FACULTY OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING
EEM 110 LINEAR ALGEBRA AND VECTOR ANALYSIS
MIDTERM EXAM
15 April 2024

1. Muhittin, who is preparing for a sporting competition, is having a breakfast containing four food types every day. The macro contents of these food types are given below.

	Carbohydrate (g)	Protein (g)	Fat (g)
Mix A	3	1	2
Mix B	1	4	9
Mix C	14	6	1
Mix D	39	2	1

Muhittin wants to take exactly 50 grams of carbohydrate, 15 grams of protein and 12.5 grams of fat in the breakfast. How many units of each food type must he take to achieve his dietary goal? Your answer should consist of **nonnegative** numbers which may or may not be integers. (There may be infinitely many solutions.) **(30 points)**

2. Solve the following linear system by using Gaussian elimination, LU factorization, inverse matrix method or Cramer's rule. **(20 points)**

$$\begin{aligned} x - y - z &= 2 \\ 2x + y + z &= 1 \\ 3x + z &= 4 \end{aligned}$$

3. Consider the linear equations $x + y + z = 1$ and $x + 2y - z = 2$. Write down a third equation in the unknowns x, y, z such that the resulting 3×3 system has a unique solution. Note that your solution should not consist of trial and error. **(20 points)**

4. Let $U = \text{Span}\{2 + x + 3x^2, 1 - x\}$ and $V = \text{Span}\{2 + x + 3x^2, 1 - x, -1 + 4x + 3x^2\}$. Are U and V the same vector space or is V a larger space than U? Please support your answer with sufficient reasoning. **(20 points)**

5. Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 4 & -1 & 7 \\ 3 & -5 & 2 \\ 0 & 1 & 3 \end{bmatrix}$. If possible, find a nonzero vector $\mathbf{b}$ such that the system $\mathbf{Ax} = \mathbf{b}$ has infinitely many solutions. If it is not possible, state briefly why it is not. **(20 points)**

6. Consider the matrix $\mathbf{D} = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & -1 \\ 1 & 3 & -11 \end{bmatrix}$. Find the dimension of the null space of $\mathbf{D}$. Then find a vector from the null space of $\mathbf{D}$ of the form $\begin{bmatrix} 25 \\ * \\ * \end{bmatrix}$. **(20 points)**

Remarks: The duration of the exam is 90 minutes. Solve only 5 problems. In order to get full marks, please make explanations whenever necessary.