Deep Learning: Algorithms and Applications
Peking University
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Homework 2

Due Date: ***June 5th***

1. Variational Auto-Encoder

VAE is the typical generative model which has a lot of applications. The encoder portion of a VAE yields an approximate posterior distribution $q(z|x)$, and is parameterized on a neural network by weights collectively denoted θ. Hence we more properly write the encoder as $q_\theta(z|x)$. Similarly, the decoder portion of the VAE yields a likelihood distribution $p(x|z)$, and is parameterized on a neural network by weights collectively denoted ϕ. Hence we more properly denote the decoder portion of the VAE as $p_\phi(x|z)$. The output of the encoder are parameters of the latent distribution, which is sampled to yield the input into the decoder.

a) **Draw the model's framework of VAE**

b) **Derive the objective function e.g. the Evidence Lower Bound (ELBO)** using KL divergence.

[*HINT: 1) Please include the re-parametrisation trick implemented in VAE*
HINT: 2) You may need to use Bayes theorem: $p(x|y) = \frac{p(y|x)p(x)}{p(y)}$]

c) **Derive the closed form VAE loss: Gaussian latents, based on question b)**. e.g. Say we choose:

$$p(z) \to \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left(-\frac{(x-\mu_p)^2}{2\sigma_p^2}\right)$$

and

$$q_\theta\left(z \mid x_i\right) \to \frac{1}{\sqrt{2\pi\sigma_q^2}} \exp\left(-\frac{(x-\mu_q)^2}{2\sigma_q^2}\right)$$

2. Policy based Reinforcement Learning

Please write down the detailed derivation of the policy gradient method in the episodic case, e.g. **Show that** $\nabla J(\boldsymbol{\theta}) \propto \mathbb{E}_\pi[q_\pi(s,a)\nabla\pi(a|s)]$

Some definitions you may need:

$$J(\boldsymbol{\theta}) = v_{\pi_\theta}(s_0)$$

where v_{π_θ} is the true state value function for π_θ, the policy determined by $\boldsymbol{\theta}$

state value function:

$$v_\pi = \mathbb{E}_\pi[G_t|S_t = s] = \sum_a \pi(a|s) q_\pi(s, a)$$

state-action value function:

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t|S_t = s, A_t = a] = \sum_{s^{'}, r} p(s^{'}, r|s, a)(r + v_\pi(s^{'}))$$

where we have the return defined as:

$$G_t = R_{t+1} + R_{t+2} + ...$$