CSE 6242 - Data and Visual Analytics
HW3: Logistic Regression

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0. Data Preprocessing

$a.DownloadtheCSVfilesfortheprovideddataset.$
$b.Readmnist_train.csvandmnist_test.csvseparately.$

```
> setwd("C:/Users/eelsayed/Google Drive/CSE6242")
> rawDatalLoaded <- TRUE
> if(file.exists("mnist_train.csv")){
+   train <- read.csv(file="mnist_train.csv", header = FALSE)
+ }else{
+   rawDatalLoaded <- FALSE
+ }
> if(file.exists("mnist_test.csv")){
+   test <- read.csv(file="mnist_test.csv", header = FALSE)
+ }else{
+   rawDatalLoaded <- FALSE
+ }
> if(!rawDatalLoaded){
+   print("Data wasn't loaded correctly.")
+ }
> train <- as.data.frame(t(train))
> names(train)[785] <- "Label"
> test <- as.data.frame(t(test))
> names(test)[785] <- "Label"
>
```

$c.Partitionthetrainingsetforclassificationof0,1and3,5classesbasedontheclasslabel(lastrow785):$ $train_{01}, train_{35}.$

```
> train_0_1 <- train[(train$Label == 0) | (train$Label == 1),]
> train_3_5 <- train[(train$Label == 3) | (train$Label == 5),]
```

$d.Dothesameforthetestset: test_{01}, test_{35}.$

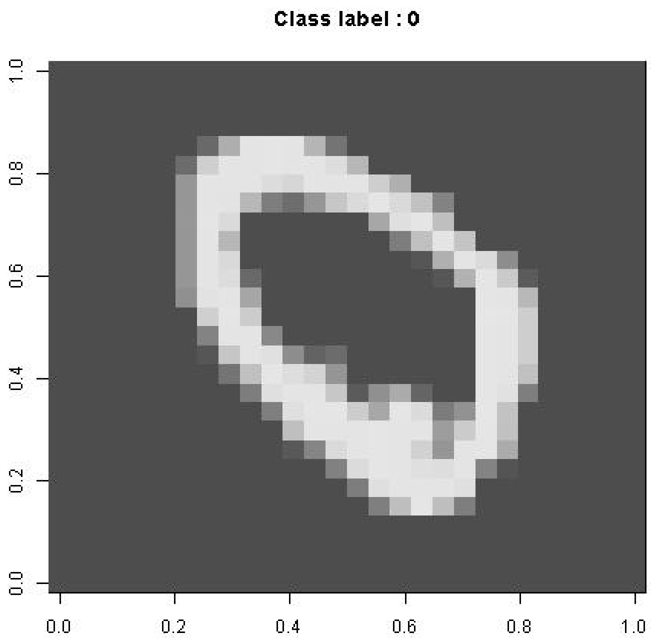
```
> test_0_1 <- test[(test$Label == 0) | (test$Label == 1),]
> test_3_5 <- test[(test$Label == 3) | (test$Label == 5),]
```

$efSeparatetheclasslabelfromallthepartitionscreated(removerow785fromtheactualdataandstoreit$ $asaseparatevector).$

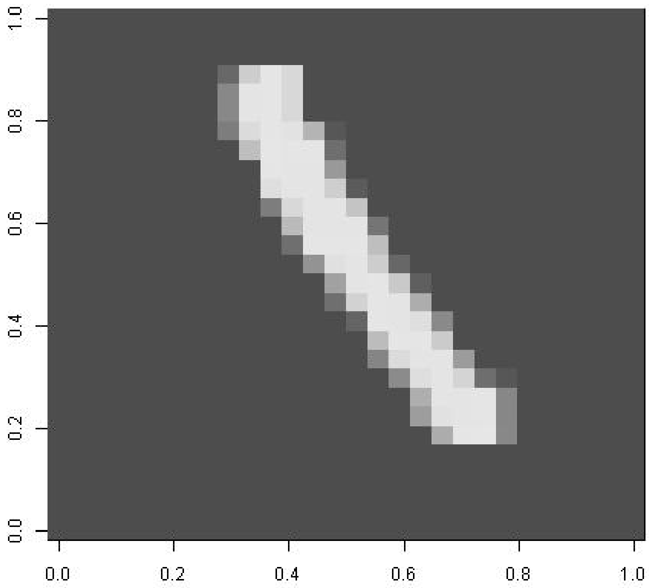
```
> true_label_train_0_1 <- train_0_1$Label
> train_0_1 <- subset(train_0_1, select = names(train_0_1) != "Label" )
> true_label_train_3_5 <- train_3_5$Label
> train_3_5 <- subset(train_3_5, select = names(train_3_5) != "Label" )
> true_label_test_0_1 <- test_0_1$Label
> test_0_1 <- subset(test_0_1, select = names(test_0_1) != "Label" )
> true_label_test_3_5 <- test_3_5$Label
> test_3_5 <- subset(test_3_5, select = names(test_3_5) != "Label" )
```

$g.Visualize1imagefromeachclasstoensureyouhavereadinthedatacorrectly.$

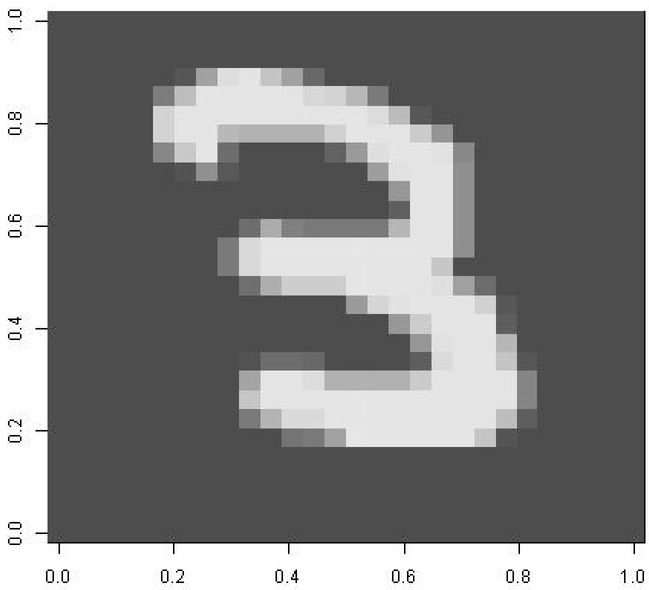
```
> save_digit_image <- function(df, digitClass, imageTitle, fileName) {
+   tmp <- df[df$Label == digitClass,]
+   m <- matrix(unlist(tmp[1,1:784]), ncol = 28, byrow = TRUE)
+
+   jpeg(filename = fileName)
+   image(z = m, col = gray.colors(256))
+   title(main = imageTitle)
+   dev.off()
+ }
> save_digit_image(train, 0, "Class label : 0", "0.jpg")
> save_digit_image(train, 1, "Class label : 1", "1.jpg")
> save_digit_image(train, 3, "Class label : 3", "3.jpg")
> save_digit_image(train, 5, "Class label : 5", "5.jpg")
```

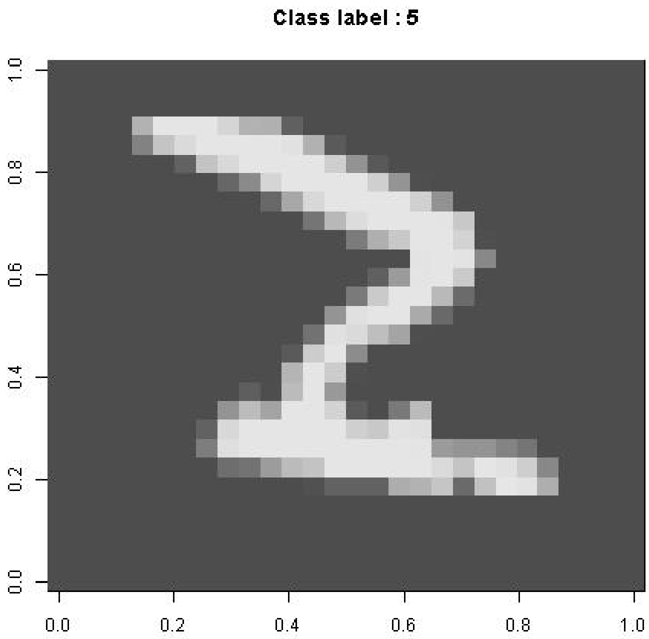


Class label : 1



Class label : 3





1 Theory

a Write down the formula for computing the gradient of the loss function used in Logistic Regression. Specify what each variable represents in the equation.

The formula for the gradient descent is:

$$\theta_j \leftarrow \theta_j - \alpha \sum_{i=1}^{n} \frac{1}{1 + \exp(-y^{(i)} < \theta, x^{(i)} >)}$$

where $x^{(i)}$ is the data point represented in a vector of features, $y^{(i)}$ is the class label, and θ is the parameter vector. The goal is to reach the θ that maximizes our likelihood function (given the data we use to train the model).

b Write pseudocode for training a model using Logistic Regression.

Data: Training data
convergence threshold: η
step size: α
for $j \leftarrow 0$ **to** d **do**
  initialize θ_j
  initialize $\Delta\theta_j$
end
for $i \leftarrow 1$ **to** n **do**
  $x_0^{(i)} = 1$
end
while $\cup_{j\in\{0,1,\ldots,d\}}|\delta\theta_j/\theta_j| > \eta$ **do**
  for $i \leftarrow 1$ **to** n **do**
    $z^{(i)} = \sum_{j=0}^{d} \theta_j x_j^{(i)}$
  end
  for $j \leftarrow 0$ **to** d **do**
    $\Delta = 0$
    for $i \leftarrow 1$ **to** n **do**
      $\Delta \leftarrow \Delta + \frac{y^{(i)}x_j^{(i)}}{1+\exp(-y^{(i)}z^{(i)})}$
    end
  end
  $\theta_j \leftarrow \theta_j - \alpha\Delta$
  $\delta\theta_j = \alpha\Delta$
end
return $\{\theta_0, ..., \theta_d\}$

c Calculate the number of operations per gradient descent iteration.
Each gradient descent update iteration requires $2n(d+1)$

2 Implementation

```
> gradient_descent <- function(X, Y, theta){
+     z <- as.matrix(X) %*% theta
+     yz_exp <- 1 / (1 + exp(-y * z))
+     tmp <- apply(X,2,function(x) x * y)
+     delta_theta <- t(tmp) %*% yz_exp
+     return(delta_theta)
+ }
> logistic_regression <- function(X, y, alpha, eta, seed = 123, threshold = 1e6) {
+   # initializations
+   set.seed(seed)
+   X$intercept <- 1
+   theta <- rnorm(ncol(X), sd = 0.5)
+   delta_theta <- rep(10000, ncol(X))
+   numIterations <- 1
+   while (any(abs(alpha * d_theta/(theta + 0.0001)) > eta) & numIterations < threshold){
+     delta_theta <- gradient_descent(X,Y,theta)
+     theta <- theta - alpha * delta_theta
```

```
+       numIterations <- numIterations + 1
+    }
+    return(theta)
+ }
```