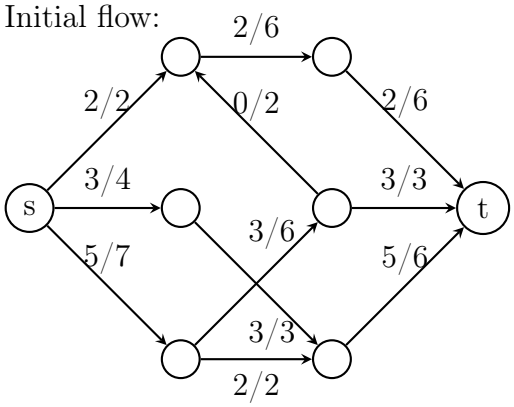
June 10, 2024 Final Exam Duration: 90 minutes

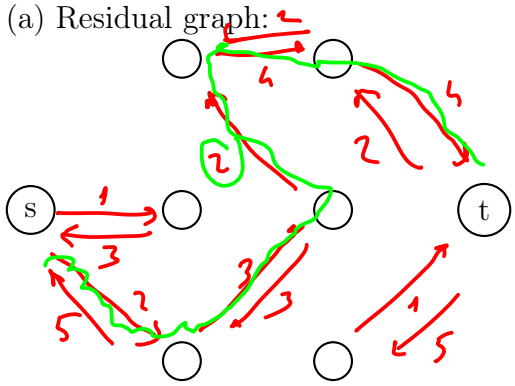
Name: Student No:

P1 [20 points] A flow network is given below. (a) Draw the residual graph and find out whether the flow can be increased, (b) Update the flow accordingly, (c) and show that it is indeed a maximum flow.

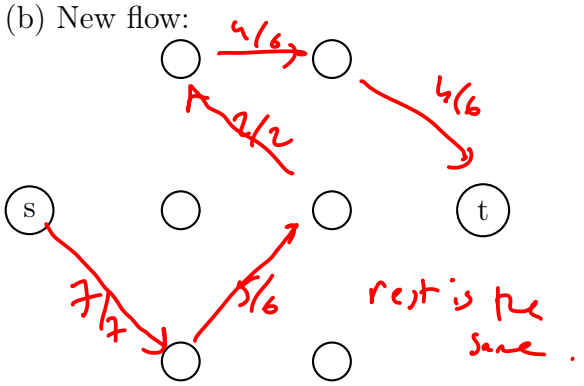
Initial flow:



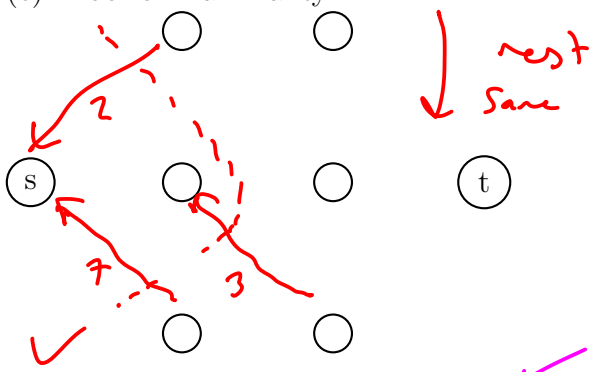
(a) Residual graph:



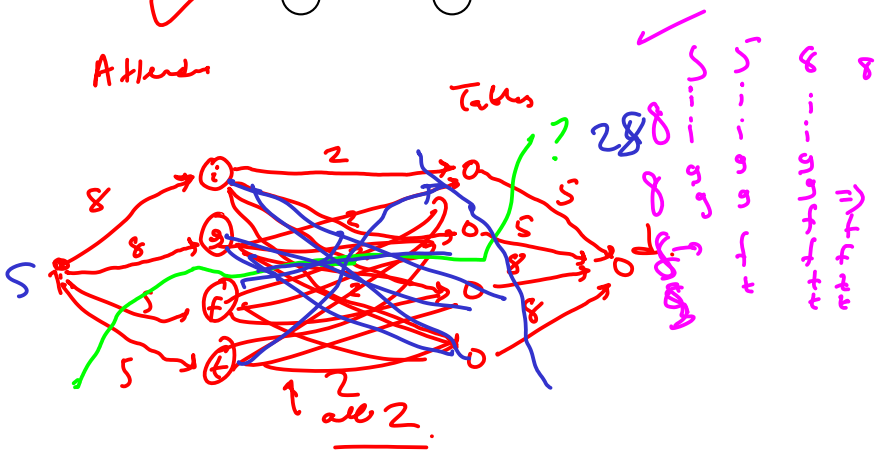
(b) New flow:



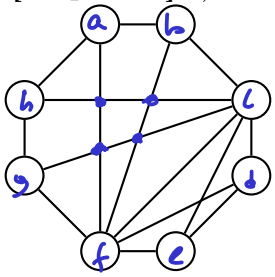
(c) Proof of maximality:

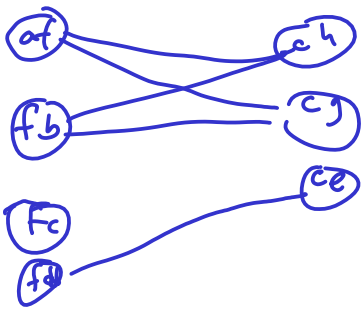


P2 [20 points] In an international conference, there are 8 Italians, 8 Germans, 5 French and 5 Turkish attendees. They will be seated to 4 tables having 5,5,8,8 chairs. The organizer wants to seat at most 2 people from the same country to each table. a) Formulate this problem as a network flow problem. b) If it is possible find such an arrangement, otherwise prove why it is impossible.



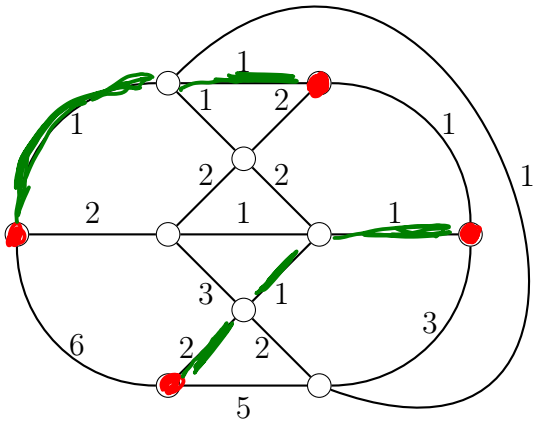
P3 [10 points] a) Is this graph planar? Circle your answer: YES / NO b) Why/why not?

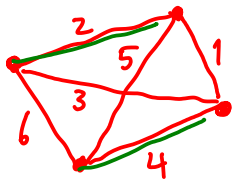




See this graph is bipartite, our orig. graph is planar.

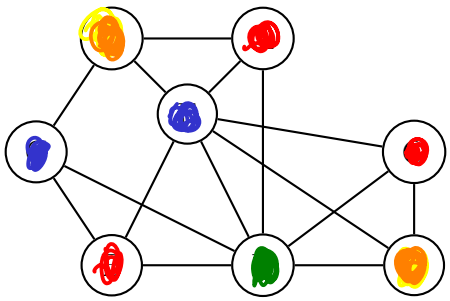
P4 [20 points] A postman has to *visit each edge* and *arrive at his starting point* in the following graph. But he wants to *minimize his cost*. The cost of each edge is given in the graph. Help the postman by finding out the optimal route. As your final answer, find out which edges will be visited twice and their total extra cost.





6+1 7
2+4 6
3+5 8

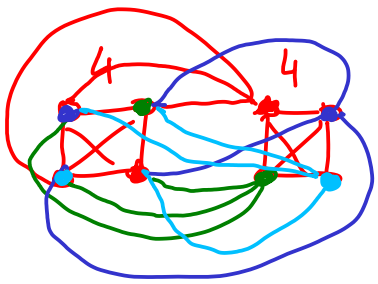
P5 [10 points] Use the GreedyColor algorithm to color the following graph with 4 colors (numbered as 1:Blue, 2:Red, 3:Yellow, 4:Green):



First line of the following table should be your random order of the nodes (a random permutation you will make up) and in the second line must contain the color codes assigned according to the Greedy Coloring algorithm.

Nodes Ordered:	F	G	E	H	A	D	B	C
Their Colors:	B	R	Y	G	R	Y	R	B

P6 [10 points] Suppose you have two copies of K_4 side by side and you will draw some more edges between them to get a larger single simple graph. *At most* how many more edges can you draw between them so that the chromatic number of the final graph turns out to be 4.



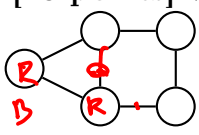
→ 12

$k(k-1)(k-2)\left((k-1)^2-(k-2)\right)$

$k^2-2k+1-k+2$

k^2-3k+3

P7 [10 points] Write the chromatic polynomial of the following graph:



$p_G(k) =$ $P(\square, k) - P(\square, k)$

$= P(\square, k) - P(\square, k) - P(\square, k)$

$= (k-2)\, P(\square, k)$

$= (k-2)\left(P(\square, k) - P(\triangle, k)\right)$

$= (k-2)\left(k(k-1)^3 - k(k-1)(k-2)\right)$