Friday 12/02/2021 Resit Exam Duration: 90 minutes

Name: Student No:

P1 [20 points] Rook Polynomials

In the following table, delete(shade) all six squares having the numbers in your parameters for P1. Then, find the rook polynomial for the remaining table.

1	2	3
4	5	6
7	8	9
10	11	12

P2 [20 points] Pigeonhole Principle

a) Consider the problem P1 above. Assuming that 6 random parameters are selected uniformly at random for each student, how many students need to enter this exam to guarantee that there would be at least one couple of students having the same parameters?

b) X people sit around a circular table. The sum of their ages is Y. What is the largest number n such that you can certainly claim that the sum of ages of at least one group of consecutive three people is at least n? Explain your answer. What are pigeons and pigeonholes?

P3 [20 points] Generating Functions You will play a game with Jeff Bezos. He tells you that he'll give you 1 million dollars if you win the game. Rules are simple: you'll choose an integer n first. Then you'll roll a classical (six sided, numbered 1 to 6) fair die three times and then roll a 12 sided fair die (12 sided, numbered from 1 to 12) three times and if the sum of all these six rolls is exactly n, you win.

a) What n will you choose to maximize your chance? (This is very easy.)

b) With the n you chose, what is your chance (probability) of winning?

P4 [20 points] Bipartite Graphs

a) Draw a graph that is 4-regular, bipartite and not planar.

b) Show that your graph is indeed not planar.

P5 [20 points] Graph Basics - MST First, write your personal parameters for this question in writing direction (left to right, top to bottom) in the given graph. Then, apply your favorite minimum spanning tree algorithm to find an MST. Write the steps of your algorithm. Also indicate the edges of the MST on the graph.