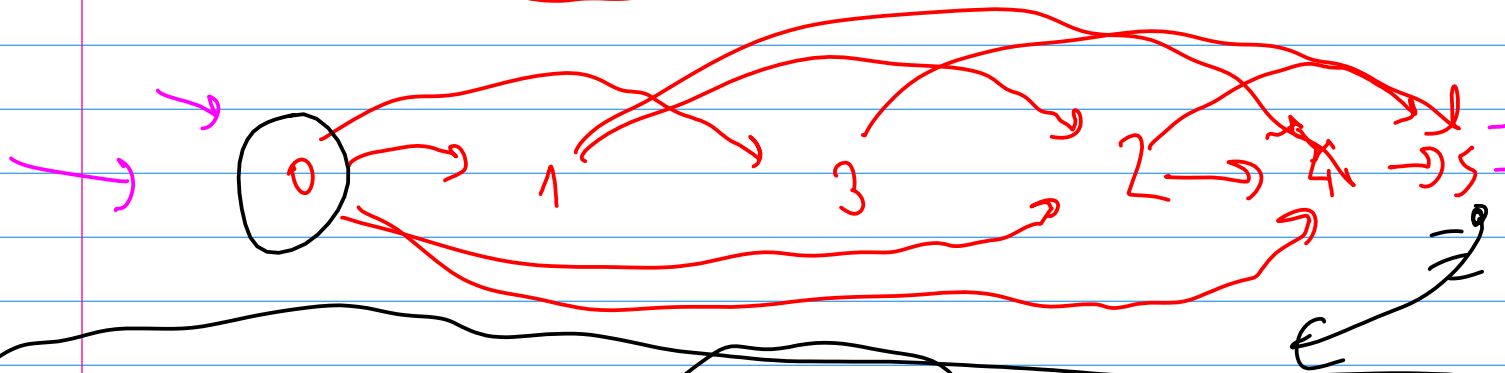
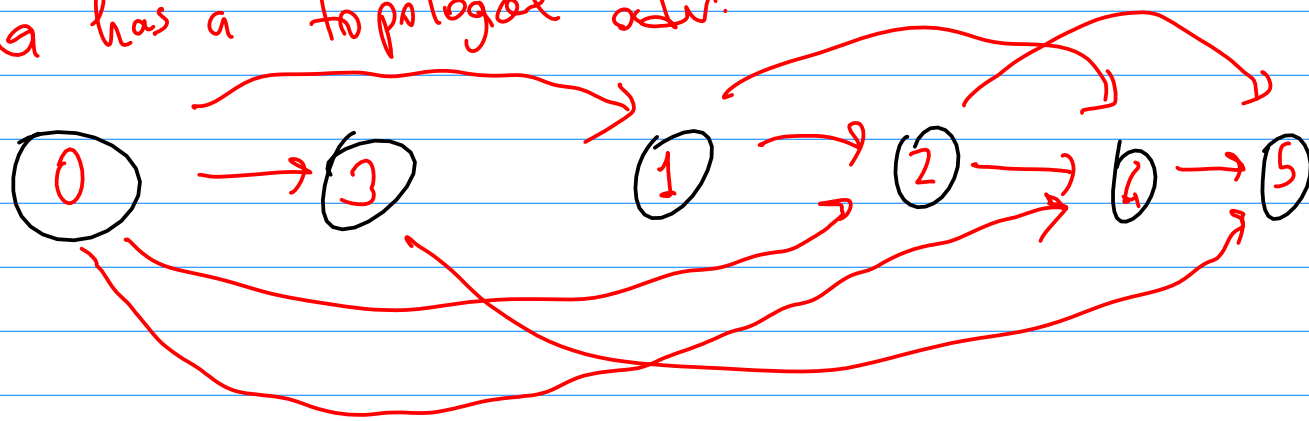


$(0, 3) \in E$

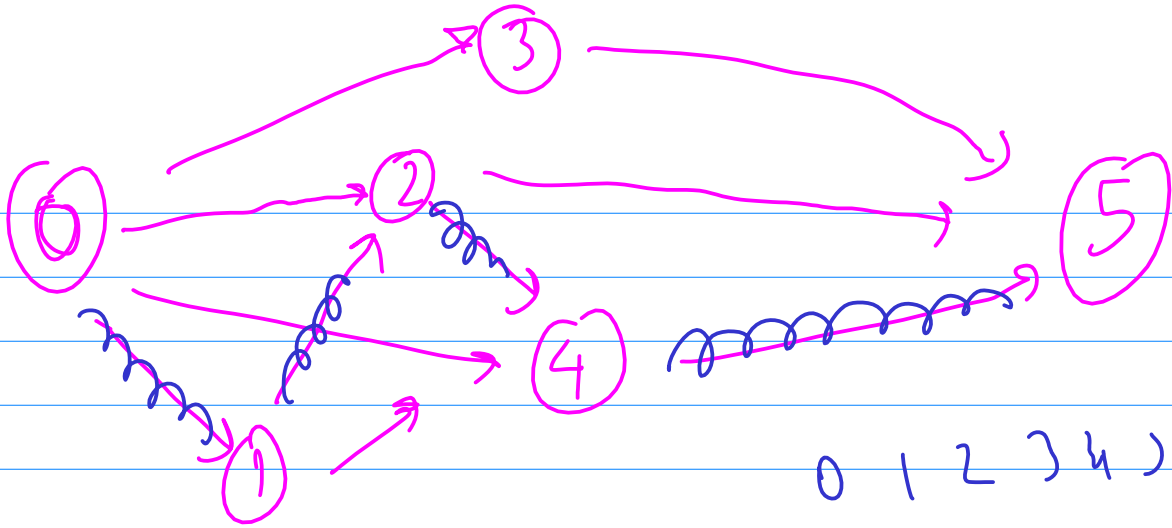
Thm. If  $\exists$  cycle in a graph  
a top. ord is imp.

$G$  has a topological order.



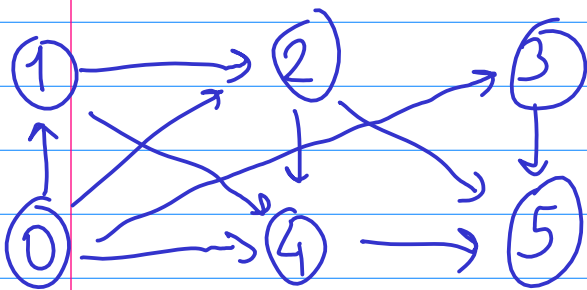
Thm.  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

Thm. For a Graph  $G=(V, E)$  if  $\forall v \in V$   $d_{in}(v) > 0$ ,  
there has to be a cycle therefore  $G$  can't have a top. order.



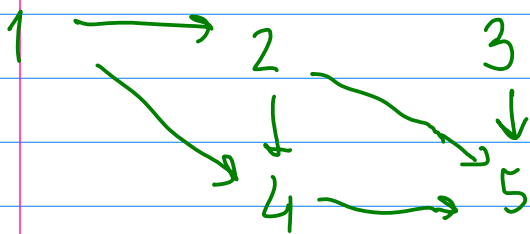
$\rightarrow 0 \times 1 \times (2) \times 4 \times 5$   
 There can be 4 different top. orders

t.o.  $\Rightarrow$ 
 ① ③ ① ② ④ ⑤  
 1 2 3 4 5 6



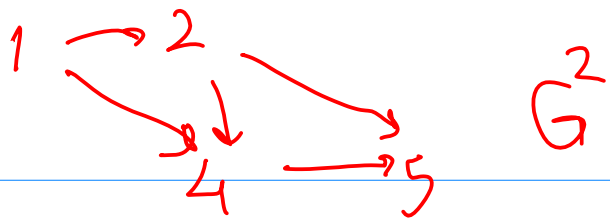
$$G^0 = G$$

$$v = 0$$



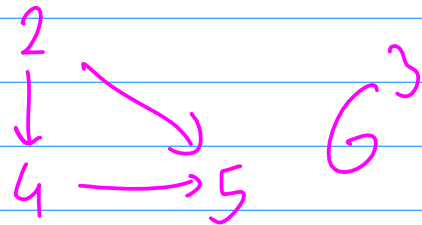
$$G^1$$

$$v = 3$$



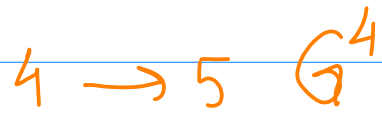
$G^2$

$v = 1$



$G^3$

$v = 2$



$G^4$

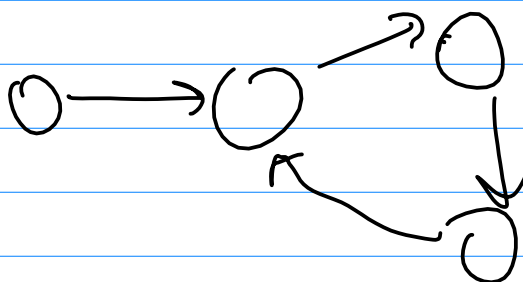
$v = 4$



$G^5$

$v = 5$

empty =  $G^6$



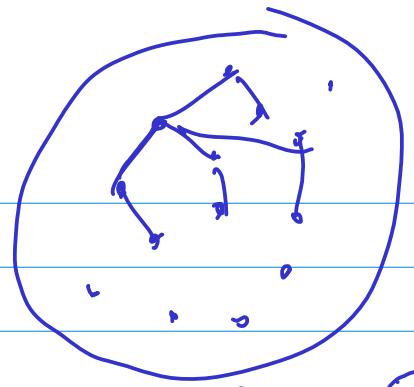
$G^0$



$G^1$

$n$  vert.

$m$  edges



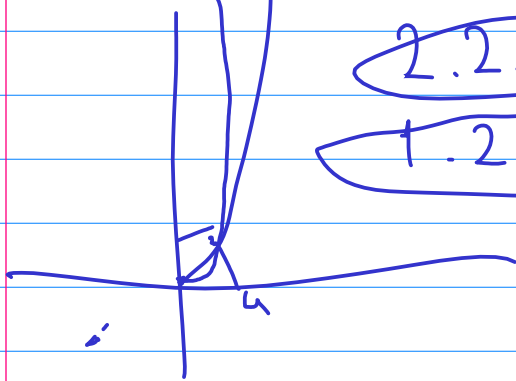
$$O(n! \cdot m)$$

$f_{n \geq 4}$

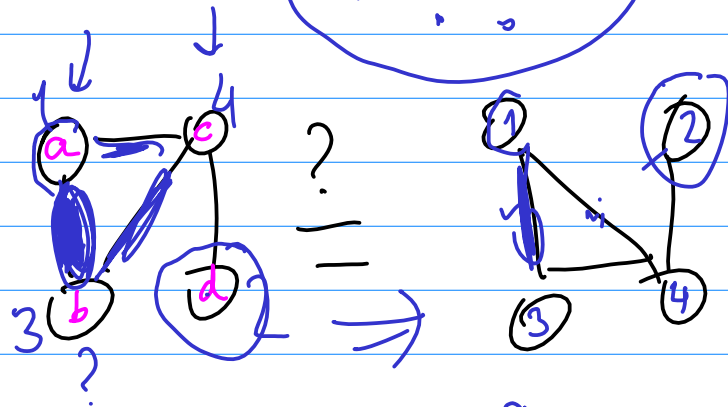
$$\underline{n! > 2^n}$$

2.2.2.

1.2.3.3.4.



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$a=1, b=2, c=3, d=4$   
 $a-c \Rightarrow 1-3 \checkmark$   
 $a-b \quad 1-2 \times$

$n!$   $a, b, c, d$

