

25/04/2024

Midterm Exam

Duration: 90 minutes

Name:

Student No:

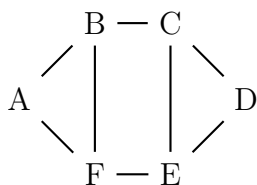
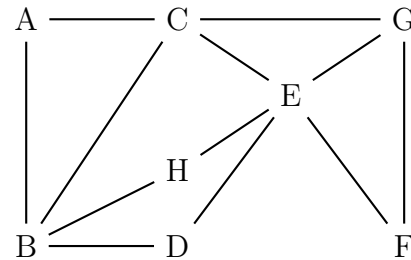
P1 [20 points]

a) Draw a 3-regular graph having 6 vertices.

b) In a complete bipartite graph with 13 vertices, what is the maximum number of edges?

P2 [20 points]a) If we draw K_6 and $K_{5,5}$ and then draw an edge from every vertex of K_6 to every vertex of $K_{5,5}$, how many edges will the final graph have?

b) In how many different ways can the following graph be labeled? Isomorphic labelings will be considered the same. [For example, if we mirror (flip around y axis) the graph, the new labeling with D on the left is the same as the original one.]

**P3 [10 points]** Find the measures for the following graph:

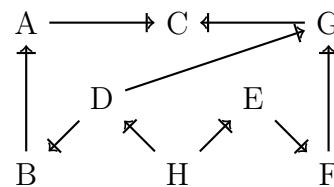
Enccentricity of G:

Enccentricity of H:

Radius:

Diameter:

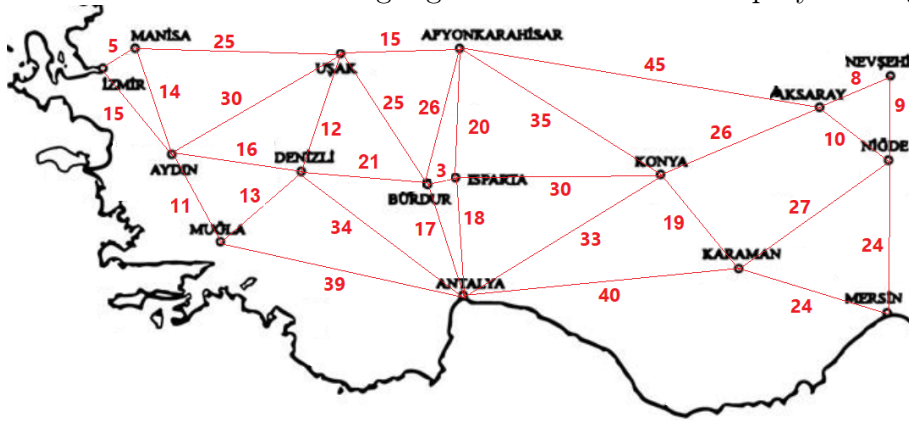
Center:

P4 [15 points] Topological Sort & Counting

Give a topological order for the graph:

How many different topological orders are there?

P5 [15 points] Minimum Spanning Tree In the map below, find a minimum spanning tree by using Prim's Algorithm starting from a random city (except Antalya) and write the cities in the order you add them to the MST. Also highlight the MST on the map by making the edges bold.



1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	
8		16	

P6 [20 points] Planar graphs

a) Can five houses be connected to two utilities without connections crossing? If yes draw it, otherwise prove why not.

b) Suppose that we have a 3-regular planar graph having 8 vertices. Into how many regions is the plane divided by a planar drawing of this graph? [Answer without drawing it. Direct answers get 0 credit, show your work.]

c) If you randomly create a 5-vertex graph by putting an edge or not with $1/2$ probability for every vertex pair, what is the probability of getting a planar graph?

d) If you randomly create a 6-vertex bipartite graph by first splitting the vertices into two groups having 3 vertices each, and then putting an edge or not with $1/2$ probability from every vertex of one group to every vertex of the other group, what is the probability of getting a planar graph?