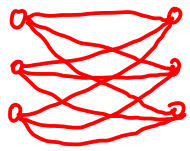
25/04/2024 | Midterm Exam | Duration: 90 minutes

Name: Solutions | Student No:

P1 [20 points]

a) Draw a 3-regular graph having 6 vertices.

$K_{3,3}$ is a simple example:



b) In a complete bipartite graph with 13 vertices, what is the maximum number of edges?

Possible such comp. bip. graphs are $K_{1,12}$, $K_{2,11}$, $K_{3,10}$, $K_{4,9}$, $K_{5,8}$ and $K_{6,7}$. Since $K_{m,n}$ has $m \cdot n$ edges, $K_{6,7}$ would give us the maximum number of edges: $6 \cdot 7 = 42$.

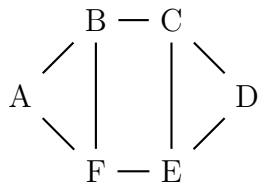
P2 [20 points]

a) If we draw K_6 and $K_{5,5}$ and then draw an edge from every vertex of K_6 to every vertex of $K_{5,5}$, how many edges will the final graph have?

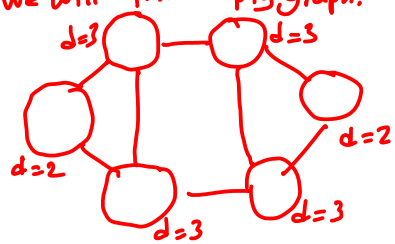
K_6 | Edges in between | $K_{5,5}$

$\binom{6}{2} + 6 \cdot 10 + 25 = 15 + 60 + 25 = 100.$

b) In how many different ways can the following graph be labeled? Isomorphic labelings will be considered the same. [For example, if we mirror (flip around y axis) the graph, the new labeling with D on the left is the same as the original one.]



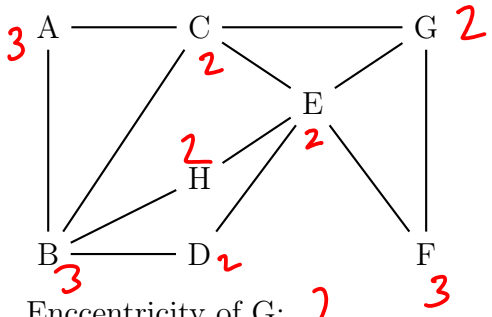
We will fill empty graph:



First choose two letters for degree 2 ones, and then place others.

$\binom{6}{2} \cdot \binom{4}{2} \cdot 2 = 15 \cdot 6 \cdot 2 = 180.$

P3 [10 points] Find the measures for the following graph:



Enccentricity of G: 2

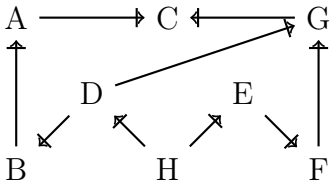
Enccentricity of H: 2

Radius: 2

Diameter: 3

Center: C, D, E, H, G

P4 [15 points] Topological Sort & Counting



Give a topological order for the graph:

H D B E F A G → C

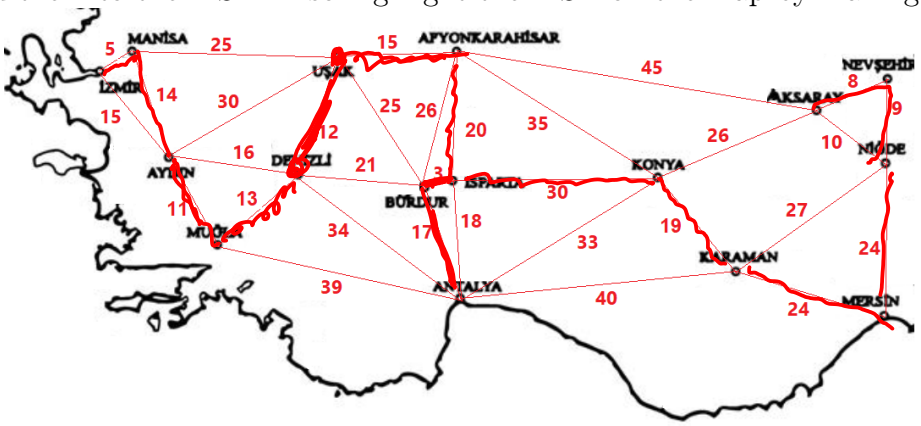
How many different topological orders are there?

H & C is places are determined: 1st and 8th.

if D → G wasn't present, we would just choose positions of D, B, A in: H _ _ _ _ _ C which is $\binom{6}{3}$. (and the rest would be E, F, G.

Since we have D → G, only invalid one among these is H E F G D B A C. So, the answer is $\binom{6}{3} - 1 = 19$.

P5 [15 points] Minimum Spanning Tree In the map below, find a minimum spanning tree by using Prim's Algorithm starting from a random city (except Antalya) and write the cities in the order you add them to the MST. Also highlight the MST on the map by making the edges bold.



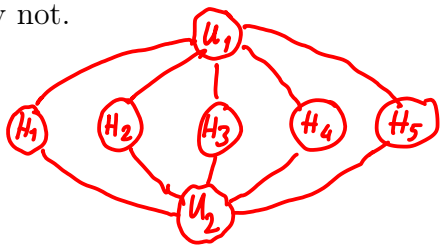
Let's choose Uşak to start:

1	Uşak	9	Burdur
2	Denizli	10	Antalya
3	Muğla	11	Konya
4	Aydın	12	Karaman
5	Manisa	13	Mersin
6	İzmir	14	Niğde
7	Afyon	15	Nevşehir
8	Isparta	16	Aksaray

P6 [20 points] Planar graphs

a) Can five houses be connected to two utilities without connections crossing? If yes draw it, otherwise prove why not.

Yes, of course:



b) Suppose that we have a 3-regular planar graph having 8 vertices. Into how many regions is the plane divided by a planar drawing of this graph? [Answer without drawing it. Direct answers get 0 credit, show your work.]

We have Euler's formula: $f = e - n + 2$ So, $f = 12 - 8 + 2 = 6$

3-reg: All 8 ver. has 3 outgoing edges.. $3 \times 8 = 24 = 2e$, $e = 12$.

We can even check it ooo

c) If you randomly create a 5-vertex graph by putting an edge or not with 1/2 probability for every vertex pair, what is the probability of getting a planar graph?

A 5-vertex graph is not planar iff it is K_5. So, to make it non-planar, we need all the possible $\binom{5}{2}$ edges. So, our probability is $1 - \frac{1}{2^{10}} = \frac{1023}{1024}$

d) If you randomly create a 6-vertex bipartite graph by first splitting the vertices into two groups having 3 vertices each, and then putting an edge or not with 1/2 probability from every vertex of one group to every vertex of the other group, what is the probability of getting a planar graph?

Such a graph is not planar iff all 3×3 edges are present giving us $K_{3,3}$. Thus, our prob. is $1 - \frac{1}{2^9} = \frac{511}{512}$.