

Index

This index uses the following conventions. Numbers are alphabetized as if spelled out; for example, “2-3-4 tree” is indexed as if it were “two-three-four tree.” When an entry refers to a place other than the main text, the page number is followed by a tag: ex. for exercise, pr. for problem, fig. for figure, and n. for footnote. A tagged page number often indicates the first page of an exercise or problem, which is not necessarily the page on which the reference actually appears.

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