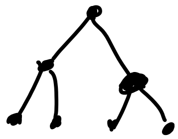
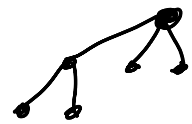
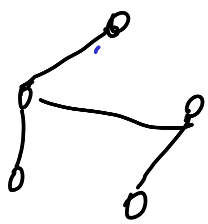


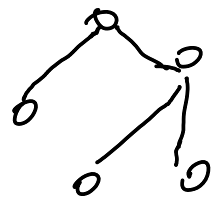


n = 5



$m = n-1$ (4)

- Tree $\Rightarrow$ #1
- #1 $\Rightarrow$ Tree.



- Prop. If G is a tree, then G is conn. and has $n-1$ edges.

Proof. Since G is a tree, it is acyclic & connected by defn. Then,

For G to be connected we need <u>at least</u> $n-1$ edges.
(Because, By adding the initial edge, we connect 2 vertices and with every additional edge we connect 1 more vertex.)

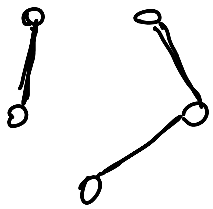
For G to be acyclic we can have <u>at most</u> $n-1$ edges. (Assume we have n edges. Then, we have to introduce a cycle bec. $n-1$ edges already make G connected so adding the n^{th} edge surely forms a cycle. So we are done □.

- Prop. If G is conn. and has $n-1$ edges. it is a tree.

Proof. G is connected already (given) we only need to prove it is acyclic.

Assume it has a cycle containing k vertices and k edges to make G connected, we need to connect all other $n-k$ vertices with $n-k$ edges.

So, we would have n edges. but we only have $n-1$. Contradiction. QED.



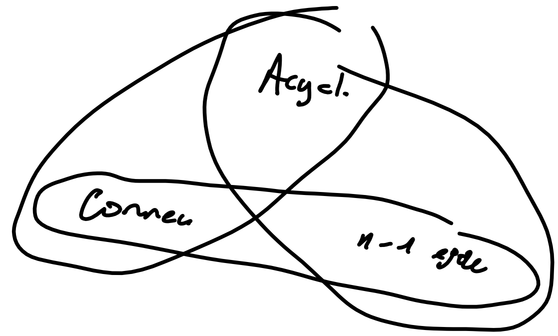
$n-1=4$ edges

keep it acycl.

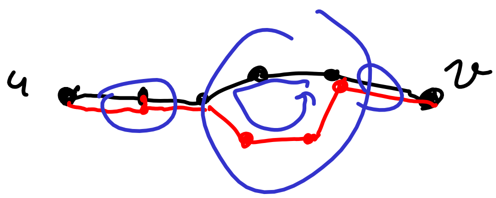
don't form a cycle

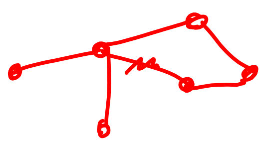
Tree $\to$ conn $\to$ $n-1$ edge

Tree $\to$ acycl. $\to$ $n-1$ edge

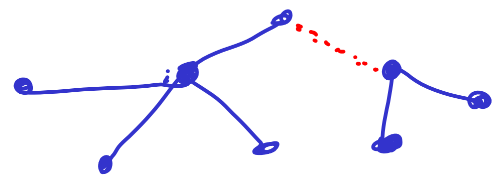


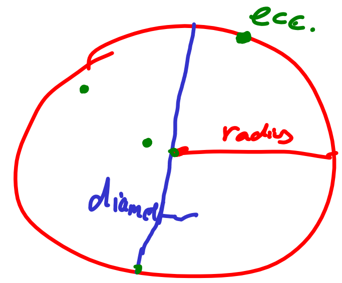
4th

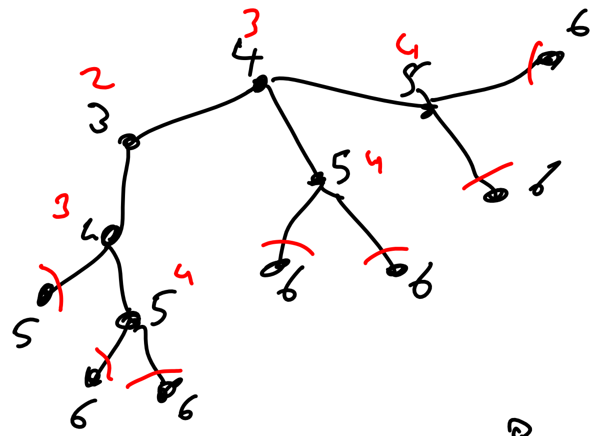


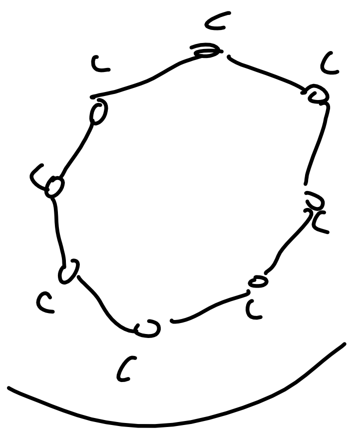


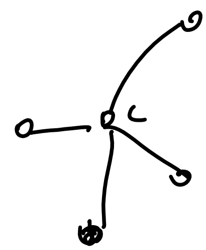
5th

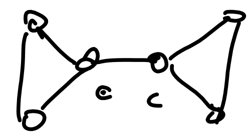


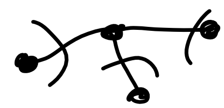


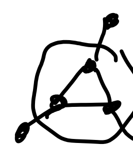






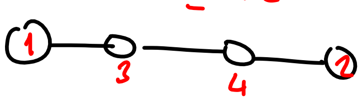


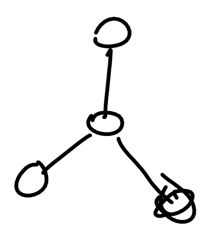




12 // + 4 // → 16

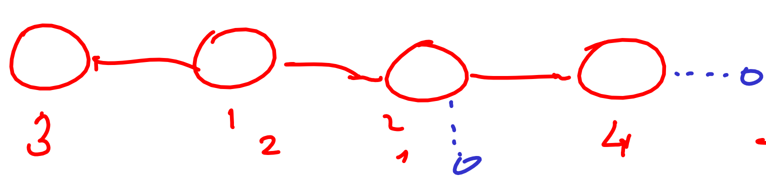
$\binom{4}{2} \cdot 2 = 12$





$\frac{4!}{2}$ $\binom{4}{2} \cdot 2.$

1, 2, 3, 4

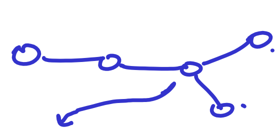


2 3 4 4 1 6 2 3

60 ? $\frac{5!}{2} = 60$

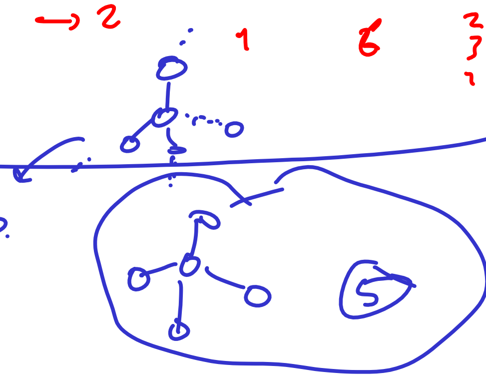
$\binom{5}{3} 3! = 10 \cdot 6 = 60$

$5 \cdot \binom{4}{2} \cdot 2$

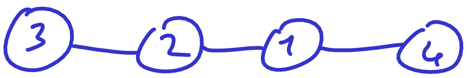


5.4.3.

60

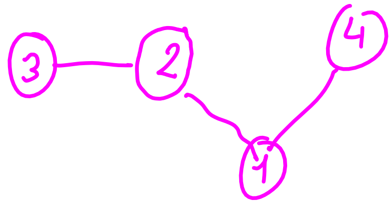


125

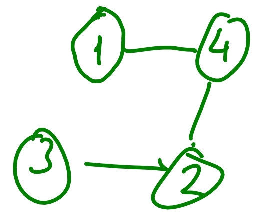


i	s_i	t_i
1	3	2
2	2	1

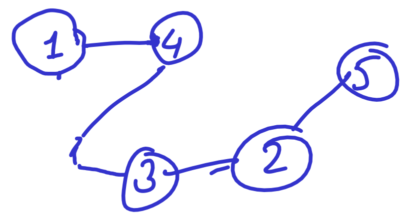
1 1 1 1 2 2 2 2 3 3 . . . 4 4
1 2 3 4 1 2 3 4 1 . . . 4



i	s_i	t_i
1	?	4
2	?	2

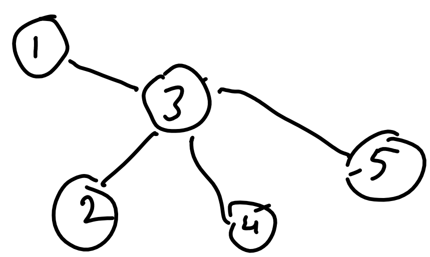


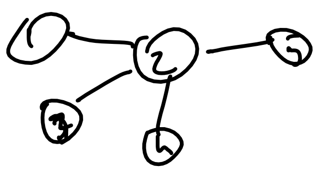
i	s_i	t_i
1	?	4
2	?	3
3	?	2



i	s_i	t_i
	?	2
	3	2
	.	2

i	s_i	t_i
1	?	3
2	?	3
3	?	3





n^{n-2}

t_i
5
5
5