MAT 222 Linear Algebra – Assignment 2

IMPORTANT: In the following problems, consider your student ID number as **a-b-c-d-e-f-g-h-i-j-k**.

Problem 1: (10 points) Consider the six data points given in the table below.

x_i	1	1.5	1.9	2.3	2.6	3
y_i	c	a	$j+1$	$k+1$	$\frac{(k+1)(j+1)}{2}$	$k \cdot j + 2$

In this problem, your task is to find the least squares regression line for this data. In the 10th week and the first lecture of the 11st week, we discussed several ways of doing this. In this assignment, your task is to use two of them, according to the instructions below.

(a) Find the least squares regression line for the given data using QR factorization of the design matrix. You are encouraged to get computer assistance in this part. For example, two python functions that can be used are `numpy.linalg.qr` and `scipy.linalg.qr`. (Hint: First you should use the given data to construct the design matrix.)

(b) In this part, you will use gradient descent method to compute the same least squares regression line numerically. Since it is not feasible to perform the necessary computation manually, a reasonable way is to write a small function in any computer program to carry out the task for you. In the web, there are many pages that describes how to do it using python; you can even make use of them. But you must first understand the mathematical formulation of the problem, of course. You can use the following parameters, although you are free to change them as long as your algorithm converges to a solution.

- Initial point: may be $\mathbf{x}_0 = (0, 0)$ or $\mathbf{x}_0 = (1, 1)$ or any randomly generated point
- Learning rate: $\alpha = 0.001$
- $\varepsilon = 0.005$
- Stopping criterion: $\|\nabla f\| < \varepsilon$

NOTE: Whenever you take computer assistance, please include the relevant prints/screenshots in your homework.

Problem 2: (5 points) Consider the following 4×4 linear system (Don't forget that j and k are the last two digits of your student id):

$$
\begin{aligned}
(11 + j)x_1 - x_2 + 2x_3 + x_4 &= 8 \\
x_1 + (11 + k)x_2 - 3x_3 - x_4 &= 10 \\
2x_1 - x_2 + (19 - j)x_3 - 2x_4 &= 0 \\
x_1 + 2x_2 + 4x_3 + (19 - k)x_4 &= 18
\end{aligned}
$$

In this problem, your goal is to approximately solve this system using Jacobi or Gauss-Seidel iteration. For this purpose you can use the following:

- Initial guess: $\mathbf{x}_0 = (0, 0, 0, 0)^T$
- Stopping criterion: $\mathbf{x}\text{TOL} = \|\mathbf{x}_n - \mathbf{x}_{n-1}\| < \varepsilon$, where $\|.\|$ is the maximum norm
- $\varepsilon = 0.01$

Your answer should include the final approximate solution $\mathbf{x}_n$ and the $\mathbf{x}$TOL value that caused your algorithm to stop. This problem can be reasonably solved by pencil and paper (possibly with some help from computer in eg. matrix multiplications), but you are again free to use a computer software/code and some websites. In case you do so, please include the relevant prints/screenshots in your work.