frac{4{,}38\ V}{270\ \Omega} = \mathbf{16{,}22\ mA};$$

$$I_1 = I_3 = \frac{U_2}{R_I} = \frac{4{,}38\ V}{300\ \Omega} = \mathbf{14{,}6\ mA}$$

$$U_1 = R_1 \cdot I_1 = 180\ \Omega \cdot 14{,}6\ mA = \mathbf{2{,}628\ V}$$

$$U_3 = R_3 \cdot I_1 = 120\ \Omega \cdot 14{,}6\ mA = \mathbf{1{,}752\ V}$$

53/21. e) Anschluss zwischen C und D:



zu 53/21.e)

$$R_I = R_1 + R_3 = 180\ \Omega + 120\ \Omega = 300\ \Omega$$

$$R_{II} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{300\ \Omega \cdot 270\ \Omega}{300\ \Omega + 270\ \Omega} = 142{,}11\ \Omega$$

$R_{III} = R_{II} + R_4 = 142{,}11\,\Omega + 150\,\Omega = 292{,}11\,\Omega$

$$R = \frac{1}{\frac{1}{R_{III}} + \frac{1}{R_5} + \frac{1}{R_6}} = \frac{1}{\frac{1}{292{,}11\,\Omega} + \frac{1}{470\,\Omega} + \frac{1}{220\,\Omega}} = \frac{1}{0{,}010096\,\frac{1}{\Omega}} = 99{,}04\,\Omega$$

$U = U_5 = U_6 = 9\,V$; $I_5 = \frac{U_5}{R_5} = \frac{9\,V}{470\,\Omega} = 19{,}15\,mA$; $I_6 = \frac{9\,V}{220\,\Omega} = 40{,}91\,mA$

$I_4 = \frac{U}{R_{III}} = \frac{9\,V}{292{,}11\,\Omega} = 30{,}81\,mA$; $U_4 = I_4 \cdot R_4 = 30{,}81\,mA \cdot 150\,\Omega = 4{,}62\,V$

$U_2 = U - U_4 = 9\,V - 4{,}62\,V = 4{,}38\,V$; $I_2 = \frac{U_2}{R_2} = \frac{4{,}38\,V}{270\,\Omega} = 16{,}22\,mA$

$I_1 = I_3 = \frac{U_2}{R_1} = \frac{4{,}38\,V}{300\,\Omega} = 14{,}6\,mA$

$U_1 = I_1 \cdot R_1 = 14{,}6\,mA \cdot 180\,\Omega = 2{,}63\,V$; $U_3 = I_1 \cdot R_3 = 14{,}6\,mA \cdot 120\,\Omega = 1{,}75\,V$

3.8.4 Spannungsteiler

Lösungen zu 3.8.4

Unbelasteter Spannungsteiler

54/1. $U_{20} = U \cdot \frac{R_2}{R_1 + R_2} = 12\,V \cdot \frac{5{,}5\,k\Omega}{1{,}5\,k\Omega + 5{,}5\,k\Omega} = 9{,}43\,V$

54/2. $U_{20} = U \cdot \frac{R_2}{R_1 + R_2} = 15\,V \cdot \frac{90\,k\Omega}{25\,k\Omega + 90\,k\Omega} = 11{,}74\,V$

54/3. $\frac{R_1}{R_2} = \frac{3}{5}$; $\frac{R_1}{R_2} = \frac{U}{U_{20}} - 1$; $U_{20} = \frac{U}{\frac{R_1}{R_2} + 1} = \frac{24\,V}{\frac{3}{5} + 1} = 15\,V$

54/4. a) $U_{AB} = U \cdot \frac{R_{AB}}{R} = 150\,V \cdot \frac{200\,\Omega}{900\,\Omega} = 33{,}3\,V$ b) $U_{AC} = U \cdot \frac{R_{AC}}{R} = 150\,V \cdot \frac{500\,\Omega}{900\,\Omega} = 83{,}3\,V$

c) $U_{BC} = U \cdot \frac{R_{BC}}{R} = 150\,V \cdot \frac{300\,\Omega}{900\,\Omega} = 50{,}0\,V$ d) $U_{BD} = U \cdot \frac{R_{BD}}{R} = 150\,V \cdot \frac{700\,\Omega}{900\,\Omega} = 116{,}7\,V$

e) $U_{CD} = U \cdot \frac{R_{CD}}{R} = 150\,V \cdot \frac{400\,\Omega}{900\,\Omega} = 66{,}7\,V$

54/5. $U_{20} = U \cdot \frac{R_2}{R_1 + R_2} \Rightarrow U = U_{20} \cdot \frac{R_1 + R_2}{R_2} = 6{,}0\,V \cdot \frac{2{,}7\,k\Omega + 8{,}1\,k\Omega}{8{,}1\,k\Omega} = 8{,}0\,V$

54/6. $\frac{R_1}{R_2} = \frac{U}{U_{20}} - 1 \Rightarrow R_1 = \frac{U \cdot R_2}{U_{20}} - R_2 = \frac{24\,V \cdot 12\,k\Omega}{9\,V} - 12\,k\Omega = 20{,}0\,k\Omega$

54/7. $N \triangleq R$; a) $N_2 = \frac{N \cdot U_{20}}{U} = \frac{640 \cdot 15\,V}{24\,V} = 400$; b) $N_2 = \frac{640 \cdot 5\,V}{24\,V} = 133{,}3$

c) $N_2 = \frac{640 \cdot 3\,V}{24\,V} = 80$; d) $N_2 = \frac{640 \cdot 6{,}3\,V}{24{,}V} = 168$

54/8. a) Ermittelt aus Schaubild Rechenbuch Elektrotechnik, Seite 73, Bild 3:
R_2 bei 20°C: $R_2 \approx 12\,k\Omega$
R_2 bei 60°C: $R_2 \approx 2{,}5\,k\Omega$

bei 20°C: $U_{20} = U \cdot \frac{R_2}{R_1 + R_2} = 12\,V \cdot \frac{12\,k\Omega}{4{,}7\,k\Omega + 12\,k\Omega}$

$U_{20} = 8{,}62\,V$

bei 60°C: $U_{20} = U \cdot \frac{R_2}{R_1 + R_2} = 12\,V \cdot \frac{2{,}5\,k\Omega}{4{,}7\,k\Omega + 2{,}5\,k\Omega}$

$U_{20} = 4{,}17\,V$

b) $\frac{R_1}{R_2} = \frac{U}{U_{20}} - 1 \Rightarrow R_2 = \frac{R_1}{\frac{U}{U_{20}} - 1} = \frac{4{,}7\,k\Omega}{\frac{12\,V}{3{,}2\,V} - 1} = 1{,}71\,k\Omega$

Ermittelt aus Schaubild Rechenbuch Elektrotechnik, Seite 73, Bild 3: $\vartheta \approx 72°C$

54/9. $\frac{U_{20}}{U} = \frac{N_2}{N}$; $U_{20} = \frac{230\,V \cdot 1}{450} = 0{,}511\,V$

54/10. $U_{20} = \frac{24\,V \cdot 1}{380} = 63{,}2\,mV$

Belasteter Spannungsteiler

55/1. a) $R_{2L} = \frac{R_2 \cdot R_L}{R_2 + R_L} = \frac{35\,\Omega \cdot 50\,\Omega}{35\,\Omega + 50\,\Omega} = 20{,}6\,\Omega$

$U_2 = U \cdot \frac{R_{2L}}{R_1 + R_{2L}} = 30\,V \cdot \frac{20{,}6\,\Omega}{100\,\Omega + 20{,}6\,\Omega} = 5{,}12\,V$

b) $I_L = \frac{U_2}{R_L} = \frac{5{,}12\,V}{50\,\Omega} = 0{,}102\,A$

55/2. a) Stellung A: $\Rightarrow$ Kurzschluss von R_{L1}, R_{L2} und R_2 $\Rightarrow$ $U_2 = 0\,V$

b) Stellung E:

$R_L = \frac{R_{L1} \cdot R_{L2}}{R_{L1} + R_{L2}} = \frac{2{,}2\,k\Omega \cdot 1{,}2\,k\Omega}{2{,}2\,k\Omega + 1{,}2\,k\Omega} = 0{,}78\,k\Omega$

$U_2 = \frac{U}{\frac{R_1 \cdot (R_L + R_2)}{R_L \cdot R_2} + 1} = \frac{6\,V}{\frac{820\,\Omega \cdot (780\,\Omega + 390\,\Omega)}{780\,\Omega \cdot 390\,\Omega} + 1} = \frac{6\,V}{3{,}15 + 1} = 1{,}44\,V$

55/3. a) $I_{q1} = q_1 \cdot I_B = 3 \cdot 1{,}0\,mA = 3{,}0\,mA$

$R_2 = \frac{U_2}{I_{q1}} = \frac{0{,}72\,V}{3{,}0\,mA} = 240\,\Omega$

$R_1 = \frac{U - U_2}{I_{q1} + I_B} = \frac{12{,}00\,V - 0{,}72\,V}{3{,}0\,mA + 1{,}0\,mA} = 2820\,\Omega$

b) $I_{q2} = q_2 \cdot I_B = 4 \cdot 1{,}0\,mA = 4{,}0\,mA$

$R_2 = \frac{U_2}{I_{q2}} = \frac{0{,}72\,V}{4{,}0\,mA} = 180\,\Omega$

$R_1 = \frac{U - U_2}{I_{q2} + I_B} = \frac{12{,}00\,V - 0{,}72\,V}{4{,}0\,mA + 1{,}0\,mA} = 2260\,\Omega$

c) $I_{q3} = q_3 \cdot I_B = 5 \cdot 1{,}0\,mA = 5{,}0\,mA$

$R_2 = \frac{U_2}{I_{q3}} = \frac{0{,}72\,V}{5{,}0\,mA} = 144\,\Omega$

$R_1 = \frac{U - U_2}{I_{q3} + I_B} = \frac{12{,}00\,V - 0{,}72\,V}{5{,}0\,mA + 1{,}0\,mA} = 1880\,\Omega$

55/4. a) $R_1 = \frac{R \cdot 3}{3+1} = \frac{280\,\Omega \cdot 3}{4} = 210\,\Omega$; $R_2 = \frac{R \cdot 1}{3+1} = \frac{280\,\Omega}{4} = 70\,\Omega$

b) $U_{20} = \frac{R_2}{R_1 + R_2} \cdot U = \frac{70\,\Omega}{280\,\Omega} \cdot 24\,V = 6\,V$

c) $U_2 = \frac{U}{\frac{R_1 \cdot (R_L + R_2)}{R_L \cdot R_2} + 1} = \frac{24\,V}{\frac{210\,\Omega \cdot (200\,\Omega + 70\,\Omega)}{200\,\Omega \cdot 70\,\Omega} + 1} = \frac{24\,V}{4{,}05 + 1} = 4{,}75\,V$

d) $I_q = \frac{U_2}{R_2} = \frac{4{,}75\,V}{70\,\Omega} = 0{,}068\,A$

e) $I_L = \frac{U_2}{R_L} = \frac{4{,}75\,V}{200\,\Omega} = 0{,}0238\,A$

f) $q = \frac{I_q}{I_L} = \frac{I_q}{U_2/R_L} = \frac{0{,}068\,A}{0{,}0238\,A} = 2{,}9$

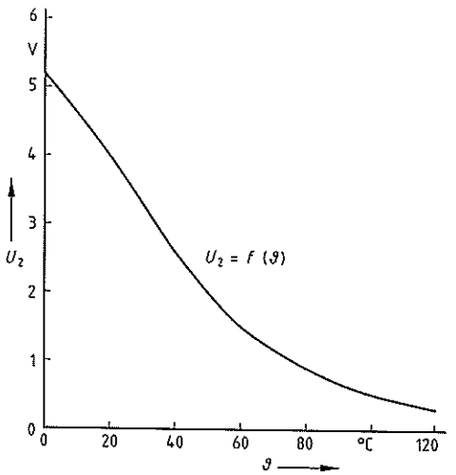
55/5. a)

ϑ in °C	R_ϑ in kΩ	$R_2 \| R_\vartheta$ in kΩ	U_2 in V
0	32	11,5	5,2
20	12,5	7,4	4,0
40	5,4	4,2	2,6
60	2,5	2,2	1,5
80	1,3	1,2	0,9
100	0,7	0,67	0,52
120	0,4	0,39	0,31

R_ϑ aus Diagramm ablesen:

$R_{2\vartheta} = \frac{R_2 \cdot R_\vartheta}{R_2 + R_\vartheta} = \frac{18\,k\Omega \cdot 32\,k\Omega}{18\,k\Omega + 32\,k\Omega} = 11{,}5\,k\Omega$

$U_2 = \frac{U \cdot R_{2\vartheta}}{R_1 + R_{2\vartheta}} = \frac{12\,V \cdot 11{,}5\,k\Omega}{15\,k\Omega + 11{,}5\,k\Omega} = 5{,}2\,V$



zu 55/5.b)

55/6. a) Siehe Schaltskizze

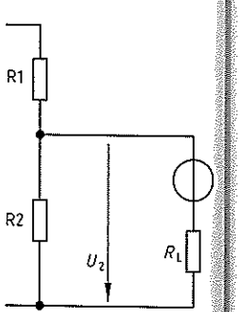
b) $U_{20} = \frac{R_2}{R_1 + R_2} \cdot U = \frac{6{,}2\,k\Omega}{(47\,k\Omega + 6{,}2\,k\Omega)} \cdot 50\,V = 5{,}83\,V$

c) P1: $R_m = R_L = r_k \cdot U = 20\frac{k\Omega}{V} \cdot 6\,V = 120\,k\Omega$

$U_2 = \frac{U}{\frac{R_1 \cdot (R_L + R_2)}{R_L \cdot R_2} + 1} = \frac{50\,V}{\frac{47\,k\Omega \cdot (120\,k\Omega + 6{,}2\,k\Omega)}{120\,k\Omega \cdot 6{,}2\,k\Omega} + 1} = 5{,}57\,V$

P2: $R_m = R_L = r_k \cdot U = 40\frac{k\Omega}{V} \cdot 6\,V = 240\,k\Omega$

$U_2 = \frac{50\,V}{\frac{47\,k\Omega \cdot (240\,k\Omega + 6{,}2\,k\Omega)}{240\,k\Omega \cdot 6{,}2\,k\Omega} + 1} = 5{,}7\,V$



zu 55/6.a)

d) Messfehler bei P1: $\Delta U = U_{20} - U_2 = 5{,}83\,V - 5{,}57\,V = 0{,}26\,V$

$\Delta U_\% = \frac{0{,}26\,V \cdot 100\%}{5{,}83\,V} = 4{,}5\%$

bei P2: $\Delta U = U_{20} - U_2 = 5{,}83\,V - 5{,}7\,V = 0{,}13\,V$; $\Delta U_\% = \frac{0{,}13\,V \cdot 100\%}{5{,}83\,V} = 2{,}2\%$

3.8.5 Abgeglichene Brückenschaltung

Lösungen zu 3.8.5

56/1. $\frac{R_x}{R_n} = \frac{R_3}{R_4}$

a) $R_x = \frac{R_3 \cdot R_n}{R_4} = \frac{100\,\Omega \cdot 14\,\Omega}{200\,\Omega} = 7\,\Omega$

b) $R_x = \frac{100\,\Omega \cdot 250\,\Omega}{200\,\Omega} = 125\,\Omega$

c) $R_x = \frac{100\,\Omega \cdot 1400\,\Omega}{200\,\Omega} = 700\,\Omega$

56/2. $R_x = \frac{l_1 \cdot R_n}{l_2} = \frac{39\,cm \cdot 100\,\Omega}{61\,cm} = 63{,}9\,\Omega$

56/3. $\frac{l_1}{l_2} = \frac{R_x}{R_n}$; $\frac{l_1}{l_2} = \frac{55\,\Omega}{220\,\Omega} = \frac{1}{4}$; $l_1 + l_2 = 1\,m$; $(1+4)$ Teile $= 1\,m \Rightarrow l_1 = 0{,}2\,m$; $l_2 = 0{,}8\,m$

56/4. $\frac{R_x}{R_n} = \frac{R_3}{R_4} \Rightarrow R_3 = \frac{R_x \cdot R_4}{R_n} = \frac{0{,}8\,k\Omega \cdot 2{,}2\,k\Omega}{1{,}7\,k\Omega} = 1{,}04\,k\Omega$

56/5. a) $I_n = I_x = \frac{U_n}{R_n} = \frac{14\,V}{680\,\Omega} = 20{,}59\,mA$

$U_3 = U_x = U - U_n = 24\,V - 14\,V = 10\,V$

$R_x = \frac{U_x}{I_x} = \frac{10\,V}{20{,}59\,mA} = 485{,}67\,\Omega$

$U_4 = U_n = 14\,V$

$I_4 = I_3 = \frac{U_4}{R_4} = \frac{14\,V}{220\,\Omega} = 63{,}64\,mA$

$R_3 = \frac{R_x}{R_n} \cdot R_4 = \frac{485{,}67\,\Omega}{680\,\Omega} \cdot 220\,\Omega = 157{,}1\,\Omega$

b) $I = I_x + I_3 = 20{,}59\,mA + 63{,}64\,mA = 84{,}23\,mA$

56/6. Bei 100°C: $R_x = R_\vartheta$

$R_\vartheta = R_{20} \cdot (1 + \alpha \cdot \Delta\vartheta) = 6{,}8\,k\Omega \cdot \left(1 + 0{,}0039\frac{1}{K} \cdot 80\,K\right) = 8{,}92\,k\Omega$

$R_n = \frac{R_\vartheta \cdot R_4}{R_3} = \frac{8{,}92\,k\Omega \cdot 470\,\Omega}{1{,}5\,k\Omega} = 2795\,\Omega$

56/7. $R_i = \frac{R_3 \cdot R_2}{R_4} = \frac{56\,\Omega \cdot 33\,\Omega}{342\,\Omega} = 5{,}4\,\Omega$

$I_{Ri} = \frac{U}{R_i + R_2} = \frac{15\,V}{5{,}4\,\Omega + 33\,\Omega} = 390{,}63\,mA$

$U_{Ri} = R_i \cdot I_{Ri} = 5{,}4\,\Omega \cdot 390{,}63\,mA = 2{,}11\,V$

56/8. Ermittelt aus Schaubild Rechenbuch Elektrotechnik, Seite 57, Bild 4: $R_4 = R_{200} \approx 175\,\Omega$

$R_2 = \frac{R_{200} \cdot R_1}{R_3} = \frac{175\,\Omega \cdot 560\,\Omega}{470\,\Omega} = 208{,}5\,\Omega$

56/9. b) 1. Lösungsweg:

$l = l_1 + l_2 \Rightarrow l_2 = l - l_1$ (1)

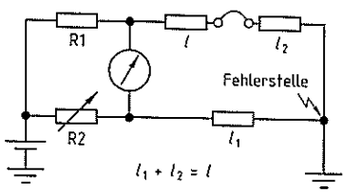
$\frac{R_1}{l + l_2} = \frac{R_2}{l_1}$ (2)

(1) in (2):

$\frac{R_1}{l + l - l_1} = \frac{R_2}{l_1} \Rightarrow \frac{R_1}{2l - l_1} = \frac{R_2}{l_1}$

$l_1 \cdot R_1 = 2l \cdot R_2 - l_1 \cdot R_2$

a)



$l_1 (R_1 + R_2) = 2l \cdot R_2$

$$l_1 = 2l \cdot \frac{R_2}{R_1 + R_2} = 2 \cdot 1500\,\text{m} \cdot \frac{280\,\Omega}{420\,\Omega + 280\,\Omega} = 1200\,\text{m}$$

2. Lösungsweg: In einer abgeglichenen Brückenschaltung verhalten sich entsprechende Widerstände (bzw. Widerstandsgruppen) gleich, also gilt:

$$\frac{R_2}{\text{Summe R linker Zweig}} = \frac{l_1}{\text{Summe R rechter Zweig}}$$

$$\frac{R_2}{R_1 + R_2} = \frac{l_1}{2l}$$

$$l_1 = 2l \cdot \frac{R_2}{R_1 + R_2} = 2 \cdot 1500\,\text{m} \cdot \frac{280\,\Omega}{420\,\Omega + 280\,\Omega} = 1200\,\text{m}$$

3.8.6 Unabgeglichene Brückenschaltung

Lösungen zu 3.8.6

57/1. a) $U_3 = U \cdot \frac{R_3}{R_3 + R_4} = 12\,\text{V} \cdot \frac{220\,\Omega}{220\,\Omega + 390\,\Omega} = 4{,}33\,\text{V}$

Masche: $U_1 + U_{AB} - U_3 = 0 \Rightarrow U_1 = U_3 - U_{AB} = 4{,}33\,\text{V} - (-1\,\text{V}) = +5{,}33\,\text{V}$

$U_2 = U - U_1 = 12\,\text{V} - 5{,}33\,\text{V} = 6{,}67\,\text{V}$

$I_1 = \frac{U_1}{R_1} = \frac{5{,}33\,\text{V}}{100\,\Omega} = 0{,}0533\,\text{A} = 53{,}3\,\text{mA}$; $R_2 = \frac{U_2}{I_1} = \frac{6{,}67\,\text{V}}{0{,}0533\,\text{A}} = \mathbf{125\,\Omega}$

b) $U_1 = U_3 - U_{AB} = 4{,}33\,\text{V} - 1\,\text{V} = 3{,}33\,\text{V}$; $U_2 = U - U_1 = 12\,\text{V} - 3{,}33\,\text{V} = 8{,}67\,\text{V}$

$I_1 = \frac{U_1}{R_1} = \frac{3{,}33\,\text{V}}{100\,\Omega} = 0{,}0333\,\text{A}$; $R_2 = \frac{U_2}{R_2} = \frac{8{,}67\,\text{V}}{0{,}0333\,\text{A}} = \mathbf{260\,\Omega}$

57/2. a) $U = 10\,\text{V}$; $R_I = R_1 + R_2 = 20\,\Omega + 30\,\Omega = 50\,\Omega$; $R_{II} = R_3 + R_4 = 200\,\Omega$

$\frac{U_1}{U} = \frac{R_1}{R_I} \Rightarrow U_1 = \frac{U \cdot R_1}{R_I} = \frac{10\,\text{V} \cdot 20\,\Omega}{50\,\Omega} = 4\,\text{V}$; $U_3 = \frac{U \cdot R_3}{R_{II}} = \frac{10\,\text{V} \cdot 70\,\Omega}{200\,\Omega} = 3{,}5\,\text{V}$

Masche: $U_1 + U_{AB} - U_3 = 0$; $\Rightarrow U_{AB} = U_3 - U_1 = 3{,}5\,\text{V} - 4\,\text{V} = \mathbf{-0{,}5\,V}$

b) $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow R_2 = R_1 \cdot \frac{R_4}{R_3} = 20\,\Omega \cdot \frac{130\,\Omega}{70\,\Omega} = \mathbf{37{,}1\,\Omega}$

57/3. a) $R_I = R_1 + R_2 = 8{,}2\,\text{k}\Omega + 5{,}6\,\text{k}\Omega = 13{,}8\,\text{k}\Omega$; $R_{II} = R_3 + R_4 = 2{,}7\,\text{k}\Omega + 3{,}9\,\text{k}\Omega = 6{,}6\,\text{k}\Omega$

$\frac{U_1}{U} = \frac{R_1}{R_I} \Rightarrow U_1 = \frac{U \cdot R_1}{R_I} = \frac{5\,\text{V} \cdot 8{,}2\,\text{k}\Omega}{13{,}8\,\text{k}\Omega} = 2{,}97\,\text{V}$; $U_3 = \frac{U \cdot R_3}{R_{II}} = \frac{5\,\text{V} \cdot 2{,}7\,\text{k}\Omega}{6{,}6\,\text{k}\Omega} = 2{,}05\,\text{V}$

Masche: $U_1 + U_{AB} - U_3 = 0$; $U_{AB} = U_3 - U_1 = 2{,}05\,\text{V} - 2{,}97\,\text{V} = \mathbf{-0{,}92\,V}$

b) $\frac{R_2}{R_4} = \frac{R_1}{R_3} \Rightarrow R_2 = \frac{R_1 \cdot R_4}{R_3} = \frac{8{,}2\,\text{k}\Omega \cdot 3{,}9\,\text{k}\Omega}{2{,}7\,\text{k}\Omega} = \mathbf{11{,}8\,k\Omega}$

57/4. a) $R_2 = \frac{R_0}{R_3} \cdot R_1 = \frac{100\,\Omega}{470\,\Omega} \cdot 470\,\Omega = \mathbf{100\,\Omega}$

b) Ermittelt aus **Rechenbuch Elektrotechnik, Seite 57, Bild 4:** $R_{400} \approx 250\,\Omega$

$U_2 = \frac{R_2 \cdot U}{R_1 + R_2} = \frac{100\,\Omega \cdot 12\,\text{V}}{470\,\Omega + 100\,\Omega} = 2{,}105\,\text{V}$

$U_{400} = \frac{R_{400} \cdot U}{R_{400} + R_3} = \frac{250\,\Omega \cdot 12\,\text{V}}{250\,\Omega + 470\,\Omega} = 4{,}167\,\text{V}$

$U_{AB} = U_2 - U_{400} = 2{,}105\,\text{V} - 4{,}167\,\text{V} = \mathbf{-2{,}06\,V}$

57/5. a) Ablesung aus Schaubild **Rechenbuch Elektrotechnik, Seite 73, Bild 3:**

ϑ in °C	0	10	20	30	40	50
R in kΩ	32	19	12,5	8,5	5,4	3,7

Für 0°C: $R_I = R_1 + R_2 = 10\,\text{k}\Omega + 32\,\text{k}\Omega = 42\,\text{k}\Omega$

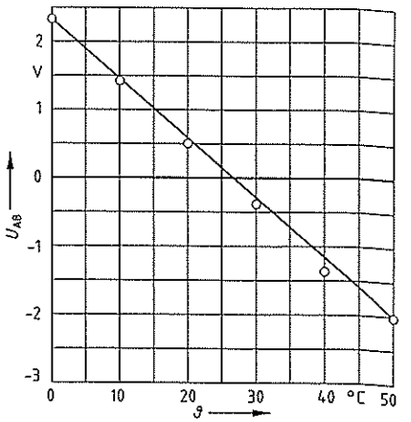
$R_{II} = R_3 + R_4 = 10\,\text{k}\Omega + 10\,\text{k}\Omega = 20\,\text{k}\Omega$

$U_1 = \frac{U \cdot R_1}{R_I} = \frac{9\,\text{V} \cdot 10\,\text{k}\Omega}{42\,\text{k}\Omega} = 2{,}14\,\text{V}$;

$U_3 = \frac{U \cdot R_3}{R_{II}} = \frac{9\,\text{V} \cdot 10\,\text{k}\Omega}{20\,\text{k}\Omega} = 4{,}5\,\text{V}$

$U_{AB} = U_3 - U_1 = 4{,}5\,\text{V} - 2{,}14\,\text{V} = 2{,}36\,\text{V}$

ϑ in °C	0	10	20
U_{AB} in V	+2,36	+1,4	+0,5
ϑ in °C	30	40	50
U_{AB} in V	−0,36	−1,34	−2,07



zu 57/5.b)

57/6. a) Die Spannung U_2 muss positiver als die Spannung U_4 sein, weil $U_{AB} = U_2 - U_4$ positiv sein soll.
Zuordnung: $R_1 \rightarrow$ DMS2 (klein), $R_2 \rightarrow$ DMS1 (groß), $R_3 \rightarrow$ DMS3 (groß), $R_4 \rightarrow$ DMS4 (klein)

b) $U_2 = \frac{R_2}{R_2 + R_1} \cdot U = \frac{358\,\Omega}{358\,\Omega + 342\,\Omega} \cdot 12\,\text{V} = 6{,}14\,\text{V}$

$U_4 = \frac{R_4}{R_4 + R_3} \cdot U = \frac{342\,\Omega}{342\,\Omega + 358\,\Omega} \cdot 12\,\text{V} = 5{,}86\,\text{V}$

$U_{AB} = U_2 - U_4 = 6{,}14\,\text{V} - 5{,}86\,\text{V} = \mathbf{0{,}28\,V}$

3.9 Elektrische Leistung und Arbeit

3.9.1 Elektrische Leistung

Lösungen zu 3.9.1

58/1. a) $R = \frac{U}{I} = \frac{12\,\text{V}}{6{,}25\,\text{A}} = \mathbf{1{,}92\,\Omega}$

b) $P = U \cdot I = 12\,\text{V} \cdot 6{,}25\,\text{A} = \mathbf{75\,W}$

58/2. a) $I = \frac{P}{U} = \frac{25\,\text{W}}{230\,\text{V}} = \mathbf{0{,}109\,A}$; $R = \frac{U}{I} = \frac{230\,\text{V}}{0{,}109\,\text{A}} = \mathbf{2110\,\Omega}$

b) $I = \frac{P}{U} = \frac{40\,\text{W}}{230\,\text{V}} = \mathbf{0{,}174\,A}$; $R = \frac{U}{I} = \frac{230\,\text{V}}{0{,}174\,\text{A}} = \mathbf{1322\,\Omega}$

58/3. $R = \frac{U^2}{P_{220}} = \frac{(220\,\text{V})^2}{1000\,\text{W}} = \mathbf{48{,}4\,\Omega}$; $P_{230} = \frac{(230\,\text{V})^2}{48{,}4\,\Omega} = \mathbf{1093\,W}$

58/4. a) $P = I^2 \cdot R = (0{,}007\,\text{A})^2 \cdot 2150\,\Omega = 0{,}105\,\text{W} = \mathbf{105\,mW}$

b) $U = R \cdot I = 2150\,\Omega \cdot 0{,}01\,\text{A} = \mathbf{21{,}5\,V}$

58/5. Wertetabellen für $I = \frac{P}{U}$:

P = 1000 mW:	U in V	20	30	40	50	80
	I in mA	50	33	25	20	12,5

P = 500 mW:	U in V	10	15	20	25	40
	I in mA	50	33	25	20	12,5

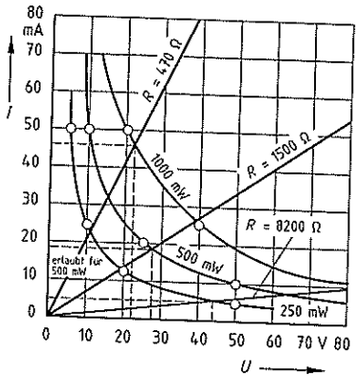
P = 250 mW:	U in V	5	10	20	50
	I in mA	50	25	12,5	5

Abgelesen aus Zeichnung für:

a) 500 mW und R = 1,5 kΩ: **U = 28 V; I = 18 mA**

b) 1000 mW und R = 470 Ω: **U = 22 V; I = 46 mA**

c) 250 mW und R = 8,2 kΩ: **U = 45 V; I = 5 mA**



zu 58/5.

58/6. a) $U = U_2 = I_2 \cdot R_2 = 4\,mA \cdot 8{,}2\,k\Omega = \mathbf{32{,}8\,V}$

b) $I_3 = \frac{U}{R_3 + R_4} = \frac{32{,}8\,V}{10\,k\Omega + 1{,}5\,k\Omega} = \mathbf{2{,}85\,mA}$

c) $\frac{1}{R} = \frac{1}{5{,}6\,k\Omega} + \frac{1}{8{,}2\,k\Omega} + \frac{1}{11{,}5\,k\Omega} = 0{,}387\,\frac{1}{k\Omega} \Rightarrow \mathbf{R = 2{,}58\,k\Omega}$

$P = \frac{U^2}{R} = \frac{(32{,}8\,V)^2}{2{,}58\,k\Omega} = \mathbf{0{,}417\,W \approx 0{,}42\,W}$

58/7. a) $U_2 = U_3 = I_2 \cdot R_2 = 9{,}9\,mA \cdot 270\,\Omega = \mathbf{2{,}67\,V}$

$I_3 = \frac{U_3}{R_3} = \frac{2{,}67\,V}{150\,\Omega} = \mathbf{0{,}0178\,A = 17{,}8\,mA}$

$I_4 = I_2 + I_3 = 9{,}9\,mA + 17{,}8\,mA = \mathbf{27{,}7\,mA}$

$U_4 = R_4 \cdot I_4 = 120\,\Omega \cdot 27{,}7\,mA = \mathbf{3{,}32\,V}$; $I_5 = \frac{U_5}{R_5} = \frac{U_2 + U_4}{R_5} = \frac{5{,}99\,V}{180\,\Omega} = \mathbf{33{,}3\,mA}$

$I = I_1 = I_4 + I_5 = 27{,}7\,mA + 33{,}3\,mA = \mathbf{61\,mA}$

$U_1 = I_1 \cdot R_1 = 61\,mA \cdot 100\,\Omega = \mathbf{6{,}1\,V}$; $U = U_1 + U_5 = 6{,}1\,V + 5{,}99\,V = \mathbf{12{,}1\,V}$

b) $P_1 = U_1 \cdot I_1 = 6{,}1\,V \cdot 61\,mA = \mathbf{373\,mW}$

$P_2 = 2{,}67\,V \cdot 9{,}9\,mA = \mathbf{26{,}4\,mW}$; $P_3 = 2{,}67\,V \cdot 17{,}8\,mA = \mathbf{47{,}5\,mW}$

$P_4 = 3{,}32\,V \cdot 27{,}7\,mA = \mathbf{92{,}0\,mW}$; $P_5 = 5{,}99\,V \cdot 33{,}3\,mA = \approx \mathbf{200\,mW}$

c) Normwerte für R_1: **0,5 W**; R_2: **0,05 W**; R_3: **0,05 W**; R_4: **0,125 W**; R_5: **0,25 W**

58/8. a) $P = U \cdot I \Rightarrow I = \frac{P}{U} = \frac{700\,W}{230\,V} = \mathbf{3{,}04\,A}$ b) $R = \frac{U}{I} = \frac{230\,V}{3{,}04\,A} = \mathbf{75{,}6\,\Omega}$

c) $U_{unter} = 230\,V \cdot 0{,}95 = 218{,}5\,V$; $P = \frac{U^2}{R} = \frac{(218{,}5\,V)^2}{75{,}6\,\Omega} = 632\,W$

$P_{\%} = \frac{632\,W \cdot 100\%}{700\,W} = 90{,}3\% \Rightarrow$ Er gibt 100% − 90,3% = **9,7%** weniger Leistung ab.

59/9. a) $A = \frac{\pi \cdot d^2}{4} = \frac{\pi \cdot (0{,}5\,mm)^2}{4} = 0{,}1963\,mm^2$; $R = \frac{\varrho \cdot l}{A} = \frac{1{,}45\,\Omega \cdot mm^2 \cdot 20{,}5\,m}{m \cdot 0{,}1963\,mm^2} = \mathbf{151{,}4\,\Omega}$

$P = \frac{U^2}{R} \Rightarrow U = \sqrt{R \cdot P} = \sqrt{151{,}4\,\Omega \cdot 2000\,W} = \mathbf{550\,V}$; b) $I = \frac{U}{\quad} = \frac{550\,V}{\quad} = 3{,}63\,A$

59/10. a) $P = \frac{U^2}{R}$; $U = \sqrt{P \cdot R} = \sqrt{5\,W \cdot 47\,\Omega} = \sqrt{235}\,V = \mathbf{15{,}3\,V}$; $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{5\,W}{47\,\Omega}} = \mathbf{326\,mA}$

b) $U = \sqrt{1\,W \cdot 330\,\Omega} = \sqrt{330}\,V = \mathbf{18{,}2\,V}$; $I = \sqrt{\frac{1\,W}{330\,\Omega}} = \mathbf{55\,mA}$

c) $U = \sqrt{0{,}125\,W \cdot 15000\,\Omega} = \sqrt{1875}\,V = \mathbf{43{,}3\,V}$; $I = \sqrt{\frac{0{,}125\,W}{15\,k\Omega}} = \mathbf{2{,}88\,mA}$

d) $U = \sqrt{0{,}125\,W \cdot 0{,}1\,M\Omega} = \sqrt{0{,}0125}\,kV = 0{,}112\,kV = \mathbf{112\,V}$; $I = \sqrt{\frac{0{,}125\,W}{100\,k\Omega}} = \mathbf{1{,}12\,mA}$

e) $U = \sqrt{0{,}5\,W \cdot 4{,}7\,M\Omega} = \sqrt{2{,}35}\,kV = 1{,}53\,kV = \mathbf{1530\,V}$; $I = \sqrt{\frac{0{,}5\,W}{4{,}7\,M\Omega}} = \mathbf{0{,}326\,mA}$

f) $U = \sqrt{0{,}25\,W \cdot 5600\,\Omega} = \sqrt{1400}\,V = \mathbf{37{,}4\,V}$; $I = \sqrt{\frac{0{,}25\,W}{5\,k\Omega}} = \mathbf{7{,}07\,mA}$

59/11. a) R_1, R_2 und R_3 in Reihe an 400 V: P = 18 kW

b) $R_1 + R_2 = \frac{U^2}{P} = \frac{(400\,V)^2}{18000\,W} = 8{,}89\,\Omega$

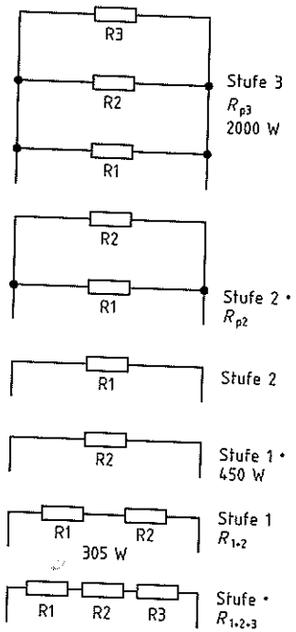
R_1 allein an 400 V: $P = 18\,kW \cdot 1{,}25 = 22{,}5\,kW$

$R_1 = \frac{(400\,V)^2}{22500\,W} = \mathbf{7{,}11\,\Omega}$; $R_2 = 8{,}89\,\Omega - 7{,}11\,\Omega = \mathbf{1{,}78\,\Omega}$

$R_1 + R_2 + R_3$ an 400 V; $P = 18\,kW \cdot 0{,}85 = 15{,}3\,kW$;

$R_1 + R_2 + R_3 = \frac{(400\,V)^2}{15300\,W} = 10{,}46\,\Omega$

$R_3 = 10{,}46\,\Omega - 8{,}89\,\Omega = \mathbf{1{,}57\,\Omega}$



zu 59/12.

59/12. a) **Stufe 3:** $I = \frac{P}{U} = \frac{2000\,W}{230\,V} = \mathbf{8{,}70\,A}$

Stufe 1 ●: $I = \frac{P}{U} = \frac{450\,W}{230\,V} = \mathbf{1{,}96\,A}$

Stufe 1: $I = \frac{P}{U} = \frac{305\,W}{230\,V} = \mathbf{1{,}33\,A}$

Widerstände:

Stufe 3: $R_{p3} = \frac{U^2}{P} = \frac{(230\,V)^2}{2000\,W} = \mathbf{26{,}5\,\Omega}$

Stufe 1 ●: $R_2 = \frac{U^2}{P} = \frac{(230\,V)^2}{450\,W} = \mathbf{118\,\Omega}$

Stufe 1: $R_{1+2} = \frac{U^2}{P} = \frac{(230\,V)^2}{305\,W} = \mathbf{173\,\Omega}$

$R_1 = R_{1+2} - R_2 = 173\,\Omega - 118\,\Omega = \mathbf{55\,\Omega}$

$\frac{1}{R_3} = \frac{1}{R_{p3}} - \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{26{,}5\,\Omega} - \frac{1}{55\,\Omega} - \frac{1}{118\,\Omega} = 0{,}01108\,\frac{1}{\Omega}$; $\mathbf{R_3 = 90{,}3\,\Omega}$

Stufe ●: $R_{1+2+3} = R_1 + R_2 + R_3 = 55\,\Omega + 118\,\Omega + 90{,}3\,\Omega = 263{,}3\,\Omega$

$I = \frac{U}{R_{1+2+3}} = \frac{230\,V}{263{,}3\,\Omega} = \mathbf{0{,}874\,A}$

Stufe 2: $I = \frac{U}{R_1} = \frac{230\,V}{55\,\Omega} = \mathbf{4{,}18\,A}$

Stufe 2 ●: $R_{p2} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{55\,\Omega \cdot 118\,\Omega}{55\,\Omega + 118\,\Omega} = \mathbf{37{,}5\,\Omega}$; $I = \frac{U}{R} = \frac{230\,V}{37{,}5\,\Omega} = \mathbf{6{,}13\,A}$

$I = 0{,}874\,A + \ldots$

b) Stufe ●: $P = U \cdot I_{●} = 230\,V \cdot 0{,}874\,A = 201\,W$
Stufe 2: $P = 230\,V \cdot 4{,}18\,A = 961\,W$
Stufe 2●: $P = 230\,V \cdot 6{,}13\,A = 1410\,W$

3.9.2 Elektrische Arbeit

Lösungen zu 3.9.2

59/1. $W = P \cdot t = 0{,}14\,kW \cdot 4{,}25\,h = 0{,}595\,kWh/Tag$
$W_{Mon} = W \cdot Tage = 0{,}595\,kWh/Tag \cdot 30\,Tage = \mathbf{17{,}9\,kWh}$

59/2. a) $W = P \cdot t;\quad t = \frac{W}{P} = \frac{1\,kWh}{0{,}015\,kW} = \mathbf{66{,}7\,h}$ b) $t = \frac{W}{P} = \frac{1\,kWh}{0{,}075\,kW} = \mathbf{13{,}3\,h}$

59/3. a) $W = P \cdot t \Rightarrow P = \frac{W}{t} = \frac{160\,Wh \cdot 60\,min}{8\,min \cdot 1\,h} = \mathbf{1200\,W}$ c) $R = \frac{U}{I} = \frac{230\,V}{5{,}22\,A} = \mathbf{44{,}1\,\Omega}$

b) $P = U \cdot I \Rightarrow I = \frac{P}{U} = \frac{1200\,W}{230\,V} = \mathbf{5{,}22\,A}$

59/4. a) $R = \frac{l}{\gamma \cdot A} = \frac{90000\,m}{56\,\frac{m}{\Omega \cdot mm^2} \cdot 50\,mm^2} = \mathbf{32{,}14\,\Omega}$

$W = P \cdot t = I^2 \cdot R \cdot t = (27\,A)^2 \cdot 32{,}14\,\Omega \cdot 24\,h = 562321\,Wh = \mathbf{562\,kWh}$

b) $\Delta U = R \cdot I = 32{,}14\,\Omega \cdot 27\,A = \mathbf{868\,V}$

59/5. a) $P = \frac{U^2}{R} = \frac{(12\,V)^2}{27\,\Omega} = \mathbf{5{,}\bar{3}\,W}$ b) $W = P \cdot t = 5{,}\bar{3}\,W \cdot \frac{50}{60}\,h = \mathbf{4{,}\bar{4}\,Wh}$

c) $I = \frac{U}{R} = \frac{12\,V}{27\,\Omega} = \mathbf{0{,}44\,A}$ d) $W = \frac{U^2 \cdot t}{R} = \frac{(12\,V)^2 \cdot 50\,min \cdot 1\,h}{39\,\Omega \cdot 60\,min} = \mathbf{3{,}08\,Wh}$

Änderung: $\Delta W = 4{,}44\,Wh - 3{,}08\,Wh = 1{,}36\,Wh;\quad 1{,}36\,Wh \triangleq \frac{1{,}36\,Wh \cdot 100\,\%}{4{,}44\,Wh} = \mathbf{31\,\%}$

3.9.3 Leistungsbestimmung mit dem Zähler

Lösungen zu 3.9.3

60/1. $n = 2 \cdot 60 = 120\,\frac{1}{h};\quad P = \frac{n}{C_z} = \frac{120\,1/h}{1200\,1/kWh} = \mathbf{0{,}1\,kW = 100\,W}$

60/2. $n = \frac{25 \cdot 60}{2} = 750\,\frac{1}{h};\quad P = \frac{n}{C_z} = \frac{750\,1/h}{600\,1/kWh} = \mathbf{1{,}25\,kW}$

60/3. $n = \frac{21 \cdot 60}{0{,}5} = 2520\,\frac{1}{h};\quad P = \frac{n}{C_z} = \frac{2520\,1/h}{120\,1/kWh} = \mathbf{21\,kW}$

60/4. $n = \frac{15 \cdot 60}{4} = 225\,\frac{1}{h};\quad C_z = \frac{n}{P} = \frac{225\,1/h}{0{,}2\,kW} = \mathbf{1125\,\frac{1}{kWh}}$

60/5. a) $P = 2 \cdot 0{,}06\,kW + 5 \cdot 0{,}009\,kW + 0{,}12\,kW = 0{,}29\,kW$

$P = \frac{n}{C_z};\quad n = P \cdot C_z = 0{,}29\,kW \cdot 750\,\frac{1}{kWh} = 218\,\frac{1}{h}$

in 2 min: $n = \frac{218\,1/h \cdot 2\,min}{60\,min/h} = \mathbf{7{,}3\,Umdr.}$ b) $I = \frac{P}{U} = \frac{290\,W}{230\,V} = \mathbf{1{,}26\,A}$

60/6. a) $P = \frac{n}{C_z};\quad n = P \cdot C_z = 3\,kW \cdot 375\,\frac{1}{kWh} = 1125\,\frac{1}{h}$

in 4 min: $n \cdot \frac{4}{60} = 1125\,\frac{1}{h} \cdot \frac{4}{60} = \mathbf{75\ Umdrehungen}$

b) $I = \frac{P}{U} = \frac{3000\,W}{230\,V} = \mathbf{13{,}0\,A}$

60/7. a) $P = 30 \cdot 0{,}04\,kW + 50 \cdot 0{,}04\,kW + 120 \cdot 0{,}069\,kW + 4 \cdot 18\,kW + 24\,kW =$
$= 1{,}2\,kW + 2\,kW + 8{,}28\,kW + 72\,kW + 24\,kW = 107{,}5\,kW$
$P_{gl} = 107{,}5\,kW \cdot 0{,}24 = 25{,}8\,kW$

$n = P_{gl} \cdot C_z = 25{,}8\,kW \cdot 60\,\frac{1}{kWh} = 1548\,\frac{1}{h};\quad$ in 30 s: $n = \frac{1548}{2 \cdot 60} = \mathbf{12{,}9\ Umdrehungen}$

b) $W = P \cdot t \Rightarrow t = \frac{W}{P} = \frac{5\,kWh}{25{,}8\,kW} = \mathbf{0{,}194\,h = 11{,}6\,min}$

c) $P = \frac{n}{C_z} \Rightarrow n = P \cdot C_z = 90\,kW \cdot 60\,\frac{1}{kWh} = 5400\,\frac{1}{h} = \frac{5400}{3600}\,\frac{1}{s} = 1{,}5\,\frac{1}{s}$

$v = \pi \cdot d \cdot n = \pi \cdot 0{,}1\,m \cdot 1{,}5\,\frac{1}{s} = \mathbf{0{,}471\,\frac{m}{s}}$

60/8. a) $P = 6 \cdot 40\,W + 7 \cdot 60\,W + 4 \cdot 69\,W + 2000\,W + 3000\,W + 3300\,W + 18000\,W + 10000\,W =$
$= 37236\,W = 37{,}2\,kW$
$P_{gl} = 37{,}2\,kW \cdot 0{,}12 = 4{,}5\,kW$

$n = P_{gl} \cdot C_z = 4{,}5\,kW \cdot 150\,\frac{1}{kWh} = 675\,\frac{1}{h} \Rightarrow$ in 1 min: $\frac{675}{60} = \mathbf{11{,}3\ Umdrehungen}$

b) $t = \frac{W}{P} = \frac{2\,kWh}{4{,}5\,kW} = \mathbf{0{,}44\,h = 26{,}4\,min}$

c) $n = P \cdot C_z = 25\,kW \cdot 150\,\frac{1}{kWh} = 3750\,\frac{1}{h} = \frac{3750}{3600}\,\frac{1}{s} = 1{,}042\,\frac{1}{s}$

$v = \pi \cdot d \cdot n = \pi \cdot 0{,}1\,m \cdot 1{,}042\,\frac{1}{s} = \mathbf{0{,}327\,\frac{m}{s}}$

3.9.4 Wirkungsgrad

Lösungen zu 3.9.4

61/1. a) $\eta = \frac{P_{ab}}{P_{zu}};\quad P_{zu} = \frac{P_{ab}}{\eta} = \frac{30\,kW}{0{,}9} = \mathbf{33{,}3\,kW};$ b) $P_v = P_{zu} - P_{ab} = 33{,}3\,kW - 30\,kW = \mathbf{3{,}3\,kW}$

61/2. $P_{zu} = P_{ab} + P_v = 18{,}5\,kW + 1{,}5\,kW = 20\,kW;\quad \eta = \frac{P_{ab}}{P_{zu}} = \frac{18{,}5\,kW}{20\,kW} = \mathbf{0{,}925 = 92{,}5\,\%}$

61/3. $\eta = \frac{P_{ab}}{P_{zu}} = \frac{U_2 \cdot I_2}{U_1 \cdot I_1} = \frac{13{,}8\,V \cdot 5\,A}{230\,V \cdot 0{,}33\,A} = \mathbf{0{,}9 = 90\,\%}$

61/4. $\eta = \frac{P_{ab}}{P_{zu}};\quad P_{ab} = \eta \cdot P_{zu} = 0{,}94 \cdot 1570\,W = \mathbf{1475{,}8\,W}$

61/5. $\eta = \eta_G \cdot \eta_M = 0{,}86 \cdot 0{,}82 = \mathbf{0{,}705 = 70{,}5\,\%}$

61/6. $P_{ab} = \frac{F \cdot s}{t} = \frac{80000\,kg \cdot 9{,}81\,N/kg \cdot 50\,m}{3600\,s} = 10900\,W = 10{,}9\,kW$

$P_{zu} = \frac{P_{ab}}{\eta} = \frac{10{,}9\,kW}{0{,}74} = 14{,}73\,kW = \mathbf{15\,kW}$

61/7. a) $P_{zu} = U \cdot I = 220\,V \cdot 68\,A = 14960\,W = \mathbf{14{,}96\,kW}$

b) $\eta = \frac{P_{ab}}{P_{zu}} = \frac{P_{ab}}{U \cdot I} = \frac{13000\,W}{220\,V \cdot 68\,A} = 0{,}869 = \mathbf{87\,\%}$

c) $P_v = P_{zu} - P_{ab} = 14960\,W - 13000\,W = \mathbf{1960\,W}$

61/8. $P_{ab} = P_{zu} \cdot \eta = 20\,kW \cdot 0{,}62 = 12{,}4\,kW$; $F = \frac{P_{ab} \cdot t}{s} = \frac{12400\,W \cdot 30\,s}{15\,m} = \mathbf{24800\,N}$

61/9. a) $P_{zuTu} = \frac{F \cdot s}{t} = \frac{110\,kg}{1\,s} \cdot 9{,}81\,\frac{N}{kg} \cdot 19\,m = 20500\,\frac{Nm}{s} = 20{,}5\,kW$

$P_{abTu} = P_{zuTu} \cdot \eta_1 = 20{,}5\,kW \cdot 0{,}73 = \mathbf{14{,}97\,kW}$

b) $P_{abGen} = P_{abTu} \cdot \eta_2 = 14{,}97\,kW \cdot 0{,}89 = \mathbf{13{,}32\,kW}$; c) $\eta_{ges} = \eta_1 \cdot \eta_2 = 0{,}73 \cdot 0{,}89 = \mathbf{0{,}65 = 65\,\%}$

61/10. a) $P_{abP} = \frac{F \cdot s}{t} = \frac{60000\,kg \cdot 9{,}81\,N/kg \cdot 26\,m}{3600\,s} = \mathbf{4251\,W = 4{,}25\,kW}$

b) $P_{zuP} = P_{abM} = \frac{P_{abT}}{\eta_P} = \frac{4{,}25\,kW}{0{,}71} = 5986\,W = \mathbf{6{,}0\,kW}$

c) $P_{zuM} = \frac{P_{abM}}{\eta_M} = \frac{6{,}0\,kW}{0{,}86} = 6{,}98\,kW = \mathbf{7{,}0\,kW}$; d) $\eta_{ges} = \eta_M \cdot \eta_P = 0{,}71 \cdot 0{,}86 = 0{,}611 = \mathbf{61\,\%}$

3.10 Wärmeenergie

3.10.1 Wärmemenge und Wassermischung

Lösungen zu 3.10.1

62/1. $Q = c \cdot m \cdot \Delta\vartheta = 4{,}19\,\frac{kJ}{kg \cdot K} \cdot 300\,kg \cdot 67\,K = \mathbf{84219\,kJ}$

62/2. $Q = c \cdot m \cdot \Delta\vartheta \Rightarrow \Delta\vartheta = \frac{Q}{c \cdot m} = \frac{3000\,kJ}{4{,}19\,\frac{kJ}{kg \cdot K} \cdot 10\,kg} = 71{,}6\,K$

$\vartheta_2 = \vartheta_1 + \Delta\vartheta = 15\,°C + 71{,}6\,K = \mathbf{86{,}6\,°C}$

62/3. $m = 3000\,l \cdot 0{,}89\,\frac{kg}{l} = 2670\,kg$; $Q = c \cdot m \cdot \Delta\vartheta = 1{,}9\,\frac{kJ}{kg \cdot K} \cdot 2670\,kg \cdot 30\,K = \mathbf{152190\,kJ}$

62/4. $Q = c \cdot m \cdot \Delta\vartheta$; $c = \frac{Q}{m \cdot \Delta\vartheta} = \frac{54500\,kJ}{78\,kg \cdot 580\,K} = \mathbf{1{,}205\,\frac{kJ}{kg \cdot K}}$

62/5. $\vartheta_m = \frac{m_k \cdot \vartheta_k + m_w \cdot \vartheta_w}{m_k + m_w} = \frac{70\,kg \cdot 9\,°C + 80\,kg \cdot 65\,°C}{70\,kg + 80\,kg} = \mathbf{38{,}9\,°C}$

62/6. $m_m \cdot \vartheta_m = m_k \cdot \vartheta_k + m_w \cdot \vartheta_w$; $\vartheta_k = \frac{m_m \cdot \vartheta_m - m_w \cdot \vartheta_w}{m_k} = \frac{400\,kg \cdot 37\,°C - 120\,kg \cdot 85\,°C}{280\,kg} = \mathbf{16{,}4\,°C}$

62/7. $m_m \cdot \vartheta_m = m_k \cdot \vartheta_k + m_w \cdot \vartheta_w \quad | \; m_k = m_m - m_w$

$m_m \cdot \vartheta_m = (m_m - m_w) \cdot \vartheta_k + m_w \cdot \vartheta_w$

$m_m \cdot \vartheta_m = m_m \cdot \vartheta_k - m_w \cdot \vartheta_k + m_w \cdot \vartheta_w$

$m_m (\vartheta_m - \vartheta_k) = m_w (\vartheta_w - \vartheta_k)$

$m_w = \frac{m_m (\vartheta_m - \vartheta_k)}{\vartheta_w - \vartheta_k} = \frac{140\,kg\,(37\,°C - 12\,°C)}{60\,°C - 12\,°C} = \frac{140\,kg \cdot 25\,K}{48\,K} = \mathbf{72{,}9\,kg \triangleq 72{,}9\,l}$

3.10.2 Elektrowärme und Wärmenutzungsgrad

Lösungen zu 3.10.2

63/1. $Q_S = P \cdot t = 0{,}08\,kW \cdot \frac{20}{60}\,h \cdot 3600\,\frac{kJ}{kWh} = \mathbf{96\,kJ}$

63/2. $P = I^2 \cdot R = (25\,A)^2 \cdot 0{,}8\,\Omega = 625\,A^2 \cdot 0{,}8\,\Omega = \mathbf{500\,W}$

$Q_S = P \cdot t = 0{,}5\,kW \cdot 0{,}25\,h \cdot 3600\,\frac{kJ}{kWh} = \mathbf{450\,kJ}$

63/3. a) $Q = P \cdot t$; $P = \frac{c \cdot m \cdot \Delta\vartheta}{\zeta \cdot t} = \frac{4{,}19\,kJ/(kg \cdot K) \cdot 10\,kg \cdot 25\,K}{0{,}97 \cdot 3600\,\frac{kJ}{kWh} \cdot \frac{1}{60}\,h} = \mathbf{18\,kW}$

b) $t = \frac{200\,l \cdot 1\,min \cdot 1\,h}{10\,l \cdot 60\,min} = 0{,}33\,h$

$W = P \cdot t = 18\,kW \cdot 0{,}33\,h = \mathbf{5{,}94\,kWh}$

63/4. a) $Q_N = c \cdot m \cdot \Delta\vartheta = 4{,}19\,\frac{kJ}{kg \cdot K} \cdot 2{,}5\,kg \cdot 53\,K = \mathbf{555\,kJ}$

b) $Q_S = P \cdot t = 2{,}2\,kW \cdot \frac{9}{60}\,h \cdot 3600\,\frac{kJ}{kWh} = \mathbf{1188\,kJ}$

c) $\zeta = \frac{Q_N}{Q_S} = \frac{555\,kJ}{1188\,kJ} = \mathbf{0{,}467 = 47\,\%}$

63/5. a) $W = \frac{c \cdot m \cdot \Delta\vartheta}{\zeta} = \frac{4{,}19\,kJ/(kg \cdot K) \cdot 80\,kg \cdot 41\,K}{0{,}97 \cdot 3600\,\frac{kJ}{kWh}} = \mathbf{3{,}94\,kWh}$

b) $W = P \cdot t$; $t = \frac{W}{P} = \frac{3{,}94\,kWh}{6\,kW} = 0{,}657\,h = \mathbf{39{,}4\,min}$

63/6. a) $P = \frac{U^2}{R}$; $R = \frac{U^2}{P} = \frac{(230\,V)^2}{2000\,W} = \mathbf{26{,}5\,\Omega}$

b) $Q = c \cdot m \cdot \Delta\vartheta = 4{,}19\,\frac{kJ}{kg \cdot K} \cdot 1{,}5\,kg \cdot 82\,K = \mathbf{515{,}4\,kJ}$

c) $\zeta = \frac{c \cdot m \cdot \Delta\vartheta}{P \cdot t} = \frac{515{,}4\,kJ}{2\,kW \cdot \frac{5}{60}\,h \cdot 3600\,\frac{kJ}{kWh}} = 0{,}859 = \mathbf{86\,\%}$

63/7. $P = \frac{c \cdot m \cdot \Delta\vartheta}{\zeta \cdot t} = \frac{4{,}19\,kJ/(kg \cdot K) \cdot 83\,K \cdot 3\,kg}{0{,}7 \cdot \frac{12{,}5}{60}\,h \cdot 3600\,\frac{kJ}{kWh}} = 1{,}99\,kW = 2\,kW$; $R = \frac{U^2}{P} = \frac{(230\,V)^2}{2000\,W} = \mathbf{26{,}5\,\Omega}$

63/8. $R_{ges} = \frac{R_1}{3} = \frac{52{,}9\,\Omega}{3} = 17{,}6\,\Omega$; $P_{ges} = \frac{U^2}{R_{ges}} = \frac{(230\,V)^2}{17{,}6\,\Omega} = 3006\,W = 3\,kW$

$Q_S = P \cdot t = 3\,kW \cdot \frac{45}{60}\,h = 2{,}25\,kWh = 8100\,kJ$;

$\zeta = \frac{Q_N}{Q_S} \Rightarrow Q_N = \zeta \cdot Q_S = 0{,}83 \cdot 8100\,kJ = 6723\,kJ$

$Q_N = c \cdot m \cdot \Delta\vartheta \Rightarrow \Delta\vartheta = \frac{Q_N}{c \cdot m} = \frac{6723\,kJ}{30\,kg \cdot 4{,}19\,kJ/(kg \cdot K)} = 53{,}5\,K$; $12\,°C \triangleq 285{,}15\,K$

$\vartheta_2 = \vartheta_1 + \Delta\vartheta = 285{,}15\,K + 53{,}5\,K = 338{,}65\,K \triangleq \mathbf{65.5\,°C}$

3.11 Spannungserzeuger

3.11.1 Galvanische Elemente

Lösungen zu 3.11.1

64/1. $U = U_0 - I \cdot R_i = 1{,}55\,V - 0{,}04\,A \cdot 0{,}15\,\Omega = \mathbf{1{,}544\,V}$

64/2. $I_k = \frac{U_0}{R_i}$; $R_i = \frac{U_0}{I_k} = \frac{4{,}68\,V}{5\,A} = \mathbf{0{,}936\,\Omega}$

64/3. $R_i = \frac{U_0}{I_k} = \frac{1{,}58\,V}{4\,A} = \mathbf{0{,}395\,\Omega}$

64/4. $I_k = \frac{U_0}{R_i} = \frac{1{,}5\,V}{0{,}13\,\Omega} = \mathbf{11{,}54\,A}$

64/5. a) $U = U_0 - I \cdot R_i$; $R_i = \frac{U_0 - U}{I} = \frac{6{,}3\,V - 6{,}1\,V}{0{,}6\,A} = \mathbf{0{,}33\,\Omega}$

b) $U = U_0 - I \cdot R_i = 6{,}3\,V - 2\,A \cdot 0{,}33\,\Omega = \mathbf{5{,}64\,V}$

c) $I_k = \frac{U_0}{R_i} = \frac{6{,}3\,V}{0{,}33\,\Omega} = \mathbf{19\,A}$ (nur kurzfristig – Sicherung)

d) Leistungsanpassung $R_i = R_L = 0{,}33\,\Omega$; $P_{max} = \frac{U_0^2}{4 \cdot R_i} = \frac{(6{,}3\,V)^2}{4 \cdot 0{,}33\,\Omega} = \mathbf{30\,W}$

64/6. a) $R_i = \frac{U_0 - U}{I} = \frac{14{,}2\,V - 14{,}0\,V}{7{,}3\,A} = \mathbf{0{,}0274\,\Omega}$ b) $I_k = \frac{U_0}{R} = \frac{14{,}2\,V}{0{,}0274\,\Omega} = \mathbf{518\,A}$

c) $I = \frac{U_0}{R_i + R_L}$; $R_L = \frac{U_0}{I} - R_i = \frac{14{,}2\,V}{7{,}3\,A} - 0{,}0274\,\Omega = \mathbf{1{,}92\,\Omega}$

$R_{L2} = \frac{R_L \cdot R_P}{R_L + R_P} = \frac{1{,}92\,\Omega \cdot 0{,}82\,\Omega}{1{,}92\,\Omega + 0{,}82\,\Omega} = 0{,}575\,\Omega$; $I_2 = \frac{U_0}{R_{L2} + R_i} = \frac{14{,}2\,V}{0{,}575\,\Omega + 0{,}0274\,\Omega} = \mathbf{23{,}6\,A}$

$U_2 = U_0 - I \cdot R_i = 14{,}2\,V - 23{,}6\,A \cdot 0{,}0274\,\Omega = \mathbf{13{,}6\,V}$

64/7. a) $U_0 = U + I \cdot R_i$ (1); $R_i = \frac{U_0}{I_k}$ (2); (1) in (2) eingesetzt:

$U_0 = \frac{U}{1 - \frac{I}{I_k}} = \frac{9\,V}{1 - \frac{0{,}7\,A}{10\,A}} = \frac{9\,V}{0{,}93} = \mathbf{9{,}68\,V}$

b) $R_i = \frac{U_0}{I_k} = \frac{9{,}68\,V}{10\,A} = \mathbf{0{,}968\,\Omega}$

3.11.2 Schaltung von Spannungserzeugern

Lösungen zu 3.11.2

Reihenschaltung von Spannungserzeugern

65/1. a) $U_0 = n \cdot U_{01} = 4 \cdot 1{,}32\,V = \mathbf{5{,}28\,V}$ b) $R_i = n \cdot R_{i1} = 4 \cdot 0{,}3\,\Omega = \mathbf{1{,}2\,\Omega}$

65/2. a) $U_0 = n \cdot U_{01} = 2 \cdot 3{,}6\,V = \mathbf{7{,}2\,V}$ b) $R_i = n \cdot R_{i1} = 2 \cdot 0{,}15\,\Omega = \mathbf{0{,}3\,\Omega}$

c) $I = \frac{U_0}{R_i + R_L} = \frac{7{,}2\,V}{0{,}3\,\Omega + 40\,\Omega} = \frac{7{,}2\,V}{40{,}3\,\Omega} = 0{,}179\,A$; $U = I \cdot R_L = 0{,}179\,A \cdot 40\,\Omega = \mathbf{7{,}16\,V}$

65/3. a) $n_{Anf} = \frac{U_0}{U_{01\,Anf}} = \frac{220\,V}{2{,}4\,V} = 91{,}7 \Rightarrow \mathbf{92\ Elemente}$

b) $n_{End} = \frac{U_0}{U_{01\,End}} = \frac{220\,V}{1{,}85\,V} = 118{,}9 \Rightarrow \mathbf{119\ Elemente}$

65/4. a) $U_0 = n \cdot U_{01} = 30 \cdot 2{,}2\,V = \mathbf{66\,V}$; $R_i = n \cdot R_{i1} = 30 \cdot 0{,}002\,\Omega = 0{,}06\,\Omega$;

$I = \frac{U_0}{R_i + R_L} = \frac{66\,V}{0{,}06\,\Omega + 6{,}5\,\Omega} = \frac{66\,V}{6{,}56\,\Omega} = 10{,}06\,A$; $U = R_L \cdot I_i = 6{,}5\,\Omega \cdot 10{,}06\,A = \mathbf{65{,}4\,V}$

b) $I_k = \frac{U_0}{R_i} = \frac{66\,V}{0{,}06\,\Omega} = \mathbf{1100\,A}$

65/5. a) $U = R_L \cdot I_i = 24\,\Omega \cdot 0{,}4\,A = \mathbf{9{,}6\,V}$

b) $I = \frac{n \cdot U_{01}}{R_L + n \cdot R_{i1}} \Rightarrow n = \frac{R_L \cdot I_i}{U_{01} - R_{i1} \cdot I} = \frac{24\,\Omega \cdot 0{,}4\,A}{1{,}5\,V - 0{,}3\,\Omega \cdot 0{,}4\,A} = 6{,}96 \Rightarrow \mathbf{7\ Elemente}$

65/6. a) $U_0 = n \cdot U_{01} = 20 \cdot 1{,}32\,V = 26{,}4\,V$; $R_i = n \cdot R_{i1} = 20 \cdot 0{,}03\,\Omega = 0{,}6\,\Omega$

$R_{L1} = \frac{U^2}{P} = \frac{(24\,V)^2}{15\,W} = 38{,}4\,\Omega$;

$I = \frac{U_0}{R_i + R_L} = \frac{26{,}4\,V}{0{,}6\,\Omega + 38{,}4\,\Omega} = \frac{26{,}4\,V}{39\,\Omega} = \mathbf{0{,}68\,A}$; $U = R_L \cdot I = 38{,}4\,\Omega \cdot 0{,}68\,A = \mathbf{26{,}11\,V}$

b) $R_L = \frac{R_{L1}}{n} = \frac{38{,}4\,\Omega}{30} = 1{,}28\,\Omega$; $I = \frac{U_0}{R_i + R_L} = \frac{26{,}4\,V}{0{,}6\,\Omega + 1{,}28\,\Omega} = \frac{26{,}4\,V}{1{,}88\,\Omega} = \mathbf{14\,A}$

$U = R_L \cdot I = 1{,}28\,\Omega \cdot 14\,A = \mathbf{17{,}9\,V}$

65/7. a) $n = \frac{R_L \cdot I}{U_{01} - R_{i1} \cdot I} = \frac{47\,\Omega \cdot 0{,}35\,A}{1{,}56\,V - 1{,}1\,\Omega \cdot 0{,}35\,A} = \frac{16{,}45\,V}{1{,}175\,V} = \mathbf{14\ Elemente}$

b) $U_0 = n \cdot U_{01\,alt} = 14 \cdot 1{,}25\,V = 17{,}5\,V$; $R_{i\,alt} = n \cdot R_{i1\,alt} = 14 \cdot 1{,}35\,\Omega = 18{,}9\,\Omega$

$I = \frac{U_{0alt}}{R_{i\,alt} + R_L} = \frac{17{,}5\,V}{18{,}9\,\Omega + 47\,\Omega} = \frac{17{,}5\,V}{65{,}9\,\Omega} = \mathbf{0{,}266\,A}$

Parallelschaltung von Spannungserzeugern

66/1. a) $U_0 = U_{01} = 1{,}5\,V$; $R_i = \frac{R_{i1}}{n} = \frac{0{,}3\,\Omega}{5} = 0{,}06\,\Omega$; $I = \frac{U_0}{R_i + R_L} = \frac{1{,}5\,V}{0{,}06\,\Omega + 0{,}8\,\Omega} = \frac{1{,}5\,V}{0{,}86\,\Omega} = 1{,}74\,A$

b) $U = R_L \cdot I = 0{,}8\,\Omega \cdot 1{,}74\,A = \mathbf{1{,}4\,V}$

66/2. a) $U_0 = U_{01} = 2{,}1\,V$; $R_i = \frac{R_{i1}}{n} = \frac{25\,m\Omega}{12} = 2{,}08\,m\Omega \approx 2{,}1\,m\Omega$

$I = \frac{U_0}{R_i + R_L} = \frac{2{,}1\,V}{2{,}1\,m\Omega + 1{,}2\,\Omega} = \frac{2{,}1\,V}{1{,}202\,\Omega} = \mathbf{1{,}75\,A}$

b) $I_k = \frac{U_0}{R_i} = \frac{2{,}1\,V}{2{,}1\,m\Omega} = \mathbf{1000\,A}$

66/3. $U_0 = R_L \cdot I + \frac{R_{i1}}{n} \cdot I \Rightarrow n = \frac{R_{i1} \cdot I}{U_{01} - R_L \cdot I} = \frac{0{,}5\,\Omega \cdot 4\,A}{1{,}5\,V - 0{,}25\,\Omega \cdot 4\,A} = \frac{2\,V}{1{,}5\,V - 1\,V} = \mathbf{4\ Elemente}$

66/4. a) $U_0 = 2 \cdot U_{01} = 2 \cdot 1{,}55\,V = \mathbf{3{,}10\,V}$

b) $R_{iges} = \frac{2 \cdot 0{,}8\,\Omega}{3} = \mathbf{0{,}533\,\Omega}$; $I = \frac{U_0}{R_{iges} + R_L} = \frac{3{,}1\,V}{0{,}533\,\Omega + 3\,\Omega} = \frac{3{,}1\,V}{3{,}533\,\Omega} = \mathbf{0{,}877\,A}$

c) $U = R_L \cdot I = 3\,\Omega \cdot 0{,}877\,A = \mathbf{2{,}63\,V}$

66/5. a) $U_{01} = U + R_{i1} \cdot I_1 = 440\,V + 0{,}12\,\Omega \cdot 80\,A = 440\,V + 9{,}6\,V = \mathbf{449{,}6\,V}$

$U_{02} = U + R_{i2} \cdot I_2 = 440\,V + 0{,}09\,\Omega \cdot 130\,A = 440\,V + 11{,}7\,V = \mathbf{451{,}7\,V}$

b) $I_A = \dfrac{\Delta U}{\Sigma R_i} = \dfrac{U_{02} - U_{01}}{R_{i1} + R_{i2}} = \dfrac{451{,}7\,V - 449{,}6\,V}{0{,}12\,\Omega + 0{,}09\,\Omega} = \mathbf{10\,A}$

66/6. $U_{01} = U + R_{i1} \cdot I_1 = 220\,V + 0{,}08\,\Omega \cdot 130\,A = 220\,V + 10{,}4\,V = \mathbf{230{,}4\,V}$

$U_{02} = U + R_{i2} \cdot I_2 = 220\,V + 0{,}09\,\Omega \cdot 210\,A = 220\,V + 18{,}9\,V = \mathbf{238{,}9\,V}$

$U_{03} = U + R_{i3} \cdot I_3 = 220\,V + 0{,}06\,\Omega \cdot 180\,A = 220\,V + 10{,}8\,V = \mathbf{230{,}8\,V}$

3.11.3 Anpassung

Lösungen zu 3.11.3

67/1. a) $R_L = R_i = 8\,\Omega$; $I = \dfrac{U_0}{R_i + R_L} = \dfrac{12\,V}{8\,\Omega + 8\,\Omega} = \mathbf{0{,}75\,A}$

b) $P = U \cdot I = 6\,V \cdot 0{,}75\,A = \mathbf{4{,}5\,W}$

c) $U_0 = \dfrac{U}{0{,}5} = \dfrac{6\,V}{0{,}5} = \mathbf{12\,V}$

d) $I_k = \dfrac{U_0}{R_i} = \dfrac{12\,V}{8\,\Omega} = \mathbf{1{,}5\,A}$

67/2. a) $I_1 = \dfrac{U_0}{R_i + R_{L1}} = \dfrac{230\,V}{0{,}5\,\Omega + 120\,\Omega} = 1{,}91\,A$; $I_2 = \dfrac{U_0}{R_i + R_{L2}} = \dfrac{230\,V}{0{,}5\,\Omega + 150\,\Omega} = 1{,}53\,A$

$U_1 = I \cdot R_{L1} = 1{,}91\,A \cdot 120\,\Omega = \mathbf{229{,}2\,V}$; $U_2 = I \cdot R_{L2} = 1{,}53\,A \cdot 150\,\Omega = \mathbf{229{,}5\,V}$

b) Die Spannungen U_1 und U_2 sind nahezu gleich ⇒ **Konstantspannungsquelle**

c) **Spannungsanpassung:** $R_L \gg R_i$

67/3. a) $R_{Lges} = R_i = \dfrac{(4\,\Omega + 8\,\Omega + 4\,\Omega) \cdot 16\,\Omega}{4\,\Omega + 8\,\Omega + 4\,\Omega + 16\,\Omega} = \dfrac{16\,\Omega \cdot 16\,\Omega}{32\,\Omega} = 8\,\Omega$; $I = \dfrac{U}{R_{Lges}} = \dfrac{25{,}3\,V}{8\,\Omega} = \mathbf{3{,}16\,A}$

b) $R_L = R_i \Rightarrow U_0 = 2 \cdot U = 2 \cdot 25{,}3\,V = 50{,}6\,V$; $P_{max} = \dfrac{U_0^2}{4 \cdot R_i} = \dfrac{(50{,}6\,V)^2}{4 \cdot 8\,\Omega} = \mathbf{80\,W}$

$P_{L1} = P_{L2} = I_1^2 \cdot R_{L1} = (1{,}58\,A)^2 \cdot 4\,\Omega = \mathbf{10\,W}$; $P_{L3} = I_1^2 \cdot R_{L3} = (1{,}58\,A)^2 \cdot 8\,\Omega = \mathbf{20\,W}$

$P_{L4} = I_2^2 \cdot R_{L4} = (1{,}58\,A)^2 \cdot 16\,\Omega = \mathbf{40\,W}$

67/4. $R_i = \dfrac{U_0 - U}{I} = \dfrac{70\,V - 12\,V}{150\,A} = \mathbf{0{,}387\,\Omega}$; $R_L = \dfrac{U_L}{I} = \dfrac{12\,V}{150\,A} = \mathbf{0{,}08\,\Omega}$

$R_L \ll R_i$ ⇒ **Stromanpassung**

67/5. a) $U_{BO} = 12\,V$

$U_{Bi} = I \cdot R_{Bi} = 18\,A \cdot 0{,}04\,\Omega = 0{,}72\,V$

Maschenregel: $\Sigma U = 0$;

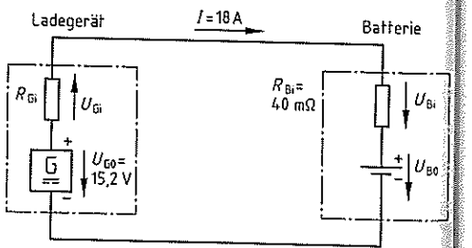
$U_{Gi} + U_{Bi} + U_{BO} - U_{GO} = 0$

$U_{Gi} = U_{GO} - U_{Bi} - U_{BO} =$

$= 15{,}2\,V - 0{,}72\,V - 12\,V = 2{,}48\,V$

$R_{Gi} = \dfrac{U_{Gi}}{I} = \dfrac{2{,}48\,V}{18\,A} = \mathbf{0{,}138\,\Omega = 137{,}8\,m\Omega}$

b) $R_{Bi} \ll R_{Gi}$ ⇒ **Stromanpassung**



zu 67/5.

67/6. a) $I = \dfrac{U_0}{R_i + R_L} = \dfrac{210\,mV}{300\,\Omega + 910\,\Omega} = 0{,}174\,mA$

$U = U_0 - I \cdot R_i = 210\,mV - 0{,}174\,mA \cdot 300\,\Omega = \mathbf{158\,mV}$

b) $P_{max} = \dfrac{U_0^2}{4 \cdot R_i} = \dfrac{(0{,}21\,V)^2}{4 \cdot 300\,\Omega} = \mathbf{36{,}75\,\mu W}$

c) $P = I^2 \cdot R_{Mic} = (0{,}174\,mA)^2 \cdot 300\,\Omega = \mathbf{9{,}1\,\mu W}$

67/7. a) $I_1 = \dfrac{U_0}{R_i + R_{L1}} = \dfrac{12\,V}{200\,\Omega + 2\,\Omega} = \mathbf{59{,}4\,mA}$

b) $I_2 = \dfrac{U_0}{R_i + R_{L2}} = \dfrac{12\,V}{200\,\Omega + 4\,\Omega} = \mathbf{58{,}8\,mA}$

c) I_1 und I_2 sind nahezu gleich (konstant) ⇒ **Konstantstromquelle**

3.11.4 Ersatzquellen

Lösungen zu 3.11.4

68/1. a) $R_i = \dfrac{R_1 \cdot R_2}{R_1 + R_2} = \dfrac{100\,\Omega \cdot 270\,\Omega}{100\,\Omega + 270\,\Omega} = 72{,}97\,\Omega \approx \mathbf{73\,\Omega}$

b) $U_{20} = U_b \cdot \dfrac{R_2}{R_1 + R_2} = 9\,V \cdot \dfrac{270\,\Omega}{100\,\Omega + 270\,\Omega} = \mathbf{6{,}57\,V}$

c) $U_L = U_{20} \cdot \dfrac{R_L}{R_i + R_L} = 6{,}57\,V \cdot \dfrac{47\,\Omega}{73\,\Omega + 47\,\Omega} = \mathbf{2{,}57\,V}$

68/2. a) $U_b = U_2 \cdot \dfrac{R_1 + R_2}{R_2} = 1{,}5\,V \cdot \dfrac{15\,k\Omega + 3{,}3\,k\Omega}{3{,}3\,k\Omega} = \mathbf{8{,}32\,V}$; $I = \dfrac{U_b - U_1}{R_1} = \dfrac{8{,}32\,V - 1{,}5\,V}{15\,k\Omega} = \mathbf{0{,}455\,mA}$

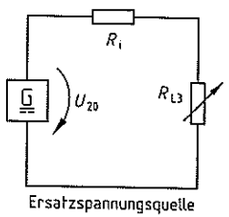
b) $R_i = \dfrac{R_1 \cdot R_2}{R_1 + R_2} = \dfrac{15\,k\Omega \cdot 3{,}3\,k\Omega}{15\,k\Omega + 3{,}3\,k\Omega} = \mathbf{2{,}70\,k\Omega}$

$I_L = I \cdot \dfrac{R_i}{R_i + R_L} = 0{,}455\,mA \cdot \dfrac{2{,}7\,k\Omega}{2{,}7\,k\Omega + 0{,}27\,k\Omega} = \mathbf{0{,}414\,mA}$

68/3. c) Leerlauf: $U_{20} = U_2 = \dfrac{U_b \cdot R_2}{R_1 + R_2} = \dfrac{10\,V \cdot 180\,\Omega}{100\,\Omega + 180\,\Omega} = \mathbf{6{,}43\,V}$

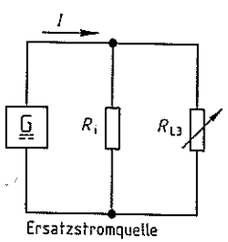
d) $R_i = \dfrac{R_1 \cdot R_2}{R_1 + R_2} = \dfrac{100\,\Omega \cdot 180\,\Omega}{100\,\Omega + 180\,\Omega} = \mathbf{64{,}3\,\Omega}$

a)



zu 68/3.a)

b)



zu 68/3.b)

68/4. a) $U_{01} = U_b \cdot \dfrac{R_3}{R_1 + R_3} = 12\,V \cdot \dfrac{470\,\Omega}{2200\,\Omega + 470\,\Omega} = \mathbf{2{,}11\,V}$

$R_{i1} = \dfrac{R_1 \cdot R_3}{R_1 + R_3} = \dfrac{2200\,\Omega \cdot 470\,\Omega}{2200\,\Omega + 470\,\Omega} = \mathbf{387\,\Omega}$

b) $U_{03} = U_b \cdot \dfrac{R_4}{R_2 + R_4} = 12\,V \cdot \dfrac{220\,\Omega}{1500\,\Omega + 220\,\Omega} = \mathbf{1{,}53\,V}$

$R_{i3} = \dfrac{R_2 \cdot R_4}{R_2 + R_4} = \dfrac{1500\,\Omega \cdot 220\,\Omega}{1500\,\Omega + 220\,\Omega} = 191{,}9\,\Omega \approx \mathbf{192\,\Omega}$

c) An der Reihenschaltung der Widerstände R_{i1}, R_L, R_{i3} liegt die Spannungsdifferenz

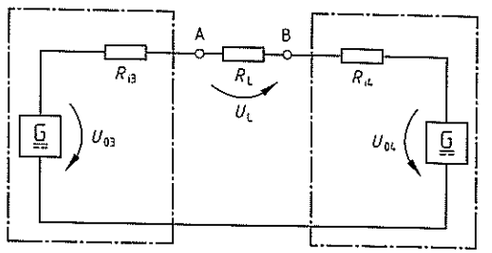
$\Delta U = U_{01} - U_{03} = 2{,}11\,V - 1{,}53\,V$

$= 0{,}58\,V$

$I_L = \dfrac{U_{01} - U_{03}}{R_{i1} + R_L + R_{i3}} =$

$= \dfrac{2{,}11\,V - 1{,}53\,V}{387\,\Omega + 120\,\Omega + 192\,\Omega} = \mathbf{0{,}83\,mA}$

$U_L = I_L \cdot R_L = 0{,}83\,mA \cdot 120\,\Omega = \mathbf{99{,}6\,mV}$



zu 68/4.c)

3.11.5 Laden und Entladen von Akkumulatoren

Lösungen zu 3.11.5

69/1. a) $t_L = a \cdot \frac{K_n}{I_L} = 1{,}2 \cdot \frac{10\,Ah}{1\,A} = \mathbf{12\,h}$

b) $t_E = \frac{K_n}{I_E} = \frac{10\,Ah}{2{,}5\,A} = \mathbf{4\,h}$

c) $K_L = I_L \cdot t_L = 1\,A \cdot 12\,h = \mathbf{12\,Ah}$

d) $K_E = I_E \cdot t_E = 2{,}5\,A \cdot 4\,h = \mathbf{10\,Ah}$

e) $\xi_{Ah} = \frac{K_E}{K_L} = \frac{10\,Ah}{12\,Ah} = \mathbf{0{,}83}$

f) $W_E = U_E \cdot I_E \cdot t_E = 6{,}3\,V \cdot 2{,}5\,A \cdot 4\,h = \mathbf{63\,Wh}$

g) $W_L = U_L \cdot I_L \cdot t_L = 7{,}05\,V \cdot 1\,A \cdot 12\,h = \mathbf{84{,}6\,Wh}$

h) $\xi_{Wh} = \frac{W_E}{W_L} = \frac{63\,Wh}{84{,}6\,Wh} = \mathbf{0{,}74}$

69/2. $K_E = 0{,}8 \cdot K_N = 0{,}8 \cdot 19{,}5\,Ah = 15{,}6\,Ah$

$K_E = I_E \cdot t_E \Rightarrow t_E = \frac{K_E}{I_E} = \frac{15{,}6\,Ah}{0{,}06\,A} = \mathbf{260\,h}$

69/3. a) $I_L = 0{,}5 \cdot \frac{1}{h} \cdot K_n = \frac{0{,}5 \cdot 1 \cdot 2000\,mAh}{h} = \mathbf{1000\,mA = 1\,A}$

b) $t_L = a \cdot \frac{K_n}{I_L} = 1{,}4 \cdot \frac{2000\,mAh}{1000\,mA} = \mathbf{2{,}8\,h}$

c) $I_E = 10 \cdot \frac{1}{h} \cdot K_n = \frac{10 \cdot 1 \cdot 2000\,mAh}{h} = 20\,000\,mA = \mathbf{20\,A}$

d) $t_E = \frac{K_n}{I_E} = \frac{2000\,mAh}{20\,000\,mA} = \mathbf{0{,}1\,h}$

e) $K_E = I_E \cdot t_E = 20\,A \cdot 0{,}1\,h = 2\,Ah$

$K_L = I_L \cdot t_L = 1\,A \cdot 2{,}8\,h = 2{,}8\,Ah$

$\xi_{Ah} = \frac{K_E}{K_L} = \frac{2\,Ah}{2{,}8\,Ah} = \mathbf{0{,}71}$

69/4. a) $I_L = 1 \cdot \frac{1}{h} \cdot K_n = \frac{1 \cdot 1 \cdot 6600\,mAh}{h} = 6600\,mA = 6{,}6\,A$

$t_L = a \cdot \frac{K_n}{I_L} = 1{,}04 \cdot \frac{6600\,mAh}{6600\,mA} = \mathbf{1{,}04\,h}$

b) $I_E = 0{,}2 \cdot \frac{1}{h} \cdot K_n = \frac{0{,}2 \cdot 1 \cdot 6600\,mAh}{h} = 1320\,mA = 1{,}32\,A$

$t_E = \frac{K_n}{I_E} = \frac{6600\,mAh}{1320\,mA} = \mathbf{5\,h}$

c) $K_E = I_E \cdot t_E = 1{,}32\,A \cdot 5\,h = 6{,}6\,Ah$

$K_L = I_L \cdot t_L = 6{,}6\,A \cdot 1{,}04\,h = 6{,}86\,Ah$

$\xi_{Ah} = \frac{K_E}{K_L} = \frac{6{,}6\,Ah}{6{,}86\,Ah} = \mathbf{0{,}96}$

3.11.6 Fotovoltaik und Solarmodul

Lösungen zu 3.11.6

70/1. a) aus Kennlinie:

$E = 1000\,W/m^2$: $U_{MPP} = 17{,}8\,V$, $I_{MPP} = 4{,}7\,A$

$E = 400\,W/m^2$: $U_{MPP} = 17{,}4\,V$, $I_{MPP} = 1{,}8\,A$

Beleuchtungsstärke $1000\,W/m^2$: $P_{MPP} = U_{MPP} \cdot I_{MPP} = 17{,}8\,V \cdot 4{,}7\,A = \mathbf{83{,}66\,W}$

$400\,W/m^2$: $P_{MPP} = 17{,}4\,V \cdot 1{,}8\,A = \mathbf{31{,}32\,W}$

b) Beleuchtungsstärke $1000\,W/m^2$: $P_{St} = E \cdot A = 1000\,\frac{W}{m^2} \cdot 0{,}98\,m^2 = \mathbf{980\,W}$

$400\,W/m^2$: $P_{St} = 400\,\frac{W}{m^2} \cdot 0{,}98\,m^2 = \mathbf{392\,W}$

c) Beleuchtungsstärke $1000\,W/m^2$: $\eta_{MPP} = \frac{P_{MPP}}{P_{St}} = \frac{83{,}66\,W}{980\,W} = 0{,}085 \triangleq \mathbf{8{,}5\,\%}$

$400\,W/m^2$: $\eta_{MPP} = \frac{31{,}32\,W}{392\,W} = 0{,}08 \triangleq \mathbf{8\,\%}$

d) aus Kennlinie:

$E = 1000\,W/m^2$: $U_0 = 22{,}9\,V$, $I_K = 5{,}3\,A$

$E = 400\,W/m^2$: $U_0 = 21\,V$, $I_K = 2{,}3\,A$

Beleuchtungsstärke $1000\,W/m^2$: $FF = \frac{P_{MPP}}{U_0 \cdot I_K} = \frac{83{,}66\,W}{22{,}9\,V \cdot 5{,}3\,A} = \mathbf{0{,}69}$

$400\,W/m^2$: $FF = \frac{31{,}32\,W}{21\,V \cdot 2{,}3\,A} = \mathbf{0{,}65}$

71/2. a) Leistungsanpassung $R_i = R_L$

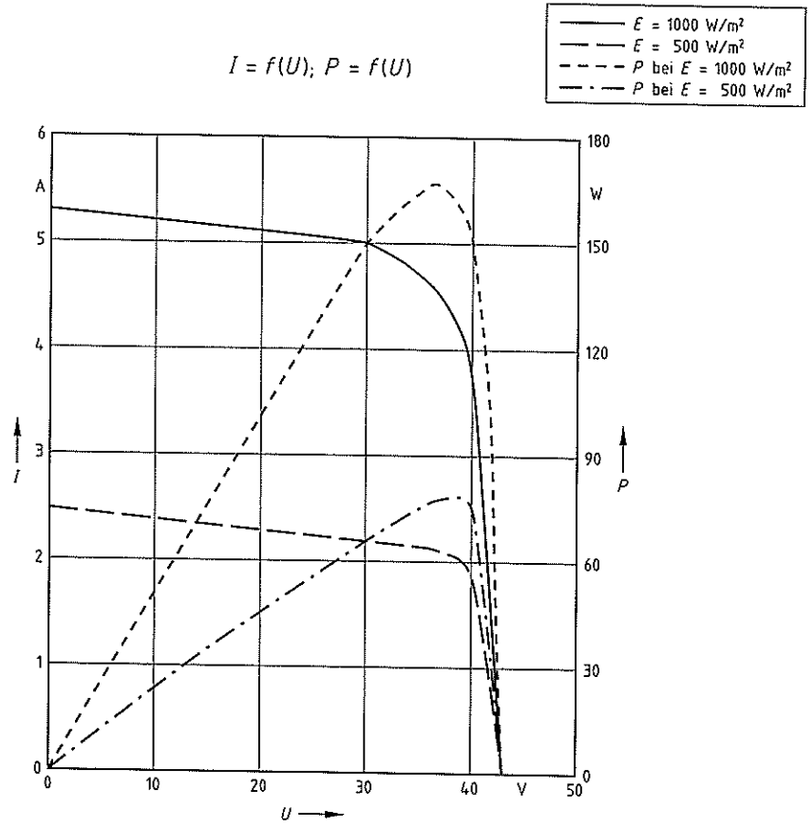
aus Kennlinie: $E = 600\,W/m^2$: $U_{MPP} = 17{,}6\,V$, $I_{MPP} = 2{,}8\,A$

$R_i = R_L = \frac{U_{MPP}}{I_{MPP}} = \frac{17{,}6\,V}{2{,}8\,A} = \mathbf{6{,}29\,\Omega}$

b) aus Kennlinie: $E = 200\,W/m^2$: $U_{MPP} = 16{,}5\,V$, $I_{MPP} = 0{,}9\,A$

$R_i = R_L = \frac{U_{MPP}}{I_{MPP}} = \frac{16{,}5\,V}{0{,}9\,A} = \mathbf{18{,}33\,\Omega}$

71/3.



71/3.a) und 3.b)

b) $P = U \cdot I$

E = 1000 W/m²								
U in V	0	10	20	30	35	37	40	43
I in A	5,3	5,2	5,1	5,0	4,7	4,5	3,8	0
P in W	0	52	102	150	164,5	166,5	152	0
E = 500 W/m²								
U in V	0	10	20	30	35	37	40	43
I in A	2,5	2,4	2,3	2,2	2,15	2,1	1,9	0
P in W	0	24	46	66	75,25	77,7	76	0

71/4. a) $R_i = \frac{\Delta U}{\Delta I} = \frac{37\,V - 35\,V}{4{,}7\,A - 4{,}5\,A} = \mathbf{10\,\Omega}$

b) $R_i = \frac{\Delta U}{\Delta I} = \frac{37\,V - 35\,V}{2{,}15\,A - 2{,}1\,A} = \mathbf{40\,\Omega}$

c) bei Leistungsanpassung $R_L = R_i$

⇒ bei E = 1000 W/m²: $R_L = R_i = \mathbf{10\,\Omega}$

bei E = 500 W/m²: $R_L = R_i = \mathbf{40\,\Omega}$

71/5. a) $P = U \cdot I$

E = 1000 W/m² (STC)								
U in V	0	10	20	30	35	37	40	43
I in A	5,3	5,2	5,1	5,0	4,7	4,5	3,8	0
P in W	0	52	102	150	164,5	166,5	152	0

aus Tabelle P in W: $P_{MPP} = \mathbf{166{,}5\,W}$

b) $P = U \cdot I$

E = 500 W/m²								
U in V	0	10	20	30	35	37	40	43
I in A	2,5	2,4	2,3	2,2	2,15	2,1	1,9	0
P in W	0	24	46	66	75,25	77,7	76	0

aus Tabelle P in W: $P_{MPP} = \mathbf{77{,}7\,W}$

c) $R_i = R_L = \frac{U_{MPP}}{I_{MPP}} = \frac{37\,V}{2{,}1\,A} = \mathbf{17{,}62\,\Omega}$

71/6. a) $FF = \frac{U_{MPP} \cdot I_{MPP}}{U_0 \cdot I_K} = \frac{17{,}5\,V \cdot 7{,}14\,A}{21{,}65\,V \cdot 7{,}69\,A} = \mathbf{0{,}75}$

b) $P_{FF} = U_0 \cdot I_K = 21{,}65\,V \cdot 7{,}69\,A = \mathbf{166{,}49\,W}$

c) $T_1 = \left(273{,}15 + \frac{\vartheta}{°C}\right) K = (273{,}15 + 60)\,K = 333{,}15\,K$

$T_2 = \left(273{,}15 + \frac{\vartheta}{°C}\right) K = (273{,}15 + 25)\,K = 298{,}15\,K$

$\Delta T = T_1 - T_2 = 333{,}15\,K - 298{,}15\,K = 35\,K$

$I_{K\vartheta} = I_K + \alpha_I \cdot \Delta T = 7{,}69\,A + \left(5{,}45\,\frac{mA}{K} \cdot 35\,K\right) = 7{,}88\,A$

$U_{0\vartheta} = U_0 + \alpha_U \cdot \Delta T = 21{,}65\,V + \left(-72\,\frac{mV}{K} \cdot 35\,K\right) = 19{,}13\,V$

$P_{FF\vartheta} = U_{0\vartheta} \cdot I_{K\vartheta} = 19{,}13\,V \cdot 7{,}88\,A = \mathbf{150{,}74\,W}$

d) $T_1 = \left(273{,}15 + \frac{\vartheta}{°C}\right) K = (273{,}15 + 40)\,K = 313{,}15\,K$

$T_2 = \left(273{,}15 + \frac{\vartheta}{°C}\right) K = (273{,}15 + 25)\,K = 298{,}15\,K$

$\Delta T = T_1 - T_2 = 313{,}15\,K - 298{,}15\,K = 15\,K$

$P_{MPP} = U_{MPP} \cdot I_{MPP} = 17{,}5\,V \cdot 7{,}14\,A = 124{,}95\,W$

$P_{MPP\vartheta} = P_{MPP} + \alpha_P \cdot \Delta T = 124{,}95\,W + \left(-440\,\frac{mW}{K} \cdot 15\,K\right) = \mathbf{118{,}35\,W}$

71/7. a) $FF = \frac{U_{MPP} \cdot I_{MPP}}{U_0 \cdot I_K} = \frac{16{,}02\,V \cdot 5{,}86\,A}{19{,}9\,V \cdot 6{,}28\,A} = \mathbf{0{,}75}$

b) $T_1 = \left(273{,}15 + \frac{\vartheta}{°C}\right) K = (273{,}15 + 50)\,K = 323{,}15\,K$

$T_2 = \left(273{,}15 + \frac{\vartheta}{°C}\right) K = (273{,}15 + 25)\,K = 298{,}15\,K$

$\Delta T = T_1 - T_2 = 323{,}15\,K - 298{,}15\,K = 25\,K$

$P_{MPP} = U_{MPP} \cdot I_{MPP} = 16{,}02\,V \cdot 5{,}86\,A = 93{,}88\,W$

$P_{MPP\vartheta} = P_{MPP} + \alpha_P \cdot \Delta T = 93{,}88\,W + \left(-440\,\frac{mW}{K} \cdot 25\,K\right) = \mathbf{82{,}88\,W}$

71/8. a) $U_{max} = U_0 \cdot n \Rightarrow n = \frac{U_{max}}{U_0} = \frac{850\,V}{21{,}65\,V} = 39{,}26$

Es können maximal 39 Module an den Wechselrichter angeschlossen werden.

b) $U_{min} = U_{MPP} \cdot n \Rightarrow n = \frac{U_{min}}{U_{MPP}} = \frac{180\,V}{17{,}5\,V} = 10{,}29$

Es sollen minimal 11 Module an den Wechselrichter angeschlossen werden.

4 Arbeiten mit Kennlinien

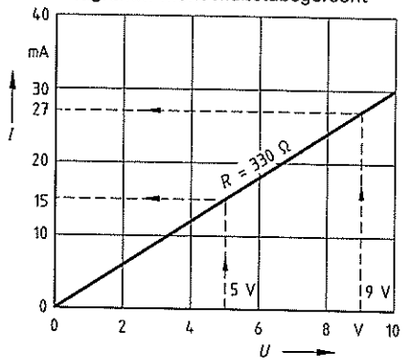
4.1 Lineare Widerstände

Lösungen zu 4.1

72/1. a) Maßstab: 1 cm ≙ 1 V
1 cm ≙ 5 mA

b) 5 V ⇒ 15 mA ⇒ R = 333 Ω
9 V ⇒ 27 mA ⇒ R = 333 Ω

Diagramm nicht maßstabsgerecht



zu 72/1.a)

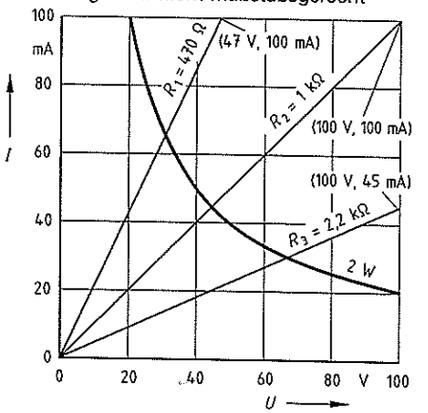
72/2. a) Maßstab: 1 cm ≙ 10 V
1 cm ≙ 10 mA

b) Leistungshyperbel/Wertetabelle $P_{tot} = 2\,W$

U in V	100	80	60	50	40	25	20
I in mA	20	25	33	40	50	80	100

c) $R_1 \Rightarrow U_{max} = 30\,V$
$I_{max} = 65\,mA$
$R_2 \Rightarrow U_{max} = 45\,V$
$I_{max} = 45\,mA$
$R_3 \Rightarrow U_{max} = 67\,V$
$I_{max} = 30\,mA$

Diagramm nicht maßstabsgerecht



zu 72/2.a)

72/3. a) R_1; U = 10 V, I = 100 mA ⇒ $R_1 = 100\,\Omega$
R_2; U = 20 V, I = 90 mA ⇒ $R_2 = 222\,\Omega$
R_3; U = 40 V, I = 82 mA ⇒ $R_3 = 488\,\Omega$
R_4; U = 60 V, I = 60 mA ⇒ $R_4 = 1{,}0\ k\Omega$
R_5; U = 100 V, I = 45 mA ⇒ $R_5 = 2{,}22\,k\Omega$
R_6; U = 100 V, I = 21 mA ⇒ $R_6 = 4{,}76\,k\Omega$

b) Widerstände nach E12
$R_1 = 100\,\Omega$, $R_2 = 220\,\Omega$, $R_3 = 470\,\Omega$,
$R_4 = 1\,k\Omega$, $R_5 = 2{,}2\,k\Omega$, $R_6 = 4{,}7\,k\Omega$

c) $U_{max} = 64\,V$, $I_{max} = 31\,mA$;
$P_{tot} = U_{max} \cdot I_{max} = 64\,V \cdot 0{,}031\,A \Rightarrow P_{tot} = 2\,W$

72/4. a) U = 12 V, I = 25 mA; $R = \frac{U}{I} = \frac{12\,V}{25\,mA} = 480\,\Omega$

b) $R_{min} = \frac{12\,V}{32\,mA} = 375\,\Omega$; $R_{max} = \frac{12\,V}{21\,mA} = 571\,\Omega$;

$\Delta R' = R_{min} - R \approx -100\,\Omega$; $\Delta R = R_{max} - R \approx 100\,\Omega$

$\Delta R\,\% = \pm \frac{100\,\Omega \cdot 100\%}{480\,\Omega}$; $\Delta R\,\% = \pm 21\,\% \approx \pm 20\%$

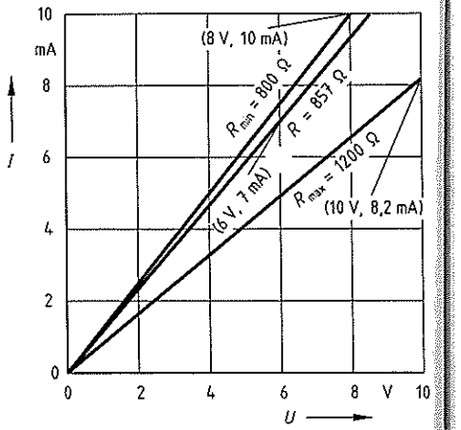
c) 20% Toleranz ⇒ E6; Gewählt: $R = 470\,\Omega$ ⇒ 3 Farbringe gelb, violett, braun

d) $P_V = \frac{U^2}{R} = \frac{20^2\,V^2}{480\,\Omega} = 833\,mW$

72/5. a) Farbringe braun, schwarz, rot
$\Rightarrow R = 1000\,\Omega \pm 20\%$
$20\% \triangleq 200\,\Omega$;
$R_{min} = 1000\,\Omega - 200\,\Omega = 800\,\Omega$;
$R_{max} = 1000\,\Omega + 200\,\Omega = 1200\,\Omega$

b) U-I-Diagramm der Widerstandsgeraden für R_{min}, R_{max} und $R = 6\,V/7\,mA$

c) Der gemessene Widerstand liegt im Toleranzbereich.



zu 72/5.b)

4.2 Logarithmische Darstellung

Lösungen zu 4.2

73/1.o. Aus Kennlinie:
a) C 990: $U_1 \Rightarrow I_1 \approx 13\,mA$
$U_2 \Rightarrow I_2 \approx 14\,mA$
b) C 960: $U_1 \Rightarrow I_1 \approx 120\,mA$
$U_2 \Rightarrow I_2 \approx 45\,mA$

73/2.o. Aus Kennlinie:
a) C 960: $I \approx 320\,mA$; $U \approx 2\,V$; $\Rightarrow R \approx 6\,\Omega$
b) C 990: $I \approx 65\,mA$; $U \approx 4\,V$; $\Rightarrow R \approx 61\,\Omega$

4.3 Nichtlineare Widerstände

Lösungen zu 4.3

73/1.u.

Temperatur	−20°C	0°C	20°C	40°C	60°C	80°C	100°C	120°C
Widerstand	1 MΩ	350 kΩ	120 kΩ	50 kΩ	20 kΩ	10 kΩ	5 kΩ	2,5 kΩ

73/2.u.

Widerstand	100 Ω	600 Ω	2 kΩ	10 kΩ
Temperatur	130°C	60°C	30°C	−8°C

74/3. a) Aus Kennlinie Bild 1: $\Rightarrow R = 1{,}1\,M\Omega$ b) $I = \frac{U}{R} = \frac{65\,V}{1{,}1\,M\Omega} = 60\,\mu A$

c) $U_1 = 80\,V$; $\Rightarrow R_1 \approx 200\,k\Omega \Rightarrow I_1 = \frac{U_1}{R_1} \approx \frac{80\,V}{200\,k\Omega} = 400\,\mu A$

$U_2 = 100\,V$; $\Rightarrow R_2 \approx 10\,k\Omega \Rightarrow I_2 = \frac{U_2}{R_2} \approx \frac{100\,V}{10\,k\Omega} = 10\,mA$

$U_3 = 150\,V$; $\Rightarrow R_3 \approx 3\,\Omega \Rightarrow I_3 = \frac{U_3}{R_3} \approx \frac{150\,V}{3\,\Omega} = 50\,A$

74/4. a) Maßstab: 1 cm ≙ 5 V; 1 cm ≙ 2 mA

b) $R_1 = \frac{U_1}{I_1} = \frac{14\,V}{10\,\mu A} = 1{,}4\,M\Omega$;

$R_2 = \frac{U_2}{I_2} = \frac{18\,V}{100\,\mu A} = 180\,k\Omega$;

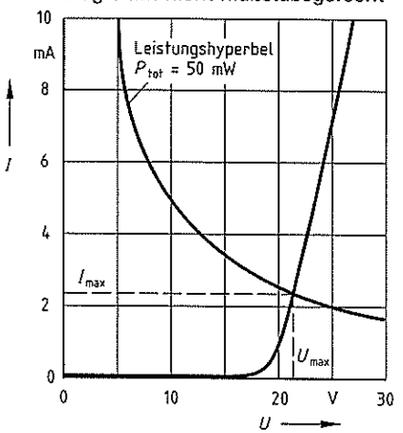
$R_3 = 20\,k\Omega$; $R_4 = 5{,}6\,k\Omega$; $R_5 = 4{,}2\,k\Omega$; $R_6 = 2{,}7\,k\Omega$

c) **Leistungshyperbel**
Wertetabelle $P_{tot} = 50\,mW$

U in V	30	25	20	15	10	5
I in mA	1,7	2	2,5	3,3	5	10

d) $U_{max} = 22\,V$, $I_{max} = 2{,}3\,mA$

Diagramm nicht maßstabsgerecht



zu 74/4.a)

74/5. a) $\vartheta = 40°C$ aus Kennlinie $R_{40} \approx 10\,k\Omega$; $I = \frac{U}{R_{40}} = \frac{6\,V}{10\,k\Omega} = 0{,}6\,mA$

b) $I_2 = 5 \cdot I_1 = 5 \cdot 0{,}6\,mA = 3\,mA$; $R_\vartheta = \frac{U}{I_2} = \frac{6\,V}{3\,mA} = 2\,k\Omega$
aus Kennlinie bei $R_\vartheta = 2\,k\Omega \Rightarrow \vartheta \approx 83°C$

74/6. a) bei 25°C: $I = \frac{U_1}{R_1} = \frac{10{,}2\,V}{2{,}2\,k\Omega} = 4{,}6\,mA$; $R_2 = \frac{U - U_1}{I} = \frac{12\,V - 10{,}2\,V}{4{,}6\,mA} = 388\,\Omega \Rightarrow R_2 = 390\,\Omega$
$\vartheta = 50°C \Rightarrow R_{50} \approx 1\,k\Omega$
bei 50°C: $I = \frac{U}{R_{50} + R_2} = \frac{12\,V}{1\,k\Omega + 390\,\Omega} = 8{,}6\,mA$; $U_1 = I \cdot R_{50} = 8{,}6\,mA \cdot 1000\,\Omega = 8{,}6\,V$

b) $\vartheta = 80°C \Rightarrow R_{80} \approx 330\,\Omega$; $I = \frac{U}{R_{80} + R_2} = \frac{12\,V}{330\,\Omega + 390\,\Omega} = 16{,}7\,mA$
$P_V = I^2 \cdot R_{80} = (16{,}7\,mA)^2 \cdot 330\,\Omega = 92\,mW$; $P_V < P_{tot} = 200\,mW \Rightarrow$ keine Überlastung

74/7. a) 60°C: $R_{60} = 200\,\Omega$; 80°C: $R_{80} = 1300\,\Omega$; $\Delta R = R_{80} - R_{60} = 1300\,\Omega - 200\,\Omega = 1100\,\Omega$

b) 60°C: $R_{60} = 200\,\Omega$; 100°C: $R_{100} = 150000\,\Omega$
$\Delta R = R_{100} - R_{60} = 150000\,\Omega - 200\,\Omega \approx 150\,k\Omega$

74/8. a) Aus Kennlinie: $R_{50} \approx 120\,\Omega$; $R_{100} \approx 150\,k\Omega$

b) $I_1 = \frac{U}{R_{50}} = \frac{1{,}2\,V}{120\,\Omega} = 10\,mA$; $I_2 = \frac{U}{R_{100}} = \frac{1{,}2\,V}{150\,k\Omega} = 8\,\mu A$

c) $R_{60} \approx 200\,\Omega$; $R_{80} \approx 1{,}3\,k\Omega$; $n = \frac{R_{80}}{R_{60}} = \frac{1{,}3\,k\Omega}{200\,\Omega} = 6{,}5$

74/9. a) $\vartheta = 80°C$ aus Kennlinie $R_{80} \approx 200\,\Omega$
$R_E = \frac{R_{80} \cdot R_2}{R_{80} + R_2} = \frac{200\,\Omega \cdot 2200\,\Omega}{200\,\Omega + 2200\,\Omega} = 183\,\Omega$; $I = \frac{U_b}{R_E} = \frac{6\,V}{183\,\Omega} = 33\,mA$

b) $\vartheta = 100°C$; $R_{100} \approx 2\,k\Omega \Rightarrow R_E = 1\,k\Omega$; $I = \frac{U_b}{\quad} = \frac{6\,V}{\quad} = 6\,mA$

4.4 Ermittlung des Arbeitspunktes

4.4.1 Reihenschaltung linearer Widerstände

Lösungen zu 4.4.1

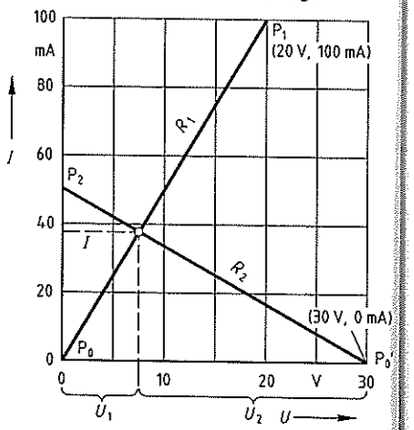
75/1. a) Maßstab: 1 cm ≙ 5 V
1 cm ≙ 20 mA

1. Widerstandsgerade R_1
R1 ⇒ Ursprungsgerade
P_0 (0 V, 0 mA); P_1 (20 V, 100 mA)

2. Arbeitsgerade für R_2
$P_0' \triangleq U_b = 30$ V; $P_2 \triangleq \frac{U_b}{R_2} = \frac{30\text{ V}}{600\ \Omega} = 50$ mA
I = 37 mA

b) $U_1 = 7{,}5$ V; $U_2 = 22{,}5$ V

Diagramm nicht maßstabsgerecht



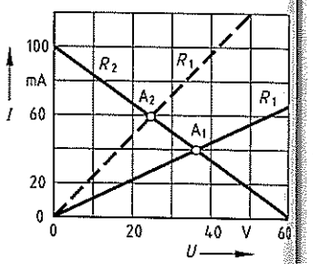
zu 75/1.a)

75/2. a) Aus der Kennlinie Arbeitspunkt A_1
I = 40 mA

b) $U_1 = 36$ V; $U_2 = 24$ V

c) Arbeitspunkt A_2 ⇒ 60 mA ⇒ 24 V
$R_1 = \frac{U}{I} = \frac{24\text{ V}}{60\text{ mA}} = 400\ \Omega$

d) R_1: $P_V = I \cdot U_1 = 60\text{ mA} \cdot 24\text{ V} = 1{,}44$ W
⇒ **keine Überlastung**
R_2: $P_V = I \cdot U_2 = 60\text{ mA} \cdot 36\text{ V} = 2{,}16$ W
⇒ **Überlastung**



zu 75/2.a)

75/3. a) Maßstab: 1 cm ≙ 0,2 V; 1 cm ≙ 0,5 A

1. Arbeitsgerade für R_i
$P_0' \triangleq U_b = 1{,}5$ V (0,75 V)
$P_2 = \frac{U_b}{R_i} = 5$ A (2,5 A)
Hilfsgerade verwenden, danach Hilfsgerade parallel verschieben an P_0'
$U_{L1} = 1{,}34$ V; $U_{L2} = 1{,}20$ V

b) $R_1 = \frac{U_{L1}}{I_1} = \frac{1{,}34\text{ V}}{0{,}5\text{ A}} = 2{,}68\ \Omega$

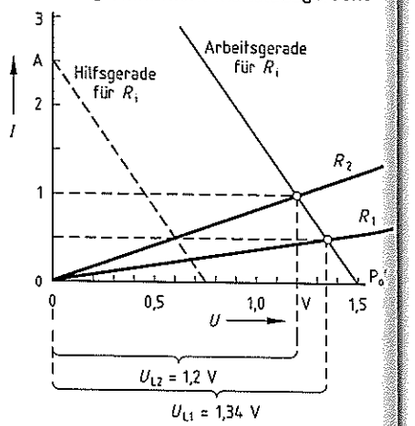
$R_2 = \frac{U_{L2}}{I_2} = \frac{1{,}20\text{ V}}{1\text{ A}} = 1{,}20\ \Omega$

c) $U_{i1} = 160$ mV; $U_{i2} = 300$ mV

d) $I_k = \frac{U_0}{R_i} = \frac{1{,}5\text{ V}}{0{,}3\ \Omega} = 5$ A

e) Der Konstruktionspunkt P_2 der Arbeitsgeraden R_i entspricht dem Kurzschlussstrom I_k

Diagramm nicht maßstabsgerecht



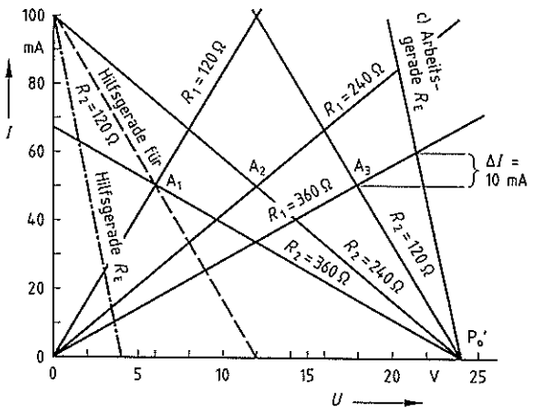
zu 75/3.a)

75/4. a) 1. $R_1 = 120\ \Omega \Rightarrow R_2 = 360\ \Omega$
(12 V, 100 mA) ($P_0' \triangleq U_b = 24$ V,
$P_2 \triangleq \frac{U_b}{R_2} = \frac{24\text{ V}}{360\ \Omega} = 67$ mA)

2. $R_1 = 240\ \Omega \Rightarrow R_2 = 240\ \Omega$
(18 V, 75 mA)
($P_0' \triangleq 24$ V, $P_2 \triangleq 100$ mA)

3. $R_1 = 360\ \Omega \Rightarrow R_2 = 120\ \Omega$
(18 V, 50 mA) ($P_0' \triangleq 24$ V; 12 V),
$P_2 \triangleq 200$ mA (100 mA))

b) A_1: $U_1 = 6$ V, $U_2 = 18$ V
A_2: $U_1 = 12$ V, $U_2 = 12$ V
A_3: $U_1 = 18$ V, $U_2 = 6$ V
} $I_1 = 50$ mA



zu 75/4.a)

c) $R_E = \frac{R_2 \cdot R_L}{R_2 + R_L} = \frac{120\ \Omega \cdot 60\ \Omega}{120\ \Omega + 60\ \Omega} = 40\ \Omega$
$R_1 = 360\ \Omega$; $R_E = 40\ \Omega$
($P_0' \triangleq 24$ V, $P_2 \triangleq \frac{U_b}{R_E} = 600$ mA)
Hilfsgerade $P_0' \triangleq 4$ V, $P_2 \triangleq 100$ mA

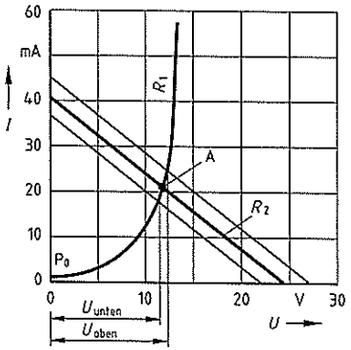
d) $U_1 = 21{,}6$ V, $U_2 = 2{,}4$ V

4.4.2 Reihenschaltung linearer und nichtlinearer Widerstände

Lösungen zu 4.4.2

76/1. a) Aus Kennlinie:
$U_{oben} = 12$ V
$U_{unten} = 11$ V

b) $\Delta U = \pm \frac{U_{oben} - U_{unten}}{2} = \pm \frac{12\text{ V} - 11\text{ V}}{2} = \pm 0{,}5$ V



zu 76/1.a)

76/2. a) NTC 891/20 k; $R_{25} = 20$ kΩ

$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow R_2 = \frac{R_1 \cdot R_4}{R_3} = \frac{20\text{ k}\Omega \cdot 33\text{ k}\Omega}{22\text{ k}\Omega} = 30\text{ k}\Omega$

$\vartheta = 40$ °C: ⇒ $R_{40} = 10$ kΩ (aus Kennlinie); $I_{1,2} = \frac{U_b}{R_{40} + R_2} = \frac{6\text{ V}}{10\text{ k}\Omega + 30\text{ k}\Omega} = 0{,}15$ mA

$U_1 = I_{1,2} \cdot R_{40} = 0{,}15\text{ mA} \cdot 10\text{ k}\Omega = 1{,}5$ V; $U_2 = 4{,}5$ V;

$I_{3,4} = \frac{U_b}{R_3 + R_4} = \frac{6\text{ V}}{22\text{ k}\Omega + 33\text{ k}\Omega} = 0{,}11$ mA

$U_3 = I_{3,4} \cdot R_3 = 0{,}11\text{ mA} \cdot 22\text{ k}\Omega = 2{,}4$ V;

$U_4 = 3{,}6$ V

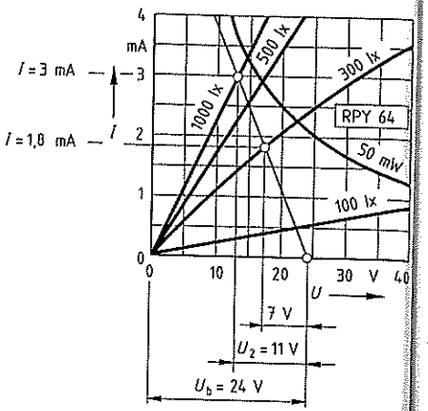
Maschenregel: $U_1 + U_{AB} - U_3 = 0$

⇒ $U_{AB} = U_3 - U_1$;

$U_{AB} = 2{,}4\text{ V} - 1{,}5 = +0{,}9$ V

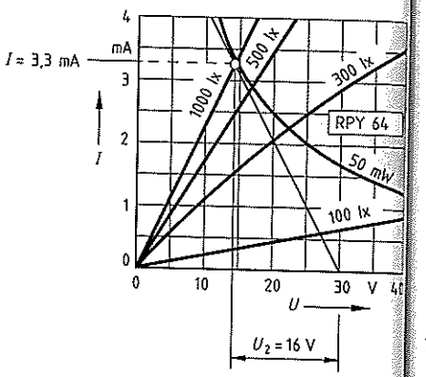
b) $\vartheta = 0\,°C$: $\Rightarrow R_0 = 60\,k\Omega$ (aus Kennlinie);
$I_{1,2} = 0{,}067\,mA \Rightarrow U_1 = 4\,V$; $U_2 = 2\,V$
$U_{AB} = U_3 - U_1 = 2{,}4\,V - 4\,V = -1{,}6\,V$

76/3. a) Aus Kennlinie: $E = 1000\,lx$
b) Aus Kennlinie: Relais fällt ab bei $E = 300\,lx$
c) $U_2 = 7\,V$



zu 76/3.a)

76/4. a) siehe 76/4.a)
b) $E = 1000\,lx$
c) $U_2 = 16\,V$; $I = 3{,}3\,mA$; $R_2 = 4{,}85\,k\Omega$
d) $E = 300\,lx$; $U_2 = 10\,V$; $I_2 = 2{,}1\,mA$
$E = 100\,lx$; $U_2 = 3\,V$; $I_2 = 0{,}6\,mA$



zu 76/4.a)

76/5. a) aus Kennlinie bei $B = 1\,T \Rightarrow \frac{R_B}{R_0} = 15$

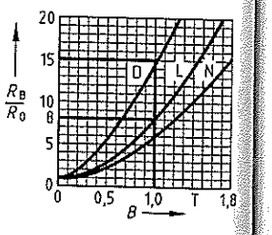
$R_B = 15 \cdot R_0 = 15 \cdot 250\,\Omega$

$R_B = 3{,}75\,k\Omega$

$$\frac{U_2}{U_b} = \frac{R_B}{R_V + R_B}$$

$$\Rightarrow U_2 = \frac{R_B}{R_V + R_B} \cdot U_b = \frac{3{,}75\,k\Omega}{2{,}2\,k\Omega + 3{,}75\,k\Omega} \cdot 12\,V = 7{,}6\,V$$

b) aus Kennlinie bei $B = 1\,T$; $\frac{R_B}{R_0} = 8 \Rightarrow R_B = 1{,}6\,k\Omega$; $U_2 = 5{,}0\,V$



zu 76/5.a)

4.5 Statischer und differentieller Widerstand

Lösungen zu 4.5

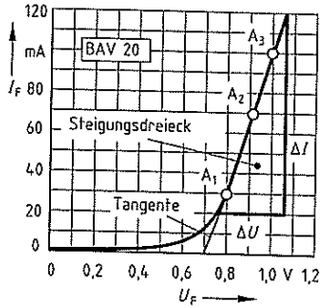
77/1. a) Aus Kennlinie
A_1: $U_{F1} = 0{,}8\,V$; $I_{F1} = 30\,mA$

$$R_F = \frac{U_F}{I_F} = \frac{0{,}8\,V}{30\,mA} = 27\,\Omega$$

A_2: $U_{F2} = 0{,}92\,V$; $I_{F2} = 70\,mA$
$R_F = 13\,\Omega$

b) A_1: $r_{F1} = \frac{0{,}85\,V - 0{,}75\,V}{50\,mA - 20\,mA} = 3{,}3\,\Omega$

A_2: $r_{F2} = \frac{1{,}0\,V - 0{,}8\,V}{100\,mA - 30\,mA} = 2{,}9\,\Omega$



zu 77/1.a)

77/2. $r = \frac{\Delta U_F}{\Delta I_F} \Rightarrow \Delta I_F = \frac{\Delta U_F}{r} = \frac{0{,}1\,V}{20\,\Omega} = 5\,mA$

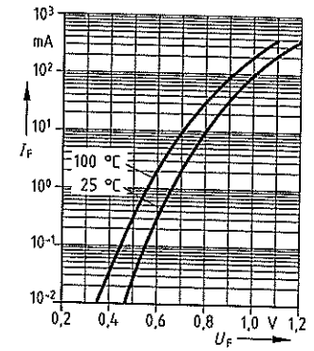
77/3. a) Aus Kennlinie für $\vartheta_u = 25\,°C$
$U_{F1} = 0{,}5\,V \Rightarrow I_{F1} \approx 25\,\mu A$

$$R_{F1} = \frac{U_{F1}}{I_{F1}} = \frac{0{,}5\,V}{25\,\mu A} \approx 20\,k\Omega$$

$U_{F2} = 0{,}8\,V \Rightarrow I_{F2} \approx 10\,mA$; $R_{F2} \approx 80\,\Omega$
$U_{F2} = 1{,}0\,V \Rightarrow I_{F3} \approx 100\,mA$; $R_{F3} \approx 10\,\Omega$

Aus Kennlinie für 100 °C
$U_{F1} = 0{,}5\,V \Rightarrow I_{F1} \approx 0{,}3\,mA \Rightarrow R_{F1} \approx 1{,}67\,k\Omega$
$U_{F2} = 0{,}8\,V \Rightarrow I_{F2} \approx 35\,mA \Rightarrow R_{F2} \approx 23\,\Omega$
$U_{F3} = 1{,}0\,V \Rightarrow I_{F3} \approx 200\,mA \Rightarrow R_{F3} \approx 5\,\Omega$

b) $r_F \approx \frac{1{,}0\,V - 0{,}9\,V}{200\,mA - 100\,mA} = 1\,\Omega$



zu 77/3.a)

77/4. a) Maßstab: 1 cm ≙ 100 mV
1 cm ≙ 20 mA

b) Wertetabelle, Leistungshyperbel $P_{tot} = 250\,mW$

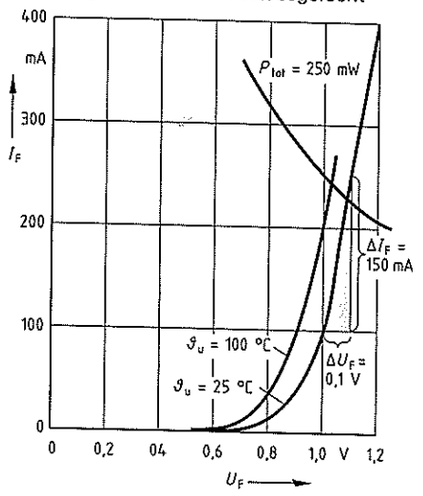
U in V	0,7	0,8	0,9	1,0	1,1	1,2	1,25
I in mA	357	312	278	250	227	208	200

c) $\vartheta_u = 25\,°C$; $U_{max} \approx 1{,}08\,V$; $I_{max} \approx 230\,mA$
$\vartheta_u = 100\,°C$; $U_{max} \approx 1{,}04\,V$; $I_{max} \approx 240\,mA$

d) 25 °C: $R_F = \frac{U_{max}}{I_{max}} = \frac{1{,}08\,V}{230\,mA}$; $R_F = 4{,}7\,\Omega$
100 °C: $R_F = 4{,}3\,\Omega$

e) $r_F = \frac{\Delta U_F}{\Delta I_F} = \frac{1{,}1\,V - 1{,}0\,V}{250\,mA - 100\,mA} = \frac{0{,}1\,V}{150\,mA}$
$r_F = 0{,}67\,\Omega$

Diagramm nicht maßstabsgerecht



zu 77/4.a)