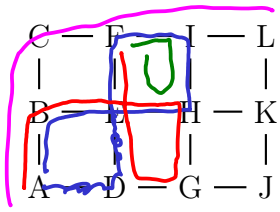
Fall 2022: CSE 221 | **Discrete Mathematics** | **Akdeniz University**

Wednesday 04/01/2023 | Final Exam | Duration: 90 minutes

Name: _____ Student No: _____

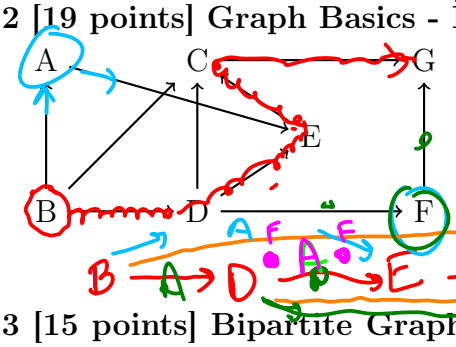
P1 [16 points] Graph Definitions - Undirected Graph



Tick **ALL** appropriate definitions for each sequence:

Sequence	Walk	Path	Circuit	Cycle
A-D-E-F-I-H-E-B-A	✓		✓	
A-B-E-H-G-D-E-F	✓			
F-I-H-E-F	✓		✓	✓
A-B-C-F-I-L	✓	✓		

P2 [19 points] Graph Basics - Directed Graph



Write the in-degrees of the vertices: B: 0 C: 3 D: 1

Write the out-degrees of the vertices: E: 1 F: 1 G: 0

Is there a cycle in this graph? Yes · No

Give a topological order for this graph: BADECFG

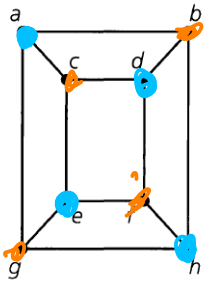
How many topological orders are possible? 7

BADFECG
BADEFCG
BADECFG
BDFAECG
BDAFECG
BDAEFCG
BDAECFG

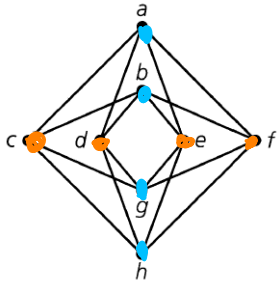
BAD → 3
DAE → 4

P3 [15 points] Bipartite Graphs Are the graphs on the right bipartite? If yes, give a partition of vertices as reds and blues (Like R: a,b,c,d B: e,f,g,h), if not explain why.

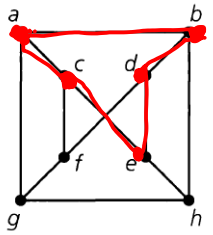
	REDS	BLUES
a)	b,c,f,g.	a,d,e,h
b)	c,d,e,f	a,b,g,h
c)	not bipartite bec. a b d e c a is a cycle of length 5.	



(a)

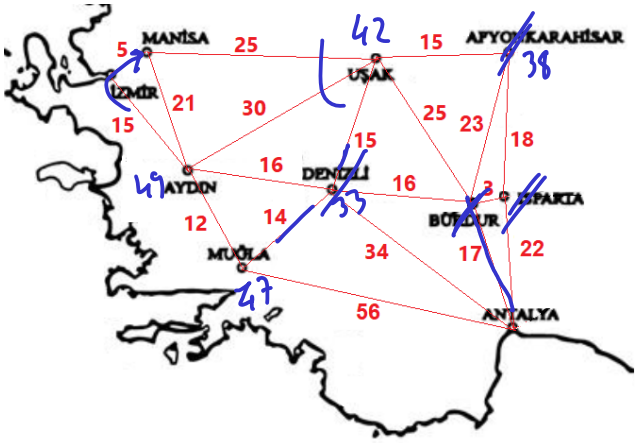


(b)



(c)

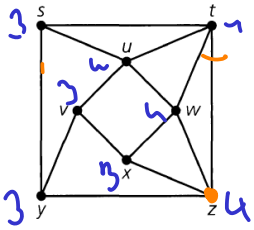
P4 [20 points] Dijkstra's Algorithm In the map below, find the shortest paths from Antalya to all other cities by using Dijkstra's Algorithm. The first line in the table is given. Fill the rest of the table. (Positions with [ ] are 1pt each, city order (leftmost column) 1pt each, the rest of the table 3pts)



Explored	Burdur	Isparta	Mugla	Denizli	Afyon	Usak	Aydin	Manisa	Izmir
Ant	17	22	56	34	∞	∞	∞	∞	∞
Bur	17	[20]	66	33	[40]	42	∞	∞	∞
Isp	17	20	[56]	33	[38]	42	∞	∞	∞
Den	17	20	[47]	33	38	42	49	∞	∞
Afy	17	20	47	33	38	42	49	∞	∞
Usa	17	20	47	33	38	42	49	67	∞
Muğ	17	20	47	33	38	42	49	67	∞
Ayd	17	20	47	33	38	42	49	67	64
Izm	17	20	47	33	38	[42]	[49]	[67]	[64]

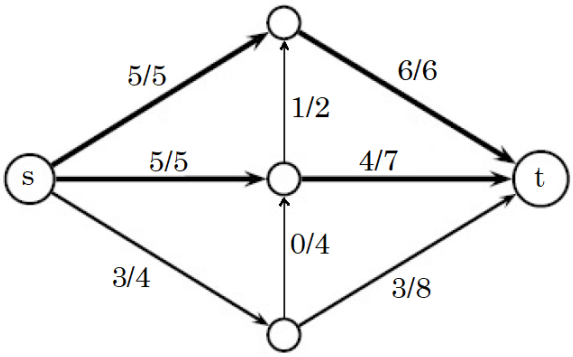
P5 [15 points] Counting paths

a) In the graph below, how many paths of length 2 are there? (You will count the paths which visit 2 edges 3 vertices. e.g. utw) (Do not count one-by-one, try to find an easy way so that you can also solve part b)



b) In a 6-regular graph with 100 vertices, how many paths of length 2 are there?

P6 [15 points] Max flow A flow network is given below. Use the allocated spaces to 1) Draw the residual graph and find out whether the flow can be further improved (increased) 2) Update the flow accordingly 3) and show that it is indeed a maximum flow.



Residual graph:

New flow:

Proof of maximality:

P7 [15 points] Inclusion-Exclusion Principle

In an exam, there are 10 questions each worth 10 points. In how many different ways can a student get 50 points? (For example, the student can get $7, 10, 6, 2, 7, 0, 8, 0, 10, 0$ from questions 1 through 10, respectively. You need to count the number of such gradings that add up to 50.)

P8 [15 points] Generating Functions

In how many ways can a farmer distribute 24 apples to four children so that each child gets at least three apples but no more than eight?

Table 1: Some generating functions that can be useful. For all $m, n \in \mathbb{Z}^+$, $a \in \mathbb{R}$

1) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$

2) $(1+ax)^n = \binom{n}{0} + \binom{n}{1}ax + \binom{n}{2}a^2x^2 + \cdots + \binom{n}{n}a^nx^n$

3) $(1+x^m)^n = \binom{n}{0} + \binom{n}{1}x^m + \binom{n}{2}x^{2m} + \cdots + \binom{n}{n}x^{nm}$

4) $(1-x^{n+1})/(1-x) = 1 + x + x^2 + x^3 + \cdots + x^n$

5) $1/(1-x) = 1 + x + x^2 + x^3 + \cdots$

6) $1/(1-ax) = 1 + ax + a^2x^2 + a^3x^3 + \cdots$

7) $1/(1+x)^n = \binom{-n}{0} + \binom{-n}{1}x + \binom{-n}{2}x^2 + \cdots = 1 + (-1)\binom{n+1-1}{1}x + (-1)^2\binom{n+2-1}{2}x^2 + \cdots$

8) $1/(1-x)^n = \binom{-n}{0} + \binom{-n}{1}(-x) + \binom{-n}{2}(-x)^2 + \cdots = 1 + (-1)\binom{n+1-1}{1}(-x) + (-1)^2\binom{n+2-1}{2}(-x)^2 + \cdots$