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a	b	c	d	e	f	g	h	i	j	k
2	0	2	0	0	8	0	8	0	1	9

$$x - 4y - z = 10$$

$$-x + 3y + 3z = u$$

$$x - 6y + 3z = \vartheta$$

$$\left[\begin{array}{ccc|c} 1 & -4 & -1 & 10 \\ -1 & 3 & 3 & u \\ 1 & -6 & 3 & \vartheta \end{array}\right] \xrightarrow[-S_1+S_3]{S_1+S_2} \left[\begin{array}{ccc|c} 1 & -4 & -1 & 10 \\ 0 & -1 & 2 & 10+u \\ 0 & -2 & 4 & \vartheta-10 \end{array}\right]$$

$$\xrightarrow{-2S_2+S_3} \left[\begin{array}{ccc|c} 1 & -4 & -1 & 10 \\ 0 & -1 & 2 & 10+u \\ 0 & 0 & \underline{\underline{0}} & -20-2u+\vartheta-10 \end{array}\right]$$

first of all to system has infinitely may solution ;

$$-20 - 2u + \vartheta - 10 = 0$$

$$\underline{\underline{\vartheta = 30 + 2u}}$$

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$$3a + b + 14c + 39d = 50$$
$$a + 4b + 6c + 2d = 15$$
$$2a + 9b + c + d = 12.5$$

$$\left[\begin{array}{cccc|c} 1 & 4 & 6 & 2 & 15 \\ 3 & 1 & 14 & 39 & 50 \\ 2 & 9 & 1 & 1 & 12.5 \end{array}\right] \xrightarrow[-2R_1+R_3]{-3R_1+R_2} \left[\begin{array}{cccc|c} 1 & 4 & 6 & 2 & 15 \\ 0 & -11 & -4 & 33 & 5 \\ 0 & 1 & -11 & -3 & -17.5 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 4 & 6 & 2 & 15 \\ 0 & 1 & -11 & -3 & -17.5 \\ 0 & -11 & -4 & 33 & 5 \end{array}\right] \xrightarrow{11R_2+R_3} \left[\begin{array}{cccc|c} 1 & 4 & 6 & 2 & 15 \\ 0 & 1 & -11 & -3 & -17.5 \\ 0 & 0 & -125 & 0 & -187.5 \end{array}\right]$$

pivots

d is a free variable (There is infinitely many solution, let's take $d = 1$)

$$-125c = -187.5$$
$$\boxed{c = 1.5}$$

$$b - 11c - 3d = -17.5$$
$$b = -17.5 + 16.5 + 1$$
$$\boxed{b = 0}$$

$$a + 4b + 6c + 2d = 15$$
$$a = 15 - 0 - 9 - 2$$
$$\boxed{a = 4}$$

mix A $\rightarrow$ 4 g

mix B $\rightarrow$ 0

mix C $\rightarrow$ 1.5 g

mix D $\rightarrow$ 1 g

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$$\left[\begin{array}{cccc|c} 1+u & 1 & 1 & 1 & 11 \\ 1 & 1-u & 1 & 1 & 11 \\ 1 & 1 & 1+v & 1 & 0 \\ 1 & 1 & 1 & 1-v & 0 \end{array}\right]$$

By the Cramer's rule ;

$$x = \frac{\begin{vmatrix} 11 & 1 & 1 & 1 \\ 11 & 1-u & 1 & 1 \\ 0 & 1 & 1+v & 1 \\ 0 & 1 & 1 & 1-v \end{vmatrix}}{\begin{vmatrix} 1+u & 1 & 1 & 1 \\ 1 & 1-u & 1 & 1 \\ 1 & 1 & 1+v & 1 \\ 1 & 1 & 1 & 1-v \end{vmatrix}} = \frac{44\cancel{v^2}}{u^2\cancel{v^2}} = \frac{44}{u^2}$$

$$\Rightarrow \begin{bmatrix} 11 & 1 & 1 & 1 \\ 11 & 1-u & 1 & 1 \\ 0 & 1 & 1+v & 1 \\ 0 & 1 & 1 & 1-v \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{bmatrix} 11 & 1 & 1 & 1 \\ 0 & -u & 0 & 0 \\ 0 & 1 & 1+v & 1 \\ 0 & 1 & 1 & 1-v \end{bmatrix} \Rightarrow \det = a_{11}A_{11}$$

$$= 11.(-1)^2 \begin{vmatrix} -u & 0 & 0 \\ 1 & 1+v & 1 \\ 1 & 1 & 1-v \end{vmatrix}$$

$$= 44v^2$$

By the Sarrus

$$\begin{vmatrix} -u & 0 & 0 \\ 1 & 1+v & 1 \\ 1 & 1 & 1-v \end{vmatrix} \quad \begin{matrix} -u & 0 & 0 \\ 1 & 1+v & 1 \end{matrix}$$

$$-u(1-v^2) - (-u) = -\cancel{u} + uv^2 + \cancel{u} = uv^2$$

$$\begin{bmatrix} 1+u & 1 & 1 & 1 \\ 1 & 1-u & 1 & 1 \\ 1 & 1 & 1+v & 1 \\ 1 & 1 & 1 & 1-v \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & -u & -u & v-u+uv \\ 0 & -u & 0 & v \\ 0 & 0 & v & v \\ 1 & 1 & 1 & 1-v \end{bmatrix}$$

$$\det = a_{41}A_{41} = 1.(-1)^5 \begin{vmatrix} -u & -u & v-u+uv \\ -u & 0 & v \\ 0 & v & v \end{vmatrix} \quad \begin{matrix} -u & -u & v-u+uv \\ -u & 0 & v \end{matrix}$$

$$= -uv(v-u+uv) - (-uv^2 + u^2v)$$

$$= -\cancel{uv^2} + \cancel{u^2v} - u^2v^2 + \cancel{uv^2} - \cancel{u^2v}$$

$$= (-1)^5 . -u^2v^2$$

$$= u^2v^2$$

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Span $\{ v_1, v_2, v_3 \dots, v_p \}$

Span $\{ v_1, v_2, v_3, \dots, v_p, u \}$

If u is a lineer combination of $v_1, v_2, v_3, \dots v_p$.

$$u = c_1 v_1 + c_2 v_2 + c_3 v_3 + \dots + c_p v_p$$

it will be redundant vector for span.

And span's dimension will be p.

But in this question ; u is not a lineer combination of $v_1, v_2, v_3, \dots, v_p$ so it is not redundant.

I mean it needs to be in the span.

let $\dim(\text{span}\{ v_1, v_2, \dots, v_p \}) = x$

(becouse we don't know they are redundant or not.)

So $\dim(\text{span}\{ v_1, v_2, \dots v_p, u \})$ will be $= \underline{\underline{x+1}}$

We know that $x < x+1$

So, I prove that

$$\dim(\text{span}\{ v_1, v_2, \dots v_p \}) < \dim(\text{span}\{ v_1, v_2, \dots v_p, u \})$$

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a b c d e f g h i j k

2 0 2 0 0 8 0 8 0 1 9

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$$2x + 2y + 8z = 1$$
$$5x + 3y + 10z = 2$$
$$ax + by + cz = k$$

to unique solution

$$\begin{vmatrix} 2 & 2 & 8 \\ 5 & 3 & 10 \\ a & b & c \end{vmatrix}$$
$$\begin{matrix} 2 & 2 & 8 \\ 5 & 3 & 10 \end{matrix}$$

$\det = 6c + 40b + 20a - 24a - 20b - 10c \neq 0$

$-4a - 20b - 4c \neq 0$

divide by -4 $\hookrightarrow$ $\underbrace{a + 5b + c}$ $\neq 0$

$\hookrightarrow$ I take their coefficient and $k \in \mathbb{R}$

let's check :

$$2x + 2y + 8z = 1$$
$$5x + 3y + 10z = 2$$
$$x + 5b + z = 3$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 8 & 1 \\ 5 & 3 & 10 & 2 \\ 1 & 5 & 1 & 3 \end{array}\right] \xrightarrow[-\frac{1}{2}R_1+R_3]{-\frac{5}{2}R_1+R_2} \left[\begin{array}{ccc|c} 2 & 2 & 8 & 1 \\ 0 & -2 & -10 & -1/2 \\ 0 & 4 & -3 & 5/2 \end{array}\right] \xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|c} 2 & 2 & 8 & 1 \\ 0 & -2 & -10 & -1/2 \\ 0 & 0 & \underline{\underline{-23}} & \underline{\underline{3/2}} \end{array}\right]$$

It has unique solution.

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$$D = \begin{bmatrix} 2 & 0 & 4 \\ 1 & -1 & -8 \\ 1 & 3 & 32 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 4 \\ 1 & -1 & -8 \\ 1 & 3 & 32 \end{bmatrix} \xrightarrow[-\frac{1}{2}R_1+R_3]{-\frac{1}{2}R_1+R_2} \begin{bmatrix} 2 & 0 & 4 \\ 0 & -1 & -10 \\ 0 & 3 & 30 \end{bmatrix} \xrightarrow{3R_2+R_3} \begin{bmatrix} 2 & 0 & 4 \\ 0 & -1 & -10 \\ 0 & 0 & 0 \end{bmatrix}$$

pivots

Column Space of D $= \left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} \right\}$

$2(j+1) = 2(2) = 4$ and $\underline{1}$ v is $\begin{bmatrix} 4 \\ 1 \\ * \end{bmatrix}$

$$c_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ * \end{pmatrix}$$

$2c_1 = 4 \longrightarrow c_1 = 2$

$c_1 - c_2 = 1 \qquad c_2 = 2 - 1 = \underline{\underline{1}}$

$c_1 + 3c_2 = *$

$* = 2 + 3 = \underline{\underline{5}}$