Fall 2022: CSE 221	Discrete Mathematics	Akdeniz University

Wednesday 04/01/2023 Final Exam Duration: 90 minutes

Name: ____________________ Student No: ____________________

P1 [16 points] Graph Definitions - Undirected Graph

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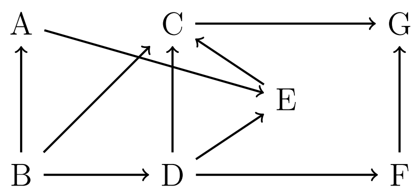
C — F — I — L
|   |   |   |
B — E — H — K
|   |   |   |
A — D — G — J
```

Tick **ALL** appropriate definitions for each sequence:

Sequence	Walk	Path	Circuit	Cycle
A-D-E-F-I-H-E-B-A				
A-B-E-H-G-D-E-F				
F-I-H-E-F				
A-B-C-F-I-L				

P2 [19 points] Graph Basics - Directed Graph



Write the in-degrees of the vertices: B: C: D:

Write the out-degrees of the vertices: E: F: G:

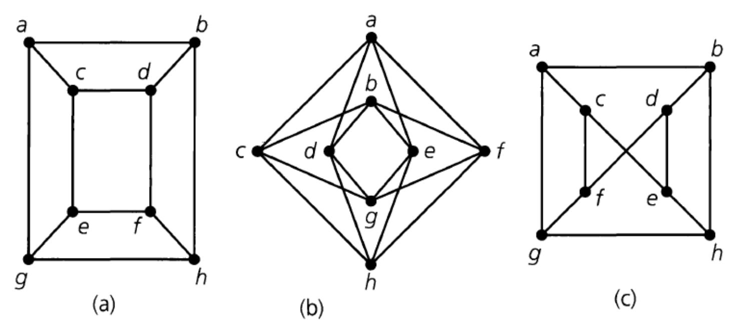
Is there a cycle in this graph? Yes · No

Give a topological order for this graph: _ _ _ _ _ _ _

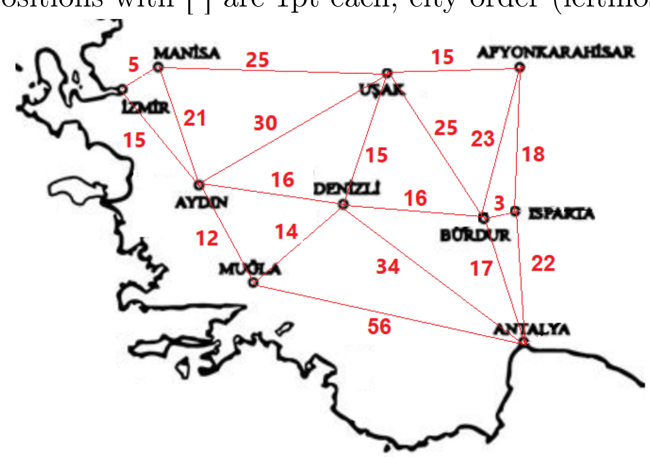
How many topological orders are possible?

P3 [15 points] Bipartite Graphs Are the graphs on the right bipartite? If yes, give a partition of vertices as reds and bluee (Like R: a,b,c,d B: e,f,g,h), if not explain why.

	REDS	BLUES
a)		
b)		
c)		



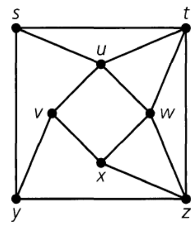
P4 [20 points] Dijkstra's Algorithm In the map below, find the shortest paths from Antalya to all other cities by using Dijkstra's Algorithm. The first line in the table is given. Fill the rest of the table. (Positions with [ ] are 1pt each, city order (leftmost column) 1pt each, the rest of the table 3pts)



Explored	Burdur	Isparta	Mugla	Denizli	Afyon	Usak	Aydin	Manisa	Izmir
Ant	<u>17</u>	22	56	34	∞	∞	∞	∞	∞
		[ ]			[ ]				
			[ ]		[ ]				
			[ ]						
						[ ]	[ ]	[ ]	[ ]

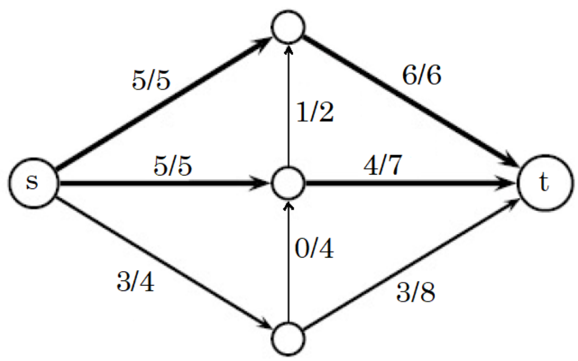
P5 [15 points] Counting paths

a) In the graph below, how many paths of length 2 are there? (You will count the paths which visit 2 edges 3 vertices. e.g. utw) (Do not count one-by-one, try to find an easy way so that you can also solve part b)



b) In a 6-regular graph with 100 vertices, how many paths of length 2 are there?

P6 [15 points] Max flow A flow network is given below. Use the allocated spaces to 1) Draw the residual graph and find out whether the flow can be further improved (increased) 2) Update the flow accordingly 3) and show that it is indeed a maximum flow.



Residual graph:

New flow:

Proof of maximality:

P7 [15 points] Inclusion-Exclusion Principle
In an exam, there are 10 questions each worth 10 points. In how many different ways can a student get 50 points? (For example, the student can get $7, 10, 6, 2, 7, 0, 8, 0, 10, 0$ from questions 1 through 10, respectively. You need to count the number of such gradings that add up to 50.)

P8 [15 points] Generating Functions
In how many ways can a farmer distribute 24 apples to four children so that each child gets at least three apples but no more than eight?

Table 1: Some generating functions that can be useful. For all $m, n \in \mathbb{Z}^+$, $a \in \mathbb{R}$

1) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$

2) $(1+ax)^n = \binom{n}{0} + \binom{n}{1}ax + \binom{n}{2}a^2x^2 + \cdots + \binom{n}{n}a^nx^n$

3) $(1+x^m)^n = \binom{n}{0} + \binom{n}{1}x^m + \binom{n}{2}x^{2m} + \cdots + \binom{n}{n}x^{nm}$

4) $(1-x^{n+1})/(1-x) = 1 + x + x^2 + x^3 + \cdots + x^n$

5) $1/(1-x) = 1 + x + x^2 + x^3 + \cdots$

6) $1/(1-ax) = 1 + ax + a^2x^2 + a^3x^3 + \cdots$

7) $1/(1+x)^n = \binom{-n}{0} + \binom{-n}{1}x + \binom{-n}{2}x^2 + \cdots = 1 + (-1)\binom{n+1-1}{1}x + (-1)^2\binom{n+2-1}{2}x^2 + \cdots$

8) $1/(1-x)^n = \binom{-n}{0} + \binom{-n}{1}(-x) + \binom{-n}{2}(-x)^2 + \cdots = 1 + (-1)\binom{n+1-1}{1}(-x) + (-1)^2\binom{n+2-1}{2}(-x)^2 + \cdots$