ECSE 543

Assignment 1

Numerical Methods in Electrical Engineering

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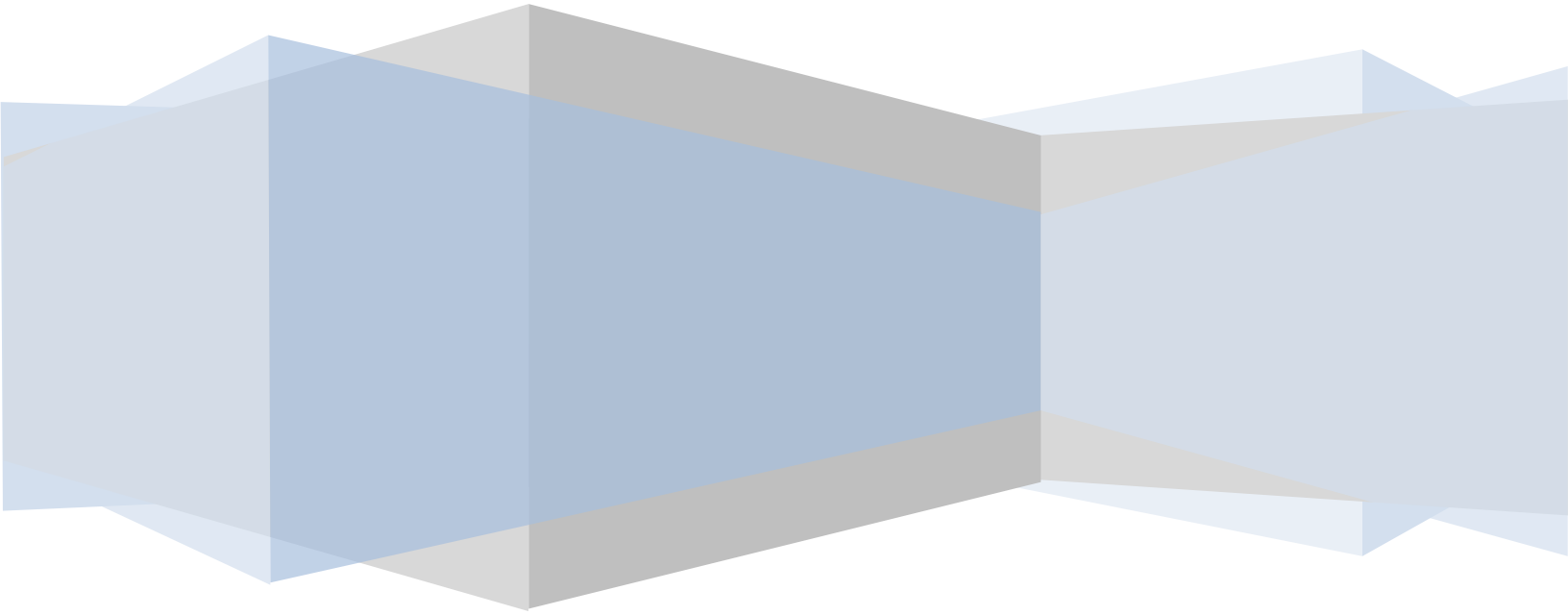

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Question 1

(a)

The Choleski decomposition was implemented according to the algorithm seen in class. The programming language used for the implementation was Python. A Matrix and MatrixElement classes were created to assist in coding the solution. The Matrix class had helper methods such as multiply (multiplies two matrices), transpose, subtract, clear, get (to get the value of an element), and set (to set the value of an element). Please see the appendix for the commented code (Choleski.py).

(b)

The real, symmetric and positive-definite matrices were constructed using the definition $A = LL^T$. In other words, we multiply an arbitrary lower matrix L by its transpose to obtain a real, symmetric and positive-definite matrix A. This was done for matrices A of size $n \times n$ with n=2, 3 and 4. See the next question for details of how the Choleski implementation was tested.

(c)

The program was tested successfully using the matrices from (b) and arbitrary x vectors. A method called testCholeski was written to take a lower matrix L and a vector x to generate the real, symmetric and positive-definite A ($A = LL^T$) and the vector b ($b = Ax$). These were then passed to the Choleski method to compute the solution. The program was made such that a visual comparison between the expected x and the resulting solution x was possible (allowing confirmation of the proper operation of the Choleski method).

(d)

The input file organizes data as follows:

Line 1: number of branches Line 2: number of nodes Next lines: J,R,E values for each branch Next lines: incidence matrix (columns separated by commas, and rows by newlines)

The program that reads the file (called circuitSolver.py) works by first reading the number of branches and nodes from the input file. It then goes on to construct the matrices Y, E, J and A (initializing them to zero) based on the number of branches and nodes.

In the next step, the program reads the J,R,E values of each branch and sets the corresponding elements in Y,E and J. Finally, it traverses the incidence matrix row by row (line by line in the file) and similarly sets the corresponding values in the matrix A.

Once Y, E, J and A are properly setup, a call to the Choleski method is made with the real, symmetric and positive-definite matrix defined as:

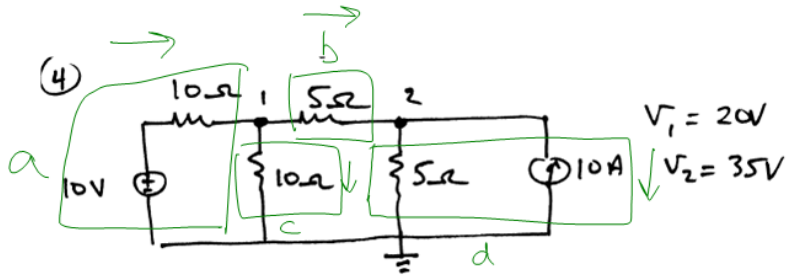
$$(AYA^T) \; or \; A.multiply(Y).multiply\big(A.transpose(\,)\big)$$

and the b vector as:

$$A(J - YE) \text{or } A.multiply\left(J.subtract\left(Y.multiply(E)\right)\right)$$

All 5 suggested networks were tested with input files and the voltage results matched the the given solution.

The figure below shows a sample circuit whose branches have been framed and whose A, Y, J and E matrices built. The corresponding input file is show below.



$$A = \begin{matrix} & a & b & c & d \\ 1 & -1 & +1 & +1 & 0 \\ 2 & 0 & -1 & 0 & +1 \end{matrix} \qquad J = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1/10 & 1/5 & 0 \\ 0 & 1/10 & 1/5 \end{bmatrix} \qquad E = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
4
2
0,10,10
0,5,0
0,10,0
10,5,0
-1,1,1,0
0,-1,0,1
```

The remaining circuits are placed in the appendix.

Question 2

(a)

Devising the algorithm to generate the input file turned out to be quite complicated.

First, computing the number of nodes and branches in the grid wasn't too hard.

$$numBranches = \frac{(N+1)N}{2} = \frac{3N^2+3N}{2}$$

$$numNodes = \frac{(N+1)(N+2)}{2} = \frac{N^2+3N+2}{2}$$

Since we are adding a test voltage source around the grid, we will take the bottom right corner to be our ground (hence removing it from the number of nodes), and incrementing the number of branches by 1. This added source branch will also have a test resistance (as R=0 breaks the Choleski algorithm implemented).

The next stage in our input file is to describe each branch line by line. This results in printing 0,1,0 for each of the grid branches and 0,1,1 for the source branch (having a voltage of 1V and test resistance of 1ohm).

The final and most complicated stage was generating the incidence matrix. This required a non-trivial algorithm. It consisted of building a list of branches where each branch is identified by coordinates (node1,node2). The nodes in the grid were numbered level by level (i.e. top vertex is 1, followed by 2 and 3 on the second level, then 4, 5 and 6 on the third level, etc.). The branches were generated based on this numbering scheme and likewise level by level. For each level, we would first save the branches linking the top level of nodes to the lower level of nodes, then follow that by the flat branches linking the nodes on the lower level. For example, in the case of N=2, we would have the following branches saved in a list in this order: (1,2),(1,3),(2,3),(2,4),(2,5),(3,5),(3,6),(4,5),(5,6).

This list of branches then allowed us to build the incidence matrix with dimensions $(numNodes - 1) \times (numBranches + 1)$. Each column represented a branch from our list of branches, and had all rows set to 0 except the two nodes defining the branch (eg. branch (3,5) would have row 1 set to +1 and row 5 set to -1). Careful consideration was taken in removing the last row (representing the ground node) and adding an extra column for the source voltage.

Once the input generated and the circuit solved, the only voltage of interest would be the one at node 1. This voltage is used in conjunction with the test resistance and the source voltage to determine the R_{in} of the grid. Voltage division is used as follows to determine the input resistance of the mesh:

$$V_1 = V_s\left(\frac{R_{in}}{R_{in}+R_{test}}\right)$$

$$R_{in} = \frac{V_1}{1-V_1}$$

N	$V_1\ (V)$	$R_{in}(ohms)$
2	0.52632	1.1111
3	0.58824	1.4286
4	0.62614	1.6748
5	0.65228	1.8759
6	0.67169	2.0459
7	0.68683	2.1932
8	0.69907	2.323
9	0.70924	2.4393
10	0.71786	2.5443

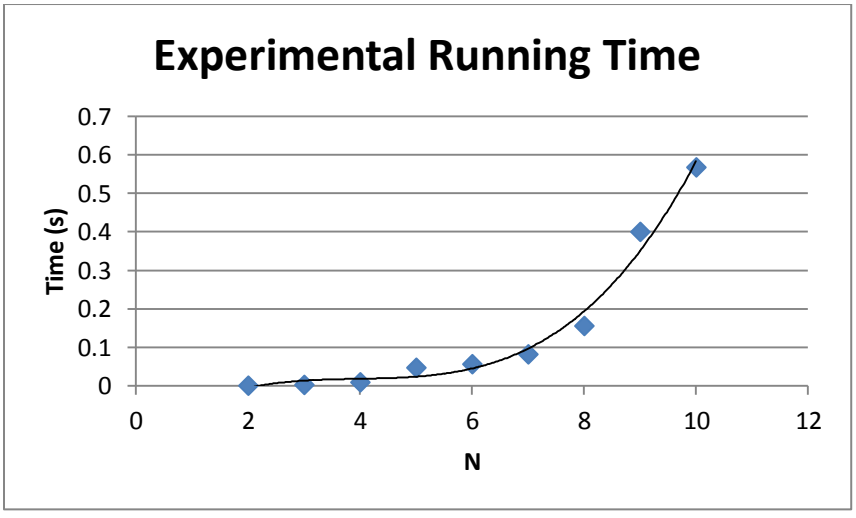
(b)

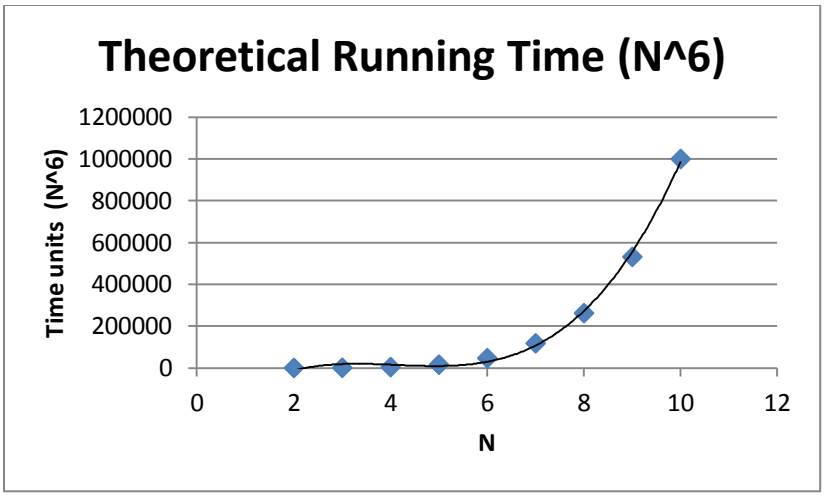
The Choleski algorithm implemented in Question 1 has three nested loops for the decomposition and elimination steps, and two nested loops for the back substitution step. This results in a total running time of $t(n) = O(n^3) + O(n^2) = O(n^3)$, where n x n is the dimension of the real, symmetric and positive definite matrix A in Ax=b.

In our triangle grid of resistors, there were $\left(\frac{N^2+3N+2}{2} - 1\right)$ nodes, resulting in a matrix A of size $\left(\frac{N^2+3N+2}{2} - 1\right) x \left(\frac{N^2+3N+2}{2} - 1\right)$. This means the Choleski algorithm for our scenario would run in $t(n) = O((N^2)^3) = O(N^6)$ where N is the number of resistors on each side of the grid.

The running time of the Choleski method was calculated using the time.clock() method in python. The starting time was registered before the execution of the algorithm, and likewise the ending time at the end of the algorithm. The elapsed time was then computed as the difference between the two.

The running time was thus collected for N=2 upto 10, and the following graphs show the results along with a comparison with the theoretical expectations.





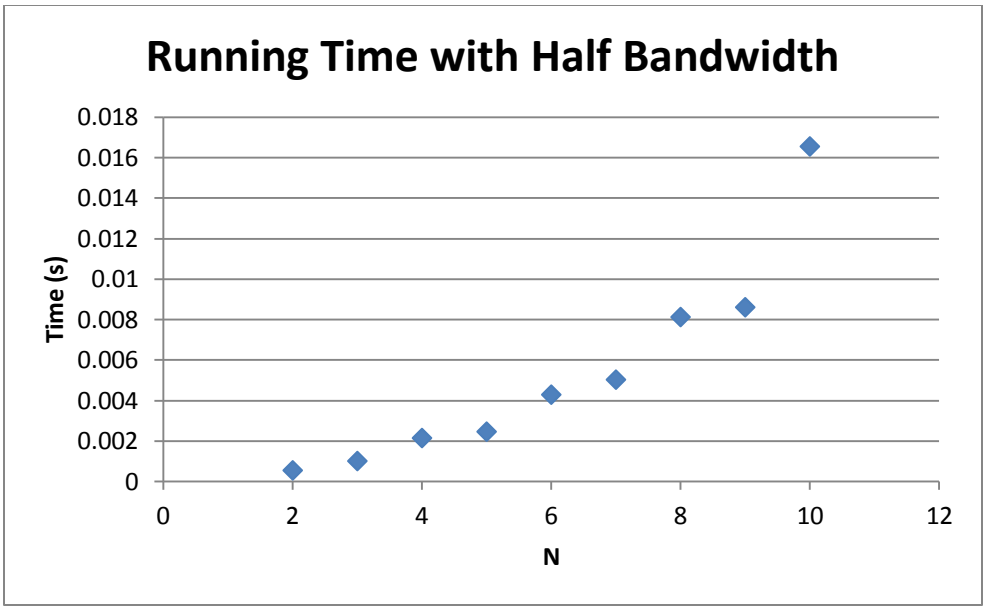
As can be seen from the shape of the plots above, the timings observed for the practical implementation are consistent with the theory. For a closer look at the actual timings, please see the time table in the next sub-question.

(c)

A method called getHalfBandwidth was written to determine the half-bandwidth of a given matrix. It works by examining every single row in the symmetric matrix and determining the number of elements from the first non-zero value to the diagonal element. The maximum such half-bandwidth is returned as the half-bandwidth of the matrix.

The Choleski algorithm was then optimized to exploit this property in sparse matrices by changing the looping indices in two instances. First, in the decomposition/elimination step, the middle nested loop was made to run from i=j+1 to the half-bandwidth (instead of n). Then, in the back substitution step, the inner loop was likewise changed to run from i=j+1 to the half-bandwidth.

These modifications resulted in significant improvement in the running time. As shown in the plot below when compared to previous experimental results, the running time went from values in the range of a tenth of a second to a hundredth of a second.



Theoretically, this improvement would have a running time of $O(b^2 n)$ as seen in the notes. In our scenario, we recall that n was $\left(\frac{N^2+3N+2}{2} - 1\right)$. Moreover, as seen in the table below, $b = N + 1$. Therefore, $O(b^2 n) = O\left((N+1)^2 \left(\frac{N^2+3N+2}{2} - 1\right)\right) = O(N^4)$. This indeed represents a significant improvement over our previous algorithm that ran in $O(N^6)$.

N	b = N + 1
2	4
3	5
4	6
5	7
6	8
7	9
8	10
9	11
10	12

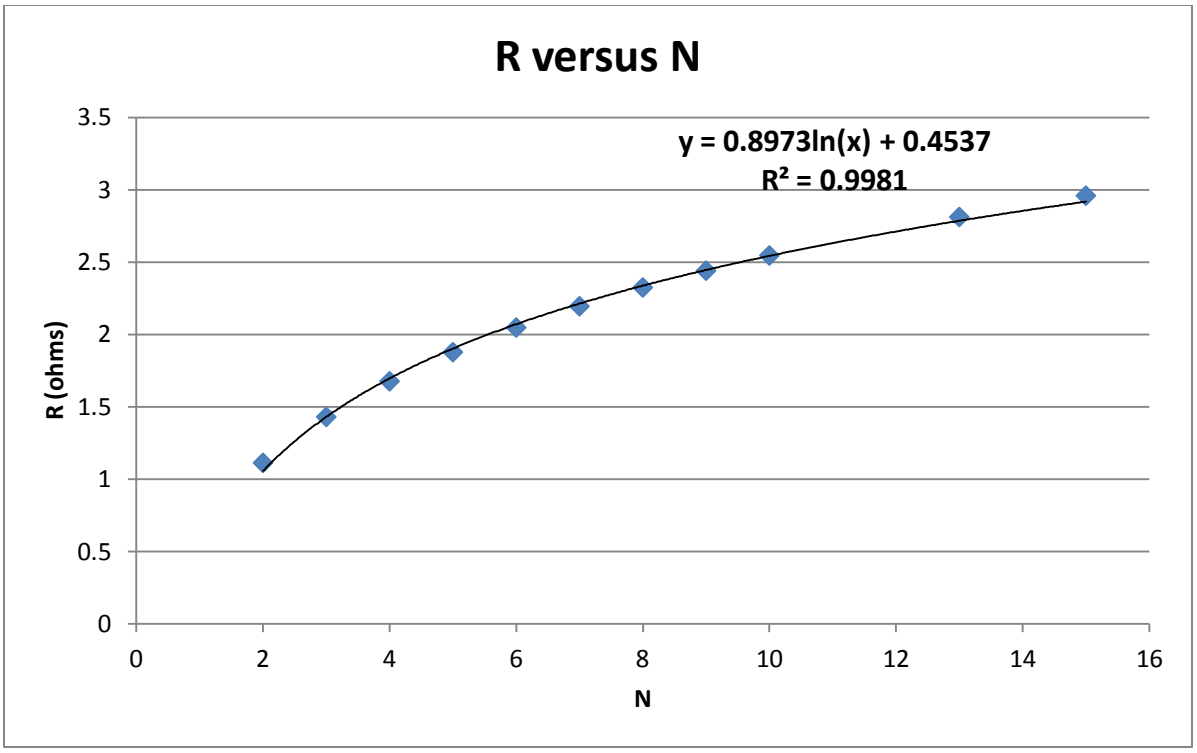
Finally, the table below affords a closer look at the timing improvements gained in the half-bandwidth approach when compared to the normal Choleski algorithm and further confirms our theoretical expectations.

N	Normal Choleski Time (s)	Half Bandwidth Time (s)
2	8.33E-04	5.56E-04
3	2.86E-03	1.02E-03
4	9.60E-03	2.15E-03
5	4.74E-02	2.47E-03

6	5.66E-02	4.30E-03
7	8.22E-02	5.04E-03
8	1.56E-01	8.13E-03
9	4.00E-01	8.61E-03
10	5.67E-01	1.66E-02

(d)

In order to obtain a more accurate fit, two additional networks were solved (N=13 and N=15) and all the points were plotted below. The function that best fits the curve is a logarithmic one and is shown in the graph.



Question 3

(a)

A Python program was written to solve the electrostatic problem using the Successive Over-Relaxation (SOR) algorithm. Translational symmetry was exploited by reducing the problem to the lower left quadrant of the structure.

The program used a hash to store the potential at each coordinate. In other words, the coordinates of a point (i,j) constituted the key in the hash where its corresponding potential was stored as the value.

The node-spacing (h) and the SOR parameter (w) were made into variables that can be easily changed for each run of the algorithm.

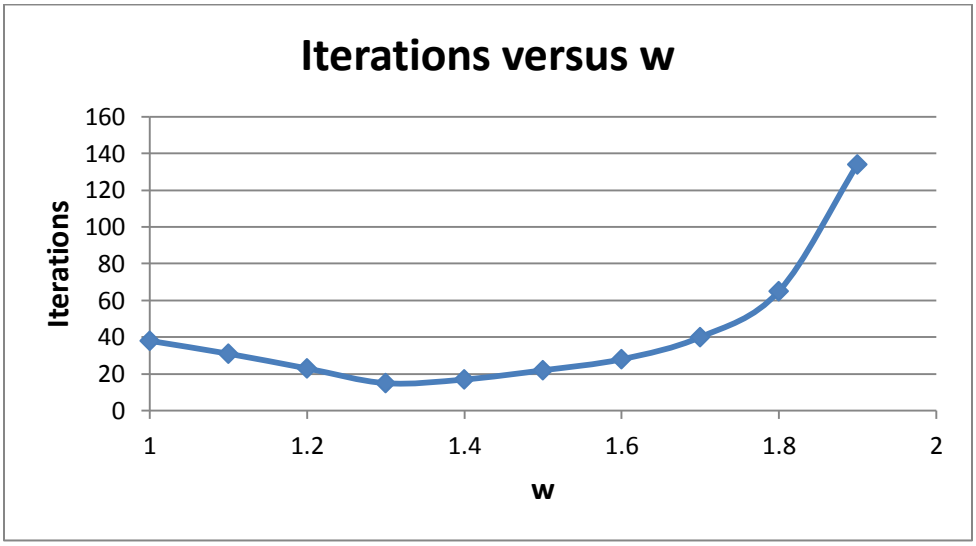
The commented code of the program can be seen in the appendix (finiteDiff.py).

(b)

With the node spacing h set to 0.02, the parameter w was varied from 1.0 to 1.9 and the number of iterations to convergence as well as the final potential at (0.06,0.04) were noted in the table below.

w	Number of iterations	(0.06,0.04) potential (V)
1.0	38	3.68421540222
1.1	31	3.68422038302
1.2	23	3.68421932593
1.3	15	3.68422663608
1.4	17	3.68422607221
1.5	22	3.68422493061
1.6	28	3.68423345287
1.7	40	3.68422493539
1.8	65	3.68423136283
1.9	134	3.68422493156

Furthermore, a graph of the number of iterations versus w was plotted below.

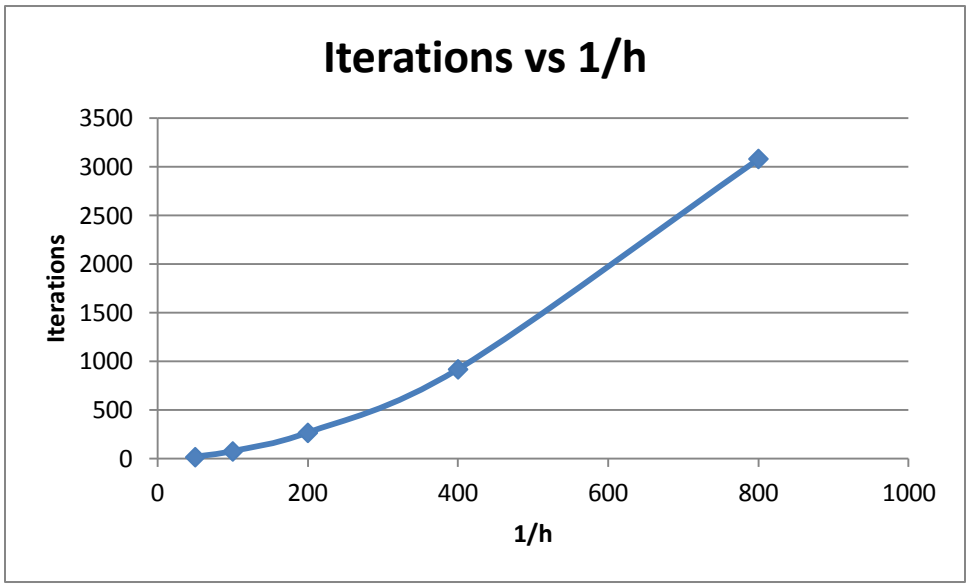


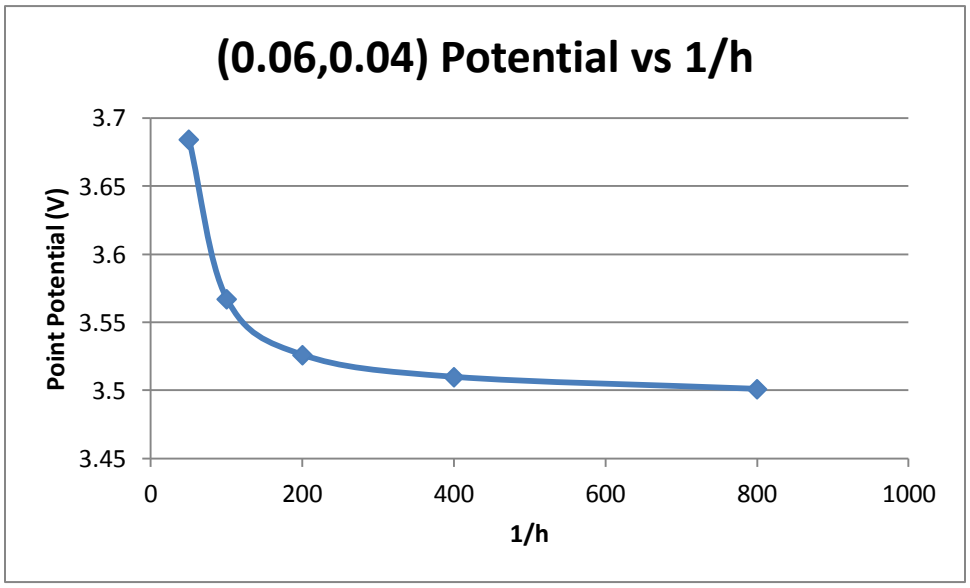
(c)

I set w=1.3 as it gives the lowest number of iterations as seen above. The table and plots below illustrate the change in iterations to convergence and potential at (0.06,0.04) as h is decreased (or equivalently, 1/h is increased).

h	1/h	Number of iterations	(0.06,0.04) potential (V)
0.02	50	15	3.68422663608
0.01	100	74	3.56707561273
0.005	200	265	3.52598765785
0.0025	400	918	3.50981240818
0.00125	800	3081	3.50098721043

Based on the results in the table above, the potential at (0.06,0.04) seems to converge towards 3.50 as h decreases. So I think the potential at that point is 3.50 (to three significant figures).



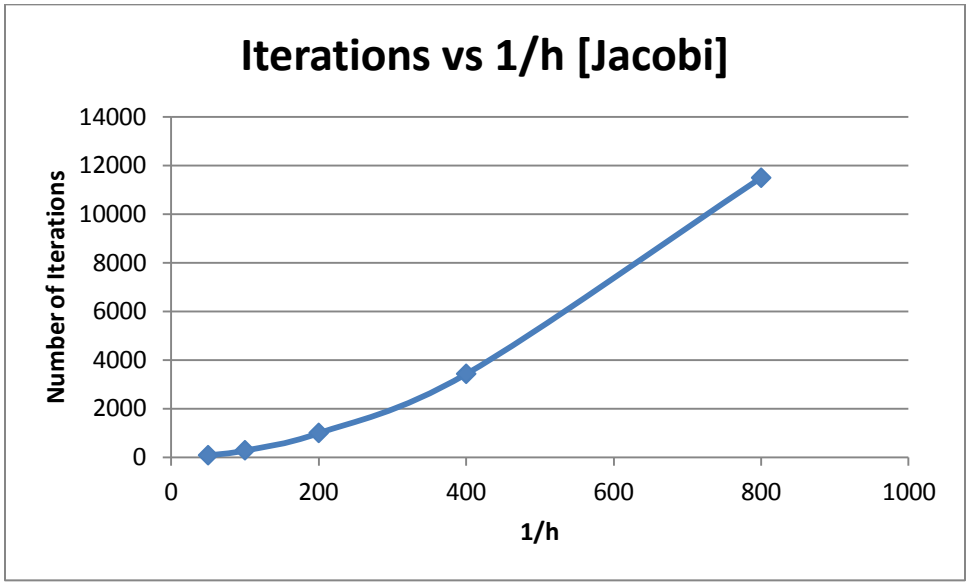


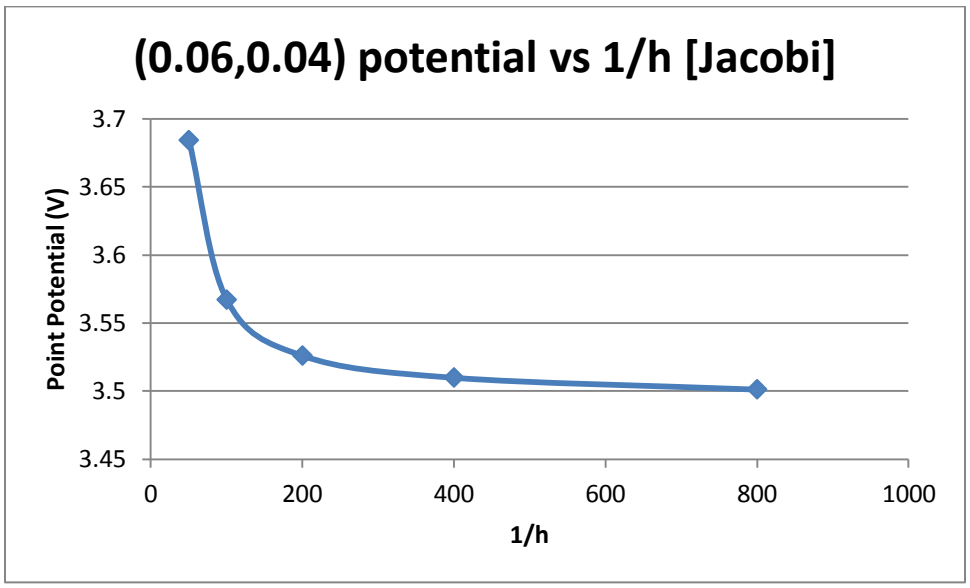
The first plot above clearly shows the exponential growth in iterations to convergence as 1/h is increased. The second plot demonstrates the convergence in accuracy of the potential at (0.06,0.04) as h is decreased (allowing us to safely hypothesize on the value of the potential to three significant figures).

(d)

The Jacobi method had a straightforward implementation given our work from part (a). The only notable point was that the code kept an extra copy of the previous state of the hash so as to be able to use potential values from the previous iteration that were overwritten in the present one. The results are shown in the table and graphs below.

h	1/h	Number of iterations	(0.06,0.04) potential (V)
0.02	50	75	3.68421752176
0.01	100	279	3.56707335729
0.005	200	991	3.52601999468
0.0025	400	3422	3.50990856262
0.00125	800	11485	3.501246871





The plots display similar curve behaviours as in the SOR method. The number of iterations seem to grow exponential with an increase of 1/h, and the point potential converges to an increasingly accurate value as 1/h increases. However, the number of iterations to convergence for the same value of h is far greater in the Jacobi method than in the SOR method. This is expected given the nature of the algorithm which takes longer to converge. It is worth noting that the time taken to run the Jacobi method with h

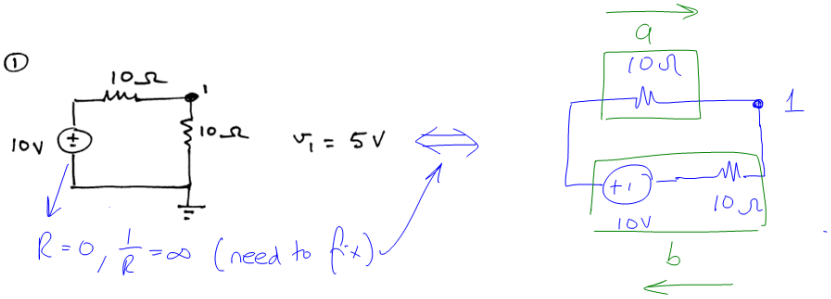
being its smallest value was over 30 minutes on an intel core i3. The Jacobi algorithm could have been optimized to avoid unnecessary copies and hence decreased the timing and space requirements (although the number of iterations would remain unchanged).

(e)

This sub-problem was not attempted due to lack of time. Completing the assignment took far more than the 13 hours assigned to it in the course outline. Nonetheless, the learning experience was definitely worth it.

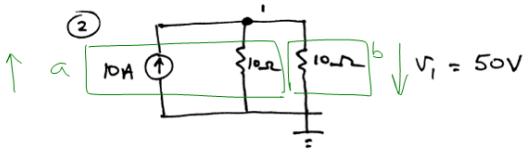
Appendix

Question 1 (d)



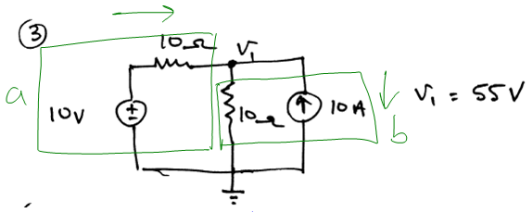
$$A = \begin{matrix} \\ 1 \end{matrix}\begin{matrix} a & b \\ \end{matrix}\begin{bmatrix} +1 & -1 \end{bmatrix} \qquad J = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1/10 & 0 \\ 0 & 1/10 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$



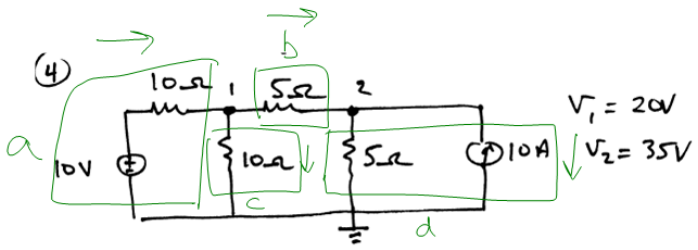
$$A = \begin{matrix} \\ 1 \end{matrix}\begin{matrix} a & b \\ \end{matrix}\begin{bmatrix} +1 & -1 \end{bmatrix} \qquad J = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1/10 & 0 \\ 0 & 1/10 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



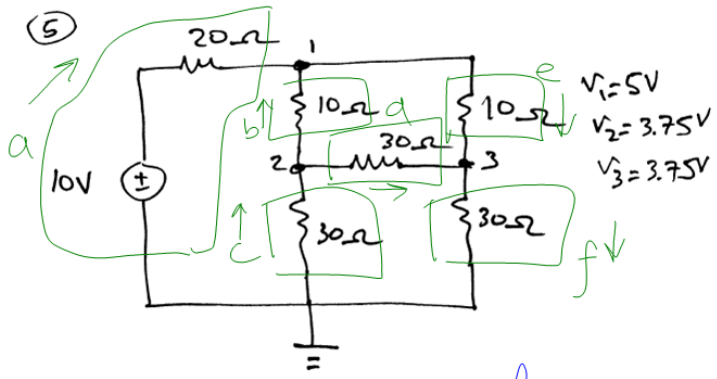
$$A = \begin{matrix} \\ 1 \end{matrix}\begin{matrix} a & b \\ \end{matrix}\begin{bmatrix} -1 & +1 \end{bmatrix} \qquad J = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1/10 & 0 \\ 0 & 1/10 \end{bmatrix} \qquad E = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$



$$A = \begin{matrix} \\ 1 \\ 2 \end{matrix} \begin{matrix} \begin{matrix} a & b & c & d \end{matrix} \\ \begin{bmatrix} -1 & +1 & +1 & 0 \\ 0 & -1 & 0 & +1 \end{bmatrix} \end{matrix} \qquad J = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1/10 & & 0 \\ & 1/5 & \\ & 1/10 & \\ 0 & & 1/5 \end{bmatrix} \qquad E = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$A = \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix} \begin{matrix} \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{bmatrix} -1 & -1 & 0 & 0 & +1 & 0 \\ 0 & +1 & -1 & +1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & +1 \end{bmatrix} \end{matrix} \qquad J = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1/20 & & & 0 \\ & 1/10 & & \\ & & 1/30 & \\ & & & 1/30 \\ 0 & & & \\ & & 1/10 & \\ & & & 1/30 \end{bmatrix} \qquad E = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Choleski.py

```
import math
import copy
from time import clock

class MatrixElement:

	def __init__(self,row,column,value):
		self.value=value
		self.i=row
		self.j=column

	def __repr__(self):
		return "("+str(self.i)+","+str(self.j)+")="+str(self.v)


class Matrix:

	#constructor can be called 2 ways
	#1) sending a list of lists of values
	#2) sending i and j to build an empty matrix of size i,j
	def __init__(self,listOfLists=None,i=None,j=None):
		self.elements={}
		self.rows=None
		self.columns=None

		if (listOfLists is None and (not i is None) and (not j is None)):
			self.rows=i
			self.columns=j

			for j in range(1,self.columns+1):
				for i in range(1,self.rows+1):
					self.elements[str(i)+","+str(j)]=MatrixElement(i,j,0)

		elif (not listOfLists is None):
			#TODO assert that listOfLists is a list
			#assert isinstance(listOfLists, List)

			self.rows=len(listOfLists)
			assert self.rows>0 #there needs to be at least one list inside the listOfLists

			#check that all the lists have same number of items & assign it to self.columns
			for i in range(0,self.rows):
				if (self.columns==None): #first iteration
					self.columns=len(listOfLists[i])
				else:
					assert len(listOfLists[i])==self.columns

			#create the matrix elements
			for i,list in enumerate(listOfLists):
				for j,value in enumerate(list):

	self.elements[str(i+1)+","+str(j+1)]=MatrixElement(i+1,j+1,value)

	#returns Matrix product of self with passed multiplier Matrix
	def multiply(self,multiplier):
		#return None if inner dimensions don't match
		if (self.columns!=multiplier.rows):
			return None

		innerDimension=self.columns

		result=Matrix(i=self.rows,j=multiplier.columns)

		for j in range(1, multiplier.columns+1):
			for i in range(1, self.rows+1):
				sum=0
				for k in range(1, innerDimension+1):
```

```
                                        sum+=self.get(i,k)*multiplier.get(k,j)

                                result.set(i,j,sum)

                return result

        #returns the self's transpose as a Matrix
        def transpose(self):
                result=Matrix(i=self.columns,j=self.rows)

                for j in range(1, self.columns+1):
                        for i in range(1, self.rows+1):
                                result.set(j,i,self.get(i,j))

                return result

        def subtract(self,subtrahend):
                if (self.rows!=subtrahend.rows or self.columns!=subtrahend.columns):
                        return None

                result=Matrix(i=self.rows,j=self.columns)

                for j in range(1, self.columns+1):
                        for i in range(1, self.rows+1):
                                result.set(i,j,self.get(i,j)-subtrahend.get(i,j))

                return result

        def get(self,i,j):
                return self.elements[str(i)+","+str(j)].value

        def set(self,i,j,value):
                self.elements[str(i)+","+str(j)].value=value

        #sets all values to 0
        def clear(self):
                for j in range(1,columns+1):
                        for i in range(1, rows+1):
                                self.set(i,j,0)

        def __repr__(self):
                string=""
                #using a list for the values that will be added to the string to be
                #printed according to our preferred format (i.e. %.2f)
                string_values=[]
                for i in range(1,self.rows+1):
                        for j in range(1,self.columns+1):
                                string+=" "

                                #negative numbers have one extra dash character (-)
                                #less one decimal place for -ve numbers to keep alignment
                                value=self.elements[str(i)+","+str(j)].value
                                if (value>=0):
                                        string+="%.5f"
                                else:
                                        string+="%.5f"

                                string_values.append(value)
                        string+="\n"
                return string % tuple(string_values)

#Solves and prints x in Ax=b given matrices A and b
def Choleski(A,b,halfBandwidth=None):
        #if no halfBandwidth is provided, then it equals columns=rows
        if (halfBandwidth is None):
                halfBandwidth=A.columns

        originalA=copy.deepcopy(A)
        originalb=copy.deepcopy(b)
```

```
        n=A.rows

        #checks to see that A is square, and b matches A
        assert n==A.columns
        assert n==b.rows

        #TODO checks to see if A is symmetric?

        startTime=clock()

        #ELIMINATION
        for j in range(1,n+1):
                if (A.get(j,j)<=0):
                        print "Choleski Error: Passed A is not real, symmetric or positive definite"
                        return -1

                A.set(j,j,math.sqrt(A.get(j,j)))
                b.set(j,1,b.get(j,1)/A.get(j,j))

                #added this to set the upper part of A (L) to 0 [optional in the algorithm]
                for i in range(1,j):
                        A.set(i,j,0)

                for i in range(j+1,halfBandwidth+1):
                        A.set(i,j,A.get(i,j)/A.get(j,j))
                        b.set(i,1,b.get(i,1)-(A.get(i,j)*b.get(j,1)))

                        for k in range(j+1,i+1):
                                A.set(i,k,A.get(i,k)-(A.get(i,j)*A.get(k,j)))

        #BACK SUBSTITUTION
        x=Matrix(i=n,j=1) #create empty matrix of specified dimensions

        #counting backwards starting from n
        for i in range(n,0,-1):
                #compute the summation
                sum=0
                for j in range(i+1,halfBandwidth+1):
                        sum+=A.get(j,i)*x.get(j,1)

                x.set(i,1,(b.get(i,1)-sum)/A.get(i,i))

        elapsedTime=clock()-startTime

        # print "================================================"
        # print "START Choleski Function Output                                                  "
        # print "================================================"
        # print ""
        # print "Given:"
        # print "--------------------------"
        # print "A ="
        # print originalA
        # print "b ="
        # print originalb
        # print ""
        print "--------------------------"
        print "Solution"
        print "--------------------------"
        print "x ="
        print x
        # print "================================================"
        # print "END Choleski Function Output                                                "
        # print "================================================"
        print ""

        return elapsedTime

#gets the half bandwidth of a symmetric matrix A
```

```
#assumes A symmetric
def getHalfBandwidth(A):
	if (A.rows!=A.columns):
		print "Matrix provided to getHalfBandwidth is not symmetric"
		return -1

	hb=0
	for i in range(1,A.rows+1):
		for j in range(1,i+1):
			if (A.get(i,j)!=0):
				#set hb=max(hb,i-j+1)
				if (hb < (i-j+1)):
					hb=(i-j+1)
					break;

	return hb

#takes lower matrix L, generates real, symmetric and positive definite A (A=LL_T)
#gets b by multiplying x by A (b=Ax)
#calls Choleski and prints expected vs obtained results
#CONDITION: x must be vector of length=rows of lower matrix L
def testCholeski(L,x):
	L_T=L.transpose()
	A=L.multiply(L_T)

	#assertion to ensure that A and x can be multipled (Ax=b)
	assert A.columns==x.rows

	b=A.multiply(x)

	print "-------------------------"
	print "Expected Solution"
	print "-------------------------"
	print "x ="
	print x

	#call Choleski
	Choleski(A,b)

	#At the output of a Choleski execution, A contains L and b contains y
	# print "L = "
	# print A

	# print "y = "
	# print b

#runs tests to ensure proper functioning of choleski fcn
#For assignment 1, Problem 1, part (b) and (c)
def runCholeskiTests():
	L=Matrix([	[2, 0],
			[4, 3]
			])

	x=Matrix([	[3],
			[23]
			])

	#x must be vector of length=rows of lower matrix L
	testCholeski(L,x)

	L=Matrix([	[-3, 0, 0],
			[3, 23, 0],
			[67, -89, 10]
			])

	x=Matrix([	[20],
			[-45],
			[46]
			])
```

```
        #x must be vector of length=rows of lower matrix L
        testCholeski(L,x)

        L=Matrix([          [345.56, 0, 0, 0],
                            [34, -423.5539, 0, 0],
                            [2958.478, -2474.8, 10, 0],
                            [123.495, 123.039, 9340.0, -2349.3]
                    ])

        x=Matrix([          [3746895.92783],
                            [-4774975.8368],
                            [29649030.764993],
                            [9793479.3469802]
                    ])

        #x must be vector of length=rows of lower matrix L
        testCholeski(L,x)


#runCholeskiTests()
```

CircuitSolver.py

```
from Choleski import Matrix, Choleski, getHalfBandwidth

#makes the appropriate call to the Choleski method given
#the incidence matrix A, matrix Y, E and J
def solveCircuit(A,Y,E,J):
        Choleski_A=A.multiply(Y).multiply(A.transpose())
        halfBandwidth=getHalfBandwidth(Choleski_A)

        elapsedTime=Choleski(Choleski_A,A.multiply(J.subtract(Y.multiply(E))))
        print "Time Taken: "+ str(elapsedTime)

def buildMatrices(filename):
        global Y, E, J, A

        #assumes proper formatting (no error detection/correction)
        f=open(filename)
        lines=f.readlines()
        f.close()

        branches=int(lines[0])
        nodes=int(lines[1])

        Y=Matrix(i=branches,j=branches)
        E=Matrix(i=branches,j=1)
        J=Matrix(i=branches,j=1)
        A=Matrix(i=nodes,j=branches)

        #NB: lines list uses array indexing (starts at 0), but Matrix class uses indexing starting at 1.

        for lineNum in range(2,(branches+1)+1): #skip first two lines already read (adjusted for array
indexing, i.e. starts at 0)
                branch=lineNum-1
                for i,v in enumerate(lines[lineNum].split(',')):
                        if (i==0):
                                J.set(branch,1,float(v))
                        elif (i==1):
                                Y.set(branch,branch,1/float(v))
                        else:
                                E.set(branch,1,float(v))

        for lineNum in range((branches+1)+1,(branches+1+nodes)+1):
                node=lineNum-(branches+1)
```

```
                    for j,v in enumerate(lines[lineNum].split(',')):
                              A.set(node,j+1,float(v))

while (True):
          filename=raw_input("Enter path to input file(q to quit): ")
          if (filename=='q'):
                    break
          buildMatrices(filename)
          solveCircuit(A,Y,E,J)
```

InputGenerator.py

```
from Choleski import Matrix

branches=[]

n=int(raw_input("Enter N: "))
output=raw_input("Enter output filename: ")

def populateBranches():
          global branches


          upperNodes=[1]
          node=1 #will hold the node number

          #build a list of branches (where a branch is identified by (node1,node2))
          #method used (start from top, going down level by level)
          for i in range(2,(n+1)+1):
                    currentNodes=[]

                    for j in range(1,i+1):
                              node+=1
                              currentNodes.append(node)

                    #branches linking upperNodes level to currentNodes level
                    for index,upperNode in enumerate(upperNodes):
                              branches.append((upperNode,currentNodes[index]))
                              branches.append((upperNode,currentNodes[index+1]))

                    #branches linking currentNodes level
                    for index,currentNode in enumerate(currentNodes):
                              if (index+1==len(currentNodes)): #stop at last node
                                        break
                              branches.append((currentNode,currentNodes[index+1]))

                    upperNodes=currentNodes



numBranches=(3*n**2+3*n)/2
numNodes=(n**2+3*n+2)/2
populateBranches()

numBranches+=1 #add 1 to the number of branches (for source branch)

#build incidence matrix from branches list
A=Matrix(i=numNodes,j=numBranches)
for j,branch in enumerate(branches):
          A.set(branch[0],j+1,1)
          A.set(branch[1],j+1,-1)

#set source branch
A.set(1,numBranches,-1)
A.set(numNodes,numBranches,1)


f=open(output,'w')
```

```
f.write(str(numBranches)+'\n')
#number of nodes (subtract one taken as ground)
f.write(str(numNodes-1)+'\n')


#print J,R,E for each branch (0,1,0)
for i in range(1,numBranches):
        f.write('0,1,0\n')
#print source branch
f.write('0,1,1\n') #1V source, 1ohm R_test

#print incidence matrix
for i in range(1,A.rows): #stop at rows, and not rows+1 as we discard last node taken as ground
        f.write(str(int(A.get(i,1))))
        for j in range(2,A.columns+1):
                f.write(','+str(int(A.get(i,j))))
        f.write('\n')

f.close()
```

inputGenerator.py

```
from Choleski import Matrix

branches=[]

n=int(raw_input("Enter N: "))
output=raw_input("Enter output filename: ")

def populateBranches():
        global branches


        upperNodes=[1]
        node=1 #will hold the node number

        #build a list of branches (where a branch is identified by (node1,node2))
        #method used (start from top, going down level by level)
        for i in range(2,(n+1)+1):
                currentNodes=[]

                for j in range(1,i+1):
                        node+=1
                        currentNodes.append(node)

                #branches linking upperNodes level to currentNodes level
                for index,upperNode in enumerate(upperNodes):
                        branches.append((upperNode,currentNodes[index]))
                        branches.append((upperNode,currentNodes[index+1]))

                #branches linking currentNodes level
                for index,currentNode in enumerate(currentNodes):
                        if (index+1==len(currentNodes)): #stop at last node
                                break
                        branches.append((currentNode,currentNodes[index+1]))

                upperNodes=currentNodes




numBranches=(3*n**2+3*n)/2
numNodes=(n**2+3*n+2)/2
populateBranches()

numBranches+=1 #add 1 to the number of branches (for source branch)
```

```
#build incidence matrix from branches list
A=Matrix(i=numNodes,j=numBranches)
for j,branch in enumerate(branches):
          A.set(branch[0],j+1,1)
          A.set(branch[1],j+1,-1)

#set source branch
A.set(1,numBranches,-1)
A.set(numNodes,numBranches,1)


f=open(output,'w')

f.write(str(numBranches)+'\n')
#number of nodes (subtract one taken as ground)
f.write(str(numNodes-1)+'\n')


#print J,R,E for each branch (0,1,0)
for i in range(1,numBranches):
          f.write('0,1,0\n')
#print source branch
f.write('0,1,1\n') #1V source, 1ohm R_test

#print incidence matrix
for i in range(1,A.rows): #stop at rows, and not rows+1 as we discard last node taken as ground
          f.write(str(int(A.get(i,1))))
          for j in range(2,A.columns+1):
                    f.write(','+str(int(A.get(i,j))))
          f.write('\n')

f.close()
```

finiteDiff.py

```
outLength=0.2
inWidth=0.08
inHeight=0.04
ERROR=10**-5

outVoltage=0
inVoltage=10

#default h and w
h=0.02
w=1

#maximum coordinate of i (where i starts at 0)
maxI=(outLength/2)/h

maxJ=(outLength/2)/h

#maximum pts when j=maxJ where we are still on neumann boundary (and not V=10)
maxI_Neumann=(outLength/2-inWidth/2)/h   -1 # -1 to remove the boundary point (which is also 10V)
maxJ_Neumann=(outLength/2-inHeight/2)/h    -1

#will contain potential values (addressable by coordinate points)
hash={}

def buildHash():
          global hash
```

```
        for i in range(0,int(maxI+1)):
                for j in range(0,int(maxJ+1)):
                        if (i==0 or j==0):
                                hash[(i,j)]=float(outVoltage)
                        # elif (i==maxI and j<=maxJ_Neumann):
                                # hash[(i,j)]=0
                        # elif (j==maxJ and i<=maxI_Neumann):
                                # hash[(i,j)]=0
                        elif (i>maxI_Neumann and j>maxJ_Neumann):
                                hash[(i,j)]=float(inVoltage)
                        else:
                                hash[(i,j)]=float(0)

def printHash():
        import sys

        for j in range(int(maxJ),-1,-1):
                for i in range(0, int(maxI)+1):
                        sys.stdout.write(str(hash[(i,j)]))
                        sys.stdout.write(',')
                print ''

def solveFDM():
        iterations=0
        repeat=True

        while(repeat):
                iterations+=1

                #working bottom->up, left->right to respect order of computations as per the algorithm
                #skip i=0 , j=0 as these are the outer boundary (dirichlet)
                for i in range(1,int(maxI)+1):
                        for j in range (1, int(maxJ)+1):
                                if (i>maxI_Neumann and j>maxJ_Neumann):
                                        continue #skip the inner conductor with fixed potential 10V
                                elif (i==maxI):
                                        hash[(i,j)]=( (1-w) * hash[(i,j)] ) + float(w)/4 * (
hash[(i-1,j)] + hash[(i,j-1)] + hash[(i-1,j)] + hash[(i,j+1)]  )#neumann
                                elif (j==maxJ):
                                        hash[(i,j)]=( (1-w) * hash[(i,j)] ) + float(w)/4 * (
hash[(i-1,j)] + hash[(i,j-1)] + hash[(i+1,j)] + hash[(i,j-1)]  ) #neumann
                                else:
                                        hash[(i,j)]= ( (1-w) * hash[(i,j)] ) + float(w)/4 * (
hash[(i-1,j)] + hash[(i,j-1)] + hash[(i+1,j)] + hash[(i,j+1)]  )



                #check if residual for each node is small enough
                repeat=False

                for i in range(1,int(maxI)+1):
                        for j in range (1, int(maxJ)+1):
                                if (i>maxI_Neumann and j>maxJ_Neumann):
                                        continue #skip the inner conductor with fixed potential 10V
                                elif (i==maxI):
                                        continue #TODO skip neumann residual checking?
                                elif (j==maxJ):
                                        continue #TODO skip neumann residual checking?
                                else:
                                        residual = (  hash[(i-1,j)] + hash[(i,j-1)] + hash[(i+1,j)] +
hash[(i,j+1)]  ) - 4 * hash[(i,j)]

                                if (residual > ERROR):
                                        repeat = True

                        if (repeat):
                                break
```

```
        return iterations

def Jacobi():
        import copy
        iterations=0
        repeat=True

        while(repeat):
                iterations+=1

                hashPrevious=copy.deepcopy(hash)

                #working bottom->up, left->right to respect order of computations as per the algorithm
                #skip i=0 , j=0 as these are the outer boundary (dirichlet)
                for i in range(1,int(maxI)+1):
                        for j in range (1, int(maxJ)+1):
                                if (i>maxI_Neumann and j>maxJ_Neumann):
                                        continue #skip the inner conductor with fixed potential 10V
                                elif (i==maxI):
                                        hash[(i,j)]= 0.25 * (  hashPrevious[(i-1,j)] +
hashPrevious[(i,j-1)] + hashPrevious[(i-1,j)] + hashPrevious[(i,j+1)]  )#neumann
                                elif (j==maxJ):
                                        hash[(i,j)]= 0.25 * (  hashPrevious[(i-1,j)] +
hashPrevious[(i,j-1)] + hashPrevious[(i+1,j)] + hashPrevious[(i,j-1)]  ) #neumann
                                else:
                                        hash[(i,j)]= 0.25 * (  hashPrevious[(i-1,j)] +
hashPrevious[(i,j-1)] + hashPrevious[(i+1,j)] + hashPrevious[(i,j+1)]  )



                #check if residual for each node is small enough
                repeat=False

                for i in range(1,int(maxI)+1):
                        for j in range (1, int(maxJ)+1):
                                if (i>maxI_Neumann and j>maxJ_Neumann):
                                        continue #skip the inner conductor with fixed potential 10V
                                elif (i==maxI):
                                        continue #TODO skip neumann residual checking?
                                elif (j==maxJ):
                                        continue #TODO skip neumann residual checking?
                                else:
                                        residual = (  hash[(i-1,j)] + hash[(i,j-1)] + hash[(i+1,j)] +
hash[(i,j+1)]  ) - 4 * hash[(i,j)]

                                if (residual > ERROR):
                                        repeat = True

                        if (repeat):
                                break

        return iterations

#Question 3(b)
# for i in range(10,20,1):
        # w=float(i)/10
        # print w
        # hash={}
        # buildHash()
        # print solveFDM()
        # print hash[(0.06/h,0.04/h)]
        # print ''

#Question 3(c)
# w=1.3

# for i in range(0,5):
        # h=0.02/(2**i)
        # print h
```

```
        # #maximum coordinate of i (where i starts at 0)
        # maxI=(outLength/2)/h

        # maxJ=(outLength/2)/h

        # #maximum pts when j=maxJ where we are still on neumann boundary (and not V=10)
        # maxI_Neumann=(outLength/2-inWidth/2)/h   -1 # -1 to remove the boundary point (which is also 10V)
        # maxJ Neumann=(outLength/2-inHeight/2)/h    -1

        # hash={}
        # buildHash()
        # print solveFDM()
        # print hash[(0.06/h,0.04/h)]
        # print ''

#Question 3(d)
for i in range(0,5):
        h=0.02/(2**i)
        print h

        #maximum coordinate of i (where i starts at 0)
        maxI=(outLength/2)/h

        maxJ=(outLength/2)/h

        #maximum pts when j=maxJ where we are still on neumann boundary (and not V=10)
        maxI_Neumann=(outLength/2-inWidth/2)/h   -1 # -1 to remove the boundary point (which is also 10V)
        maxJ_Neumann=(outLength/2-inHeight/2)/h    -1

        hash={}
        buildHash()
        print Jacobi()
        print hash[(0.06/h,0.04/h)]
        print ''
```