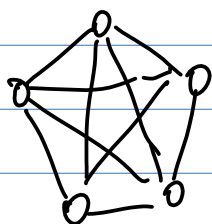


K_5

$|V| = 5$

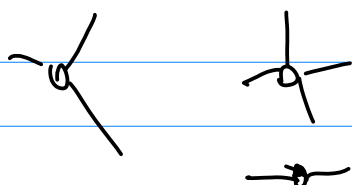
$|E| = \binom{5}{2} = 10$



$e \leq 3n - 6$

$$\underline{10} \leq 15 - 6 = 9$$

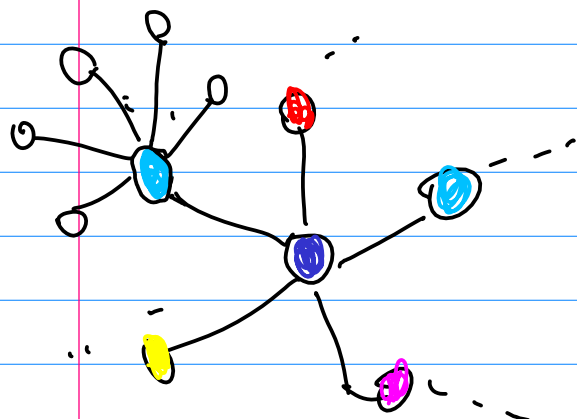
α



$\sum d = 2e$

$d_{avg} = \frac{2e}{n}$

$\frac{2e}{n} \leq \frac{n-6}{n} - \frac{12}{n}$

 $K_{3,3}$ $|V| = 6$

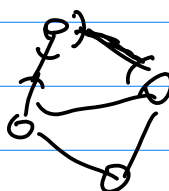
$|E| = 9 = 3 \cdot 3$

 $K_{3,3}?$

$g \leq 3 \cdot 6 - 6 = 12$

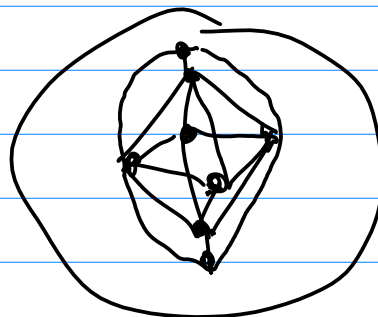
$e \leq 2n - 4$

$g \leq 12 - 4 = 8$

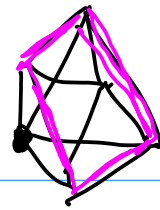
 α 

$$\frac{2e}{n} \leq \frac{n-6}{n} - \frac{12}{n}$$

$$= 6 - \frac{12}{n}$$



k degree of any vertex



5

3 eq.

l # of edges of each face

e # edges

n # vertices

f # faces

$$n \cdot k = 2e$$

$$f \cdot l = 2e$$

$$n + f = e + 2$$

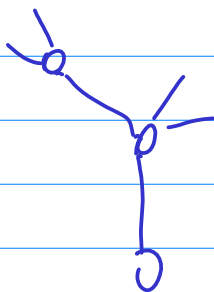
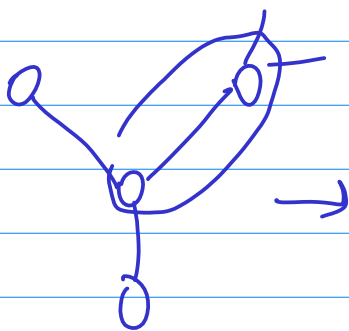
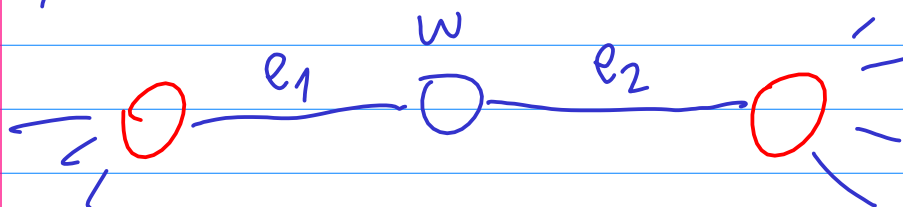
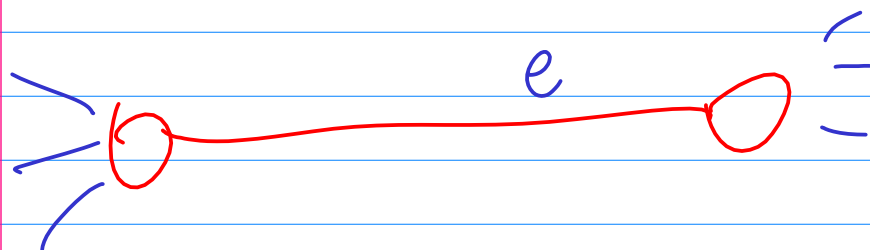
$$f = e - n + 2$$

$$\frac{2e}{k} + \frac{2e}{l} - e = 2$$
$$e \left(\frac{2}{k} + \frac{2}{l} - 1 \right) = 2$$

$$kl - 2k - 2l < 0$$

$$kl < 2k + 2l$$

$$1 < \frac{2}{l} + \frac{2}{k}$$



$$f = e - n + 2$$

faces

edges

vert.

$$n = e - f + 2$$

orig.

f

e

n

$$f = e - n + 2$$

dual

n

e

f