

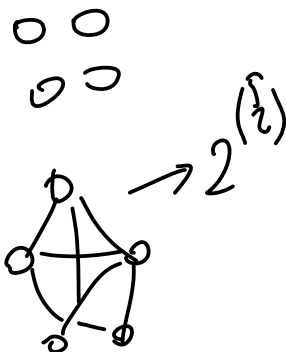
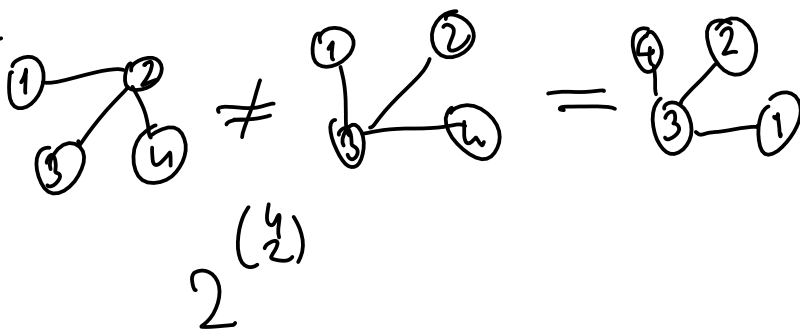
$$n=4$$



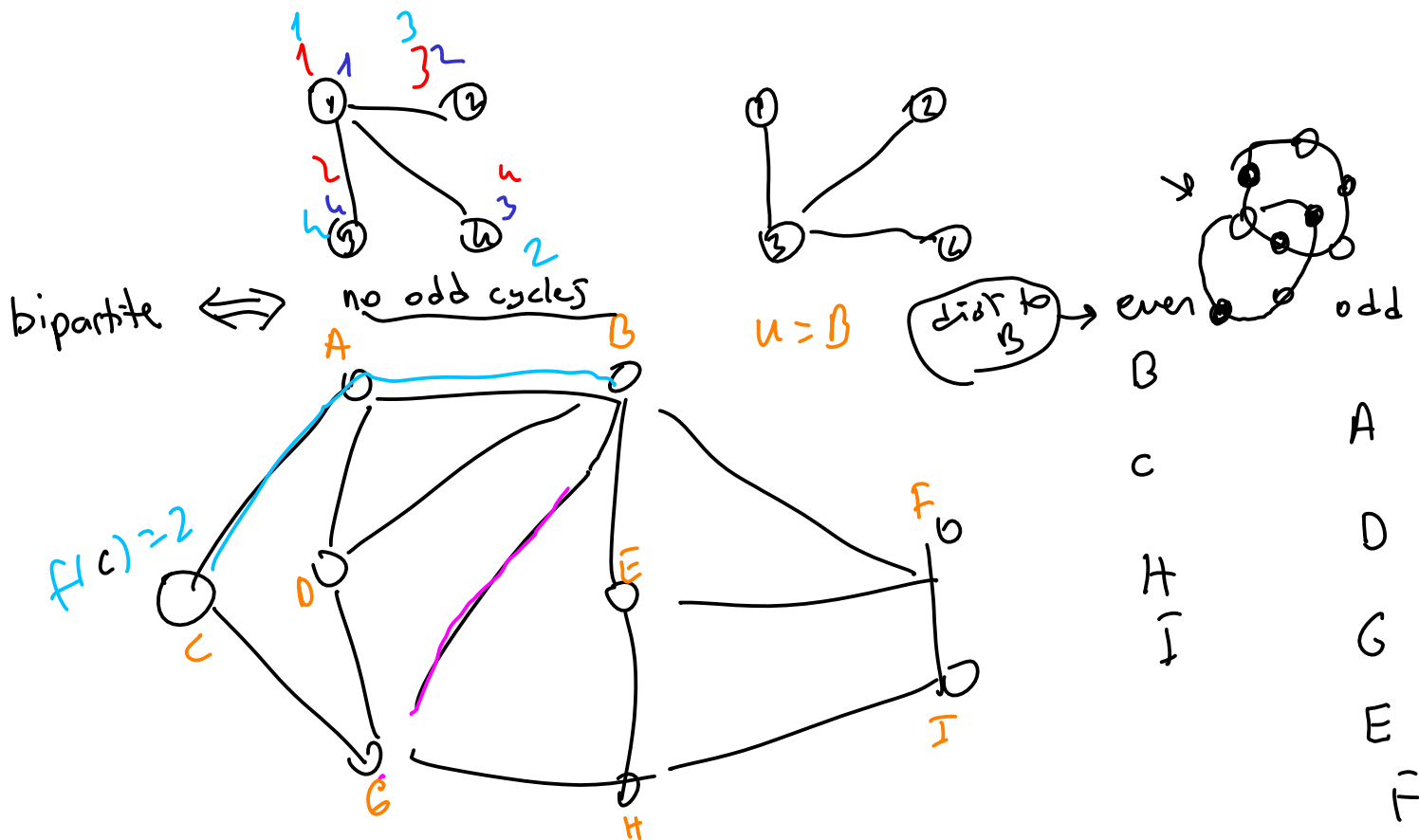
$$\binom{4}{2} = 6$$

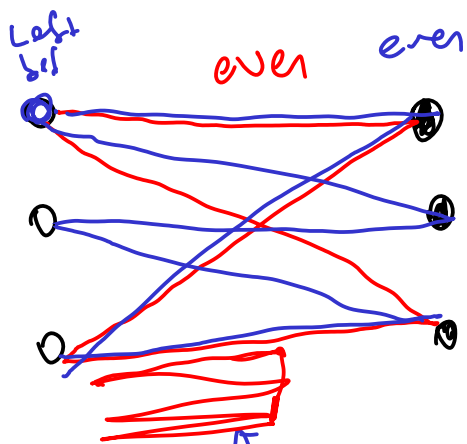
$\binom{n}{2} \rightarrow$ # of possible edges in graph with n vertices

labelled graph



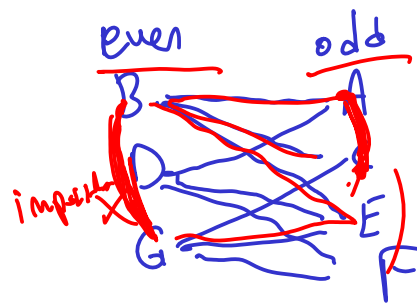
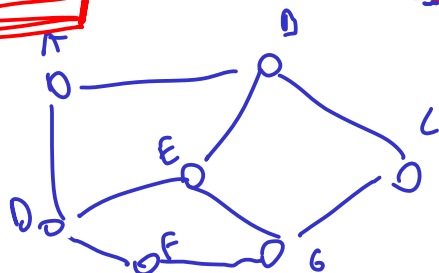
There are 11 different unlabelled graphs with 4 vertices due to isomorphism.





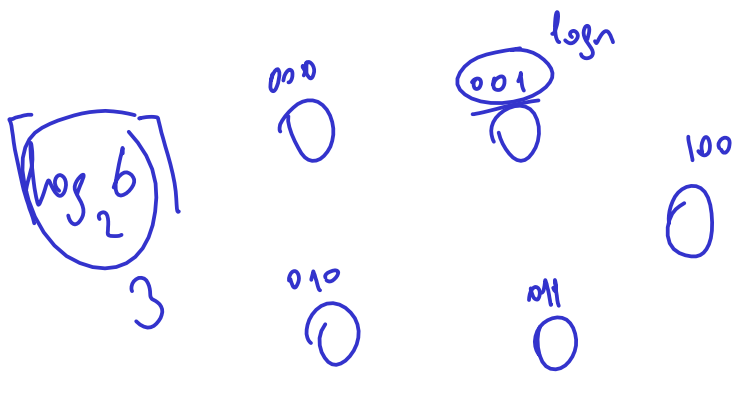
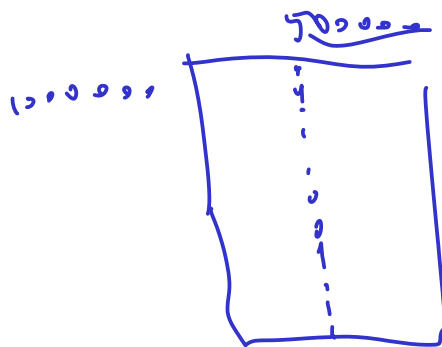
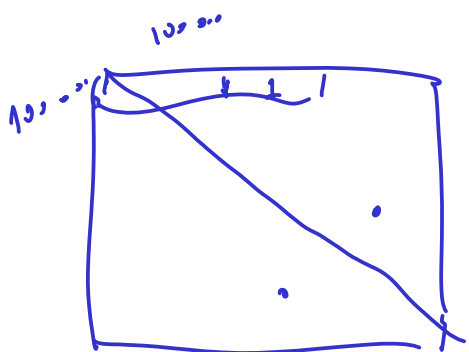
trivial.
bipartite \Rightarrow no odd cycle

no odd cycle \Rightarrow bipartite
 $u=B$



1000000 vertices
500000 edges.

sparse or dense?
secret yogurt



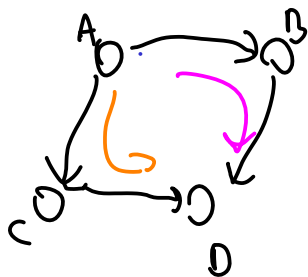
$\begin{matrix} n & m \\ \underline{000} : & \underline{010}, \underline{011} \\ \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{matrix}$
 $|V| = n$
 $|E| = m$

$\begin{bmatrix} \underline{\quad} \end{bmatrix} \begin{bmatrix} \underline{\quad} \end{bmatrix} = \begin{bmatrix} \vdots \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

0

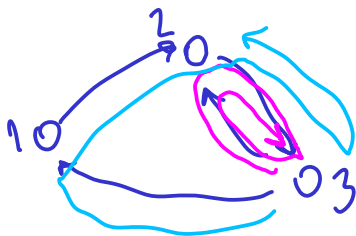
$$\begin{matrix} n \\ m \end{matrix} \cdot \begin{matrix} r \\ n \end{matrix} = \begin{matrix} r \\ m \end{matrix}$$



	A	B	C	D
A	0	1	1	0
B	0	0	0	1
C	0	0	0	1
D	0	0	0	0

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \\ \dots \\ \dots \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix}$$



3 → 2
length = 4 (2)



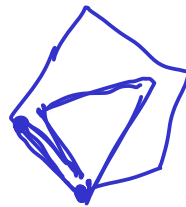
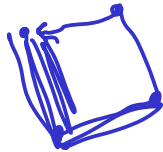
$m=4$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{S_0} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \textcircled{2} & 1 & 1 & 1 \\ 1 & \textcircled{2} & 1 & 1 \\ 1 & 1 & \textcircled{1} & 0 \\ 1 & 1 & 0 & \textcircled{3} \end{bmatrix}$$

$\text{tr}(A) = 0$

$\text{tr}(A^2) = 8$
 $\frac{8}{2} = 4$ degrees

$\sum \frac{d(v)}{2} = |E|$ ✓



$$\begin{bmatrix} \textcircled{2} & 1 & 1 & 1 \\ 1 & \textcircled{2} & 1 & 1 \\ 1 & 1 & \textcircled{1} & 0 \\ 1 & 1 & 0 & \textcircled{3} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \textcircled{2} & 3 & 1 & 4 \\ 3 & \textcircled{2} & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 4 & 4 & 3 & 2 \end{bmatrix}$$

$\rightarrow 6/6 = 1$