Assignment ④

Q1

LDR r1,[r2]

LDR r3, [r4]

ADD r7, r7, 14 ⟶ MUL r5, r3, #12

ADD r3, r3, r1

SUB r6, r6, #4

ADD r1, r1, #4

STR r3, [r4]

MUL r7, r1, #5

STR r1, [r2]

P2

a)

cycle	Processor ①		Processor ②	
	Memory access ①	Integer operation	Memory access ②	Integer opera ②
	LDR r3, [r6]			ADD r0, r0, #4
	STR r3, [r6]	ADD r6, 6, #4		ADD r8, r8, r3

Q2)

a)

Cycle	Processor 1		Processor 2	
	Memory Access 1	Intege Operation 1	Memory Access 2	Integer Operation 2
1	LDR r3, [r0			ADD r0, r0, #4
2	STR r3, [r6]	ADD r6, r6, #4		ADD r8, r8, r3
3			LDR r4, [r1	ADD r1, r1, #4
4		ADD r4, r4, r3		
5	STR r4, [r2]	ADD r4, r4, r4		
6		SUBS r5, r5, #1		ADD r2, r2, #4

b)

Cycle	Processor 1		Processor 2	
	Memory Access 1	Integer Operation 1	Memory Access 2	Memory Operation 2
1	LDR r3, [r0]	ADD r0, r0, #4	LDR r4, [r1]	
2	STR r3, [r6]	ADD r6, r6, #4		ADD r8, r8, r3
3		ADD r1, r1, #4		ADD r4, r4, r3
4	STR r4, [r2]	ADD r4, r4, r4		ADD r2, r2, #4
5		SUBS r5, r5, #1		

∴ Out-of-order saves one cycle compared to in-order.

Assignment ③

Q1

$[D_1] \leftarrow [D_1] + [D_0]$

$G_{D_0} = 1$

$C_{L_1} = 1$

$G_{D_1} = 1$

$CL_2 = 1$

$F_2, F_1, F_0 = 1, 1, 0$

$C_{D_1} = 1$

Q2

a) time = $300 + 150 + 250 + 350 + 700 + 200 = 1950$ PS

b) time = $(\text{slowest} + 20) \times 6 = (700 + 20) \times 6 = 4320$ PS

c) $\text{slowest} + 20 = 700 + 20 = 720$ PS

d) time = $((1 - 0.25) \times 720) + 720 = 1260$ PS

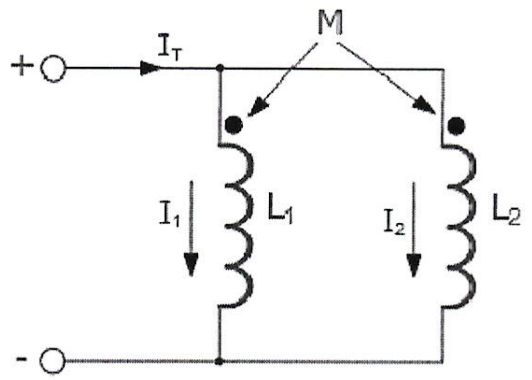
Q3

Electromagnetism PHYS74100
Assignment 6 – Inductors, emf

Name: ______________________________

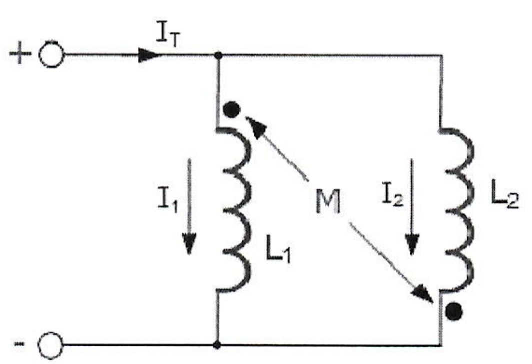
1. Show that the equivalent inductance is

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



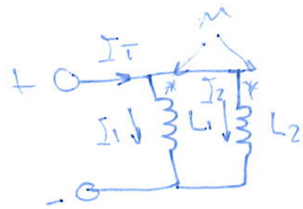
2. Show that the equivalent inductance is

$$L_T = \frac{\Phi_1}{I_1 + I_2} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



Assignment 6

Q1)



$$L_T = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

$I_T = I_1 + I_2$, $\phi_1 = L_1 I_1 + M I_2$

$\phi_2 = L_2 I_2 + M I_1$

$I_1 = \frac{\phi}{L_1}$

$\because \phi_1 = \phi_2 = \phi_T$

$\Rightarrow L_1 = \frac{\phi}{I_1}$

$\Rightarrow L_1 I_1 + M I_2 = L_2 I_2 + M I_1$

$\Rightarrow L_1 I_1 - M I_1 = L_2 I_2 - M I_2 \Rightarrow I_1 (L_1 - M) = I_2 (L_2 - M)$

$\Rightarrow I_2 = \frac{(L_1 - M)}{(L_2 - M)} I_1$

$\Rightarrow L_T = \frac{\phi_1}{L_1} + \frac{\phi_2}{L_2} \Rightarrow L_T = \frac{\phi_T}{L_1 + L_2}$ $\left(L_1 = \frac{\phi_1}{I_1} , L_2 = \frac{\phi_2}{I_2} \right)$

$\Rightarrow L_T = \frac{\phi_T}{I_1 + I_2}$ $\left(\text{sub in } \phi_T = \phi_1 , I_2 = \left(\frac{L_1 - M}{L_2 - M} \right) I_1 \right)$

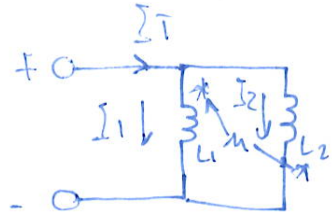
$\Rightarrow L_T = \frac{L_1 I_1 + M \left(\frac{L_1 - M}{L_2 - M} \right) I_1}{I_1 + \left(\frac{L_1 - M}{L_2 - M} \right) I_1} \Rightarrow L_T = \frac{\cancel{I_1} \left(L_1 + M \left(\frac{L_1 - M}{L_2 - M} \right) \right)}{\cancel{I_1} \left(1 + \left(\frac{L_1 - M}{L_2 - M} \right) \right)}$ $\left(\times \frac{L_2 - M}{L_2 - M} \right)$

$\Rightarrow L_T = \frac{L_1(L_2 - M) + M L_1 - M^2}{L_2 - M + L_1 - M} = \frac{L_1 L_2 \cancel{- L_1 M} \cancel{+ M L_1} - M^2}{L_1 + L_2 - 2M}$

$$\Rightarrow \boxed{L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

Q2

$$L_T = \frac{\phi_1}{I_1 + I_2} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



$$L_1 = \frac{\phi}{I_1}$$

$$I_T = I_1 + I_2$$

$$\phi_1 = I_1 L_1 - M I_2 \, , \quad \phi_2 = I_2 L_2 - M I_1 \, , \quad \phi_1 = \phi_2 = \phi_T$$

$$\Rightarrow \quad L_1 I_1 - M I_2 = L_2 I_2 - M I_1$$

$$\Rightarrow \quad L_1 I_1 + M I_1 = L_2 I_2 + M I_2 \Rightarrow I_1 (L_1 + M) = I_2 (L_2 + M)$$

$$\Rightarrow I_2 = \left(\frac{L_1 + M}{L_2 + M}\right) I_1$$

$$\Rightarrow L_T = \frac{\phi_T}{L_1 + L_2} = \frac{\phi_T}{I_1 + I_2} \qquad \left(\text{sub in } \phi_T = \phi_1 \, , \; I_2 = \frac{L_1 + M}{L_2 + M} I_1\right.$$

$$\Rightarrow L_T = \frac{I_1 L_1 - M\left(\frac{L_1 + M}{L_2 + M}\right) I_1}{I_1 + \frac{L_1 + M}{L_2 + M} I_1} \Rightarrow L_T = \frac{\cancel{I_1}\left(L_1 - M\left(\frac{L_1 + M}{L_2 + M}\right)\right)}{\cancel{I_1}\left(1 + \frac{L_1 + M}{L_2 + M}\right)} \quad \left(\times \frac{L_2 + M}{L_2 + M}\right)$$

$$\Rightarrow L_T = \frac{(L_2 + M) L_1 - M(L_1 + M)}{L_2 + M + L_1 + M}$$

$$\Rightarrow L_T = \frac{L_2 L_1 + \cancel{M L_1} - \cancel{M L_1} - M^2}{L_1 + L_2 + 2M} \Rightarrow \boxed{L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

Q3) $\vec{u} = \hat{z}\, 5\ m/s$, $I = 10\ A$, $l = 30\ cm$, $V_{12} = ?$

Since it is not a close circuit, Faraday's law is not applied.

$\vec{F} = q(\vec{u} \times \vec{B})$, $\vec{F} = \vec{E}\, q$

$\Rightarrow q(\vec{u} \times \vec{B}) = \vec{E}\, q$

$\Rightarrow \vec{u} \times \vec{B} = \vec{E}$, $V_{12} = \int_1^2 (\vec{u} \times \vec{B}) \cdot d\vec{l}$

$\Rightarrow V_{12} = \int_2^1 (\vec{u} \times \vec{B}) \cdot d\vec{l}$

$= \int_{40}^{10} \left(\hat{z}\, 5 \times \left(-\hat{\phi}\, \frac{\mu_0 I}{2\pi r}\right)\right) \cdot \hat{r}\, dr$

$\Rightarrow V_{12} = -\frac{5 \mu_0 I}{2\pi} \int_{40}^{10} \frac{dr}{r}$

$= -\frac{5 \mu_0 I}{2\pi} \left[\ln(10) - \ln(40)\right] = -\frac{5 \mu_0 I}{2\pi} \ln\left(\frac{10}{40}\right)$

$= -\frac{5 \times 4\pi \times 10^{-7} \times 10}{2\pi} \times \ln\left(\frac{10}{40}\right)$

$\Rightarrow \boxed{V_{12} = 1.38 \times 10^{-5}\ V}$

Assignment (7)

Q1)

$\vec{B}_1 = \hat{x}\,4 - \hat{y}\,6 + \hat{z}\,8$, $\mu = 5000\mu_0$

$(z \geq 0)$

Find $\vec{B}_2$ when $(z \leq 0)$

$H_1 = \frac{B_1}{\mu_1} = \frac{\hat{x}4 - \hat{y}6 + \hat{z}8}{\mu_1}$

z is the normal component to the boundary

and $z = 0 \Rightarrow B_{2z} = B_{1z} = 8$

Also

① $H_{2x} = H_{1x} = \frac{4}{\mu_1} \Rightarrow B_{2x} = \mu_2 H_{2x} = \left(\frac{\mu_2}{\mu_1}\right) 4$

② $H_{2y} = H_{1y} = \frac{-6}{\mu_1} \Rightarrow B_{2y} = \mu_2 H_{2y} = -\left(\frac{\mu_2}{\mu_1}\right) 6$

$\Rightarrow \mu = (\mu_r)(\mu_0) \Rightarrow \mu_r = \frac{\mu_2}{\mu_1} = 5000$

$\Rightarrow B_{2x} = 20000$

$\Rightarrow B_{2y} = -30000$

$\Rightarrow B_{2z} = 8$ (already proved from above)

$\Rightarrow \boxed{B_2 = \hat{x}\,20000 - \hat{y}\,30000 + \hat{z}\,8}$

Q2

$\vec{E} = E_m \sin(\omega t - \beta z)\hat{a}_y$, find $\vec{D}$, $\vec{B}$, $\vec{H}$

$\vec{D} = \varepsilon\vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$ (since it is in free space $\varepsilon_r = 1$)

$\Rightarrow \boxed{\vec{D} = \varepsilon_0 E_m \sin(\omega t - \beta z)\hat{a}_y}$ C/m^2

Now, using Max equation

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\because$ Only E_y has an electric field

$\Rightarrow E_x, E_z = 0$, and $E_y = E_m \sin(\omega t - \beta z)$

$\therefore -\frac{\partial E_y}{\partial z}\hat{a}_x + \frac{\partial E_y}{\partial x}\hat{a}_z = -\frac{\partial \vec{B}}{\partial t}$

but $\vec{E}$ is changing with respect to z

$\Rightarrow -\frac{\partial E_y}{\partial z}\hat{a}_x = -\frac{\partial \vec{B}}{\partial t}$

$\Rightarrow -E_m \frac{\partial \sin(\omega t - \beta z)}{\partial z}\hat{a}_x = -\frac{\partial \vec{B}}{\partial t}$

$\Rightarrow \left(-E_m \beta \cos(\omega t - \beta z)\hat{a}_x\right) = -\frac{\partial \vec{B}}{\partial t}$

$\Rightarrow \frac{\partial \vec{B}}{\partial t} = -E_m \beta \cos(\omega t - \beta z)\hat{a}_x$ (integrate wrt to t)

$\Rightarrow \vec{B} = -E_m \beta \int \cos(\omega t - \beta z)\hat{a}_x \, dt$

$\Rightarrow \boxed{\vec{B} = -\frac{E_m \beta}{\omega} \sin(\omega t - \beta z)\hat{a}_x}$ $\frac{Wb}{m^2}$

$\Rightarrow \vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$, $\mu_r = 1$ in free space

$\Rightarrow \vec{H} = \frac{\vec{B}}{\mu_0} \Rightarrow \boxed{\vec{H} = -\frac{E_m \beta}{\omega \mu_0} \sin(\omega t - \beta z)\hat{a}_x}$ A/m

Assignment (8)

Q1 $\varepsilon_r = 9$

$$\vec{E} = [\hat{y}\, 3\cos(\pi \times 10^7 t + kx) - \hat{z}\, 2\cos(\pi \times 10^7 t + kx)] \text{ V/m}$$

$(+kx)$ which means that $\vec{E}$ is traveling in the negative x-direction

lossless medium

$$\Rightarrow \vec{S}_{av} = -\hat{x}\frac{1}{2\eta}\left(|E_{y0}|^2 + |E_{z0}|^2\right) = -\hat{x}\frac{(3^2)+(-2)^2}{2\eta}$$

$$\Rightarrow \vec{S}_{av} = -\hat{x}\frac{9+4}{2\eta} \Rightarrow \vec{S}_{av} = -\hat{x}\frac{13}{2\eta}$$

lossless medium $\Rightarrow \eta = \sqrt{\frac{\mu}{\varepsilon}}$, in a vaccum $\Rightarrow \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$, $\eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$

$$\Rightarrow \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\frac{\sqrt{\mu_r}}{\sqrt{\varepsilon_r}} \Rightarrow \eta = \frac{120\pi}{3} \Rightarrow \eta = 40\pi$$

$$\Rightarrow \vec{S}_{av} = -\hat{x}\frac{13}{2(40\pi)} \Rightarrow \boxed{\vec{S}_{av} = -\hat{x}\frac{13}{80\pi}}$$

P2 $\mu = \mu_0$, $\varepsilon_r = 9$, $f = 10\text{MHz}$ lossless medium $\varepsilon_0 = 1$

a) $v_p = \frac{c\omega}{k} = \frac{c\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu\varepsilon}}$, $\mu = \mu_0\mu_r \Rightarrow \mu_r = \frac{\mu}{\mu_0} = 1$, $\varepsilon = \varepsilon_0\varepsilon_r \Rightarrow \varepsilon = \varepsilon_r$

$$\Rightarrow v_p = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{3\times10^8}{\sqrt{9}} = \frac{3\times10^8}{3} = \boxed{1\times10^8 \text{ m/s}}$$

b)

$$k = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu_r\varepsilon_r} = 2\pi\times 10\sqrt{9} = 18.8$$

c) $\lambda = \frac{v_p}{f} = \frac{1\times10^8}{10\times10^6} = 10 \text{ m}$

d) $\eta = \sqrt{\frac{\mu}{\varepsilon}}$, $\mu = \mu_0\mu_r \Rightarrow \mu_r = \frac{\mu}{\mu_0} = 1$, $\varepsilon = \varepsilon_0\varepsilon_r$ and $\varepsilon_0 = 1$, $\sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$$\Rightarrow \eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1}{9}} \Rightarrow \eta = \frac{120\pi}{3} \Rightarrow \boxed{\eta = 40\pi}\ \Omega$$

Q3

$\vec{E} = \hat{z}\, 10\, e^{-j4\pi y}$, $\eta = 188.5\,\Omega$

a)

$$\vec{H} = \hat{k}\,\frac{\vec{E}}{\eta}$$

$\because$ $-j4\pi y \Rightarrow \hat{k}$ is in the direction of $+y$

$$\Rightarrow \vec{H} = \hat{y}\,\frac{\vec{E}}{\eta}$$

$$\Rightarrow \vec{H} = \hat{y} \times \hat{z}\,\frac{10 e^{-j4\pi y}}{188.5\,\Omega} \times 10^{-3}\,\left(\frac{V}{m}\right) \qquad (\hat{y} \times \hat{z} = \hat{x})$$

$$\Rightarrow \boxed{\vec{H} = \hat{x}\, 5.3 \times 10^{-5}\, e^{-j4\pi y}\ A/m}$$

b)

$$E(y,t) = Re\left[\hat{z}\, 10 e^{-j4\pi y}\right]$$

$$= Re\left[\hat{z}\, 10 e^{-j4\pi y} e^{j\omega t}\right]$$

$$= \hat{z}\, 10 \cos(\omega t - 4\pi y)\ \frac{mV}{m}$$

$\therefore$ The instentaneous expression for $\vec{E}(y,t)$ is

$$\boxed{E(z,t) = \hat{z}\, 10 \cos(\omega t - 4\pi y)\ \frac{mV}{m}}$$