

16/04/2025

Midterm Exam

Duration: 90 minutes

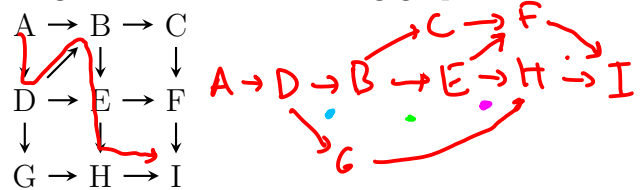
Name:

Student No:

P1 [10 points] Draw a 3-regular graph having 7 vertices, if possible. If not possible *briefly* prove why.

We know that $\sum d = 2e$
 \downarrow
 $3 \cdot 7 = 21 = 2e$
 is impossible.

P4 [15 points] How many different topological orderings exist for the following graph?



1 But C & F after H: G has 3 pos.
 2 only F after H: G has 4 pos.
 2 both before H: G has 5 pos.
 $3 + 8 + 10 = 21 \checkmark$

P2 [10 points] At most how many edges can a planar graph with 15 vertices have?

We need to have triangles everywhere to get max num of edges.

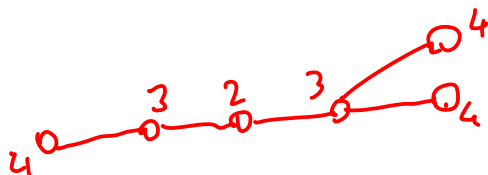
all faces are triangles

$f \cdot 3 = 2e$ "always"
 $f = e - n + 2$

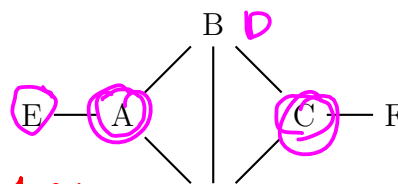
$$3f = 3e - 3n + 6$$

$$3e - 6 = e \quad 3 \cdot 15 - 6 = 39$$

P3 [10 points] Draw a graph with 6 vertices with eccentricities 4,4,4,3,3,2.



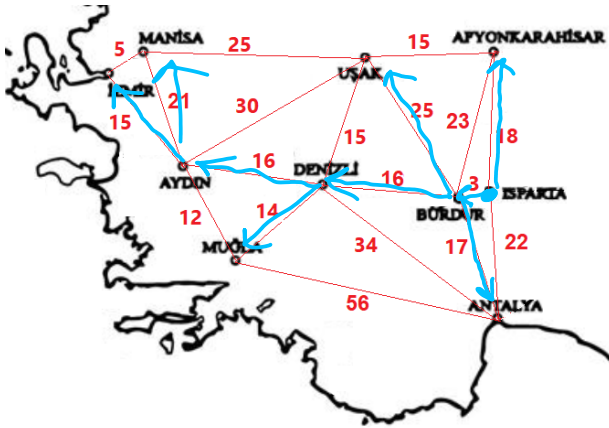
P5 [15 points] In how many distinct ways can the vertices of the following graph be labeled using labels A, B, C, D, E, F? Labelings that are isomorphic (can be obtained from one another by rotation or reflection) are considered the same.



deg 1 pos.

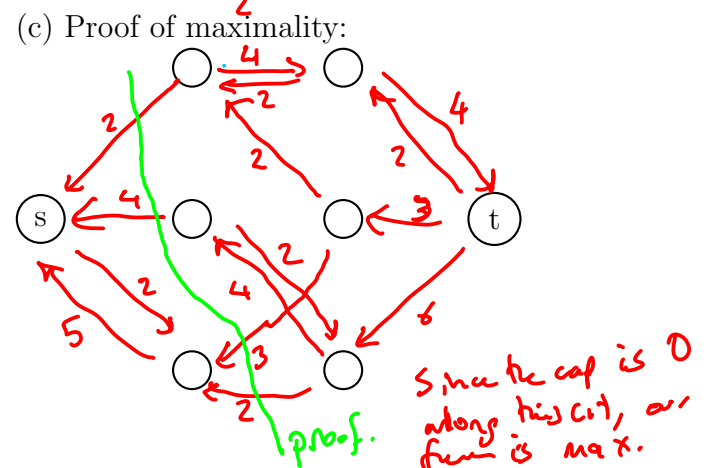
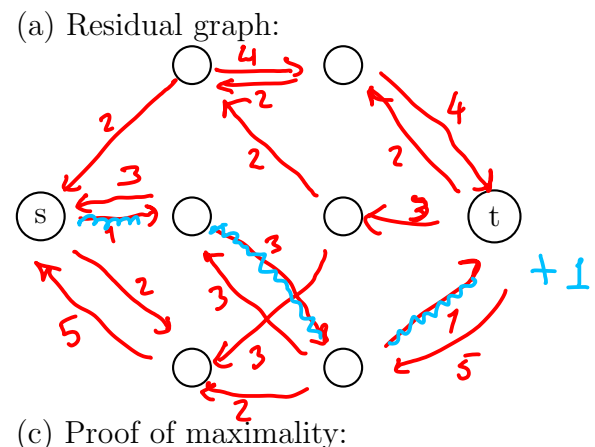
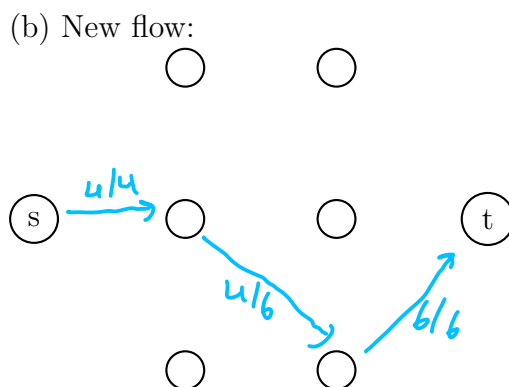
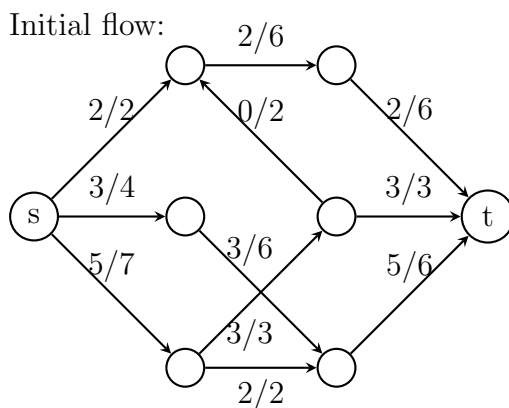
w.l.o.g. we can place alph. first one to the left
 $(6) \cdot 4 \cdot 3 = \frac{6 \cdot 5}{2} \cdot 3 = 180$
 $(4) \cdot 2$
 $6! / 2 / 2$ also correct

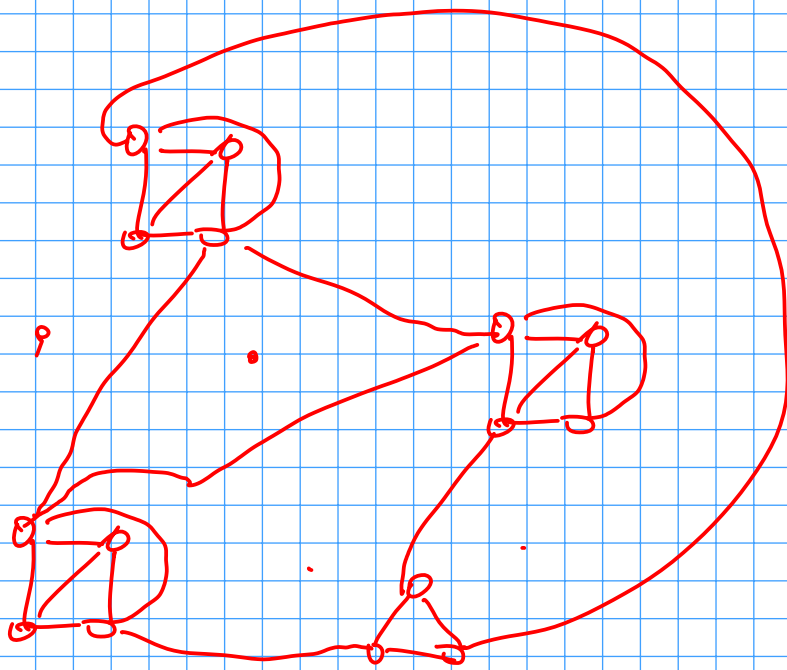
P6 [20 points] Dijkstra's Algorithm In the map below, find the shortest paths from Isparta to all other cities by using Dijkstra's Algorithm. The first line in the table is given. Fill the rest of the table. (Positions with [] are 2 points each, city order (leftmost column) 1 point each)

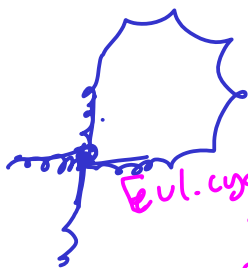


| Explored | Burdur | Afyon | Antalya | Denizli | Mugla | Usak | Aydin | Manisa | Izmir |
|----------|----------|-------|---------|----------|----------|----------|----------|----------|----------|
| Isp | <u>3</u> | 18 | 22 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| Bur | 3 | 18 | [20] | 19 | ∞ | 28 | ∞ | ∞ | ∞ |
| Af | 3 | 18 | [20] | 19 | ∞ | 28 | ∞ | ∞ | ∞ |
| Den | 3 | : | 20 | 19 | 33 | 28 | 35 | ∞ | ∞ |
| Ant | 3 | : | 20 | : | 33 | 28 | 35 | ∞ | ∞ |
| Uş | 3 | : | : | : | 33 | 28 | 35 | 53 | ∞ |
| Muğ | 3 | 1 | : | : | 33 | : | 35 | 53 | ∞ |
| Ayd | 3 | | 1 | : | : | : | 35 | 53 | 50 |
| Izm | 3 | | | : | : | [28] | [35] | [53] | [50] |

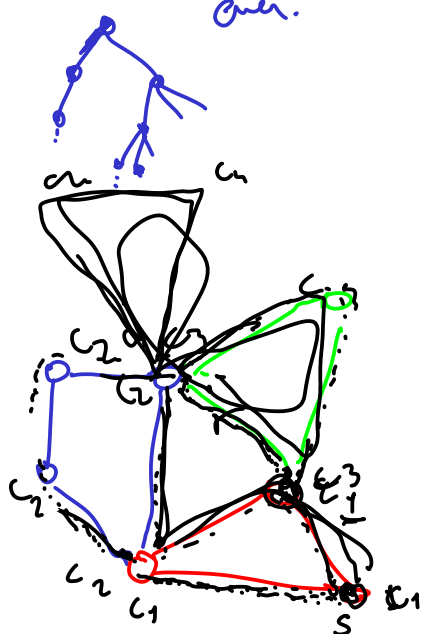
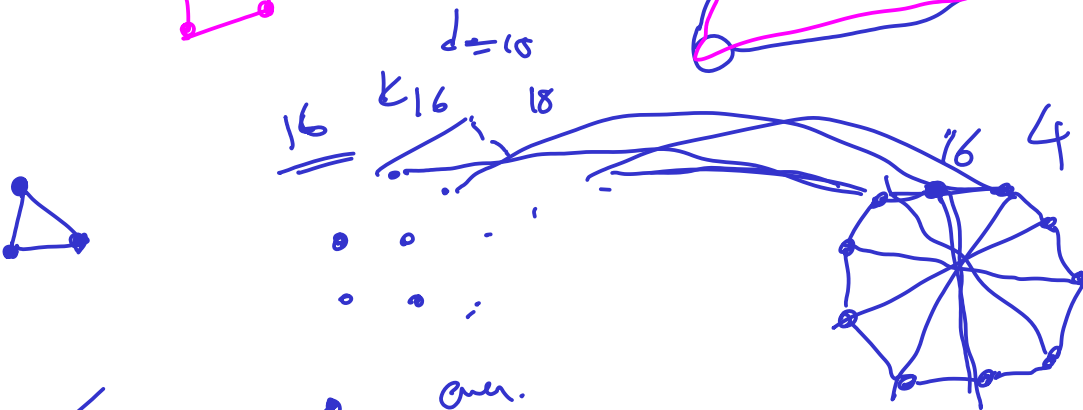
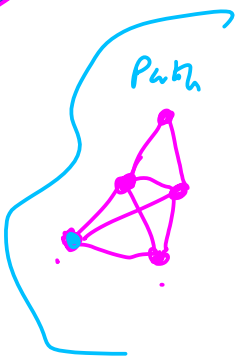
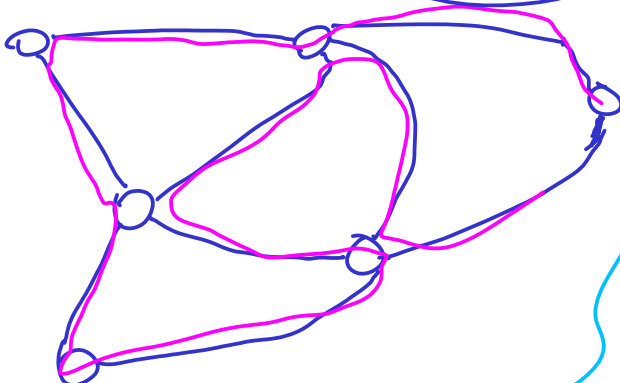
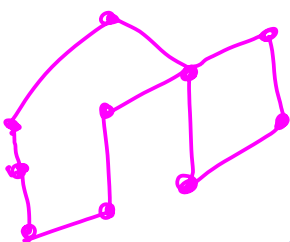
P7 [20 points] A flow network is given below. (a) Draw the residual graph and find out whether the flow can be increased, (b) Update the flow accordingly, (c) and show that it is indeed a maximum flow.







all even deg \Rightarrow Eulerian graph



directed in.

r b g
 C_1 C_2 C_3
 \perp \emptyset \emptyset

\downarrow \downarrow \downarrow
 r b g

