

25/04/2024

Midterm Exam

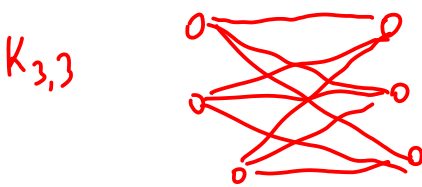
Duration: 90 minutes

Name:

Student No:

**P1 [20 points]**

- a) Draw a 3-regular graph having 6 vertices.



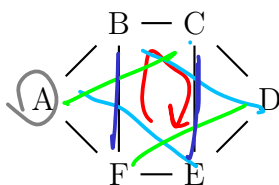
- b) In a complete bipartite graph with 13 vertices, what is the maximum number of edges?

**P2 [20 points]**

- a) If we draw
- $K_6$
- and
- $K_{5,5}$
- and then draw an edge from every vertex of
- $K_6$
- to every vertex of
- $K_{5,5}$
- , how many edges will the final graph have?



- b) In how many different ways can the following graph be labeled? Isomorphic labelings will be considered the same. [For example, if we mirror (flip around y axis) the graph, the new labeling with D on the left is the same as the original one.]

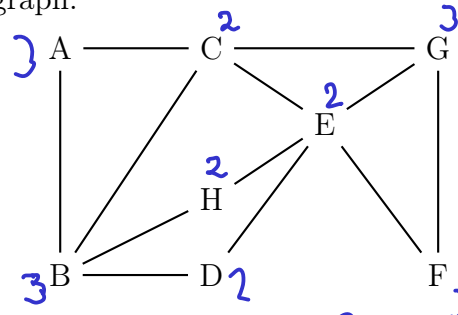


$$\binom{6}{2} \cdot \binom{4}{2} \cdot 2$$

$$15 \cdot 6 \cdot 2$$

$$\frac{4!}{2} = 12$$

$$60 \cdot 3 = 180$$

**P3 [10 points]** Find the measures for the following graph:

Eccentricity of G:

3

Eccentricity of H:

2

Radius:

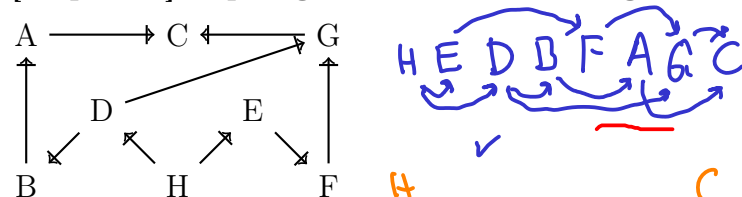
2

Diameter:

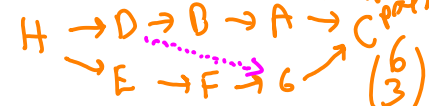
3

Center:

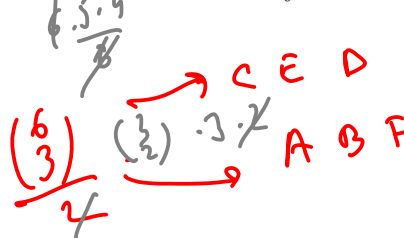
C, E, H, D

**P4 [15 points]** Topological Sort & Counting

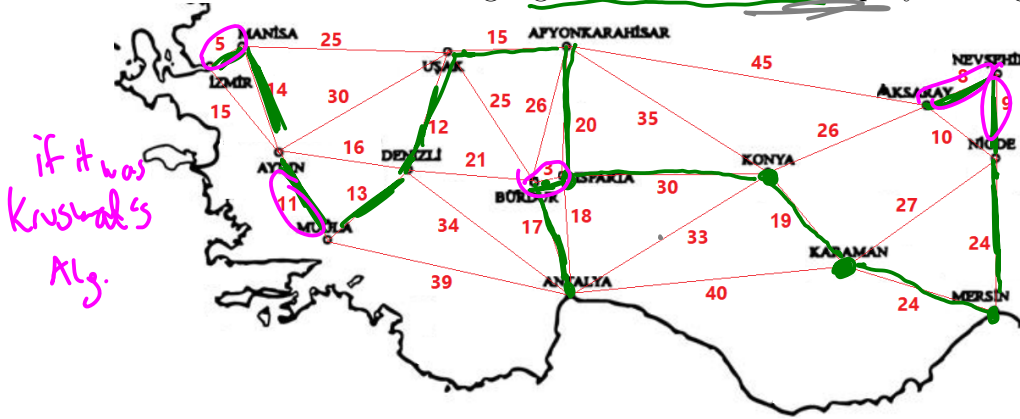
- 5 Give a topological order for the graph:



How many different topological orders are there?



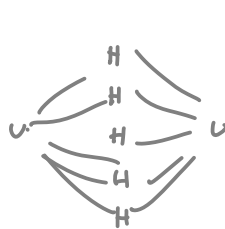
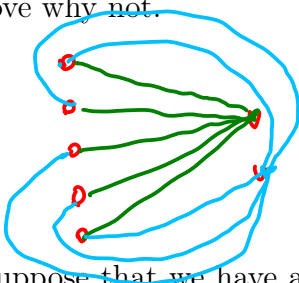
**P5 [15 points] Minimum Spanning Tree** In the map below, find a minimum spanning tree by using Prim's Algorithm starting from a random city (except Antalya) and write the cities in the order you add them to the MST. Also highlight the MST on the map by making the edges bold.



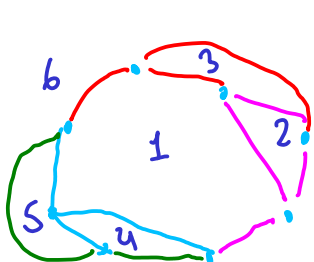
1	Karaman	9	Antalya
2	Konya	10	Akşaray
3	Mersin	11	Uşak
4	Niğde	12	Denizli
5	Nevşehir	13	Niğde
6	Aksaray	14	Ayşin
7	Isparta	15	Manisa
8	Bursa	16	Izmir

**P6 [20 points] Planar graphs**

a) Can five houses be connected to two utilities without connections crossing? If yes draw it, otherwise prove why not.



b) Suppose that we have a 3-regular planar graph having 8 vertices. Into how many regions is the plane divided by a planar drawing of this graph? [Answer without drawing it. Direct answers get 0 credit, show your work.]



$$f = e - n + 2$$

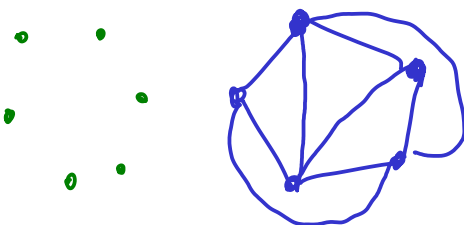
$$= 12 - 8 + 2$$

$$= 6$$

$$\sum d = 2e$$

$$38 = 2 \cdot e$$

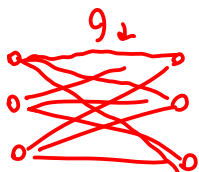
c) If you randomly create a 5-vertex graph by putting an edge or not with  $1/2$  probability for every vertex pair, what is the probability of getting a planar graph?



$$P(\text{not planar}) = \left(\frac{1}{2}\right)^{\binom{5}{2}} = \left(\frac{1}{2}\right)^{10}$$

$$P(\text{planar}) = 1 - \frac{1}{2^{10}} = \frac{1023}{1024}$$

d) If you randomly create a 6-vertex bipartite graph by first splitting the vertices into two groups having 3 vertices each, and then putting an edge or not with  $1/2$  probability from every vertex of one group to every vertex of the other group, what is the probability of getting a planar graph?



$$P(\text{planar}) = 1 - \frac{1}{2^9}$$