ECSE 543A NUMERICAL METHODS IN ELECTRICAL ENGINEERING
Assignment 2

Set: 12-Oct-2011 Due: 09-Nov-2011

1) Figure 1 shows two first-order triangular finite elements used to solve the Laplace equation for electrostatic potential. Find a local **S**-matrix for each triangle, and a global **S**-matrix for the mesh that consists of just these two triangles. The local (disjoint) and global (conjoint) node-numberings are shown in Figure 1(a) and (b), respectively. Also, Figure 1(a) shows the (x, y)-coordinates of the element vertices in meters.

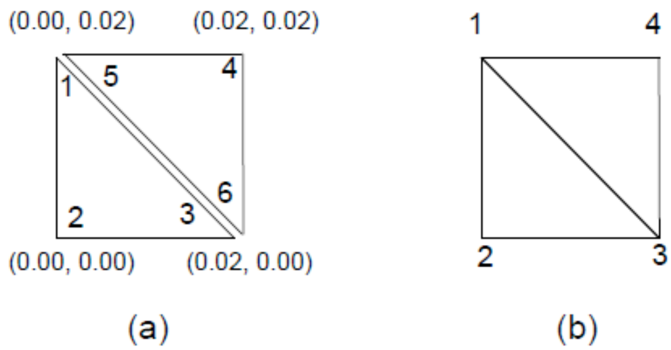
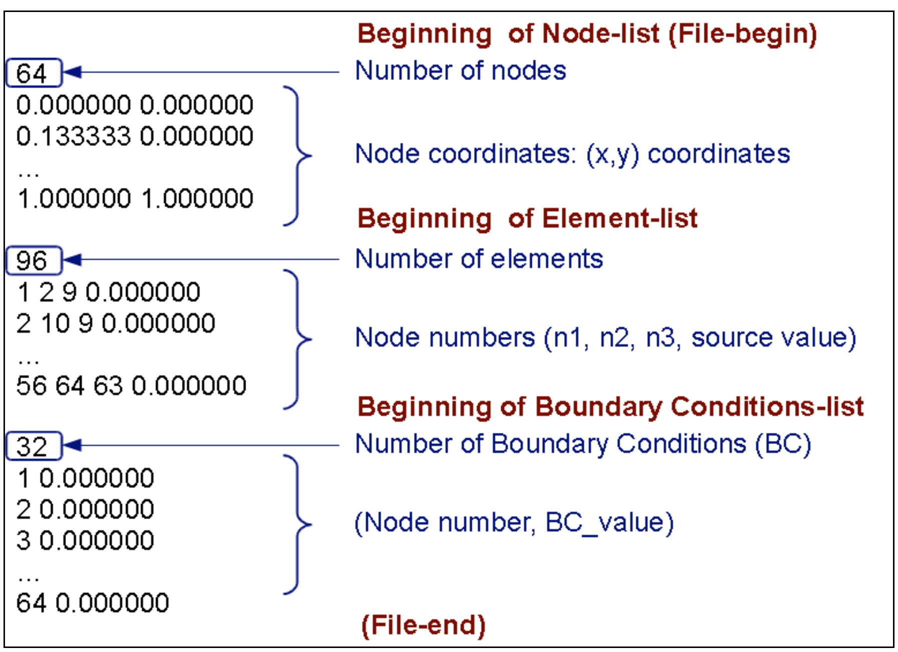


Figure 1

2) Figure 2 shows the cross-section of an electrostatic problem with translational symmetry: a rectangular coaxial cable. The inner conductor is held at 10 volts and the outer conductor is grounded. (This is similar to the system considered in Question 3, Assignment 1). Full triangular element mesh files of the proposed geometry are provided on WebCT for different size meshes (squareCoax3.msh-squareCoax10.msh) in a zip file. The file format for these meshes is as follows:



(a) Write a program that reads a mesh file with the given format and builds the corresponding data structures for ***Nodes***, ***Elements*** and ***Boundary_Conditions (BC)***. Note that in the file description presented above the node numbers in the ***Element-list*** and ***BC-list*** refer to a pair of (x,y) coordinates in the ***Node-list*** that begins with the node number 1. Programming languages that use *0-based* indexing must take this into account.

(b) Write another program that takes as inputs these three data structures and first assembles the global **S**-matrix (for first-order triangular elements), and then applies the BC's to assemble the corresponding *reduced* matrix system (including its right hand side-RHS).

(c) Use the Cholesky decomposition direct solver program that you wrote for Question 1 of Assignment 1 to compute the solution to the SPD systems obtained in part (b) corresponding to the files squareCoax3, squareCoax4, ..., squareCoax10.

(d) Determine the potential at $(x, y) = (0.05, 0.05)$ from the data in the output file of the program for each of the meshes considered in part (c) and plot a graph of this potential vs. the number of unknowns (free nodal potentials). (Hint: you may need to identify if the point is in a triangle)

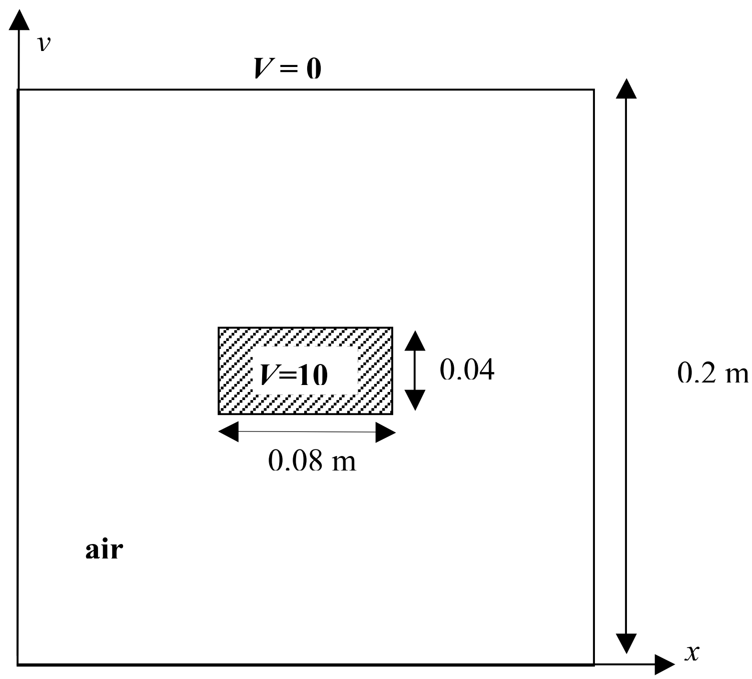


Figure 2.

3) Write a program implementing the conjugate gradient (CG) method (un-preconditioned). Solve the matrix equations corresponding to the files squareCoax3, to squareCoax10, using the full mesh information (again no symmetry is required to be exploited). Use a starting solution of zero.
Give the following results:

(a) Plot a graph of the infinity norm and the 2-norm of the residual vector vs. the number of iterations for each mesh.

(b) What is the potential at $(x, y) = (0.05, 0.05)$ for each mesh, and how does it compare with the value you computed in Question 2(d) above.

(c) Compute and plot the electrostatic energy per unit length of the system as a function of the number of unknowns.

(d) If you are using MatLab, also implement the incomplete Cholesky (IC) decomposition algorithm given in the course notes and use it to modify your CG program so that it now becomes a preconditioned conjugate gradient (PCG) method solver. Determine the value for the same point given in part (b) and plot the 2-norm of the residual vs. the number of iterations. Comment on the results obtained and compare the rate of convergence with respect to the un-preconditioned CG implementation.