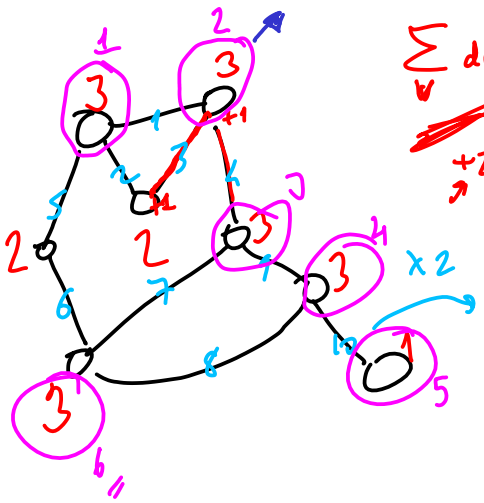
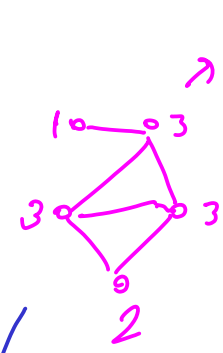


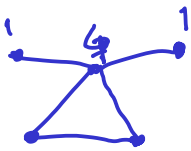
$\sum_v d(v) = 2|E|$ # of edges.

+2

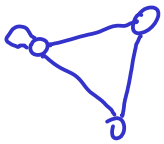
×2

20



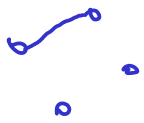


$5+3+7 = 15$



Proof by cases

e e → (oo)
e o → oe
o e → eo
o o → ee



# of odd	# of even
0	5
2	3

Proof by Contradiction

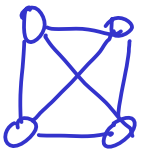
Assume that number of vert. with odd deg is odd.

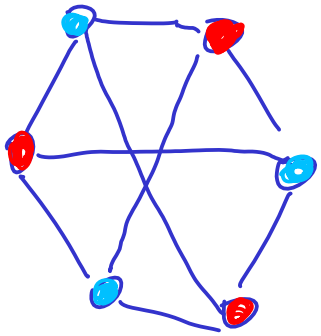
$\sum_v d(v) = 2|E|$

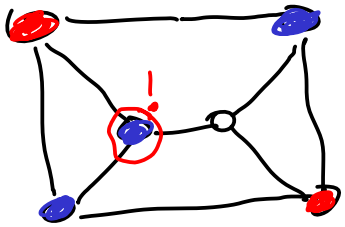
even

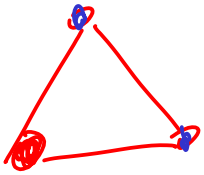
$\sum_{\text{odd } v} d(v) + \sum_{\text{even } v} d(v) = 0$ + even = 0

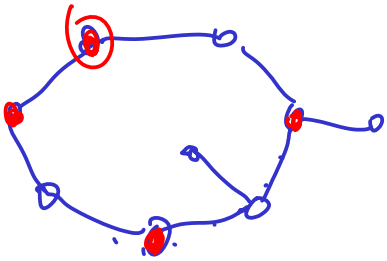
planar

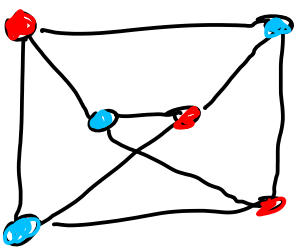




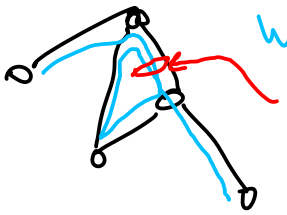








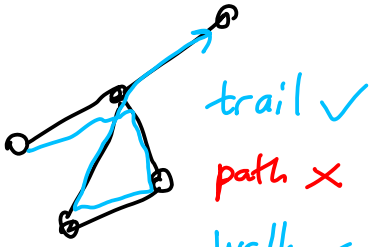
$k=3$



walk ✓

trail ✗

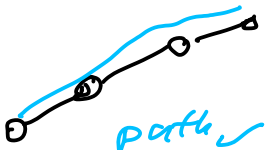
path ✗



trail ✓

path ✗

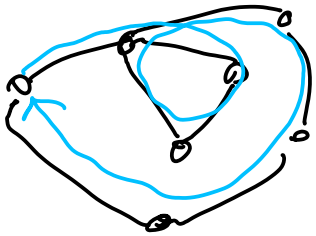
walk ✓



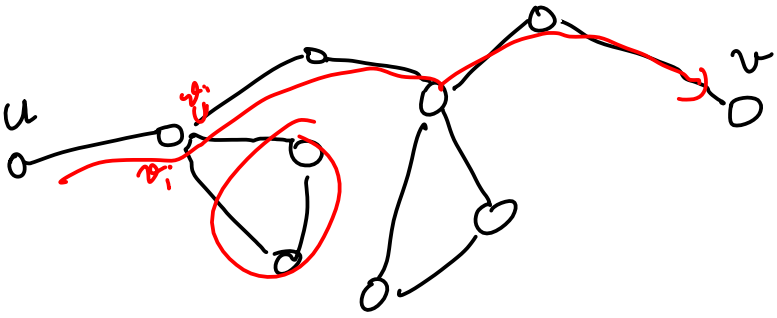
path ✓

trail ✓

walk ✓



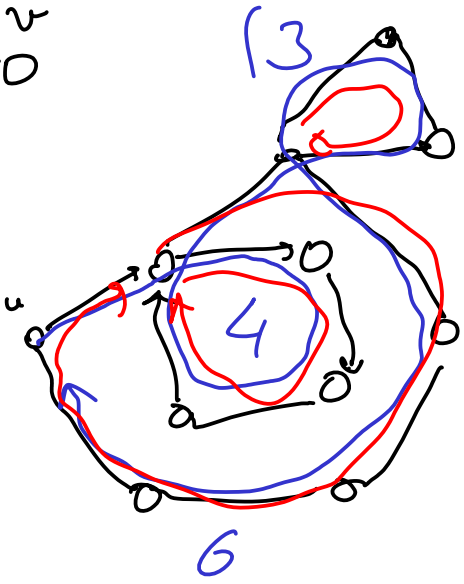
closed walk.

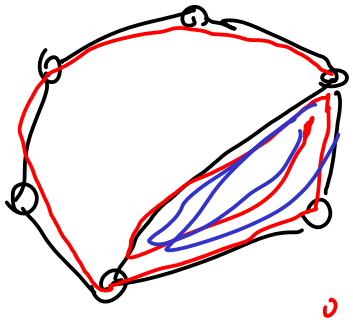


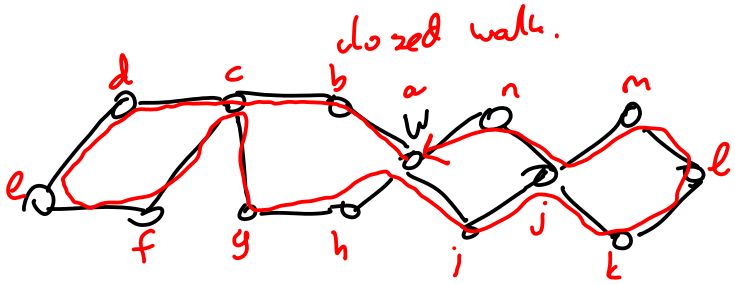
$v_0 \quad \dots v_i \{ \dots v_j \} \dots \quad v_n$

remove

$v_i := v_j$







0 1 2 3 4 5 6 7

a → 0 d → 3 c → 6
b → 1 e → 4
c → 2 f → 5

a b c ~~d e f~~ g h a i j k l m j n a