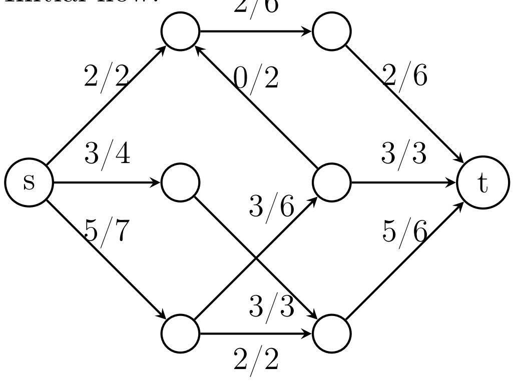
Spring 2024: CSE 222	Discrete Mathematics II	Akdeniz University

June 10, 2024 — Final Exam — Duration: 90 minutes

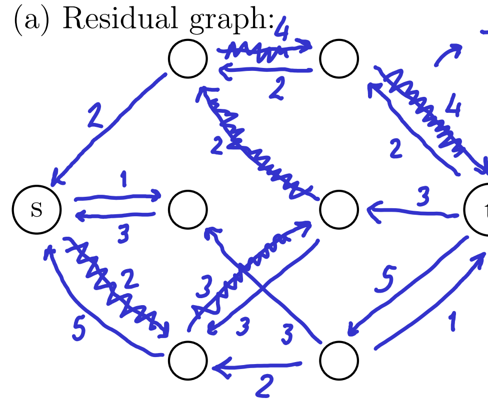
Name: ______________________ Student No: ______________________

P1 [20 points] A flow network is given below. (a) Draw the residual graph and find out whether the flow can be increased, (b) Update the flow accordingly, (c) and show that it is indeed a maximum flow.

Initial flow:

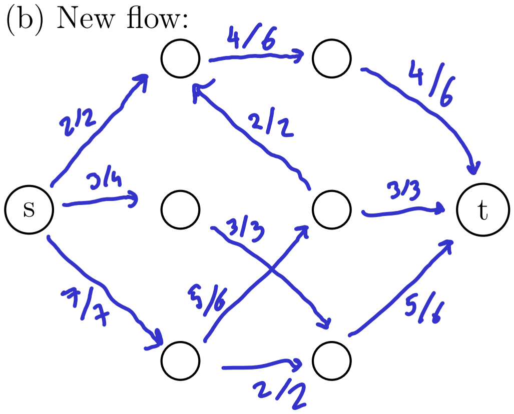


(a) Residual graph:

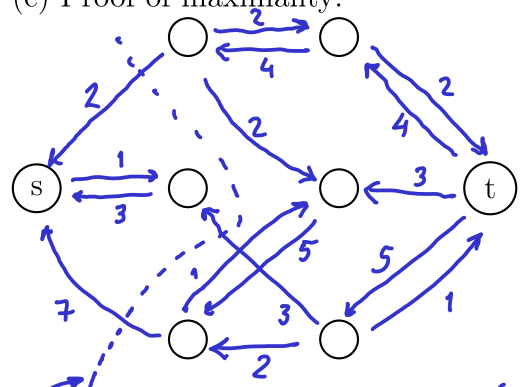


This aug. path means we can increase the flow. Min cap alongside this aug. path is 2.

(b) New flow:



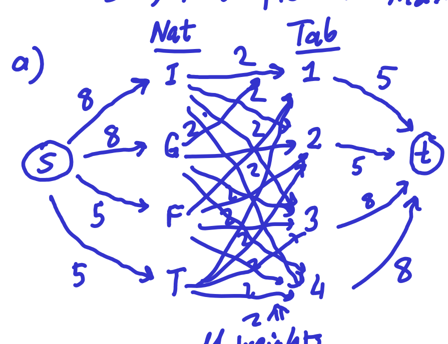
(c) Proof of maximality:



Now, this cut has no capacity left from s to t. So, this flow is maximal.

P2 [20 points] In an international conference, there are 8 Italians, 8 Germans, 5 French and 5 Turkish attendees. They will be seated to 4 tables having 5,5,8,8 chairs. The organizer wants to seat at most 2 people from the same country to each table. a) Formulate this problem as a network flow problem. b) If it is possible find such an arrangement, otherwise prove why it is impossible.

a)

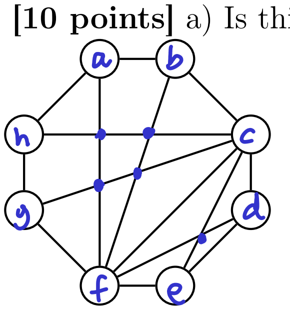


all weights are 2 here

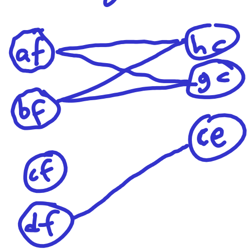
b) Yes, a valid arrangement is possible

T1	T2	T3	T4
I	I	I	I
I	I	I	I
G	G	G	G
G	G	G	G
F	T	F	F
		F	F
		T	T
		T	T

P3 [10 points] a) Is this graph planar? Circle your answer: YES / NO b) Why/why not?

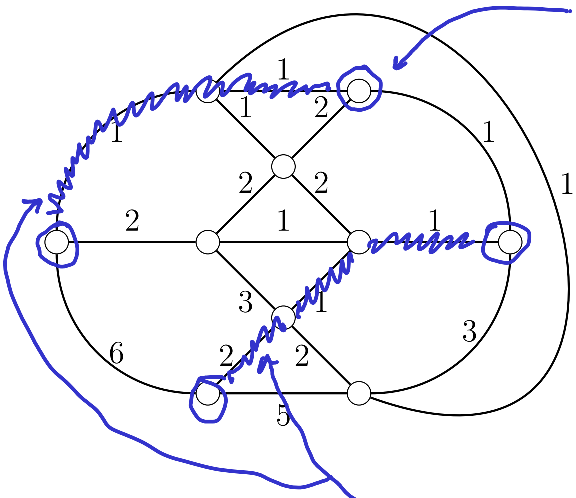


Since this graph has a Ham. Cycle, we can check its planarity by checking whether this graph is bipartite:



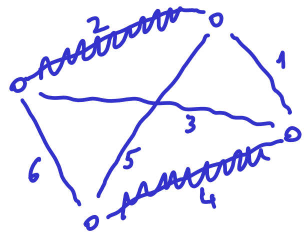
Yes, it is bipartite, so our original graph is planar.

P4 [20 points] A postman has to *visit each edge* and *arrive at his starting point* in the following graph. But he wants to *minimize his cost*. The cost of each edge is given in the graph. Help the postman by finding out the optimal route. As your final answer, find out which edges will be visited twice and their total extra cost.



First, we find odd-degree vertices

Second, we find shortest paths between them:



Third we look for the cheapest perfect matching in this graph.

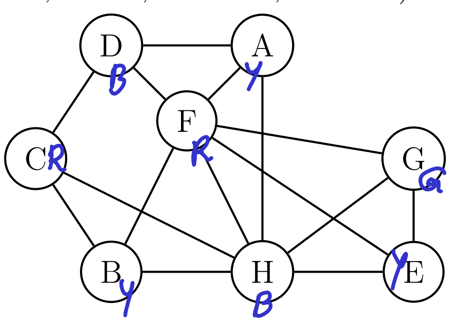
$6+1=7$

$2+4=6$ ← cheapest

$5+3=8$

This means the edges that give us these shortest paths must be visited one more time.

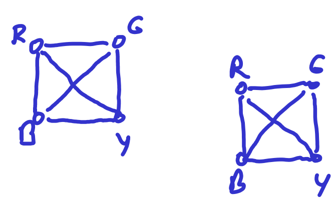
P5 [10 points] Use the GreedyColor algorithm to color the following graph with 4 colors (numbered as 1:Blue, 2:Red, 3:Yellow, 4:Green):



First line of the following table should be your random order of the nodes (a random permutation you will make up) and in the second line must contain the color codes assigned according to the Greedy Coloring algorithm.

Nodes Ordered:	H	F	D	E	A	C	G	B
Their Colors:	B	R	B	Y	Y	R	G	Y

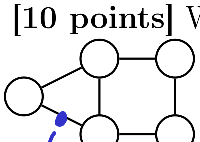
P6 [10 points] Suppose you have two copies of K_4 side by side and you will draw some more edges between them to get a larger single simple graph. *At most* how many more edges can you draw between them so that the chromatic number of the final graph turns out to be 4.



K_4 itself requires 4 colors.
So, lets assume we color both K_4 s with colors R, G, B, Y

Now, we can also connect all different-color pairs for every color on the first K_4, we can connect that vertex with 3 vertices from the second K_4 having another color. So, we can add at most $4 \times 3 = 12$ edges.

P7 [10 points] Write the chromatic polynomial of the following graph:



let's select this edge:

$p_G(k) = P_{\boxminus'}(k) - P_{\boxminus}(k)$ where $P_{\boxminus'}(k) = P_{\cdot\boxminus}(k) - P_{\boxminus}(k)$

edge removed version — edge contracted version

where $P_{\cdot\boxminus}(k) = k \cdot P_{\boxminus}(k)$.

So, $P_G(k) = P_{\cdot\boxminus}(k) - 2P_{\boxminus}(k) = (k-2)\,P_{\boxminus}(k)$. Now, $P_{\boxminus}(k) = P_{\square\!-}(k) - P_{\triangle}(k)$

$P_{\square\!-}(k) \Downarrow k(k-1)^3$ $\quad P_{\triangle}(k) \Downarrow k(k-1)(k-2)$

Thus, $P_G(k) = (k-2)\left(k(k-1)^3 - k(k-1)(k-2)\right) = k(k-1)(k-2)\left((k-1)^2 - (k-2)\right)$

$= k(k-1)(k-2)(k^2-3k+3)$ //