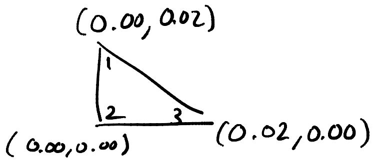
LOCAL S-MATRIX

TRIANGLE 1-2-3



$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.02 \\ 1 & 0 & 0 \\ 1 & 0.02 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\underline{U} = X \quad \underline{a}$$

$$\therefore \quad \underline{a} = X^{-1} \quad \underline{U}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -50 & 50 \\ 50 & -50 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \left(X^{-1}\right)^T \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 50 \\ 1 & -50 & -50 \\ 0 & 50 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$$

$$\alpha_1 = 50y, \quad \alpha_2 = 1 - 50x - 50y, \quad \alpha_3 = 50x$$

Let $\underline{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$

$$\nabla \underline{\alpha} = \begin{bmatrix} 0 \\ -50 \\ 50 \end{bmatrix} \alpha_i + \begin{bmatrix} 50 \\ -50 \\ 0 \end{bmatrix} \alpha_j$$

$$S_{ij} = \int \nabla \alpha_i \nabla \alpha_j \, dS$$

$$\therefore S = A\left(\begin{bmatrix}0\\-50\\50\end{bmatrix}\begin{bmatrix}0 & -50 & 50\end{bmatrix} + \begin{bmatrix}50\\-50\\0\end{bmatrix}\begin{bmatrix}50 & -50 & 0\end{bmatrix}\right)$$

where A is the Area $= \frac{1}{2}bh = \frac{1}{2}(0.02\,m)(0.02\,m)$

$$= 200\,\mu m^2$$

$$S = 200\mu\left(\begin{bmatrix}0 & 0 & 0\\0 & 2500 & -2500\\0 & -2500 & 2500\end{bmatrix} + \begin{bmatrix}2500 & -2500 & 0\\-2500 & 2500 & 0\\0 & 0 & 0\end{bmatrix}\right)$$

$$S = 200\mu\begin{bmatrix}2500 & -2500 & 0\\-2500 & 5000 & -2500\\0 & -2500 & 2500\end{bmatrix}$$

$$\boxed{S = \begin{bmatrix}0.5 & -0.5 & 0\\-0.5 & 1 & -0.5\\0 & -0.5 & 0.5\end{bmatrix}}$$

TRIANGLE 4-5-6

Similarly,

$$\begin{bmatrix}U_4\\U_5\\U_6\end{bmatrix} = \begin{bmatrix}1 & 0.02 & 0.02\\1 & 0 & 0.02\\1 & 0.02 & 0\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$$\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}-1 & 1 & 1\\50 & -50 & 0\\50 & 0 & -50\end{bmatrix}\begin{bmatrix}U_4\\U_5\\U_6\end{bmatrix}$$

$$\begin{bmatrix}\alpha_4\\\alpha_5\\\alpha_6\end{bmatrix} = \begin{bmatrix}-1 & 50 & 50\\1 & -50 & 0\\1 & 0 & -50\end{bmatrix}\begin{bmatrix}1\\x\\y\end{bmatrix}$$

$$\begin{bmatrix} U_4 \\ U_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 1 & 0.02 & 0.02 \\ 1 & 0 & 0.02 \\ 1 & 0.02 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 50 & -50 & 0 \\ 50 & 0 & -50 \end{bmatrix} \begin{bmatrix} U_4 \\ U_5 \\ U_6 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} = \begin{bmatrix} -1 & 50 & 50 \\ 1 & -50 & 0 \\ 1 & 0 & -50 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$$

$$\nabla \underline{\alpha} = \begin{bmatrix} 50 \\ -50 \\ 0 \end{bmatrix} \alpha_i + \begin{bmatrix} 50 \\ 0 \\ -50 \end{bmatrix} \alpha_j$$

same Area

$$\therefore S = 200\mu \left(\begin{bmatrix} 50 \\ -50 \\ 0 \end{bmatrix} \begin{bmatrix} 50 & -50 & 0 \end{bmatrix} + \begin{bmatrix} 50 \\ 0 \\ -50 \end{bmatrix} \begin{bmatrix} 50 & 0 & -50 \end{bmatrix} \right)$$

$$S = 200\mu \left(\begin{bmatrix} 2500 & -2500 & 0 \\ -2500 & 2500 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2500 & 0 & -2500 \\ 0 & 0 & 0 \\ -2500 & 0 & 2500 \end{bmatrix} \right)$$

$$S = 200\mu \begin{bmatrix} 5000 & -2500 & -2500 \\ -2500 & 2500 & 0 \\ -2500 & 0 & 2500 \end{bmatrix}$$

$$\boxed{\therefore S = \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0 & 0.5 \end{bmatrix}}$$

$$S_{dis} = \begin{bmatrix} 0.5 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 & 0 & 0 \\ 0 & -0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -0.5 & -0.5 \\ 0 & 0 & 0 & -0.5 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0 & 0.5 \end{bmatrix}$$

$$U_{dis} = C\, U_{con} \qquad \begin{pmatrix} \text{vertex } 1 = \text{vertex } 5 \\ \text{vertex } 3 = \text{vertex } 6 \end{pmatrix}$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

$$\Rightarrow S_{con} = C^T S_{dis} C$$

$$= \underset{4\times 6}{\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}} \underset{6\times 6}{\begin{bmatrix} 0.5 & -0.5 & 0 & & & \\ -0.5 & 1 & -0.5 & & 0 & \\ 0 & -0.5 & 0.5 & & & \\ & & & 1 & -0.5 & -0.5 \\ & 0 & & -0.5 & 0.5 & 0 \\ & & & -0.5 & 0 & 0.5 \end{bmatrix}} C$$

$$= \underset{4\times 6}{\begin{bmatrix} 0.5 & -0.5 & 0 & -0.5 & 0.5 & 0 \\ -0.5 & 1 & -0.5 & 0 & 0 & 0 \\ 0 & -0.5 & 0.5 & -0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & -0.5 & -0.5 \end{bmatrix}} \underset{6\times 4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}$$

$$\boxed{S_{global} = \begin{bmatrix} 1 & -0.5 & 0 & -0.5 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & -0.5 & 1 & -0.5 \\ -0.5 & 0 & -0.5 & 1 \end{bmatrix}}$$