

The Cosmic Echo Problem of the Andromeda Observatory

Story

Scientists at the Andromeda Observatory are recording cosmic signals from a distant galaxy using a single, sensitive receiver. Theoretically, these signals are expected to arrive at the receiver in increasing order of their **Energy Levels**; that is, low-energy signals should precede high-energy ones. However, gravitational anomalies encountered during the signals' journey through the fabric of space-time cause some signals to arrive earlier or later than expected, disrupting the sequence. This disruption creates an effect known as a "Cosmic Echo." A Cosmic Echo occurs whenever a signal with a **higher energy level is recorded before a signal with a lower energy level**. The observatory's mission is to map the gravitational instability of that region of the universe by measuring the total number of Cosmic Echoes in the recorded signal sequence. This measurement must be performed with extreme efficiency on large datasets.

Task (Problem Definition)

You are given a sequence of N integers, where each integer represents the Energy Level of a signal. This sequence represents the order in which the signals arrived at the observatory. Your goal is to calculate the total number of **Cosmic Echoes** in this sequence. A Cosmic Echo is defined as any pair of indices (i, j) that satisfies the condition $i < j$ and $\text{Array}[i] > \text{Array}[j]$.

- **Solution Requirement:** The time complexity of your solution must strictly be $O(n \log n)$. Simple nested-loop solutions with $O(n^2)$ complexity will not be accepted as they will time out under the given constraints.
- **Required Output:**
 - The total number of **Cosmic Echoes** in the signal sequence.

Example Scene

Scene 1 Signal Sequence: [8 , 4 , 2 , 1] Expected Total Cosmic Echoes: 6. *Explanation (Cosmic Echo Pairs):*

- The signal (8) has a higher energy level than the subsequent signals (4), (2), and (1). **(3 Echoes)**
 - (8, 4)
 - (8, 2)
 - (8, 1)
- The signal (4) has a higher energy level than the subsequent signals (2) and (1). **(2 Echoes)**
 - (4, 2)
 - (4, 1)

- The signal (2) has a higher energy level than the subsequent signal (1). **(1 Echo)**
 - (2, 1)
- For the signal (1), there are no subsequent signals with a lower energy level. **(0 Echoes)**

Final Total Cosmic Echoes: $3 + 2 + 1 = 6$ $3+2+1=6$.